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THE PARTNERED CORE OF AN ECONOMY

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The Partnered Core of an Economy*

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Abstract

An allocation is in the partnered core if it admits no asymmetric dependencies between players and groups of players. The partnered core of a game was introduced in Reny, Winter and Wooders (1993) and Reny and Wooders (1993). In this paper, in the context of a model with arbitrary consumption sets, we introduce the partnered core of an economy and establish that no unbounded arbitrage – a condition limiting gains from trade within groups and diversity of preferences – is sufficient for the nonemptiness of the partnered core. Under a condition of "extreme desirability" (including strict convexity as a special case), no unbounded arbitrage is necessary and sufficient for nonemptiness of the partnered core. We also establish that with strict convexity of preferences, the Bennett and Zame (1988) result that a competitive payoff is partnered extends to situations with arbitrary consumption sets. From Werner (1987) and the above results it follows that with strict convexity, no unbounded arbitrage is necessary and sufficient for existence of a partnered competitive equilibrium and nonemptiness of the partnered core.

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1 Introduction

A natural property of a distribution rule for an economy is that it has the ability to prevent "asymmetric dependencies". For example, if one player needs to trade with another player to realize his core payoff but the other player does not need to trade with the first, then there is an asymmetric dependency. An allocation which does not exhibit asymmetric dependencies is called a *partnered allocation*.

An allocation is in the partnered core if it is in the core and if, additionally, there are no asymmetric dependencies between any pair of players. The partnered core of a game with side payments was introduced in Reny, Winter and Wooders (1993) and the partnered core of a game without side payments was introduced in Reny and Wooders (1993). In this paper we introduce the partnered core of an economy with arbitrary consumption sets and show that the Page-Werner condition of no unbounded arbitrage[1] is sufficient for the existence of at least one partnered core allocation. No unbounded arbitrage is the condition that no group of agents can engage in unbounded, utility nondecreasing and rational trades. For "strictly reconcilable economies," including those with strictly convex preferences, we show that no unbounded arbitrage is necessary and sufficient for nonemptiness of the partnered core of the economy. Since the partnered core can significantly refine the core, our results are stronger than analogous results for the core in Page and Wooders (1993).

The competitive payoff is not necessarily in the partnered core. This is a significant weakness of the concept. If a group of players is dependent on another group of players at the competitive prices, it is reasonable to suppose that the group in the stronger position, instead of taking prices as given, will attempt to gain a more favorable outcome for its members. Bennett and Zame (1988, Theorem 3), however, demonstrate the important result that with strict concavity of preferences, the competitive outcome has the partnership property. We demonstrate that their result extends to economies with arbitrary consumption sets and possibly nonmonotonicities.

[1]Page (1984,1987) and Werner (1987).

The model used in this paper is sufficiently similar to that of Werner (1987) so that we can appeal to his proof for existence of an equilibrium. Also, we can readily provide a sharpening of Werner's result that no unbounded arbitrage is necessary for existence of competitive equilibrium. Thus, for the model treated in this paper no unbounded arbitrage is both necessary and sufficient for nonemptiness of the partnered core and existence of equilibrium. From our results on the partnership property of the competitive equilibrium it follows that, with a strengthening of our condition of "extreme desirability" to strict convexity of preferences, no unbounded arbitrage is necessary and sufficient for both the existence of a partnered competitive equilibrium and nonemptiness of the partnered core.

Further motivation for the partnered core is provided by Reny and Wooders (1995b) who use the concept of partnership to provide an explanation of the division of organizations into not-necessarily-self-sufficient states, as in a "commonwealth." For additional motivation of partnership, examples, and discussion of the literature we refer the reader to the literature referenced in the current paper.

Before proceeding, we briefly place our work in the context of the literature. Necessary and sufficient conditions for nonemptiness of the core of a game appear in Bondareva (1962), Shapley (1967), Kaneko and Wooders (1982), Keiding and Thorlund-Peterson (1987), Keiding (1993), Page and Wooders (1993,1994) and Chichilnisky (1994).[2] As has been established in the literature, conditions limiting arbitrage opportunities have an intimate connection with the competitive equilibrium. Both necessary and sufficient conditions for existence of equilibrium appear in Green (1973), Hart (1974), Grandmont (1982), Page (1982), Hammond (1983), Werner (1987), Page and Schlesinger (1993), Chichilnisky (1995), and Page and Wooders (1993,1994). A detailed discussion of arbitrage in these papers and the relationship of the conditions limiting arbitrages appears in Page and Wooders (1994).

[2]Chichilnisky has referenced earlier Columbia and Stanford University Discussion papers treating the core and arbitrage, but we have been unable to locate the papers in the referenced sources and also unable to obtain copies.

2 An Economy with Arbitrary Consumptions Sets and Nonmonotonicities

Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ denote an *exchange economy*. Each agent j has a *consumption set* $X_j \subset \Re^L$ and an *endowment* $\omega_j \in X_j$. The j^{th} agent's preferences over X_j are specified via a *utility function* $u_j(\cdot) : X_j \longrightarrow \Re$.[3]

The set of *individually rational allocations* is given by

$$A = \{(x_1, ..., x_n) \in X_1 \times ... \times X_n :$$
$$\textstyle\sum_{j=1}^n x_j = \sum_{j=1}^n \omega_j \text{ and } u_j(x_j) \geq u_j(\omega_j) \text{ for all } j\}.$$

The corresponding set of *utility possibilities* is given by

$$U(A) = \{(u_1, ..., u_n) \in \Re^n : \text{ for some } (x_1, ..., x_n) \in A,\ u_j = u_j(x) \text{ for all } j\}.$$

Given any individually rational allocation $x = (x_1, ..., x_n) \in A$, let $pr_j(x) = x_j$.

For each $x \in X_j$ the j^{th} agent's *preferred set* is given by

$$P_j(x_j) := \{x' \in X_j : u_j(x') > u_j(x_j)\}.$$

The following assumptions on the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ will be maintained.[4]

(A-1)(a) For each $j = 1, ..., n$, X_j is closed and convex and $\omega_j \in intX_j$ where "*int*" denotes "interior".

(A-1)(b) For each $j = 1, ..., n$, $u_j(\cdot)$ is continuous and concave with $P_j(x_j) \neq \emptyset$ for all $x_j \in pr_j(A)$.

Thus, preferences are continuous, concave, and non-satiated at individually rational allocations.[5]

[3]We use utility representations of preferences for ease of discussion. The results of this paper, however, also hold for ordinal preferences where only quasi-concavity is required, as in Page and Wooders (1993).

[4]Our assumptions are chosen for brevity and clarity; in other research we relax several of the assumptions of this paper.

[5]This assumption does not imply local nonsatiation. Since arbitrage is a global rather than local concept, such satiation is not relevant.

Given prices $p \in \Re^L$, the cost of a consumption vector $x = (x_1, ..., x_L) \in X_j$ is $\langle x, p \rangle = \sum_{\ell=1}^{L} x_\ell \cdot p_\ell$. The *budget set* for the jth agent is given by $B(p, \omega_j) = \{x \in X_j : \langle p, x \rangle \leq \langle p, \omega_j \rangle\}$. Without loss of generality we can assume that commodity prices are contained in the unit ball $\mathcal{B} := \{p \in \Re^L : \|p\| \leq 1\}$.

An *equilibrium* for the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ is an $(n+1)$-tuple of vectors $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ such that

(i) $(\overline{x}_1, ..., \overline{x}_n) \in A$ (the allocation is feasible);

(ii) $\overline{p} \in B \backslash \{0\}$ (prices are in the unit ball and not all prices are zero);

(iii) and for each j,

(a) $\langle \overline{p}, \overline{x}_j \rangle = \langle \overline{p}, \omega_j \rangle$ (budget constraints are satisfied) and

(b) $P_j(\overline{x}_j) \cap B(\overline{p}, \omega_j) = \emptyset$ (there are no affordable preferred net trades).

2.1 Unbounded arbitrage

Given a subset $S \subset \Re^L$, we say that $y \in \Re^L$ is a *direction of recession for* S if $x + \lambda y \in S$ for all $\lambda \geq 0$ and $x \in S$. We shall denote by $\mathcal{R}(S)$ the set of all recession directions of S. If S is a closed convex set, then $\mathcal{R}(S)$ is a closed convex cone containing the origin (Rockafellar (1970), Section 8). Thus, the *recession cone* $\mathcal{R}(X_j)$ corresponding to the consumption set X_j is a closed convex cone containing the origin.

Now let

$$CP_j = \{y \in \mathcal{R}(X_j) : \text{ for some } x \in X_j$$
$$u_j(x + \lambda y) \text{ is nondecreasing in } \lambda \text{ for } \lambda \geq 0\}$$

and let

$$\mathcal{I}_j(x) = \{y \in \mathcal{R}(X_j) : u_j(x + \lambda y) \text{ is increasing in } \lambda \text{ for } \lambda \geq 0\}.$$

For each j, CP_j is the set of net trades that are feasible and utility nondecreasing on any scale starting at some consumption vector $x \in X_j$ while $\mathcal{I}_j(x)$ is the set of net trades that are feasible and utility *increasing* on any scale starting at a particular consumption vector $x \in X_j$. For each agent j, CP_j is a closed convex cone containing the origin. We shall refer to CP_j as *the j^{th} agent's arbitrage cone.* As indicated by the notation, the set of utility-increasing arbitrages $\mathcal{I}_j(x)$ depends on x but arbitrage cones are independent of x.[6]

2.1.1 Reconcilable and strictly reconcilable economies

We will sometimes assume *extreme desirability*, that is,

(A-2) $CP_j \backslash \{0\} = \mathcal{I}_j(x)$ for all x in X_j.

If an agent's utility function is strictly concave,then(A-2) holds automatically.

An economy satisfying (A-1) is said to be *reconcilable* – the diverse wants of agents can be reconciled by a price system. An economy satisfying (A-1) and (A-2) is said to be *strictly reconcilable.*

2.2 No unbounded arbitrage

As discussed by Page (1987,1989), in an exchange economy with heterogenous agents and possibly unbounded consumption sets, unbounded arbitrages arise due to differences in agents' preferences. To ensure the boundedness of the set of utility nondecreasing arbitrages, we utilize the Page-Werner condition of no unbounded arbitrage (see Page (1984,1987) and Werner (1985,1987)). No unbounded arbitrage is a similarity assumption on preferences, eliminating the possibility that any one trader can find a mutually compatible trading partner (or group of trading partners) with whom to engage in unbounded and possibility utility increasing trades. An economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ satisfies *no unbounded arbitrage* if :

$$\begin{aligned} &\text{whenever } \textstyle\sum_{J=1}^n y_j = 0 \text{ and } y_j \in CP_j \text{ for all agents } j \\ &\text{it holds that } y_j = 0 \text{ for all agents } j. \end{aligned} \tag{1}$$

It is important to note that no unbounded arbitrage does not imply an absence of bounded arbitrages for individual agents. (See Page (1989) and Page and Schlesinger

[6]In contrast, with only the requirement of quasi-concavity as in Page and Wooders (1993) rather than concavity as in this paper, arbitrage cones may well depend on x.

(1993) for further discussion of the distinction between bounded and unbounded arbitrage.)

No unbounded arbitrage is equivalent to the condition that agents' recession cones are *strictly* on one side of a hyperplane through the origin. The following characterization is an immediate consequence of the classical Dubovitski-Milyutin Theorem.[7] (See Page (1987) or Page and Wooders (1994).)

Proposition 1. *(Characterization of no unbounded arbitrage.)* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy. Then the following are equivalent:

(a) No unbounded arbitrage (1) holds.

(b) There exists a nonzero vector of prices $p \in \mathcal{B}$ such that for any agent j and any vector of net trades $y \in CP_j$ it holds that $\langle p, y \rangle > 0$.

Thus, no unbounded arbitrage is equivalent to the condition that there exists a vector of prices at which no individual has an opportunity for unbounded arbitrage.

With the Dubovitski-Milyutin Theorem in hand, we can now note that (b) is a special case of the condition introduced in Page (1984) for a general equilibrium model with price dependent preferences. Werner (1987) also uses (b). Page (1987), again for a model with price-dependent preferences, assumes (a). Page and Wooders (1993) prove the equivalence of (a) and (b) from the Hahn-Banach Theorem.

The intimate connection between no unbounded arbitrage and competitive equilibrium is summarized via the following result due to Werner (1987).[8]

Theorem. *In a strictly reconcilable economy, no unbounded arbitrage is necessary*

[7] We are grateful to Erik Balder for bringing the Dubovitski-Milyutin Theorem to our attention.

[8] In a general equilibrium model Werner shows that condition (b) in Proposition 1 is sufficient for the existence of equilibrium and notes that if indifference curves contain no half-lines then (b) is also necessary for existence. In an asset market model with no redundant assets and sufficiently risk averse agents, Page (1982) establishes that (b) is necessary and sufficient for existence of equilibrium. Other authors, in particular, Hart (1974), Hammond (1983), and Chichilnisky (1995) obtain related results; these are reviewed in detail and related to the Dubovitski-Milyutin Theorem in Page and Wooders (1994).

and sufficient for existence of equilibrium. Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a strictly reconcilable economy. Then the following are equivalent:

(a) $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ has an equilibrium.

(b) There exists a nonzero vector of prices $p \in \mathcal{B}$ such that for any agent j and any nonzero vector of net trades $y \in CP_j$ it holds that $\langle p, y \rangle > 0$.

The proof that (b) implies (a) can be found in Werner (1987). To see that (a) implies (b) let $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ be an equilibrium for the economy and suppose that condition (b) is *not* satisfied. Then for some agent, say j, there is a nonzero vector of net trades $\overline{y}_j$ contained in CP_j such that $\left\langle p, \overline{y}_j \right\rangle \leq 0$. Since the economy is strictly reconcilable, $u_j(\overline{x}_j + \lambda \overline{y}_j)$ is increasing in λ. Thus, for any $\lambda > 0, \overline{x}_j + \lambda \overline{y}_j \in P_j(\overline{x}_j) \cap B(\omega_j, \overline{p})$, contradicting the fact that $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is an equilibrium.[9]

3 The Partnered Core of an Economy

3.1 Partnered collections of players

Let $N = \{1, ..., n\}$ be a finite set of players and let P be a collection of subsets of N. For each i in N let $P_i = \{S \in P : i \in S\}$. The collection P has *the partnership property* (for N) if for each i in N the set P_i is nonempty and for each pair of players i and j in N the following requirement is satisfied:

$$\text{if } P_i \subset P_j \text{ then } P_j \subset P_i\ ;$$

i.e. if all the coalitions in P that contain player i also contain player j then all the coalitions that contain j also contain i. Two players i and j are *partners* (or *i is partnered with j*) if $P_i = P_j$. The interpretation of a partnered collection of sets will be that there are no asymmetric dependencies. If i "needs" j, that is, if j is in all the sets in the collection containing player i, then j needs i in the same sense.

[9]This is actually a strengthening of Werner (1987) since Werner assumes that there are no half-lines in indifference surfaces; extreme desirability is a weaker condition.

Note that the set of partners of an agent could be just the agent himself, or it could be as large as the total agent set. The concept of partnership is further discussed in Bennett and Zame (1988), Reny, Winter, and Wooders (1993), and Reny and Wooders (1993,1995b) and in papers referenced therein.

3.2 The game induced by an economy

For any coalition $S \subset N$, the set of $S-$ *allocations* is given by

$$A(S) = \{(x_j)_{j\in S} : \ \sum_{j\in S} x_j = \ \sum_{j\in S} \omega_j \text{ and } \\ \text{for each } j \in S,\ x_j \in X_j \text{ and } u_j(x_j) \geq u_j(\omega_j)\}.$$

Corresponding to the set of S-allocations is a set of *utility possibilities* given by

$$U(A(S)) = \{(u_j)_{j\in S} : \text{ for some } (x_j)_{j\in S} \text{ in } A(S) \text{ it holds that } u_j = u_j(x_j) \text{ for each } j \in S\ \}.$$

Now for each coalition of agents $S \subset N$ define

$$V(S) = \{(u_1, ..., u_n) : \text{there exists } (u'_j)_{j\in S} \in U(A(S)) \\ \text{such that } u_j \leq u'_j \text{ for all } j \in S\}.$$

The pair (N, V) is *the game induced by the economy* $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$. The *core of the game* (N, V) is defined as

$$C(N, V) = \{u \in V(N) : \text{ there does notexist a coalition } S \subset N \text{ and} \\ \text{a } u' \in V(S) \text{ such that } u'_j > u_j \text{ for all } j \in S\}.$$

3.3 The partnered core

Let $u \in \Re^n$ be such that $\mathcal{S}(u) := \{S \subset N : u \in V(S)\} \neq \emptyset$. A utility vector $u \in \Re^n$ is *partnered* if the collection of coalitions $\mathcal{S}(u)$ is partnered. Let $P(N, V) = \{u \in \Re^n : \mathcal{S}(u) \neq \emptyset$ and $\mathcal{S}(u)$ is partnered$\}$. The *partnered core* of a game (N, V), denoted by $C^*(N, V)$, is given by

$$C^*(N, V) = P(N, V) \cap C(N, V).$$

In interpretation, a payoff $u \in \Re^n$ is in the partnered core of the game if it is in the core and if there are no asymmetric dependencies. If some pair of players i and j,

i is in all the coalitions in which j can realize his core utility u_j then j is in all the coalitions in which i can realize his core utility u_i. For games without side payments, the partnered core may constitute a "small" subset of the core – a single point, for example, in a continuum. (See Example 1 in Reny and Wooders (1993)).

Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be an economy. An allocation $(x_1, ..., x_n) \in A$ is in the *partnered core of the economy* if the utility vector $(u_1(x_1), ..., u_n(x_n)) \in C^*(N, V)$. Let

$$C^*((X_j, \omega_j, u_j(\cdot))_{j=1}^n)$$

denote the partnered core of the economy.

Theorem 1. *No unbounded arbitrage is sufficient for nonemptiness of the partnered core.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy. If no unbounded arbitrage holds, then there exists an allocation $x^* = (x_1^*, ..., x_n^*) \in A$ in the partnered core of the economy and $u^* := (u_1(x_1^*), ..., u_n(x_n^*))$ is in the partnered core of the game induced by the economy.

To show non-emptiness of the core in Page and Wooders (1993) we were able to appeal to the Debreu-Scarf (1967) result that an equilibrium is in the core. The competitive equilibrium is not, in general, partnered. Thus, our proof of Theorem 1 appeals to the Reny and Wooders (1993) result that a balanced game has a nonempty partnered core.[10] All proofs are contained in Section 5.

Theorem 2. *In a strictly reconcilable economy, no unbounded arbitrage is necessary and sufficient for nonemptiness of the partnered core.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a strictly reconcilable economy. Then the following three statements are equivalent:

(a) The partnered core of the economy, $C^*((X_j, \omega_j, u_j(\cdot))_{j=1}^n))$, is nonempty.

(b) The partnered core of the game induced by the economy, $C^*(N, V)$, is nonempty.

[10]Since the partnered core is a relatively new concept, the mathematical tools that would appear to be required to obtain a direct proof of our result have not yet been established. Further research aimed in this direction is in progress; see Reny and Wooders (1995a) and Kannai and Wooders (1995).

(c) The economy satisfies no unbounded arbitrage (1).

The next result implies that Theorem 3 of Bennett and Zame (1988) holds in an exchange economy with arbitrary closed, convex consumption sets and nonmonotonicities. Note, however, that we cannot use Theorem 3 to prove Theorem 1 since Theorem 3 requires strict concavity (or strict quasi-concavity). Let $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ be an equilibrium for the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$. We say that $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is a *partnered competitive equilibrium* if

(a) $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is an equilibrium and

(b) $\mathcal{S}(\overline{x}_1, ..., \overline{x}_n) := \{S \subset N : \sum_{j \in S} \overline{x}_j = \sum_{j \in S} \omega_j\}$is partnered.

Theorem 3. *The partnership property of the competitive equilibrium.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy such that, for each agent j, $u_j(\cdot)$ is strictly concave. If $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is an equilibrium for the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ then $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is a partnered competitive equilibrium.

The following Corollary is an immediate consequence of the fact that strict convexity of preferences implies extreme desirability, the proof of sufficiency of Werner (1987), and Theorem 3.

Corollary. *Necessary and sufficient conditions for existence of a partnered competitive equilibrium and nonemptiness of the partnered core.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy such that, for each agent j, $u_j(\cdot)$ is strictly concave. Then the following statements are equivalent:

(a) The partnered core of the economy, $C^*((X_j, \omega_j, u_j(\cdot))_{j=1}^n))$, is nonempty.

(b) The partnered core of the game induced by the economy, $C^*(N, V)$, is nonempty.

(c) The economy satisfies no unbounded arbitrage (1).

(d) The economy has a partnered competitive equilibrium.

The reader may note that in all cases considered that the partnered core of the game and the partnered core of the economy are related in the same way as in economies without short sales; there is a (possibly) many to one mapping from the partnered core of the economy onto the partnered core of the game. This is a result of the particular assumptions on our model. In a further paper we show that without these assumptions, this relationship between the core of the economy and the core of the game, appropriately defined, may be broken.

4 Arbitrage and Boundedness

Our next results provide the building blocks for the proofs of our main Theorems.[11] To state these results, we require the notion of a bounded economy. Given a positive integer k, a *k-bounded economy* is denoted by $(X_{kj}, \omega_j, u_j(\cdot))_{j=1}^n$ where $X_{kj} := B_k(\omega_j) \cap X_j$ and where $B_k(\omega_j)$ is the closed ball of radius k centered at the agent's endowment ω_j.

The set of *individually rational $S-$allocations for the $k-$bounded economy* is given by $A_k(S) = \{(x_j)_{j\in S} : \sum_{j=1}^n x_j = \sum_{j=1}^n \omega_j$ and for each $j \in S$, $x_j \in X_{kj}$ and $u_j(x_j) \geq u_j(\omega_j)\}$. The corresponding set of utility possibilities is given by $U(A_k(S)) = \{(u_j)_{j\in S} :$ for some $(x_j)_{j\in S}$ in $A_k(S)$, $u_j = u_j(x_j)$ for each $j \in S\}$.

Theorem 4. *No unbounded arbitrage is necessary and sufficient for compactness of the set of individually rational allocations.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy. The following statements are equivalent:

(a) $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ satisfies no unbounded arbitrage.

[11]Results in the literature on existence of equilibrium in economies with unbounded short sales typically proceed by showing some sort of boundedness – see the discussion in Page (1992). Except for Page and Wooders (1993), our boundedness results do not appear in the prior literature. In particular, they do not appear in Chichilnisky (1994,1995) since she uses a different economic model, requiring a condition on norms of gradients to utility functions. Moreover, she uses a different condition limiting arbitrages.

(b) There exists an integer $\overline{k}$ such that for any coalition $S \subset N$, $A(S) = A_k(S)$ for all $k \geq \overline{k}$.

The following Corollary is an immediate consequence of the Theorem. It is important to note that the corollary does not claim that compactness of the utility possibility set implies no unbounded arbitrage. Even if the utility possibility set is compact, there may exist trivial arbitrages – those that are not utility increasing –and no unbounded arbitrage may be violated. The assumption that the economy is strictly reconcilable rules out trivial arbitrages.

Corollary. *Compactness of the utility possibility set*: Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy satisfying no unbounded arbitrage. Then for any coalition $S \subset N$ the set of utility possibilities $U(A(S))$ is compact.

Theorem 5. *In a strictly reconcilable economy no unbounded arbitrage is necessary and sufficient for compactness of the sets of utility possibilities.* Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a strictly reconcilable economy. The following statements are equivalent:

(a) $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ satisfies no unbounded arbitrage.

(b) There exists an integer $\overline{k}$ such that for any coalition S, $U(A(S)) = U(A_k(S))$ for all $k \geq \overline{k}$.

It follows directly from the Corollary that if the economy is reconcilable, then no unbounded arbitrage implies that for any coalition $S \subset N$, $V(S) \cap \Re^N_+$ is compact.[12] It follows directly from Theorem 5 that in a strictly reconcilable economy, no unbounded arbitrage and compactness of $V(S) \cap \Re^N_+$ for every coalition S are equivalent.

[12]For the game induced by the economy to satisfy the conditions of Scarf's (1967) Theorem, this compactness is necessary. In preliminary versions of Page and Wooders (1993), the well-definedness of the induced game was used to show that no unbounded arbitrage is necessary and sufficient for nonemptiness of the core.

5 Proofs

We first prove Theorem 4. Before proving the Theorem, we require three Lemmas.

Lemma 1. (Theorem 8.2, Rockafellar (1970)). Let S be a closed convex set. The following statements are equivalent:

(a) $y \in \mathcal{R}(S)$.

(b) y is a cluster point of some sequence $\{\lambda_\nu x_\nu\}_\nu \subset S$ where $\{x_\nu\} \subset S$ and $\{\lambda_\nu\}$ is a sequence of positive real numbers converging to zero.

Lemma 2. Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a reconcilable economy. The following statements are true for all $j = 1, ..., n$ and $x \in X_j$.

(a) $\mathcal{R}(P_j(x)) = \mathcal{R}(clP_j(x))$.

(b) $\mathcal{R}(\{x' \in X_j : u_j(x') \geq u_j(x)\}) = \mathcal{C}P_j$.

(c) $\mathcal{R}(P_j(x)) = \mathcal{C}P_j$.

Proof. (a) is an immediate consequence of Corollary 8.3.1 in Rockafellar (1970) and (b) is an immediate consequence of Theorem 8.7 in Rockafellar. To prove (c), first note that $\mathcal{R}(P_j(x)) \subset \mathcal{R}(\{x' \in X_j : u_j(x') \geq u_j(x)\})$. Let $y \in \mathcal{R}(\{x' \in X_j : u_j(x') \geq u_j(x)\})$. By (b) and the definition of $\mathcal{C}P_j$, $u_j(x+\lambda y)$ is nondecreasing in $\lambda \geq 0$ for any $x \in X_j$. Suppose now that $y \notin \mathcal{R}(P_j(x))$. Then for some $x' \in P_j(x)$ and λ' sufficiently large, $u_j(x' + \lambda' y) \leq u_j(x)$. But since $u_j(x') > u_j(x)$, this contradicts the fact that $u_j(x' + \lambda' y)$ is nondecreasing in $\lambda \geq 0$. ■

The following Lemma can be proven using elementary facts concerning sequences.

Lemma 3: Let $\{x^k\}_k = \{(x_1^k, ..., x_n^k)\}_k \subset A$ be a sequence of individually rational allocations such that $\sum_{j=1}^n \left\|x_j^k\right\| \to \infty$ as $k \to \infty$. Then for any cluster point $(y_1, ..., y_n)$ of the sequence $\{\lambda^k x^k\}_k$ where $\lambda^k = \left(\sum_{j=1}^n \left\|x_j^k\right\|\right)^{-1}$ it holds that $\sum_{j=1}^n y_j = 0$ and $\sum_{j=1}^n \|y_j\| = 1$.

Proof of Theorem 4: The proof that (b) implies (a) is obvious. To see that (a) implies (b), it suffices to show that (a) implies that for some integer $\overline{k}$, $U(A) =$

$U(A_k)$ for all $k \geq \overline{k}$. To see that this implication holds, consider the following. Let $\{\overline{x}^k\}_k = \{(\overline{x}_1^k, ..., \overline{x}_n^k)\}_k \subset A$ be a sequence of individually rational allocations such that for each k, $(\overline{x}_1^k, ..., \overline{x}_n^k) \notin A_k$. This implies that $\sum_{j=1}^n \left\|\overline{x}_j^k\right\| \to \infty$ as $k \to \infty$. Let $(\overline{y}_1, ..., \overline{y}_n) \in \Re^L \times ... \times \Re^L$ be a cluster point of the sequence

$$\{(\lambda^k \overline{x}_1^k, ..., \lambda^k \overline{x}_n^k)\} \text{ where } \lambda^k = \left[\sum_{j=1}^n \left\|\overline{x}_j^k\right\|\right]^{-1}.$$

Since $\overline{x}_j^k \in \{x \in X_j : u_j(x) \geq u_j(\omega_j)\}$ for all k it follows from Lemmas 1 and 2(b) that $\overline{y}_j \in CP_j$ for each j. By Lemma 3, $\sum_{j=1}^n \overline{y}_j = 0$ and $\sum_{j=1}^n \left\|\overline{y}_j\right\| = 1$. Thus for some j, $\overline{y}_j \neq 0$, contradicting the condition of no unbounded arbitrage. ■

Proof of Theorem 5: Since a strictly reconcilable economy is reconcilable, (a) implies (b) follows immediately from Theorem 4. To prove the other direction, suppose that there exists an integer $\overline{k}$ such that for all $k \geq \overline{k}$, $U(A) = U(A_k)$ and the no unbounded arbitrage condition (1) does not hold. Then there exists $(y_1, ..., y_n) \neq (0, ..., 0)$ in $\Re^L \times \cdots \times \Re^L$ such that $\sum_j y_j = 0$ and $y_j \in CP_j$ for each j. Thus, given any $(x_1, ..., x_n) \in A$, $(x_1 + \lambda y_1, ..., x_n + \lambda y_n) \in A$ for all $\lambda \geq 0$ and since $CP_j \backslash \{0\} = I_j(x_j)$, it holds that $u_j(x_j + \lambda y_j)$ is increasing in λ provided $y_j \neq 0$.

Now let $\delta = (\delta_1, ..., \delta_n)$ be a vector in $\Re^L$ with strictly positive components. Let $u^k \in U(A_k)$ be such that:

$$\left\langle \delta, u^k \right\rangle = \sup\{\langle \delta, u \rangle : u \in U(A_k)\}. \qquad (*)$$

Since $U(A_k)$ is compact for each k there is a u^k in $U(A_k)$ satisfying $(*)$. Now suppose $k \geq \overline{k}$ and let $(x_1^k, ..., x_n^k) \in A_k$ be such that $u_j^k = u_j(x_j^k)$ for each j and k. Since the economy is strictly reconcilable, for any agent j with $y_j \neq 0$, $u_j(x_j^k + \lambda y_j) > u_j(x_j^k)$ for all $\lambda > 0$. Therefore, for all $\lambda > 0$, $\sum_j \delta_j u_j(x_j^k + \lambda y_j) > \sum_j \delta_j u_j(x_j^k) = \left\langle \delta, u^k \right\rangle = \sup\{\langle \delta, u \rangle : u \in U(A_k)\}$. But $\left(u_1(x_1^k + \lambda y_1), ..., u_n(x_n^k + \lambda y_n)\right) \in U(A)$ for all $\lambda > 0$, and since $U(A) = U(A_k)$, we have a contradiction. ■

The compactness of $U(A)$ follows from continuity of the utility functions (reconcilability) and compactness of $X_k(\omega_j)$ for each k and j. ■

Proof of Theorem 1. To prove Theorem 1 we need only observe that from Theorem 4 it follows that the game derived from the economy is well defined. In particular, condition (1.4) of Reny and Wooders (1993) is satisfied. Since the game is derived from an economy with concave utilities, it is balanced. (See, for example, Scarf (1967) or Hildenbrand and Kirman (1988, 128-132)). With a normalization of utilities so that the utility of each agent from his endowment is greater than zero, it then follows by standard methods that the derived game satisfies all of the conditions of Reny and Wooders (1993) and the partnered core is nonempty. ■

Proof of Theorem 2. (a) implies (b) follows from Theorem 1. To show that (b) implies (a) consider the following. Let $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ be a strictly reconcilable economy and let $\overline{x} = (\overline{x}_1, ..., \overline{x}_n)$ be an allocation in the partnered core of the economy. Suppose that no unbounded arbitrage is not satisfied. Then there is an n-tuple of net trades $(y_1, ..., y_n) \neq (0, ...0)$ in $\Re^L \times \cdots \times \Re^L$ such that $\sum_j y_j = 0$ and $y_j \in CP_j$ for each j. By extreme desirability $(A-2)$, $u_j(x + \lambda y_j)$ is increasing in λ for $\lambda > 0$ and $x \in X_j$. Let $S \neq \emptyset$ be a subset of agents such that for each $j \in S$, $y_j \neq 0$. For each agent $j \in S$, $u_j(\overline{x}_j + \lambda^k y_j) > u_j(\overline{x}_j)$ for any $\lambda > 0$ and for each agent $j \in \{1, ..., n\} \backslash S$, $u_j(\overline{x}_j + \lambda^k y_j) = u_j(\overline{x}_j)$ for any $\lambda > 0$. Since $\sum_{j=1}^n y_j = 0$ and $\sum_{j=1}^n \overline{x}_j = \sum_{j=1}^n \omega_j$, $\sum_{j=1}^n (\overline{x}_j + \lambda y_j) = \sum_{j=1}^n \omega_j$ for all $\lambda > 0$. Thus, $(\overline{x}_1 + \lambda y_1, ...,$ $\overline{x}_n + \lambda y_n)$ is an individually rational allocation and this allocation Pareto dominates ($\overline{x}_1, ..., \overline{x}_n)$, contradicting the supposition that $(\overline{x}_1, ..., \overline{x}_n)$ is an allocation in the core.

The equivalence of (b) and (c) follows from the definition of the partnered core of the economy. ■

In our next proof we will use the following characterization of partnered utility vectors.

Lemma 4. Let (N, V) be the game corresponding to the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ where for each agent j, $u_j(\cdot)$ is strictly quasi-concave.[13] Let $u \in C(N, V)$. The following are equivalent:

(a) $S(u) \neq \emptyset$ is partnered.

[13]The utility function $u_j(\cdot)$ is *strictly quasi-concave* if for each pair x' and x'' in X_j with $x' \neq x''$ and each α with $0 < \alpha < 1$, it holds that $u_j(\alpha x' + (1-\alpha)x'') > \min\{u_j(x'), u_j(x'')\}$.

(b) Given any proper subcoalition $S \subset N$, $S \neq \emptyset$, if $u \in V(S)$ then $u \in V(N\backslash S)$.[14]

Proof of Lemma 4. Let $u \in C(N,V)$ and let S be a coalition satisfying $u \in V(S)$. It is immediate that (b) implies (a) since, whenever there is are agents i and j such that $j \in S$ and $j \notin S$ then there is a coalition, specifically, N/S, containing j but not i and such that $u \in V(N\backslash S)$. To show that (a) implies (b), suppose the u satisfies the conditions of the Lemma. Then there is an allocation $x \in A(N)$ such that for each j it holds that $u_j(x_j) = u_j$. Since $u \in V(S)$ it holds that there is an S-allocation $(x'_j)_{j\in S}$ such that $u_j(x'_j) = u_j$. Since u is in the core, and preferences are strictly convex, it follows that $\sum_{j\in S} x'_j = \sum_{j\in S} x_j = \sum_{j\in S} \omega_j$; otherwise we would have a contradiction to the assumption that u is in the core. But then it follows that $\sum_{j\in N|S} x_j = \sum_{j\in N|S} \omega_j$ so $u \in V(N\backslash S)$. ■

Proof of Theorem 3. The method of proof is similar to that used by Bennett and Zame (1989, Theorem 3). Let $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ be an equilibrium for the economy $(X_j, \omega_j, u_j(\cdot))_{j=1}^n$ and consider the vector $\overline{u} = (u_1(\overline{x}_1), ..., u_n(\overline{x}_n))$. We know that $\overline{u} \in C(N,V)$. By Lemma 3, to show that $\overline{u} \in P(N,V)$ we will show that if $\overline{u} \in V(S)$ then $\overline{u} \in V(N\backslash S)$. So suppose that $\overline{u} \in V(S)$ for some subcoalition $S \subset N$. Let $\overline{x} \in A$ be such that $\overline{u}_j = u_j(\overline{x}_j)$ and let $z \in A(S)$ be such that $\overline{u}_j = u_j(z_j)$ for $j \in S$. If for some $j \in S$, $\overline{x}_j \neq z_j$ we have, by the strict concavity of $u_j(\cdot)$, $u_j((1-t)\overline{x}_j + tz_j) > u_j(\overline{x}_j)$ for $0 < t < 1$. Since $(\overline{x}_1, ..., \overline{x}_n, \overline{p})$ is an equilibrium, $\langle \overline{p}, (1-t)\overline{x}_j + tz_j \rangle > \langle \overline{p}, \omega_j \rangle$. Thus, $\langle \overline{p}, z_j \rangle > \langle \overline{p}, \omega_j \rangle$. But now we have a contradiction because $\sum_{j\in S} z_j + \sum_{j\in N\backslash S} \omega_j = \sum_{j\in N} \omega_j$, yet $\left\langle \overline{p}, \sum_{j\in S} z_j + \sum_{j\in N\backslash S} \omega_j \right\rangle > \left\langle \overline{p}, \sum_{j\in N} \omega_j \right\rangle$. Therefore $\overline{x}_j = z_j$ for all $j \in S$. Hence, $\sum_{j\in N\backslash S} \overline{x}_j = \sum_{j\in N\backslash S} \omega_j$ and $u_j(\overline{x}_j) \geq u_j(\omega_j)$ for all $j \in S$. And therefore $\overline{u} \in V(N\backslash S)$. ■

6 Conclusions

The results of this paper serve as an introduction to the partnered core of an economy and the relationship between no unbounded arbitrage and the partnered core. In on-

[14] A similar observation was used in Bennett and Zame (1988) to show that the competitive payoff is partnered.

going research, Page and Wooders (1994), we extend the condition of no unbounded arbitrage and establish further relationships between no unbounded arbitrage, partnership, and equilibrium.

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