Monetary and Fiscal Policy Interaction and Government Debt Stabilization

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Abstract

In many developing and developed countries, government debt stabilization is an important policy issue. This paper models the strategic interaction between the monetary authorities who control monetization and the fiscal authorities who control primary fiscal deficits. Government debt dynamics are driven by the interest payments on outstanding debt and the part of the primary fiscal deficits that is not monetized. Modelling the interaction as a differential game, we compare the cooperative equilibrium and the non-cooperative Nash open-loop equilibrium. The well-known unpleasant monetarist arithmetic is reinterpreted in this differential game framework. We consider also the effects of making the Central Bank more independent.

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1. Introduction

During the 1980s, many developed and developing countries experienced substantial increases in government indebtedness and problems in stabilizing government debt. Therefore, government debt stabilization became a prominent issue in policy discussions. Recent interest in the issue of government debt stabilization is related also to the debt target of 60% in the Maastricht Treaty. Recently, the OECD (1994) surveyed the fiscal stance in its member countries and expressed its concern regarding the development of public debt in several countries. Projections of current fiscal policies show that in several countries debt stabilizes only far beyond the year 2000 at levels that are some 30 percentage points of GDP higher than current levels, which are already fairly high.

A conflict between fiscal and monetary authorities typically arises on whether fiscal or monetary instruments should be adjusted to stabilize government debt. To formalize this conflict, Tabellini (1986) has formulated a differential game between a fiscal and a monetary authority. The current analysis extends Tabellini's work by allowing the fiscal authority to care also about monetary objectives and by introducing a specific debt target. Moreover, we determine and interpret the various externalities the players impose on each other. We also calculate the transitional and steady-state solutions for fiscal deficits, inflation and government debt, providing new interpretations for these solutions as the outcome of the conflict between monetary and fiscal authorities. Furthermore, the effects of changes in the objective functions of monetary and fiscal authorities are derived. In particular, we consider the effects of making monetary authorities more independent and we reinterpret the unpleasant monetarist arithmetic of Sargent and Wallace (1981) as the outcome of strategic interaction between monetary and fiscal policymakers.

Section 2 introduces the differential game between fiscal and monetary authorities. In section 3 we provide the solutions for both the first-best cooperative equilibrium and the second-best non-cooperative Nash open-loop equilibrium. Section 4 compares these two equilibria. The effects of changes in the preference functions of both players are discussed in section 4. Section 5 provides a nume-rical example that illustrates the results found in the theoretical part.

2. A differential game on government debt stabilization.

Primary fiscal deficits have to be financed either by base-money creation or by accumulation of government debt. In many cases decisions on primary fiscal deficits are delegated to the Treasury and management of monetary policy to a central bank. While monetary and fiscal policies are delegated to different institutions, their policies are interdependent because of the dynamic government budget constraint. The dynamic government budget constraint namely shows the relation between primary fiscal deficits, $f(t)$, monetization or seignorage[3], $m(t)$, interest payments on government debt, $rd(t)$, and government debt accumulation $\dot{d}$, where a dot above a variable refers to its time derivative:

$$\dot{d} = r\,d(t) + f(t) - m(t) \tag{1}$$

$d(t)$, $f(t)$ and $m(t)$ are expressed as fractions of GDP. r represents the rate of interest on outstanding government debt minus the growth rate of output and is assumed to be exogenous and therefore independent of the level of government indebtedness. (1) can be interpreted in nominal as well as real terms, obtained if nominal variables are divided by $P(t)$, the aggregate price level.

If the fiscal deficit, $f(t)+rd(t)$, exceeds the revenues from money creation of the monetary authorities, $m(t)$, government debt accumulation allows policymakers to shift the adjustment burden associated with the fiscal deficit to the future. The dynamic government budget constraint thus reveals that the interaction between the monetary and fiscal authorities has both an *intra*temporal and an *inter*temporal dimension. The latter implies a link between monetary and fiscal policies and the accumulation of government debt. The initial stock of outstanding government debt $d(0)$ and the (net of output growth) real interest rate play an important role in the process of fiscal consolidation. With a large initial stock of debt and a high interest rate on debt, government debt stabilization requires larger efforts than in a situation with a low initial stock of debt and low interest rates.

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[3] In Appendix A, we define $m(t)$ in such a way that the inflation tax from unexpected inflation on holders of government debt is also included in $m(t)$. In practice, the inflation tax on outstanding government debt is more important than the inflation tax on base money. Appendix A shows how $m(t)$ and the rate of inflation, $\pi=\dot{P}$, are related.

Government solvency is ensured if we assume that the following transversality condition -generally referred to as the no-Ponzi game condition- is met:

$$\lim_{t \to \infty} d(t)\, e^{-r t} = 0 \qquad (2)$$

Stabilization of government debt can be achieved in two alternative ways: by decreasing primary fiscal deficits or by increasing base money creation. Policy conflicts arise if fiscal and monetary policies are controlled by different institutions that assign different weights to various objectives, including inflation, government debt stabilization, and public spending. Following Tabellini (1986), we formalize the strategic interaction between monetary and fiscal authorities by specifying instruments and objectives of the policymakers within a formal game structure.

Consider the following intertemporal loss function of the fiscal authority, which depends on the time profiles of the primary fiscal deficit, inflation, and government debt[4]:

$$L^F(t_0) = \frac{1}{2}\int_{t_0}^{\infty} \{(f(t) - \bar{f})^2 + \eta\,(m(t) - \bar{m})^2 + \lambda\,(d(t) - \bar{d})^2\} e^{-\delta(t - t_0)} dt \qquad (3)$$

Fiscal authorities manage primary fiscal deficits to minimize the intertemporal loss function, subject to the dynamic government budget constraint (1), the transversality condition (2), and the initial stock of government debt, $d(0)$. $\bar{f}$, $\bar{m}$ and $\bar{d}$ represent exogenous policy targets for inflation, the primary fiscal deficit and public debt. These "blisspoints" reflect the institutional and political structures in which decisionmaking on macroeconomic policies takes place[5]. The subjective rate of time preference, δ, determines how much policymakers discount future losses[6].

As in Tabellini (1986), government debt features in the loss function because higher

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[4] Tabellini considers a special case with η=0 and $\bar{d}$=0.

[5] It is possible to interpret $\bar{f}$ as preferred government expenditures, given an exogenous path for taxes, or alternatively, as preferred taxes given an exogenous path for government expenditures. In the open economy, the monetary target can be interpreted also as an exchange rate target.

[6] A high rate of time preference is sometimes associated with a high degree of political instability.

levels of debt imply larger tax distortions in order to service the interest payments. Moreover, the larger the stock of public debt, the larger the required adjustments in taxes associated with fluctuations in the real rate of interest and real output. If Ricardian equivalence does not hold, high levels of public debt are likely to also crowd out private investment and induce undesirable intergenerational redistributions of wealth.

Monetary authorities exhibit a similar loss function:

$$L^{CB}(t_0) \;=\; \frac{1}{2}\int_{t_0}^{\infty}\{(m(t)-\bar{m})^2+\kappa\,(d(t)-\bar{d})^2\}\,e^{-\delta\,(t-t_0)}\,dt \tag{4}$$

The preference parameters $\{\eta,\lambda,\kappa,\bar{m},\bar{f},\bar{d}\}$ are important determinants of the dynamics of the fiscal deficit, the inflation rate and government debt. λ and κ determine how the adjustment burden of government debt stabilization is distributed over the fiscal and monetary policymakers in the form of, respectively, low primary fiscal deficits and a high level of money creation. If κ is high and λ low, high money creation rather than small primary fiscal deficits resolve the tension between the Treasury and the Central Bank on government debt stabilization. Hence, this situation implies a strong fiscal player and a weak Central Bank. If both κ and λ are small, neither player is willing to substantially adjust its policy in order to stabilize government debt. Hence, the adjustment burden is shifted mainly towards the future by accumulating even more public debt.

The difference between $\bar{f}+r\bar{d}$ and $\bar{m}$, which is assumed to be positive, is an important determinant of government debt accumulation: it measures the gap between the desired financing by the fiscal player, $\bar{f}+r\bar{d}$, and the desired accommodation by the monetary authorities, $\bar{m}$. Accordingly, a larger difference between these two objectives intensifies the conflict between the two authorities. Also a large initial stock of government debt, $d(0)$, or a low debt target intensify the conflict. In the remainder of the analysis, we assume that the initial stock of debt exceeds the target, i.e. $d(0)>\bar{d}$.

Another important factor is the difference between the rate of time preference, δ, and the net real interest rate, r. If $\delta>r$ and public debt does not directly feature in the objective functions (i.e. $\lambda=\kappa=0$), the subjective benefits of additional government debt are lower than its objective costs and policymakers would always prefer additional government debt so that government debt would accumulate without bound.

3. Solving the differential game.

Two elements are crucial in the dynamic interaction between monetary and fiscal authorities: namely, first, whether policies are coordinated and, second, the information structure. Coordination of macroeconomic policies internalizes the positive externalities on the other player from efforts to stabilize government debt. The cooperative equilibrium is thus Pareto efficient. Hence, we refer to the cooperative equilibrium as the Pareto equilibrium. This equilibrium can serve as a benchmark to determine the inefficiency associated with non-cooperative equilibria. In the case of the cooperative game, ω and 1 are the weights attached to the objectives of the fiscal and monetary authorities, respectively. The weights are the outcome of an earlier bargaining process and are assumed to be given[7].

Regarding the information structure two aspects are crucial. First, we need to distinguish equilibria in which players can credibly commit to a sequence of future actions from equilibria in which players cannot do so. With commitment, *open-loop* equilibria result. Lack of commitment, in contrast, yields *closed-loop* equilibria. Open-loop strategies involve an optimal time path of policy variables taking the policy actions of the other player as given. Closed-loop strategies, in contrast, take into account the reaction of the other player at each point in time. Second, non-cooperative equilibria in which one player acts as a leader have to be distinguished from equilibria in which both player act simultaneously. When one player assumes leadership, Stackelberg equilibria result, whereas Nash equilibria emerge without leadership of one player.

Commitment problems arise in the current framework, because policies aimed at reducing government debt are time-inconsistent: both players have an incentive to deviate from commitments on debt stabilization. We do not solve the Nash-closed loop equilibrium analytically because of complexity. The inability to commit typically aggravates inefficiencies compared to the Nash open-loop equilibrium in the current framework, as shown in Tabellini (1986). The complexity of closed-loop strategies requires the use of numerical techniques. In order to compare analytical results, we focus here only on the differences between the first-best cooperative equilibrium and the second-best Nash open-loop equilibrium, discarding the

[7] It is possible to consider ω to be the Nash-bargaining solution associated with the coalition formation.

Nash closed-loop and the Stackelberg open- and closed-loop equilibria.

The Pareto equilibrium is found by minimizing the following present-value Hamiltonian,

$$H^P(t) = \frac{\omega}{2}(f(t)-\bar{f})^2 + \frac{(\omega\eta+1)}{2}(m(t)-\bar{m})^2 + \frac{(\omega\lambda+\kappa)}{2}(d(t)-\bar{d})^2 + \mu^P(t)(rd(t)+f(t)-m(t)) \tag{5}$$

with respect to the available instruments $\{f(t),m(t)\}$. $\mu^P(t)$ is the co-state variable of the dynamic constraint (1). The first-order conditions of this dynamic optimization problem are:

$$\begin{aligned} f(t) &= \bar{f} - \frac{\mu^P(t)}{\omega} \\ m(t) &= \bar{m} + \frac{\mu^P(t)}{\omega\eta+1} \\ \dot{\mu}^P &= (\delta-r)\,\mu^P(t) - (\omega\lambda+\kappa)(d(t)-\bar{d}) \end{aligned} \tag{6}$$

The Nash open-loop equilibrium is found by separately maximizing the present-value Hamiltonians of the fiscal and monetary authorities. The Hamiltonian of the fiscal authorities is given by:

$$H^F(t) = \frac{1}{2}(f(t)-\bar{f})^2 + \frac{\eta}{2}(m(t)-\bar{m})^2 + \frac{\lambda}{2}(d(t)-\bar{d})^2 + \mu^F(t)(rd(t)+f(t)-m(t)) \tag{7}$$

which gives rise to the following first-order conditions:

$$\begin{aligned} f(t) &= \bar{f} - \mu^F(t) \\ \dot{\mu}^F(t) &= (\delta-r)\,\mu^F(t) - \lambda\,(d(t)-\bar{d}) \end{aligned} \tag{8}$$

Maximization of the present-value Hamiltonian of the monetary authorities,

$$H^{CB}(t) = \frac{1}{2}(m(t)-\bar{m})^2 + \frac{\kappa}{2}(d(t)-\bar{d})^2 + \mu^{CB}(t)(rd(t)+f(t)-m(t)) \tag{9}$$

yields the following optimality conditions:

$$\begin{aligned} m(t) &= \bar{m} + \mu^{CB}(t) \\ \dot{\mu}^{CB} &= (\delta-r)\,\mu^{CB}(t) - \kappa(d(t)-\bar{d}) \end{aligned} \tag{10}$$

The optimization of quadratic objective functions produces linear dynamic systems of government debt and the co-state variables associated with government debt, $\mu^i(t)$. These dynamic systems $\{d(t),\mu^i(t)\}$ are replicated in appendix B. The dynamic systems are assumed to display saddlepoint stability in order to rule out explosive government indebtedness and thus violation of the transversality constraint (2)[8].

If initial government debt, $d(0)$, exceeds steady-state government debt, $d(\infty)$, an instantaneous upward jump in $\mu^i(0)$ induces a decrease of the primary fiscal deficit to $f(0)$ and an increase of money growth to $m(0)$. Such changes in policies place the system on the unique converging trajectory and take the system to its new steady-state equilibrium $\{d(\infty),\mu^i(\infty)\}$. The stable root of the dynamic system in $\{d(t),\mu^i(t)\}$ determines the transient dynamics of the saddlepoint stable system: with a negative sign it measures the adjustment speed towards steady- state. The adjustment speed is denoted by h.

The system dynamics can be written in the form:

$$\begin{aligned} d(t) &= (d(0) - d(\infty))\,e^{-ht} + d(\infty) \\ \mu^i(t) &= (\mu^i(0) - \mu^i(\infty))\,e^{-ht} + \mu^i(\infty) \\ f(t) &= (f(0) - f(\infty))\,e^{-ht} + f(\infty) \\ m(t) &= (m(0) - m(\infty))\,e^{-ht} + m(\infty) \end{aligned} \tag{11}$$

where ∞ refers to the steady-state and 0 to the initial state of a variable. Expression (11) reveals that the dynamics of any variable can be characterized by three elements: the initial state, the adjustment speed, h, and the steady-state solution. The next section derives these three elements analytically for both the cooperative Pareto equilibrium and the non-cooperative Nash open-loop equilibrium and compares both equilibria.

[8] The dynamic systems exhibit saddlepoint stability as long as the number of negative eigenvalues of the adjustment matrix equals the number of backward-looking variables and the number of positive eigenvalues equals the number of forward-looking variables. The state variable, $d(t)$, is backward-looking. Hence, its initial value $d(0)$ is given by history. The co-state variables, $\mu^i(t)$, in contrast, are forward-looking variables: they jump if new information arrives to ensure that $d(t)$ is placed on the unique convergent path that is consistent with the transversality condition (2).

4. A comparison of the cooperative and non-cooperative Nash open-loop equilibria.

The cooperative Pareto equilibrium and the non-cooperative Nash open-loop equilibria can be solved analytically. Appendix B describes how one can obtain the initial value, the adjustment speed and the steady-state of both equilibria. It is shown how the initial value, the adjustment speed and the steady-state can be expressed in the following form:

Table 4.1 General formulation of solutions.

$$m(0) = \bar{m} + (1-\alpha)[(1-h\beta)(\bar{f} + r\bar{d} - \bar{m}) + (h+r)(d(0) - \bar{d})]$$

$$f(0) = \bar{f} - \alpha[(1-h\beta)(\bar{f} + r\bar{d} - \bar{m}) + (h+r)(d(0) - \bar{d})]$$

$$f(0) - m(0) = h\beta(\bar{f} + r\bar{d} - \bar{m}) - h(d(0) - \bar{d}) - r d(0)$$

$$m(\infty) = \bar{m} + (1-\alpha)[(1+r\beta)(\bar{f} + r\bar{d} - \bar{m})]$$

$$f(\infty) = \bar{f} - \alpha[(1+r\beta)(\bar{f} + r\bar{d} - \bar{m})]$$

$$f(\infty) - m(\infty) = -r\bar{d} - r\beta(\bar{f} + r\bar{d} - \bar{m})$$

$$d(\infty) = \bar{d} + \beta(\bar{f} + r\bar{d} - \bar{m})$$

$$f(\infty) - f(0) = -\alpha(h+r)[\beta(\bar{f} + r\bar{d} - \bar{m}) - (d(0) - \bar{d})]$$

$$m(\infty) - m(0) = (1-\alpha)(h+r)[\beta(\bar{f} + r\bar{d} - \bar{m}) - (d(0) - \bar{d})]$$

$$d(\infty) - d(0) = \beta(\bar{f} + r\bar{d} - \bar{m}) + (\bar{d} - d(0))$$

$$h = -\frac{\delta}{2} + \frac{\sqrt{\delta^2 + 4\Delta}}{2}$$

$$\beta = \frac{(\delta - r)}{\Delta} = \frac{(\delta - r)}{h(h+\delta)}$$

Table 4.1 implies that short-run and long-run inflation and primary fiscal deficit and long-run debt can be expressed in terms of two elements: first, a parameter indicating the *intra*temporal distribution of the adjustment burden associated with government debt stabilization, α[9], and, second, a parameter indicating the *inter*temporal distribution of that burden, β,

[9] α and $(1-\alpha)$ are closely related to the "feedback"-coefficients θ_1 and π_1 used by Tabellini. These coefficients measure by how much the monetary and fiscal authorities adjust their policies to changes in the current stock of debt. Since the game is linear-quadratic, Tabellini proposes the following feedback relations: $m(t)=\theta_0+\theta_1 d(t)$ and $f(t)=\pi_0-\pi_1 d(t)$. Table 4.1 and (11) together imply: $\theta_0=m(0)-(1-\alpha)(h+r)d(0)$, $\theta_1=-\alpha(h+r)$, $\pi_0=f(0)+\alpha$-

which is inversely related to the adjustment speed, *h*. Both the cooperative and the non-cooperative games yield the same expressions for initial and steady-state inflation, primary fiscal deficits and steady-state debt in terms of α and Δ. Δ and α differ in the Pareto and Nash open-loop equilibrium as follows:

Table 4.2 Δ and α in both equilibria.

Pareto	*Nash open-loop*
$\Delta^P = h^P(h^P+\delta)$ $= (\omega\lambda+\kappa)\left(\frac{1}{\omega}+\frac{1}{\omega\eta+1}\right) - r(\delta - r)$ $\alpha^P = \frac{\frac{1}{\omega}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}} = \frac{\omega\eta+1}{\omega\eta+1+\omega}$	$\Delta^O = h^O(h^O+\delta)$ $= (\lambda+\kappa) - r(\delta - r)$ $\alpha^O = \frac{\lambda}{\lambda+\kappa}$

The parameter α indicates how the *intra*temporal adjustment burden is distributed over the two authorities. A large α indicates relatively weak fiscal authorities who bear most of the adjustment burden. In particular, the primary fiscal deficit, $f(t)$, is much below its bliss point, $\bar{f}$, while money growth is relatively close to its target value, $\bar{m}$. If the fiscal authorities are strong and the Central Bank is weak (i.e. α is small), in contrast, debt stabilization is achieved mainly through monetization of fiscal deficits.

The parameter β reveals how the adjustment burden is shifted intertemporally. This parameter is zero if authorities are patient (i.e. $\delta=r$). This indicates that the conflict between fiscal and monetary policies is resolved without shifting the adjustment burden over time. However, adjustment is largely shifted to the future if β is large, which occurs if policymakers are impatient (i.e. $\delta>r$) and attach a low weight to debt stabilization. In that case the adjustment speed, *h*, is low[10]. The impact of β on short-run and long-run policy variables

$(h+r)d(0)$ and $\pi_1=(1-\alpha)(h+r)$.

[10] If weights attached to debt stabilization are low, Δ is small, according to Table 4.2. According to Table 4.1, this implies that *h* is small as well.

reflects the *inter*temporal distribution of the adjustment burden. A higher value of β implies higher deficits and lower money growth in the short run, but lower deficits and higher money growth in the long run[11]. In other words the short run features underadjustment, causing government debt to accumulate. The associated interest payments on government debt require overadjustment in the long run.

Saddlepoint stability requires Δ^P and Δ^O to be positive. The expressions in Table 4.2 reveal that positive values for Δ require that policymakers attach a sufficiently high priority to government debt stabilization (i.e. high values of λ and κ) as long as δ exceeds r. Intuitively, in order to avoid explosive debt dynamics, authorities need to attach a sufficiently high priority to debt stabilization in order to offset their impatience. Adjustment is slow if the dynamic system is close to being unstable, i.e. the value of Δ is small. In particular, debt stabilization is a time-consuming process if authorities are impatient (i.e. δ exceeds r by a large margin) and at the same time care little about debt stabilization (i.e. λ and κ are small).

The expressions for α show that the intratemporal share of the adjustment burden that falls on the fiscal authorities is inversely related to ω in the Pareto case. At the same time, a higher weight of inflation in the objective function of the fiscal authorities increases α. In the Nash open-loop case, α depends only on the relative weights attached to debt stabilization by the monetary and fiscal authorities. In particular, if the monetary authorities value debt stabilization more than the fiscal authorities do (i.e. when κ/λ is large), they bear most of the adjustment burden associated with the conflict between monetary and fiscal policies.

A comparison of the Pareto and Nash open-loop equilibria leads to the following proposition:

Proposition 4.1:

The speed of adjustment is higher and steady-state debt lower in the cooperative equilibrium, if either (a) fiscal authorities attach less weight to inflation stabilization relative to debt stabilization than the monetary authorities do, i.e. if $\lambda/\eta>\kappa$, or (b) fiscal authorities have limited bargaining power, ω, in the cooperative case.

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[11] Note that we assume that $\bar{f}+r\bar{d}-\bar{m}>0$ and $d(0)>\bar{d}$.

Proof :

According to Table 4.1, $h^P = -\frac{1}{2}\delta + \sqrt{\delta^2 + 4\Delta^P}$ and $h^O = -\frac{1}{2}\delta + \sqrt{\delta^2 + 4\Delta^O}$. Therefore, $h^P > h^O$ if $\Delta^P > \Delta^O$. If we insert the definitions of Δ^P and Δ^O from Table 4.2 the inequality becomes:

$$(\omega\lambda + \kappa)\left(\frac{1}{\omega} + \frac{1}{\omega\eta + 1}\right) > \lambda + \kappa \Leftrightarrow (\lambda - \eta\kappa)\frac{\omega}{\omega\eta + 1} + \frac{\kappa}{\omega} > 0 \qquad (12)$$

The inequality $(\lambda - \eta\kappa) > 0$ is a sufficient condition for the second inequality (12) to hold. If, $\omega \downarrow 0$ the second term on the LHS of the second inequality (12) dominates the first term. Hence, the inequality also holds. According to the definition of β in Table 4.1, β and h are negatively related. Moreover, a positive relationship between β and $d(\infty)$ exists according to the definition of $d(\infty)$ in Table 4.1. Steady-state government debt, therefore, is inversely related to the adjustment speed. Hence, $h^P > h^O$ implies $d^P(\infty) < d^O(\infty)$. □

The first term on the LHS of the second inequality in (12) reflects the positive externality on 'fiscal' welfare if monetary authorities raise money growth to stabilize government debt. The second term measures the positive spillover on 'monetary' welfare if the fiscal authorities decrease the primary fiscal deficit to stabilize government debt. The cooperative equilibrium internalizes the positive spillover on the monetary policy-maker of reductions in the primary fiscal deficit and the positive spillover on the fiscal policy-maker from increases in the rate of money growth. Both actions reduce government debt accumulation, thereby lowering the steady-state stock of debt and speeding up adjustment.

The information of Table 4.1 and Table 4.2 reveals that short-term fiscal policies are typically too loose, while monetary policies are too tight in the non-cooperative equilibrium. In particular, $(f(0)-m(0))$ is smaller with cooperation than without cooperation, since $h^P\beta^P$ is smaller than $h^O\beta^O$ [12] and h^P is larger than h^O [13]. However, $(f(\infty)-m(\infty))$ is larger under cooperation than under non cooperation, since β^P is smaller than β^O. Regarding steady-state primary fiscal deficits and money creation in both equilibria, we can show the following:

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[12] Note that $\bar{f} + r\bar{d} - \bar{m} > 0$.

[13] Note that $d(0) > \bar{d}$.

Proposition 4.2:

Steady-state primary fiscal deficits are lower in the cooperative Pareto equilibrium than in the non-cooperative Nash open-loop, if:

$$\kappa - \omega(\lambda - \kappa\eta) > \frac{r(\delta - r)\kappa}{\omega\lambda + \kappa}(\omega\eta + 1) \tag{13a}$$

Steady-state money creation is lower in the cooperative Pareto equilibrium than in the non-cooperative Nash open-loop, if:

$$\kappa - \omega(\lambda - \kappa\eta) > -\frac{r(\delta - r)(\lambda - \kappa\eta)}{\omega\lambda + \kappa}\omega^2 \tag{13b}$$

Proof:

From Table 4.1 it follows that $f^P(\infty)<f^O(\infty)$ if $\alpha^P(1+r\beta^P)>\alpha^O(1+r\beta^O)$. With the definitions for α^i and Δ^i from Table 4.2, this inequality can be rewritten as:

$$\frac{\frac{1}{\omega}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}\left(\frac{(\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa)}{(\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa)-r(\delta-r)}\right) > \frac{\lambda}{\lambda+\kappa-r(\delta-r)}$$

Dividing both the numerator and the denominator of the LHS by $\omega\lambda+\kappa$ and at the RHS by λ, we arrive at:

$$\frac{1}{\omega\,[(\frac{1}{\omega}+\frac{1}{\omega\eta+1})-\frac{r(\delta-r)}{(\omega\lambda+\kappa)}]} > \frac{1}{1+\frac{\kappa}{\lambda}-\frac{r(\delta-r)}{\lambda}}$$

which implies,

$$\omega\,[(\frac{1}{\omega}+\frac{1}{\omega\eta+1})-\frac{r(\delta-r)}{(\omega\lambda+\kappa)}] < 1+\frac{\kappa}{\lambda}-\frac{r(\delta-r)}{\lambda}$$

Multiplying both sides of this inequality by $(\omega\eta+1)\lambda$ and collecting terms yields (13a). In a similar vein, $m^P(\infty)<m^O(\infty)$ if $(1-\alpha^P)(1+r\beta^P)>(1-\alpha^O)(1+r\beta^O)$. Substituting the expressions for α^i and Δ^i from Table 4.2, we find:

$$\frac{\frac{1}{\omega\eta+1}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}\left(\frac{(\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa)}{(\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa)-r(\delta-r)}\right) > \frac{\kappa}{\lambda+\kappa-r(\delta-r)}$$

which implies,

$$(\omega\eta+1)\left[(\frac{1}{\omega}+\frac{1}{\omega\eta+1})-\frac{r(\delta-r)}{(\omega\lambda+\kappa)}\right] < 1+\frac{\lambda}{\kappa}-\frac{r(\delta-r)}{\kappa}$$

Multiplying both sides of this inequality by $\omega\kappa$ and collecting terms yields (13b). □

If $\delta=r$, the policy conflict is resolved without intertemporal shifting and β equals 0. In that case, coordinated policies, compared to uncoordinated policies, are more disciplined in the long run (i.e. inflation and primary fiscal deficits are lower with cooperation), if $\alpha^P>\alpha^O$. This is the case if the bargaining weight ω of the fiscal player is small. Intuitively, the coordinated equilibrium is dominated by the player caring more about low inflation and less about high fiscal deficits.

If the Central Bank is dependent (i.e. κ large), policies are likely to be more disciplined in the cooperative case. Intuitively, in that case, the externality of the fiscal authorities on the monetary authorities (i.e. the 'fiscal externality') dominates the externality of the monetary authorities on the fiscal authorities (i.e. the 'monetary' externality). With the fiscal externality dominating the monetary externality, it is the fiscal rather than the monetary authority that has to conduct most of the adjustment in the coordinated equilibrium. If the Central Bank is rather independent (small κ), policies are more disciplined in the absence of cooperation. The reason is that without cooperation the Central Bank is free to chose its own restrictive monetary policy without paying much attention to the consequences on public debt accumulation. Accordingly, coordination worsens discipline in this case. In other words, in preserving discipline, independence acts as a substitute for coordination.

If policymakers are impatient (i.e. δ exceeds r), the condition for tighter fiscal policy in the coordinated case than in the uncoordinated case becomes stronger than $\alpha^P>\alpha^O$. The condition for tighter monetary policies in the Pareto case, in contrast, weakens. The reason is that coordinated policies result in less accumulation of public debt. The associated lower long-run adjustment burden allows larger steady-state deficits and lower steady-state money growth. If the weight of the fiscal authorities is the same under Pareto and Nash open-loop (i.e.

$\alpha^P=\alpha^O$), coordination implies smaller primary fiscal deficits in the short run and larger primary fiscal deficits in the long run. Intuitively, coordination implies that the adjustment burden is shifted less to the future as the authorities value policies reducing debt accumulation. Accordingly, monetary growth is higher in the short run but lower in the long run.

5. Comparative dynamics.

In the preceding section we explored the features of the initial state, adjustment speed and steady-state of the Pareto and Nash open-loop equilibria. Here we examine how both equilibria are affected by changes in both the preference parameters and the initial stock of government debt. The partial derivatives are collected in Appendix C. Table 5.1 provides the effects of parameter changes on the initial state $\{m(0),f(0)\}$ and the adjustment speed, h:

Table 5.1 Effects on the initial state and the adjustment speed.

	$m^P(0)$	$f^P(0)$	h^P	$m^O(0)$	$f^O(0)$	h^O
λ	+	-	+	?	-	+
κ	+	-	+	+	?	+
$\bar{m}$	+	+	o	+	+	o
$\bar{f}$	+	+	o	+	+	o
$\bar{d}$	-	+	o	-	+	o
$d(0)$	+	-	o	+	-	o

An increase of κ raises money creation by the monetary authorities in the initial state of both equilibria and speeds up the adjustment. An increase of λ has similar effects as it reduces the initial primary fiscal deficit and increases the adjustment speed. In the Nash open-loop equilibrium the effect of an increase of κ on the initial fiscal deficit and of an increase in λ on the initial money creation, are ambiguous of sign. With coordination, an increase in κ decreases $f^P(0)$ and an increase in λ increases $m^P(0)$. In the cooperative equilibrium a higher priority to debt stabilization attached by one player also induces the other player to act more active regarding debt stabilization. In the non-cooperative case the opposite reaction is likely to appear: more efforts of one player to reduce government debt induce the other player to reduce its own effort. In both equilibria, a higher primary fiscal deficit target raises the initial primary fiscal deficit, and a lower inflation target reduces initial inflation. A higher debt target lowers initial inflation and raises the initial primary fiscal deficit, whereas a high initial stock of government debt has the opposite effect.

The effects of changes in the preference parameters on the steady-state, $\{m(\infty),f(\infty),d(\infty)\}$ of both equilibria are found in Table 5.2:

Table 5.2 Steady-state effects.

	$d^P(\infty)$	$m^P(\infty)$	$f^P(\infty)$	$d^O(\infty)$	$m^O(\infty)$	$f^O(\infty)$
λ	- (o)	- (o)	+ (o)	- (o)	-	?(-)
κ	- (o)	- (o)	+ (o)	- (o)	?(+)	+
$\bar{m}$	- (o)	? (+)	+	- (o)	?(+)	+
$\bar{f}$	+ (o)	+	? (+)	+ (o)	+	?(+)
$\bar{d}$	+	+	-	+	+	-
$d(0)$	o		o	o	o	o

An increase in κ and λ lowers steady-state debt in both equilibria. In the Pareto case, increases in κ and λ allow for lower steady-state inflation and higher steady-state primary fiscal deficits, because of the lower stock of steady-state debt that needs to be financed. In the Nash open-loop, however, the effect of an increase of κ on steady-state inflation and the effect of an increase in λ on steady-state money creation are ambiguous. An explanation for this ambiguity is provided in Proposition 5.2.

Furthermore, a higher primary fiscal deficit target, $\bar{f}$, or a lower monetary target, $\bar{m}$, increase steady-state government debt. The impact of a change in the inflation target on steady-state inflation is ambiguous as is the effect of a higher primary fiscal deficit target on steady-state primary fiscal deficits. We explore this ambiguity in more detail in proposition 5.1. A higher debt target induces, in both equilibria, a higher steady-state level of government debt and hence requires higher money growth and lower primary fiscal deficits in the long run.

If different from the case of $\delta > r$, we have indicated the signs of the partial derivatives if $\delta = r$ between parentheses in Tables 5.1 and 5.2. If $\delta = r$, the conflict between monetary and fiscal authorities is resolved without accumulation of government debt. In that case, steady-state debt is not affected by changes in the preference parameters, since β is equal to 0. If $\delta=r$, ambiguities of steady-state effects on the primary fiscal deficit and inflation disappear. In particular, in the open-loop equilibrium, an increase in κ increases steady-state inflation. Furthermore, an increase in λ decreases steady-state primary fiscal deficits. Note that these effects contrast with to the cooperative equilibrium where such changes imply the opposite. Moreover, an increase in the primary fiscal deficit target and the inflation target increase viz. steady-state primary fiscal deficits and steady-state inflation in the Nash open-loop equilibrium.

The intertemporal shifting of the adjustment burden of government debt stabilization can give rise to unpleasant monetarist arithmetic of the type introduced by Sargent and Wallace (1981). Unpleasant monetarist arithmetic occurs when disinflationary monetary policies have to be reversed because of higher debt accumulation that such policies induce. If fiscal authorities do not cut primary fiscal deficits to reduce government debt accumulation, i.e. if the fiscal player is strong, a large part of the debt adjustment burden will be shifted eventually to the monetary authority, resulting in higher inflation in the long run. Conversely, fiscal expansions, e.g. in the form of tax cuts, have to be reversed in the long run if the fiscal player is weak compared to the monetary player. The adjustment burden from larger government debt that such policies produce is for the most part shifted back to the fiscal authorities in the long run, if the Central Bank does not monetize the additional debt. We can formulate the following proposition:

Proposition 5.1

A lower target value for inflation reduces inflation in the short run. However, it raises long-run inflation if (a) $\lambda<r(\delta-r)-\kappa/\omega$ *with policy coordination and (b)* $\lambda<r(\delta-r)$ *without policy coordination. A higher target value for primary fiscal deficits raises short-run deficits. However, it reduces long-run primary fiscal deficits if (c)* $\kappa<r(\delta-r)(\omega\eta+1)-\omega\lambda$ *with policy coordination and (d)* $\kappa<r(\delta-r)$ *without policy coordination.*

Proof:

The partial derivatives of $m(\infty)$ and $f(\infty)$ w.r.t. $\bar{m}$ and $\bar{f}$ are given in Appendix C. A decrease of $\bar{m}$ induces an instantaneous drop in $m(0)$ in both the Pareto and Nash open-loop equilibrium since $1-(1-\alpha^P)(1-h^P\beta^P)$ and $1-(1-\alpha^O)(1-h^O\beta^O)$ are both positive. The partial derivatives of $m^P(\infty)$ and $m^O(\infty)$ w.r.t. $\bar{m}$ imply that a decrease in $\bar{m}$ induces an increase in $m(\infty)$ if (a) $1-(1-\alpha^P)(1+r\beta^P)<0$ in the Pareto equilibrium, and (b) $1-(1-\alpha^O)(1+r\beta^O)<0$ in the Nash open-loop equilibrium. With the definitions of α and β in Table 4.1 and 4.2, we can rewrite (a) as $1-\left(\frac{\frac{1}{\omega\eta+1}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}\right)\left(\frac{(\omega\lambda+\kappa)(\frac{1}{\omega}+\frac{1}{\omega\eta+1})}{\Delta^P}\right)<0$ and (b) as $1-\frac{\kappa}{\lambda+\kappa}\frac{\lambda+\kappa}{\Delta^O}<0$. Rewriting both inequalities and using the definitions of Δ^P and Δ^O from Table 4.2, we find for (a) $\frac{\lambda+\frac{\kappa}{\omega}-r(\delta-r)}{\Delta^P}<0$ - whereas (b) can be rewritten as $\frac{\lambda-r(\delta-r)}{\Delta^O}<0$ Conditions (a) and (b) of the proposition then follow. In a similar vein, we find that an increase in $\bar{f}$ induces an instantaneous increase in $f(0)$ in both equilibria, since $(1-\alpha^P)(1-h^P\beta^P)$ and $(1-\alpha^O)(1-h^O\beta^O)$ are positive. According to the partial derivatives of $f^P(\infty)$ and $f^O(\infty)$ w.r.t. $\bar{f}$, an increase in $\bar{f}$ causes a permanent decrease in $f(\infty)$ if (c) $1-\left(\frac{\frac{1}{\omega}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}\right)\left(\frac{(\omega\lambda+\kappa)(\frac{1}{\omega}+\frac{1}{\omega\eta+1})}{\Delta^P}\right)<0$ in the Pareto equilibrium and (d) $1-\frac{\lambda}{\lambda+\kappa}\frac{\lambda+\kappa}{\Delta^O}<0$ in the Nash open-loop equilibrium. (c) and (d) can be rewritten as $\dfrac{\frac{\omega\lambda+\kappa}{\omega\eta+1}-r(\delta-r)}{\Delta^{P}}<0$ and $\dfrac{\kappa-r(\delta-r)}{\Delta^{O}}<0$ respectively, from which conditions (c) and (d) of the proposition follow directly. □

Unpleasant monetarist arithmetic implies that the initial disinflation is not sustainable in the long run and that the new steady-state is instead characterized by a higher rate of inflation. This becomes more likely if the fiscal authorities are strong, i.e. λ is small, impatience is high, i.e. $\delta \gg r$, and if the bargaining power of the fiscal authorities is high in the case of cooperation, i.e. if ω is large. Unpleasant fiscal arithmetic, on the other hand, occurs if monetary authorities are strong, i.e. κ is small, impatience is high, and if the bargaining power of the fiscal authorities is low or their inflation aversion is high under cooperation, i.e. if ω is large and η is high.

Unpleasant monetary and fiscal arithmetic can occur only if the discount rate substantially exceeds the interest rate. Indeed, a larger fiscal blisspoint raises the long-run primary fiscal deficit, if authorities are patient (i.e. $\delta=r$). If authorities are impatient, in contrast, the long-run deficit may decline if the weight attached to debt stabilization in the objective function of monetary authorities is small. Intuitively, the burden of adjustment associated with larger short-run primary deficits is not met through monetization. Instead, it is shifted to the future through debt accumulation. With the monetary authority being strong, the burden is eventually paid by the fiscal authorities in terms of lower primary deficits. Indeed, the expression for the long-run primary deficit in Table 4.1 reveals that unpleasant fiscal arithmetic occurs only if both β and α are large (i.e. $\alpha(1+r\beta)>1$): a high value of β indicates substantial intertemporal shifting, while a high α reflects a strong position of the monetary authorities. If we assume that monetary authorities care more about inflation than fiscal authorities, i.e. that $\lambda/\eta>\kappa$, unpleasant fiscal arithmetic is less likely to occur with coordination: the faster adjustment implies less debt accumulation as compared to the Nash open-loop equilibrium (see proposition 4.1).

Fiscal consolidation possesses a political-economy dimension because expenditure cuts and increases in ordinary taxes and the inflation tax typically affect groups of constituents in a different manner. Left-wing governments are generally believed to prefer more active fiscal and monetary policies than do right-wing governments. The replacement of a right-wing government by a left-wing government could be represented by a simultaneous increase in $\overline{f}$ and $\overline{m}$ and perhaps even an increase in the debt target, $\overline{d}$. The effects on steady-state fiscal deficits, inflation and government debt according to Table 4.1 will depend in particular on the change of ($\overline{f}$+$r\overline{d}$-$\overline{m}$) that results from such a change in orientation of the policymakers.

The parameter κ, which models the weight monetary authorities attach to debt stabilization, is inversely related to the independence of the Central Bank. The issue of central bank independence has encountered a lot of interest, both in politics and academics. Cukierman (1992) summarizes the current insights into this issue. Central in the literature on central bank independence stands the interaction with the private sector, an aspect that does not feature in our analysis. Public finance aspects and strategic interaction with a fiscal player are, however, neglected in the literature on central bank independence. Only recently, Jensen (1994) introduced seignorage revenues into the Barro-Gordon type of models that dominate the literature on central bank independence.

The degree of central bank independence has important effects in our model. In particular, more Central Bank independence implies that the fiscal authorities face a larger adjustment burden from debt stabilization than they did before. This yields the following proposition:

Proposition 5.2

More independence of the monetary authorities, (a) reduces the adjustment speed and increases steady-state debt. (b) Under policy coordination, more Central Bank independence decreases steady-state primary fiscal deficits and increases steady-state inflation if policymakers are impatient, (i.e. if $\delta > r$). (c) In the open-loop case, in contrast, a more independent Central Bank reduces both steady-state primary fiscal deficits and inflation if $\lambda > r(\delta - r)$, i.e. if fiscal authorities are not too strong.

Proof :

According to the definitions in Table 4.1 and 4.2, a decrease in κ implies that the adjustment speed of the cooperative equilibrium and the Nash open-loop equili-brium, h^P and h^O, decrease. Since steady-state debt is negatively related to the adjustment speed (cf. Table 4.1), a decrease of κ increases steady-state debt (given our assumptions that $(\bar{f}+r\bar{d}-\bar{m})>0$ and $\delta>r$). This proves (a). In the Pareto equilibrium, κ does not affect α^P according to Table 4.2. Hence, changes in κ affect $f^P(\infty)$ and $m^P(\infty)$ only by changing β^P. If $\delta=r$, β^P equals 0 and changes of κ have no effect at all. If $\delta>r$, a decrease in κ reduces the adjustment speed implying an increase in β^P. A higher β^P implies higher steady-state government debt and, consequently, lower steady-state primary fiscal deficits and higher steady-state inflation, as stated in (b). In the Nash open-loop equilibrium, a decrease in κ increases both α^O and β^O. According to the definition in Table 4.1, the long run primary fiscal deficit, $f^O(\infty)$ decreases. Long run inflation, $m^O(\infty)$, decreases if κ is lowered, as long as the decrease in $(1-\alpha^O)$ exceeds the increase in $(1+r\beta^O)$. A decrease of κ, decreases $(1-\alpha^O)$ by $\frac{\lambda}{(\lambda+\kappa)^2}$ and to an increase in $(1+r\beta^O)$ of $\frac{r(\delta-r)}{(\lambda+\kappa)^2}$ The first effect dominates the second effect if $\frac{\lambda}{(\lambda+\kappa)^2}-\frac{r(\delta-r)}{(\lambda+\kappa)^2}>0$ This leads to part (c) of the proposition.□

Without coordination, a more independent Central Bank may be counterproductive in reducing long-run inflation if authorities are impatient and at the same time the fiscal authorities are strong (in the sense that they attach a low priority to stabilizing debt).[14] Hence, making a central bank more independent is not enough to ensure low inflation rates. Low inflation rates are sustainable only if fiscal authorities are disciplined (i.e. they attach substantial weight to debt stabilization) and patient (i.e. δ does not exceed r by a large margin). This suggests that an independent CB needs to be complemented by fiscal reforms to ensure low inflation rates. Whereas a more independent Central Bank may raise inflation in the long run, it succeeds in strengthening fiscal discipline by reducing fiscal deficits in the

[14] Note that the condition for a more independent central bank to raise long-run inflation is the same condition for a lower target value for money growth to raise long-run inflation, in the open-loop case.

long run.[15] With coordination, a more independent Central Bank raises the intertemporal shifting of the adjustment burden to the future. This allows larger fiscal deficits and lower money growth in the short run, but requires lower deficits and higher money growth in the long run.

6. Numerical simulations with the model.

The analytical results from the preceding section can be illustrated with a stylized example. Consider an economy characterized by the following initial situation:

$d(0)= 1$	$\bar{m}=0.00$
$\delta=0.10$	$\lambda=0.03$
$r=0.02$	$\eta=0.5$
$\bar{d}= 0.6$	$\kappa=0.02$
$\bar{f}=0.075$	$\omega=1$

The initial situation is characterized by a high stock of government debt and policymakers featuring a high rate of time preference. However, a fairly high primary deficit target is combined with a conservative inflation target. A value of κ of 0.02, implies that the Central Bank is not entirely independent. Cooperative policies give a relatively large weight to fiscal objectives. A debt target of 60% of GDP is chosen for this economy, the entrance criterium for the European Monetary Union. Doubts are often raised whether the EC will satisfy the fiscal convergence criteria in 1999, when the introduction of the common currency is planned[16]. With this set of model parameters the Pareto and open-loop Nash equilibria display the following character:

[15] Only with positive public assets in the initial equilibrium (i.e. $d(0)$ negative) may fiscal policy become more expansionary in the short run.

[16] A steady-state debt ratio of 60% of GDP results from primary fiscal deficits of 3% of GDP and a net of output growth real interest rate of 5%, as the Delors Committee assumed when advocating such debt and deficit targets. As the simulations will show, however, it can take a fairly long time before new steady- states are achieved. The quick convergence implicitly assumed by the Delors Committee implicitly seems to be rather on the optimistic side. Cosetti and Roubini (1993) test the sustainability of current government debt accumulation in the EC. They find that the current process of government debt accumulation in Italy, Belgium and Ireland is not sustainable in the long run.

Table 6.1 Government debt stabilization as a differential game.

	Pareto	Nash open-loop
$m(0)$	0.068	0.056
$f(0)$	-0.027	-0.009
$f(0)+rd(0)$	-0.007	0.011
h	0.240	0.176
$d(\infty)$	0.685	0.744
$m(\infty)$	0.035	0.036
$f(\infty)$	0.022	0.021
$f(\infty)+rd(\infty)$	0.036	0.036
α	0.60	0.60
β	0.979	1.653
$L^F(t_0)$	0.264	0.283
$L^{CB}(t_0)$	0.156	0.171

The adjustment paths of government, money creation, primary and total fiscal deficit are displayed in graphs (6.1a)-(6.1d):

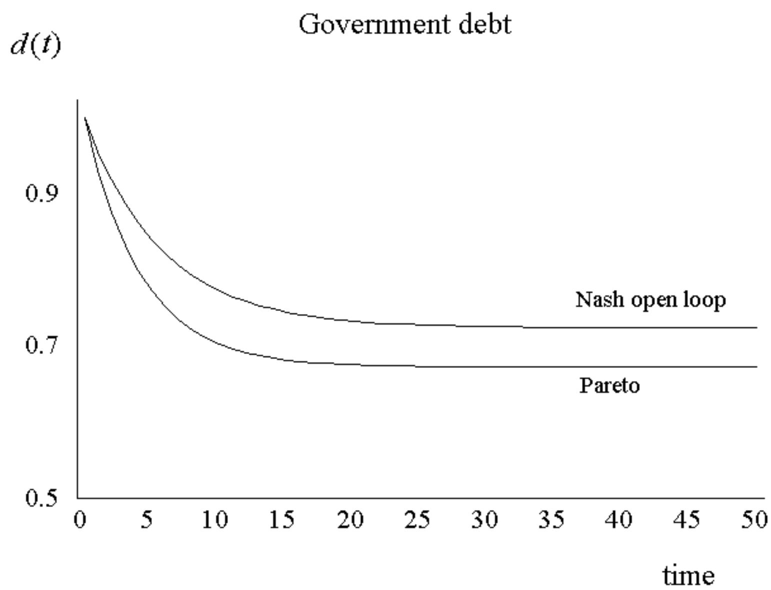

Figure 6.1a Government debt dynamics

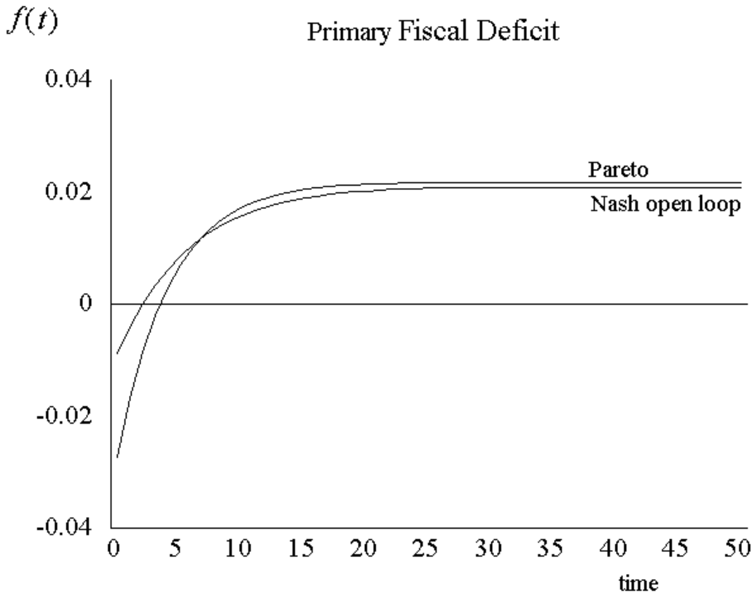

Figure 6.1b Primary fiscal deficit dynamics

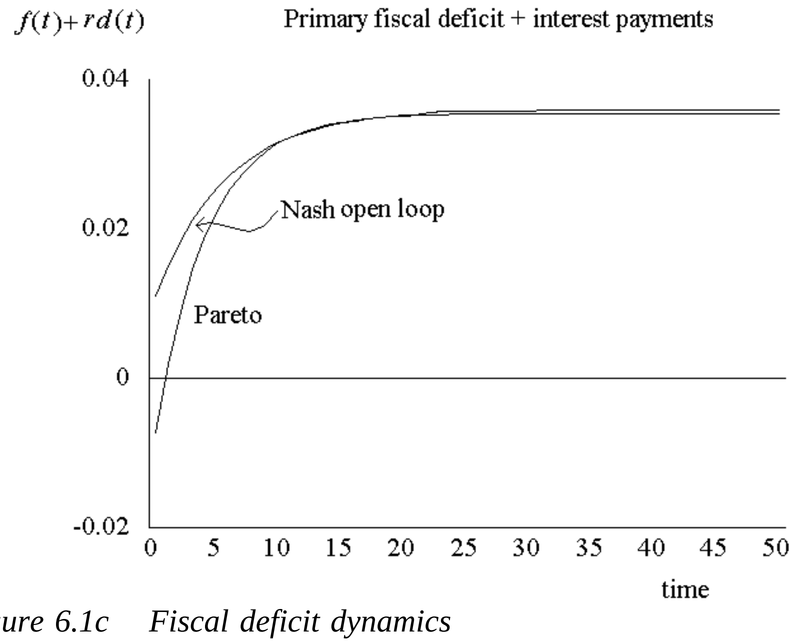

Figure 6.1c Fiscal deficit dynamics

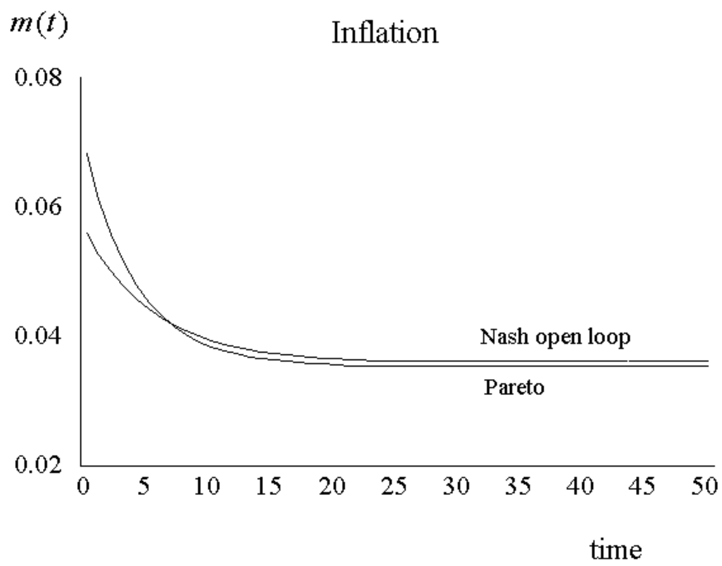

Figure 6.1d Inflation dynamics

$\{\lambda,\kappa,\omega,\eta\}$ have been chosen in such a manner that the intratemporal shares of the adjustment burden from government debt stabilization are the same in both equilibria, $\alpha^P=\alpha^O$. The differences between both equilibria are caused, therefore, by differences in the intertemporal allocation of the adjustment burden, as measured by β^i. In the cooperative equilibrium less of the adjustment is postponed to the future (β^P is smaller than β^O): inflation is higher in the short run than it is under non-cooperative policies, whereas fiscal deficits are smaller in the short run. Coordination of policies is more effective in bringing about a reduction in steady-state government debt, as we proved in proposition 4.1. According to the calculated losses, both players benefit from coordination of monetary and fiscal policy.

Consider the possibility of making the Central Bank more independent. If we decrease κ from 0.02 to 0, the following picture emerges:

Table 6.2 Government debt stabilization with a more independent Central Bank.

	Pareto	Nash open-loop
$m(0)$	0.056	0.000
$f(0)$	-0.009	-0.039
$f(0)+rd(0)$	0.011	-0.019
h	0.176	0.126
$d(\infty)$	0.744	0.845
$m(\infty)$	0.036	0.000
$f(\infty)$	0.021	-0.017
$f(\infty)+rd(\infty)$	0.036	0.036
α	0.60	1
β	1.653	2.817
$L^F(t_0)$	0.283	0.375
$L^{CB}(t_0)$	0.021	0.000

Higher Central Bank independence slows down the adjustment speed considerably and leads to a higher steady-state level of government debt, as we indicated in part (a) of proposition 5.2. Especially in the non-cooperative case, the decrease in steady-state government debt is limited. Also the effects mentioned in part (b) and (c) of proposition 5.2 show up, although the effects on steady-state deficits and inflation are very small under coordination. Fiscal authorities face a much higher adjustment burden than they did before especially in the non-cooperative equilibrium. In the Nash open-loop case the share of the fiscal authorities in the adjustment burden increases from 0.6 to 1 with an independent Central Bank.

A decrease in the rate of output growth, hampers debt stabilization because lower economic growth increases the interest burden of outstanding debt. The adjustment burden from government debt stabilization, therefore, is higher if the initial stock of government debt is high. A drop of g by 2% increases r from 2 to 4% and brings about the following situation:

Table 6.3 A drop in real output growth.

	Pareto	Nash open-loop
$m(0)$	0.077	0.065
$f(0)$	-0.041	-0.023
$f(0)+rd(0)$	-0.001	0.017
h	0.239	0.174
$d(\infty)$	0.673	0.725
$m(\infty)$	0.041	0.042
$f(\infty)$	0.014	0.013
$f(\infty)+rd(\infty)$	0.041	0.042
α	0.6	0.6
β	0.741	1.261
$L^F(t_0)$	0.280	0.297
$L^{CB}(t_0)$	0.161	0.175

Government debt stabilization is achieved at the cost of a permanent decrease in primary fiscal deficits and an increase in inflation. As compared to the initial situation in Table 6.1, less of the adjustment burden is shifted to the future, since β is lower, both in the cooperative and non-cooperative case.

Conclusions

This paper extended the analysis of Tabellini (1986) concerning the conflict between monetary and fiscal authorities on debt stabilization. The differential game framework on the interaction between fiscal and monetary authorities was reconsidered. We derived explicit solutions of the dynamics of the fiscal deficit, inflation and government debt in the cooperative and Nash open-loop equilibria. A comparison between both equilibria provided interesting results regarding differences in transitional dynamics and steady-state characteristics of both equilibria. Coordination of fiscal and monetary policies internalizes the positive spillover from debt stabilization efforts and thus induces a higher adjustment speed and lower steady-state debt. Cooperation in policies, however, can lead to higher steady-state inflation if the Central Bank is weak.

The unpleasant monetarist arithmetic of Sargent and Wallace was reformulated in the context of the differential game between the Treasury and Central Bank. A strong fiscal player -in the sense of not taking any responsibility in debt stabilization- was seen to be the cause of the unpleasant monetarist arithmetic. Moreover, we considered the effects of measures to strengthen the independence of the Central Bank in both equilibria. In the cooperative equilibrium a more independent Central Bank causes higher long run inflation, basically because of the higher government debt accumulation that such a measure induces. In the Nash open-loop equilibrium, more Central Bank independence reduces long run inflation, if the fiscal player is not too strong at least. Finally, we used a numerical example to illustrate our main findings.

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Appendix A The dynamic government budget constraint.

The nominal primary fiscal deficit, $F(t)$, and nominal interest payments on government debt, $i(t)D(t)$, can be financed either by government debt accumulation, $\dot{D}$, or base money creation $\dot{B}$:

$$\dot{D} = i(t)D(t) + F(t) - \dot{B} \qquad \text{(A.1)}$$

Assume that the nominal interest rate, $i(t)$, behaves according to the Fisher hypothesis:

$$i(t) = \rho(t) + \pi^e \qquad \text{(A.2)}$$

in which $\rho(t)$ denotes the real interest rate and π^e the expected rate of inflation. In real terms and expressed in fractions of real GDP, the dynamic government budget constraint can be written as follows:

$$\dot{d} = (\rho + \pi^e - \pi - g)d(t) + f(t) - \dot{b} = (\rho - g)d(t) + f(t) - (\dot{b} + (\pi - \pi^e)d(t)) \qquad \text{(A.3)}$$

where lower-case variables denote real variables as fractions of GDP. It has been assumed for simplicity that the real interest rate, the growth rate of real output, g, and inflation expectations are exogenously given and constant. (A.3) reappears in the main text as (1), where $(\rho\text{-}g)$ is replaced by r and $(\dot{b}+(\pi\text{-}\pi^e)d(t))$ by $m(t)$.

If we assume for simplicity that demand for base money, $B(t)$, is of the constant velocity type: $B(t)=k.P(t)y(t)$, velocity will not be not influenced by the rate of inflation. Real money creation as a fraction of GDP, $\dot{b}$, is equal to $(\pi+g)b(t)=(\pi+g)k$. πk is the inflation tax on real base money in circulation whereas gk reflects structural growth in real base money from higher demand for transaction purposes because of economic growth. Seignorage revenues $m(t)$ arise from increases in the amount of real base money in circulation, $\dot{b}$, and the inflation tax on holders of nominal government debt, $(\pi\text{-}\pi^e)d(t)$. $m(t)$ is a positive linear function of inflation, given the growth rate of real output, g, velocity of money, k, inflation expectations, π^e, and the outstanding stock of government debt at time t, $d(t)$:

$$m(t) = (\pi + g)k + (\pi - \pi^e)d(t)$$

Appendix B. Analytical solutions of the Pareto and Nash open-loop equilibria.

The $\{d(t),\mu^i(t)\}$ systems that result from the interaction between monetary and fiscal authorities in the Pareto and Nash open-loop case are linear dynamic systems of the form,

$$\dot{x} \;=\; \boldsymbol{A}x + \boldsymbol{b} \tag{B.1}$$

The steady-state of the system, $x(\infty)$, is found by solving:

$$x(\infty) \;=\; -\boldsymbol{A}^{-1}\boldsymbol{b} \tag{B.2}$$

The adjustment speed, *h*, of the dynamic systems is defined as the absolute value of the stable eigenvalue of *A*. The eigenvalues are the roots from the characteristic polynomial that is found by solving det(*A*-*wI*)=0 where w is the vector of eigenvalues. If the dynamic systems are saddlepoint stable, the co-state variables $\mu^i(t)$ will take initial values $\mu^i(0)$ such that the system is placed on its unique converging dynamic trajectory, given the initial stock of debt *d*(0). The initial value of the forward-looking variables, $\mu^i(0)$, is found by applying the method proposed by Judd (1982):

- take the Laplace[17] transform $L[(.),s]$ of the dynamic system, $\dot{x}=Ax+b$:

$$\boldsymbol{L}[\dot{x},s] = \boldsymbol{A}.\boldsymbol{L}[x,s] + \boldsymbol{L}[\boldsymbol{b},s] \tag{B.3}$$

- use the fact that $L[\dot{x},s]= s.L[x,s]-x(0)$. Equating both expressions for $L[\dot{x},s]$ yields:

$$L[x\,,s] \;=\; (sI - A)^{-1}\,[\,L[b,s] + x(0)\,] \tag{B.4}$$

- impose the condition that $\mu^i(0)$ adjusts in such a manner that saddlepoint stability of the dynamic system is ensured, given the initial stock of government debt, *d*(0). This implies that $L[b,z]+x(0)=0$, where *z* is any unstable eigenvalue of A. Imposing this condition gives $\mu^i(0)$.

I The Pareto equilibrium

The cooperative equilibrium is characterized by the following $\{d(t),\mu^i(t)\}$ system:

[17] The Laplace transform of *f*(*t*), *F*(*s*) is defined as: $F(s) = \int_{t_0}^{\infty} f(t)\, e^{-s(t_0-t)}\, dt$.

$$\begin{bmatrix}\dot{d}\\ \dot{\mu}^P\end{bmatrix} = \begin{bmatrix} r & -(\frac{1}{\omega}+\frac{1}{\omega\eta+1}) \\ -(\omega\lambda+\kappa) & \delta - r\end{bmatrix}\begin{bmatrix}d(t)\\ \mu^P(t)\end{bmatrix} + \begin{bmatrix}\bar{f}-\bar{m}\\ (\omega\lambda+\kappa)\bar{d}\end{bmatrix} \tag{B.5}$$

The inverse of the matrix A of (B.5) is equal to:

$$A^{-1} = \frac{1}{-\Delta^P}\begin{bmatrix}\delta - r & (\frac{1}{\omega}+\frac{1}{\omega\eta+1}) \\ \omega\lambda+\kappa & r\end{bmatrix} \tag{B.6}$$

The stable eigenvalue of A determines the adjustment speed, h^P, of the average system (B.5):

$$h^P = -\frac{1}{2}\delta + \frac{1}{2}\sqrt{\delta^2+4\Delta^P} = -\frac{1}{2}\delta + \frac{1}{2}\sqrt{\delta^2+4[(\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa)-r(\delta-r)]} \tag{B.7}$$

where the determinant of A, Δ^P , is equal to the product of the adjustment speed, h^P, and the unstable eigenvalue of the average system, $z^P=h^P+\delta$:

$$\Delta^P = h^P(h^P+\delta) = (\frac{1}{\omega}+\frac{1}{\omega\eta+1})(\omega\lambda+\kappa) - r(\delta-r) \tag{B.8}$$

Note that z^P is equal to $h^P+\delta$, since the trace of A, δ, has to be equal to the sum of $-h^P$ and z^P. If we define β^P as $\frac{\delta-r}{\Delta^P}$, we can write the steady-state of the system as:

$$d^P(\infty) = \bar{d} + \beta^P(\bar{f}+r\bar{d}-\bar{m})$$

$$\mu^P(\infty) = \frac{(\omega\lambda+\kappa)(\bar{f}+r\bar{d}-\bar{m})}{\Delta^P} \tag{B.9}$$

Using the first order conditions in (5) we can write $f^P(\infty)$ and $m^P(\infty)$ as:

$$f^P(\infty) = \bar{f} - \frac{\mu^P(\infty)}{\omega} = \bar{f} - \frac{(\omega\lambda+\kappa)(\bar{f}+r\bar{d}-\bar{m})}{\omega\Delta^P}$$

$$m^P(\infty) = \bar{m} + \frac{\mu^P(\infty)}{\omega\eta+1} = \bar{m} + \frac{(\omega\lambda+\kappa)(\bar{f}+r\bar{d}-\bar{m})}{(\omega\eta+1)\Delta^P} \tag{B.10}$$

If we define $\alpha^P = \frac{\frac{1}{\omega}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}$ and $(1-\alpha^P) = \frac{\frac{1}{\omega\eta+1}}{\frac{1}{\omega}+\frac{1}{\omega\eta+1}}$ and note furthermore that

$\frac{(\omega\lambda+\kappa)(\frac{1}{\omega}+\frac{1}{\omega\eta+1})}{\Delta^P} = 1 + r\beta^P$, we can rewrite (B.10) as:

$$f^P(\infty) = \bar{f} - \alpha^P (1 + r\beta^P)(\bar{f} + r\bar{d} - \bar{m})$$
$$m^P(\infty) = \bar{m} + (1 - \alpha^P)(1 + r\beta^P)(\bar{f} + r\bar{d} - \bar{m}) \tag{B.11}$$

Subtracting $f^P(\infty)$ and $m^P(\infty)$, we arrive at:

$$f^P(\infty) - m^P(\infty) = \bar{f} - \bar{m} - (1 + r\beta^P)(\bar{f} + r\bar{d} - \bar{m}) = -r\bar{d} - r\beta^P(\bar{f} + r\bar{d} - \bar{m}) \tag{B.12}$$

To determine the initial state of the Pareto equilibrium, take the Laplace transform of (B.5),

$$\begin{bmatrix} L(d^P, s) \\ L(\mu^P, s) \end{bmatrix} = \frac{1}{|sI - A|} \begin{bmatrix} s - (\delta - r) & -(\frac{1}{\omega} + \frac{1}{\omega\eta + 1}) \\ -(\omega\lambda + \kappa) & s - r \end{bmatrix} \begin{bmatrix} \frac{\bar{f} - \bar{m}}{s} + d(0) \\ \frac{(\omega\lambda + \kappa)\bar{d}}{s} + \mu^P(0) \end{bmatrix} \tag{B.13}$$

A solution to (B.13) remains bounded if $\mu^P(0)$ satisfies:

$$(z^P - (\delta - r))(\frac{\bar{f} - \bar{m}}{z^P} + d(0)) - (\frac{1}{\omega} + \frac{1}{(\omega\eta + 1)})(\frac{(\omega\lambda + \kappa)\bar{d}}{z^P} + \mu^P(0)) = 0 \tag{B.14}$$

The unstable eigenvalue z^P of the adjustment matrix A is equal to $h^P + \delta$. Therefore, we can rewrite (B.14) as,

$$\mu^P(0) = \frac{(\frac{h^P + r}{h^P + \delta})(\bar{f} - \bar{m}) + (h^P + r)d(0)}{\frac{1}{\omega} + \frac{1}{\omega\eta + 1}} - \frac{(\omega\lambda + \kappa)\bar{d}}{h^P + \delta} \tag{B.15}$$

According to the first order conditions of (5), we write the short-run changes in $f(t)$ and $m(t)$ as:

$$f^P(0) = \bar{f} - \frac{\mu^P(0)}{\omega} = \bar{f} - \frac{1}{\omega}\left[\frac{(h^P + r)(\frac{\bar{f} - \bar{m}}{h^P + \delta} + d(0))}{\frac{1}{\omega} + \frac{1}{\omega\eta + 1}} - \frac{(\omega\lambda + \kappa)\bar{d}}{h^P + \delta}\right]$$
$$m^P(0) = \bar{m} + \frac{\mu^P(0)}{\omega\eta + 1} = \bar{m} + \frac{1}{\omega\eta + 1}\left[\frac{(h^P + r)(\frac{\bar{f} - \bar{m}}{h^P + \delta} + d(0))}{\frac{1}{\omega} + \frac{1}{\omega\eta + 1}} - \frac{(\omega\lambda + \kappa)\bar{d}}{h^P + \delta}\right] \tag{B.16}$$

Using the definitions of α^P,β^P and Δ^P, we can rewrite (B.16) as:

$$f^P(0) = \bar{f} - \alpha^P[(1-h^P\beta^P)(\bar{f}+r\bar{d}-\bar{m}) + (h^P+r)(d(0)-\bar{d})]$$
$$m^P(0) = \bar{m} + (1-\alpha^P)[(1-h^P\beta^P)(\bar{f}+r\bar{d}-\bar{m}) + (h^P+r)(d(0)-\bar{d})] \qquad \text{(B.17)}$$

Subtracting $f^P(0)$ and $m^P(0)$ yields:

$$f^P(0) - m^P(0) = h^P\beta^P(\bar{f}+r\bar{d}-\bar{m}) + h^P(\bar{d}-d(0)) - r\,d(0) \qquad \text{(B.18)}$$

II The Nash open-loop equilibrium

The dynamic system of the Nash open-loop equilibrium is given by:

$$\begin{bmatrix} \dot{d} \\ \dot{\mu}^F \\ \dot{\mu}^{CB} \end{bmatrix} = \begin{bmatrix} r & -1 & -1 \\ -\lambda & \delta-r & 0 \\ -\kappa & 0 & \delta-r \end{bmatrix} \begin{bmatrix} d(t) \\ \mu^F(t) \\ \mu^{CB}(t) \end{bmatrix} + \begin{bmatrix} \bar{f}-\bar{m} \\ \lambda\bar{d} \\ \kappa\bar{d} \end{bmatrix} \qquad \text{(B.19)}$$

The three eigenvalues of A in (B.19) are $-h^O$, $z^O=h^O+\delta$ and δ-r. The adjustment speed of (B.19) is equal to:

$$h^O = -\frac{1}{2}\delta + \frac{1}{2}\sqrt{\delta^2+\Delta^O} = -\frac{1}{2}\delta + \frac{1}{2}\sqrt{\delta^2+4[\lambda+\kappa-r(\delta-r)]} \qquad \text{(B.20)}$$

The inverse of A is equal to

$$A^{-1} = \frac{1}{-\Delta^O(\delta-r)} \begin{bmatrix} (\delta-r)^2 & \delta-r & \delta-r \\ \lambda(\delta-r) & r(\delta-r)-\kappa & \lambda \\ \kappa(\delta-r) & \kappa & r(\delta-r)-\lambda \end{bmatrix} \quad \Delta^O = h^O(h^O+\delta) = \lambda+\kappa-r(\delta-r) \qquad \text{(B.21)}$$

If we define β^O as $\frac{\delta-r}{\Delta^O}$, we can write the Nash open-loop equilibrium as:

$$d^O(\infty) = d + \beta^O(f + r\,d - \bar{m})$$
$$\mu^F(\infty) = \frac{\lambda}{\Delta^O}(\bar{f}+r\bar{d}-\bar{m}) \qquad \text{(B.22)}$$
$$\mu^{CB}(\infty) = \frac{\kappa}{\Delta^O}(\bar{f}+r\bar{d}-\bar{m})$$

Using the first order conditions in (6) and (7), we can write $f^O(\infty)$ and $m^O(\infty)$ as:

$$f^O(\infty) = \bar{f} - \mu^F(\infty) = \bar{f} - \frac{\lambda}{\Delta^O}(\bar{f}+r\bar{d}-\bar{m})$$
$$m^O(\infty) = \bar{m} + \mu^{CB}(\infty) = \bar{m} + \frac{\kappa}{\Delta^O}(\bar{f}+r\bar{d}-\bar{m}) \qquad \text{(B.23)}$$

Defining $\alpha^O = \frac{\lambda}{\lambda+\kappa}$ and $(1-\alpha^O) = \frac{\kappa}{\lambda+\kappa}$, we can rewrite (B.23) as:

$$f^O(\infty) = \bar{f} - \alpha^O(1+r\beta^O)(\bar{f}+r\bar{d}-\bar{m})$$
$$m^O(\infty) = \bar{m} + (1-\alpha^O)(1+r\beta^O)(\bar{f}+r\bar{d}-\bar{m}) \quad \text{(B.24)}$$

Subtracting $f^O(\infty)$ and $m^O(\infty)$, we find:

$$f^O(\infty) - m^O(\infty) = \bar{f} - \bar{m} - (1+r\beta^O)(\bar{f}+r\bar{d}-\bar{m}) = -r\bar{d} - r\beta^O(\bar{f}+r\bar{d}-\bar{m}) \quad \text{(B.25)}$$

Find the initial state of the Nash open-loop equilibrium by taking the Laplace transform of (B.19):

$$\begin{bmatrix} L(d^O,s) \\ L(\mu^F,s) \\ L(\mu^{CB},s) \end{bmatrix} = \frac{1}{|sI-A|}\begin{bmatrix} (s-(\delta-r))^2 & -(s-(\delta-r)) & -(s-(\delta-r)) \\ -\lambda(s-(\delta-r)) & (s-r)(s-(\delta-r))-\kappa & \lambda \\ -\kappa(s-(\delta-r)) & \kappa & (s-r)(s-(\delta-r))-\lambda \end{bmatrix}\begin{bmatrix} \frac{\bar{f}-\bar{m}}{s}+d(0) \\ \frac{\lambda\bar{d}}{s}+\mu^F(0) \\ \frac{\kappa\bar{d}}{s}+\mu^{CB}(0) \end{bmatrix} \quad \text{(B.26)}$$

A solution to (B.26) remains bounded as long as for given values of the unstable eigenvalues δ-r and z^O=h^O+δ, $\mu^F(t)$ and $\mu^{CB}(t)$ take such initial values that both of the following conditions hold,

$$(h^O+r)\left(\frac{\bar{f}-\bar{m}}{h^O+\delta}+d(0)\right) - \frac{\lambda\bar{d}}{h^O+\delta} - \mu^F(0) - \frac{\kappa\bar{d}}{h^O+\delta} - \mu^{CB}(0) = 0$$
$$\kappa\,\mu^F(0) - \lambda\,\mu^{CB}(0) = 0 \quad \text{(B.27)}$$

hold. Combining both conditions, we arrive at:

$$\mu^F(0) = \frac{\lambda}{\lambda+\kappa}\left[\left(\frac{h^O+r}{h^O+\delta}\right)(\bar{f}-\bar{m}) + (h^O+r)d(0) - \left(\frac{\lambda+\kappa}{h^O+\delta}\right)\bar{d}\right]$$
$$\mu^{CB}(0) = \frac{\kappa}{\lambda+\kappa}\left[\left(\frac{h^O+r}{h^O+\delta}\right)(\bar{f}-\bar{m}) + (h^O+r)d(0) - \left(\frac{\lambda+\kappa}{h^O+\delta}\right)\bar{d}\right] \quad \text{(B.28)}$$

Substituting (B.28) into the first order conditions (6) and (7), gives:

$$f^O(0) = \bar{f} - \mu^F(0) = \bar{f} - \frac{\lambda}{\lambda+\kappa}\left[\left(\frac{h^O+r}{h^O+\delta}\right)(\bar{f}-\bar{m}) + (h^O+r)d(0)) - \left(\frac{\lambda+\kappa}{h^O+\delta}\right)\bar{d}\right]$$
$$m^O(0) = \bar{m} + \mu^{CB}(0) = \bar{m} + \frac{\kappa}{\lambda+\kappa}\left[\left(\frac{h^O+r}{h^O+\delta}\right)(\bar{f}-\bar{m}) + (h^O+r)d(0)) - \left(\frac{\lambda+\kappa}{h^O+\delta}\right)\bar{d}\right] \quad \text{(B.29)}$$

With the definitions of α^O and β^O, and noting that $\frac{h^O+r}{h^O+\delta} = 1 - h^O\beta^O$ and $\frac{\lambda+\kappa}{h^O+\delta} = (1+r\beta^O)h^O$

[18] (B.29) can be rewritten as:

$$f^O(0) = \bar{f} - \alpha^O[(1 - h^O\beta^O)(\bar{f} + r\bar{d} - \bar{m}) + (h^O + r)(d(0) - \bar{d})]$$
$$m^O(0) = \bar{m} + (1 - \alpha^O)[(1 - h^O\beta^O)(\bar{f} + r\bar{d} - \bar{m}) + (h^O + r)(d(0) - \bar{d})] \tag{B.30}$$

If we subtract $f^O(0)$ and $m^O(0)$, we find:

$$f^O(0) - m^O(0) = h^O\beta^O(\bar{f} + r\bar{d} - \bar{m}) + h^O(\bar{d} - d(0)) - r\,d(0) \tag{B.31}$$

Appendix C Comparative dynamics of the Pareto and Nash open-loop equilibria.

If we calculate the partial derivatives of initial values, steady-states and the adjustment speed w.r.t. the model parameters, we can infer the effects of changes in the model parameters. If we assume that $\delta > r$[19], $\bar{f} + r\bar{d} > \bar{m}$ and $d(0) > \bar{d}$ we can sign the partial derivatives in the Pareto and Nash open-loop equilibria as follows:

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[18] These follow since: $\frac{h^O + r}{h^O + \delta} = 1 - \frac{(\delta - r)}{h^O + \delta} = 1 - \frac{h^O(\delta - r)}{\Delta^O}$ and $\frac{\lambda + \kappa}{h^O + \delta} = \frac{h^O(\lambda + \kappa)}{\Delta^O} = h^O[1 - \frac{r(\delta - r)}{\Delta^O}]$.

[19] The signs of the various partial derivatives in the case where $\delta = r$ are found by noting that $\delta = r$ implies that β^P and β^O are equal to zero.

- of the initial money creation, $m(0)$:

$$\frac{\partial m^P(0)}{\partial \lambda} = \frac{(1-\alpha^P)}{\alpha^P\sqrt{\delta^2+4\Delta^P}}\,[(\frac{\delta-r}{(h^P+\delta)^2})(\bar{f}+r\bar{d}-\bar{m})+(d(0)-\bar{d})] > 0$$

$$\frac{\partial m^O(0)}{\partial \lambda} = -\frac{(1-\alpha^O)}{\lambda+\kappa}[(1-h^O\beta^O-\frac{(\delta-r)(\lambda+\kappa)}{(h^O+\delta)^2\sqrt{\delta^2+4\Delta^O}})(\bar{f}+r\bar{d}-\bar{m})+(r+h^O-\frac{\lambda+\kappa}{\sqrt{\delta^2+4\Delta^O}})(d(0)-\bar{d})] >< 0$$

$$\frac{\partial m^P(0)}{\partial \kappa} = \frac{(1-\alpha^P)}{\omega\,\alpha^P\sqrt{\delta^2+4\Delta^P}}[(\frac{\delta-r}{(h^P+\delta)^2})(\bar{f}+r\bar{d}-\bar{m})+(d(0)-\bar{d})] > 0$$

$$\frac{\partial m^O(0)}{\partial \kappa} = \frac{\alpha^O}{\lambda+\kappa}[(1-h^O\beta^O)(\bar{f}+r\bar{d}-\bar{m})+(h^O+r)(d(0)-\bar{d})]+\frac{(1-\alpha^O)}{\sqrt{\delta^2+4\Delta^O}}[(\frac{\delta-r}{(h^O+\delta)^2})(\bar{f}+r\bar{d}-\bar{m})+(d(0)\cdot$$

$$\frac{\partial m^P(0)}{\partial \bar{f}} = (1-\alpha^P)(1-h^P\beta^P) > 0$$

$$\frac{\partial m^P(0)}{\partial \bar{m}} = \alpha^P+(1-\alpha^P)h^P\beta^P > 0$$

$$\frac{\partial m^P(0)}{\partial \bar{d}} = -(1-\alpha^P)(1+r\beta^P)h^P < 0$$

$$\frac{\partial m^P(0)}{\partial d(0)} = (1-\alpha^P)(h^P+r) > 0$$

- of the initial fiscal deficit, $f(0)$:

$$\frac{\partial f^P(0)}{\partial \lambda} = -\frac{1}{\sqrt{\delta^2+4\Delta^P}}[(\frac{\delta-r}{h^P+\delta})(\bar{f}+r\bar{d}-\bar{m})+(d(0)-\bar{d})] < 0$$

$$\frac{\partial f^O(0)}{\partial \lambda} = -\frac{(1-\alpha^O)}{\lambda+\kappa}[(1-h^O\beta^O)(\bar{f}+r\bar{d}-\bar{m})+(r+h^O)(d(0)-\bar{d})]-\frac{\alpha^O}{\sqrt{\delta^2+4\Delta^O}}[\frac{\delta-r}{(h^O+\delta)^2}(\bar{f}+r\bar{d}-\bar{m})+(d(0)$$

$$\frac{\partial f^P(0)}{\partial \kappa} = -\frac{1}{\omega\sqrt{\delta^2+4\Delta^P}}[\frac{\delta-r}{h^P+\delta}(\bar{f}+r\bar{d}-\bar{m})+(d(0)-\bar{d})] < 0$$

$$\frac{\partial f^O(0)}{\partial \kappa} = \frac{\alpha^O}{\lambda+\kappa}[(1-h^O\beta^O-\frac{(\lambda+\kappa)(\delta-r)}{(h^O+\delta)^2\sqrt{\delta^2+4\Delta^O}})(\bar{f}+r\bar{d}-\bar{m})+(h^O+r-\frac{(\lambda+\kappa)}{\sqrt{\delta^2+4\Delta^O}})(d(0)-\bar{d})] <> 0$$

$$\frac{\partial f^P(0)}{\partial \bar{f}} = (1-\alpha^P)(1-h^P\beta^P) > 0$$

$$\frac{\partial f^P(0)}{\partial \bar{m}} = \alpha^P(1-h^P\beta^P) > 0$$

$$\frac{\partial f^P(0)}{\partial \bar{d}} = \alpha^P(1+r\beta^P)h^P > 0$$

$$\frac{\partial f^P(0)}{\partial d(0)} = -\alpha^P(h^P+r) < 0$$

- of the adjustment speed, *h*:

$$\frac{\partial h^P}{\partial \lambda} = \frac{1}{\alpha^P \sqrt{\delta^2 + 4\Delta^P}} > 0 \qquad \frac{\partial h^O}{\partial \lambda} = \frac{1}{\sqrt{\delta^2 + 4\Delta^O}} > 0$$

$$\frac{\partial h^P}{\partial \kappa} = \frac{1}{\omega \alpha^P \sqrt{\delta^2 + 4\Delta^P}} > 0 \qquad \frac{\partial h^O}{\partial \kappa} = \frac{1}{\sqrt{\delta^2 + 4\Delta^O}} > 0$$

$$\frac{\partial h^P}{\partial \bar{f}} = 0 \qquad \frac{\partial h^O}{\partial \bar{f}} = 0$$

$$\frac{\partial h^P}{\partial \bar{m}} = 0 \qquad \frac{\partial h^O}{\partial \bar{m}} = 0$$

$$\frac{\partial h^P}{\partial \bar{d}} = 0 \qquad \frac{\partial h^O}{\partial \bar{d}} = 0$$

$$\frac{\partial h^P}{\partial d(0)} = 0 \qquad \frac{\partial h^O}{\partial d(0)} = 0$$

- of steady-state government debt:

$$\frac{\partial d^P(\infty)}{\partial \lambda} = -\frac{\beta^P}{\Delta^P \alpha^P}(\bar{f} - \bar{m} + r\bar{d}) < 0 \qquad \frac{\partial d^O(\infty)}{\partial \lambda} = -\frac{\beta^O}{\Delta^O}(\bar{f} - \bar{m} + r\bar{d}) < 0$$

$$\frac{\partial d^P(\infty)}{\partial \kappa} = -\frac{\beta^P}{\omega \Delta^P \alpha^P}(\bar{f} - \bar{m} + r\bar{d}) < 0 \qquad \frac{\partial d^O(\infty)}{\partial \kappa} = -\frac{\beta^O}{\Delta^O}(\bar{f} - \bar{m} + r\bar{d}) < 0$$

$$\frac{\partial d^P(\infty)}{\partial \bar{f}} = \beta^P > 0 \qquad \frac{\partial d^O(\infty)}{\partial \bar{f}} = \beta^O > 0$$

$$\frac{\partial d^P(\infty)}{\partial \bar{m}} = -\beta^P < 0 \qquad \frac{\partial d^O(\infty)}{\partial \bar{m}} = -\beta^O < 0$$

$$\frac{\partial d^P(\infty)}{\partial \bar{d}} = (1 + r\beta^P) > 0 \qquad \frac{\partial d^O(\infty)}{\partial \bar{d}} = (1 + r\beta^O) > 0$$

$$\frac{\partial d^P(\infty)}{\partial d(0)} = 0 \qquad \frac{\partial d^O(\infty)}{\partial d(0)} = 0$$

- of steady-state money creation:

$$\frac{\partial m^P(\infty)}{\partial \lambda} = -\frac{(1-\alpha^P)}{\alpha^P}\frac{r\beta^P}{\Delta^P}(\bar{f}-\bar{m}+r\bar{d}) < 0 \qquad \frac{\partial m^O(\infty)}{\partial \lambda} = -\frac{\kappa}{\Delta^{O^2}}(\bar{f}-\bar{m}+r\bar{d}) < 0$$

$$\frac{\partial m^P(\infty)}{\partial \kappa} = -\frac{(1-\alpha^P)}{\alpha^P}\frac{r\beta^P}{\omega\,\Delta^P}(\bar{f}-\bar{m}+r\bar{d}) < 0 \qquad \frac{\partial m^O(\infty)}{\partial \kappa} = \frac{\lambda - r(\delta - r)}{\Delta^{O^2}}(\bar{f}-\bar{m}+r\bar{d}) <> 0$$

$$\frac{\partial m^P(\infty)}{\partial \bar{f}} = (1-\alpha^P)(1+r\beta^P) > 0 \qquad \frac{\partial m^O(\infty)}{\partial \bar{f}} = (1-\alpha^O)(1+r\beta^O) > 0$$

$$\frac{\partial m^P(\infty)}{\partial \bar{m}} = 1-(1-\alpha^P)(1+r\beta^P) <> 0 \qquad \frac{\partial m^O(\infty)}{\partial \bar{m}} = 1-(1-\alpha^O)(1+r\beta^O) <> 0$$

$$\frac{\partial m^P(\infty)}{\partial \bar{d}} = r(1-\alpha^P)(1+r\beta^P) > 0 \qquad \frac{\partial m^O(\infty)}{\partial \bar{d}} = r(1-\alpha^O)(1+r\beta^O) > 0$$

$$\frac{\partial m^P(\infty)}{\partial d(0)} = 0 \qquad \frac{\partial m^O(\infty)}{\partial d(0)} = 0$$

- of steady-state fiscal deficit:

$$\frac{\partial f^P(\infty)}{\partial \lambda} = \frac{r\beta^P}{\Delta^P}(\bar{f}-\bar{m}+r\bar{d}) > 0 \qquad \frac{\partial f^O(\infty)}{\partial \lambda} = -\frac{(\kappa - r(\delta - r))}{\Delta^{O^2}}(\bar{f}-\bar{m}+r\bar{d}) <> 0$$

$$\frac{\partial f^P(\infty)}{\partial \kappa} = \frac{r\beta^P}{\Delta^P\omega}(\bar{f}-\bar{m}+r\bar{d}) > 0 \qquad \frac{\partial f^O(\infty)}{\partial \kappa} = \frac{\lambda}{\Delta^{O^2}}(\bar{f}-\bar{m}+r\bar{d}) > 0$$

$$\frac{\partial f^P(\infty)}{\partial \bar{f}} = 1-\alpha^P(1+r\beta^P) <> 0 \qquad \frac{\partial f^O(\infty)}{\partial \bar{f}} = 1-\alpha^O(1+r\beta^O) <> 0$$

$$\frac{\partial f^P(\infty)}{\partial \bar{m}} = \alpha^P(1+r\beta^P) > 0 \qquad \frac{\partial f^O(\infty)}{\partial \bar{m}} = \alpha^O(1+r\beta^O) > 0$$

$$\frac{\partial f^P(\infty)}{\partial \bar{d}} = -r\alpha^P(1+r\beta^P) < 0 \qquad \frac{\partial f^O(\infty)}{\partial \bar{d}} = -r\alpha^O(1+r\beta^O) < 0$$

$$\frac{\partial f^P(\infty)}{\partial d(0)} = 0 \qquad \frac{\partial f^O(\infty)}{\partial d(0)} = 0$$