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COMMITMENT ROBUST EQUILIBRIA AND ENDOGENOUS TIMING

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Commitment Robust Equilibria and Endogenous Timing*

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Abstract

This paper investigates which outcomes result in a game when the order of the moves is endogenous. To that end we study a model in which players can move in one of two periods, i.e. players face the trade-off between committing early and forcing the opponent to best respond, and moving late so as to be able to play a best response against the opponent. It is shown that most mixed strategy equilibria of the original game are not viable when the sequencing of the moves is endogenous, but that any pure strategy equilibrium is a perfect equilibrium outcome of the timing game. More refined equilibrium concepts with an evolutionary flavor, however, allow the conclusion that only equilibria in which no player has an incentive to move first are viable.

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1 Introduction

One of the most important ideas in game theory is the value of commitment, the idea that it may be advantageous to constrain one's own behavior in order to induce others to behave in a way that is favorable to oneself. Schelling's (1960) classic *The Strategy of Conflict* is filled with examples that illustrate this idea that it might pay to reduce one's flexibility, that it may be optimal to burn one's bridges behind oneself. The simplest commitment possibility that Schelling discusses (and what he calls the "pure unconditional commitment") is equivalent to obtaining the first move: to preempt one's rivals by choosing and communicating one's action before they do. That there might be a first-mover advantage has also been pointed out by other authors. For example, in the economics literature it has been known at least since Von Stackelberg's (1934) *Marktform und Gleichgewicht* that in a quantity setting duopoly game, the leader has higher profits, hence, that each duopolist will want to move before the other.

Various authors have argued that the Cournot equilibrium is somewhat suspect exactly because of this fact that each duopolist has an incentive to move earlier than his opponent. Of course, if the duopolists are indeed forced to move simultaneously (as the standard game model of the duopoly situation assumes) then there is nothing wrong with the Cournot equilibrium, but one may wonder whether in real situations the rules are indeed that rigid as to prevent commitments from being made. Hence, it is a natural question to ask which Nash equilibria are still viable when players have the opportunity to commit themselves. This question has been addressed in Rosenthal (1991). That paper defines an equilibrium of a 2-player game to be commitment robust if there is no player who can improve his payoff by moving first and by communicating this move to the opponent before the latter moves. Hence, according to Rosenthal a Nash equilibrium is commitment robust only if it is also a Stackelberg equilibrium of each of the two games where one of the players is allowed to move first.

While Rosenthal's definition is attractive, it also is not entirely compelling. The simultaneous move game is compared with the two perfect information sequential move games in which the leader is exogenously specified, but it is not clear why the latter games are relevant to the study of the former situation. The original problem derives from the possibility that each player might or might not choose to move before the other, hence, one would like to see the order of the moves being determined endogenously. Rosenthal is aware of this problem. He writes: "In defining commitment-robustness, one might require consideration of more than just the alternative games G_I and G_{II}; after all there could be opportunities for both players to invest in commitment possibilities. It seems best, therefore, to think of the defining conditions here as being in the nature of necessary conditions." The issue we would like to raise is whether Rosenthal's definition indeed gives necessary conditions. Namely, if the order of the moves is endogenous could it not be that, even though each player could profit from moving first, no player dares to move first since he fears that the opponent might commit simultaneously, hence, could it not be that, as a consequence, the players end up in the Nash equilibrium after all? Certainly, the latter seems a possibility for games in which the two Stackelberg equilibria differ like in the ordinary quantity setting duopoly game.

The question we address in this paper is which outcomes will arise when the order of the moves is endogenous, hence, when each player has the opportunity to commit himself. The formal model we use to resolve this issue is the 2-stage game of action commitment that was introduced in Hamilton and Slutsky (1990). The rules of this game are as follows: There are two periods and each player has to move in exactly one of these periods. Choices are simultaneous, but, if one player chooses to move early while the other moves late, the latter is informed about the former's choice before making his decision. It is assumed that players can commit only to pure actions, if a player chooses to randomize in the first period, the realised action is revealed to the opponent. The program of this paper is to investigate which equilibria of the original game can arise as "sensible" equilibrium outcomes of the action commitment game. Say that a player has an incentive to move first at the Nash equilibrium s if this player's payoff is higher when she is allowed to act as a Stackelberg leader and to commit herself. The question that the paper addresses is whether indeed only equilibria at which no player has an incentive to move first are viable. Will a player that profits when she is the only one that can commit herself actually make a commitment when the opponent can commit at the same point in time?

A first important result is that a mixed strategy equilibrium indeed is viable only if no player has an incentive to move first at this equilibrium. Only in this case does the action commitment game have a Nash equilibrium producing the same outcome. Hence, endogenous timing will eliminate most mixed strategy equilibria. The intuition is quite straightforward: The actions actually resulting from the players' mixed strategy need not be in equilibrium, ex post players have an incentive to deviate. Hence, each player will have an incentive to wait, thereby guaranteeing that he is best responding no matter which action the opponent is actually choosing. But if a player waits, the opponent frequently has the incentive to move first, i.e. to commit to his Stackelberg leader strategy.

For pure strategy equilibria, the situation is much different and the answer to the question of which equilibria are viable with endogenous timing depends crucially on which equilibria of the action commitment game one considers to be "sensible." In particular, the answer depends on whether one adopts an evolutionary or an eductive interpretation of equilibria. A first result is that any pure strategy equilibrium can arise as the outcome of a subgame perfect equilibrium of the associated game of action commitment. However, some of these subgame perfect equilibria appear fragile. To eliminate such "implausible" equilibria we have to work with more refined equilibrium notions. As these are more readily defined in the normal form we take a normal form perspective in most of this paper.

It turns out that, under a mild regularity condition, an immediate commitment to a pure equilibrium of the original game constitutes a perfect (hence, undominated) equilibrium in the normal form of the action commitment game. Hence, requiring perfectness does not allow one to conclude that only pure equilibria in which no player has an incentive to move first are viable when the timing is endogenous. Intuitively, if each player expects an unattractive outcome in case the timing game reaches the second stage, then it is optimal for each player to commit to the pure equilibrium immediately if he expects his opponent to do the same. Still, it turns out that the intuition that only equilibria at which no player has an incentive to move first are viable, can be formalized, provided one is willing to accept a consistency requirement on top of the perfection requirement. In this paper we define an equilibrium s^* to be commitment robust if, whenever players are sure that they will play s^* if they come to the second period, there is a perfect equilibrium of the first stage game that induces the outcome s^*. We show that indeed only equilibria in which no player has an incentive to move first are commitment robust.

A major part of this paper is devoted to the question of whether in a game with a unique and pure commitment robust equilibrium, this equilibrium is the unique "sensible" equilibrium of the 2-stage action commitment game. We have already seen that the answer has to be no if "sensible" is defined as "perfect". It turns out that the answer also is no if one substitutes "proper" or "stable" for "sensible." A main result of this paper, however, is that the answer is in the affirmative for a variety of set-valued solution concepts that have an evolutionary flavor, i.e. these concepts seem to correspond more closely to the interpretation of an equilibrium as a fixed point of an unspecified dynamic process, than to the interpretation of an equilibrium as a self-enforcing agreement. Specifically, we show that the concepts of persistent equilibria (Kalai and Samet (1984)), curb-equilibria and curb*-equilibria (Basu and Weibull (1991)) force players to coordinate on the commitment robust equilibrium whenever this is unique and pure. Hence, if we accept curb or persistent equilibria as the relevant solution concept, the results in this paper allow us to identify two classes of games for which we can unambiguously determine the outcome if the order of the moves is endogenous, viz. zero-sum games and games with common interest. While the result concerning zero-sum games is not surprising, it is remarkable that one has to turn to very restrictive equilibrium notions to "justify" playing the Pareto-efficient equilibrium in common interest games.

The remainder of the paper is organized as follows. In Section 2 we give an example of a coordination game to show that the intuition that only equilibria in which no player has an incentive to move first when the timing is endogenous might not be correct. In Section 3 we introduce notation as well as the regularity assumptions that we impose on the underlying game. The 2-stage game of action commitment is formally introduced in Section 4. In Section 5 we show that a mixed strategy equilibrium is viable with endogenous timing only if it yields each player at least his Stackelberg leader payoff. In contrast it is shown in Section 6 that requiring perfectness in the timing game does not eliminate pure equilibria of the underlying game. In this section we also discuss the related work of Hamilton and Slutsky (1990). In Section 7, commitment robust equilibria are defined and characterized, and we also compare our approach to that of Rosenthal (1991). Section 8 investigates whether players will automatically coordinate on a commitment robust equilibrium when the order of the moves is endogenous. Section 9 concludes.

2 A coordination game

In this section we discuss a simple example to show that the intuition that, when the timing of the moves is endogenous, only equilibria in which no player has an incentive to move first are viable is not completely reliable. We show that one needs to employ quite restrictive solution concepts in order to arrive at this conclusion. Specifically, the following example demonstrates that even if a game has a strict equilibrium that is also the unique Stackelberg equilibrium outcome in each of the two games where one of the players is forced to move first, it is by no means obvious that players will necessarily coordinate on this equilibrium when the order of the moves is determined endogenously. Consider the common interest coordination game g given in Table 1a. When players have to move simultaneously, there are three equilibria: (T, L), (B, R) and a mixed equilibrium that yields each player the payoff $\frac{2}{3}$. When one of the players is forced to move first, with this player's choice being revealed to the other, the outcome is (T, L), irrespective of which player has the right to move first. According to Rosenthal's definition only the equilibrium (T, L) is commitment robust.

	L		R	
T	2	2	0	0
B	0	0	1	1

Table 1a: A coordination game.

	L^1		L^2		R^2		R^1	
T^1	2	2	2	2	2	2	0	0
T^2	2	2	2	2	0	0	1	1
B^2	2	2	0	0	1	1	1	1
B^1	0	0	1	1	1	1	1	1

Table 1b: The (reduced) normal form of the 2-period game of endogenous timing associated with the game from Table 1a.

Let us investigate whether this equilibrium is the unique "sensible" outcome of the 2-stage game of action commitment, described in the introduction, in which the order of the moves is endogenously determined. The reduced normal form of the 2-period game derived from the coordination game of Table 1a is given in Table 1b. Here X^1 denotes the pure strategy "commit to X in period 1", while X^2 denotes "wait till period 2 and play the unique best response against the opponent's action if the latter has already moved in period 1, otherwise, i.e. if the opponent has not yet moved, play X." Hence, the question to be addressed is whether "2,2" is the unique "sensible" outcome in the game of Table 1b or whether perhaps one can also give good arguments in favor of "1,1", or in favor of the mixed equilibrium.

Inspection shows that the game from Table 1b has three Nash equilibrium outcomes: There is a connected set of equilibria with payoff (2,2), there is also a connected set of equilibria with payoff (1,1) and there is a completely mixed equilibrium in which each player plays (1,2,4,8)/15. The latter yields each player the payoff 14/15. Hence, we see that endogenous timing allows both pure equilibria of the coordination game to survive as Nash equilibrium outcomes, but that it eliminates the mixed strategy equilibrium. The intuition for the latter is simple. If my opponent commits himself to the mixed equilibrium in the first stage, I have an incentive to wait since in that case I guarantee that we either coordinate on (T, L) or (B, R), i.e. I prevent a disequilibrium outcome. However, if I wait, my opponent does better by committing to his Stackelberg leader strategy.

Having eliminated the mixed equilibrium from Table 1a, let us now turn to the "inferior" pure strategy equilibrium. A glance at Table 1b shows that this is not as easily eliminated. Namely, B^1 and R^1 are undominated strategies in Table 1b so that (B^1, R^1) is a perfect equilibrium. Even stronger, (B^1, R^1) is a proper equilibrium (Myerson (1978)): For any $\varepsilon > 0$ that is sufficiently small the strategy pair in which each player plays the completely mixed strategy $(\varepsilon^3, \varepsilon^2, \varepsilon, 1-\varepsilon-\varepsilon^2-\varepsilon^3)$ is a 2ε-proper equilibrium in Table 1b. In fact, one may show that, in the game of Table 1b, the connected set of equilibria with payoff "1,1" contains a stable set as defined in Kohlberg and Mertens (1986). (Details are available from the authors upon request.) Intuitively, what is going on is that, if each player expects mistakes to occur with a relatively large probability in the second period, or if players believe that the unattractive mixed strategy equilibrium would be played in case the second period would be reached, then each player has a strong incentive to commit to the equilibrium "1,1" in the first period if he expects his opponent to do the same. Hence, it seems that Rosenthal's (1991) conclusion that only (T, L) makes sense in the game of Table 1a when timing aspects are taken into account was premature.

Nevertheless, there is a sense in which the equilibrium (B^1, R^1) of Table 1b is fragile. Namely, if players would be sure that they would continue with (B, R) if they both still had to move in the second stage, they would both find it optimal to deviate to (T, L) in the first stage. Namely, if players are sure of (B, R) in the second stage, then playing this equilibrium already in the first stage becomes dominated: By moving in the first stage, one risks a coordination failure, which, by assumption, does not exist in the second period. Therefore, each player can count on the opponent not playing the inferior action in the first period and each can safely commit to the preferred equilibrium action. (Formally, being sure of continuing with (B, R) in the second stage corresponds to eliminating T^2 and L^2 from Table 1b, and the unique perfect equilibrium of the resulting reduced game is (T^1, L^1).) In Section 6 of this paper we elaborate this idea. We define an equilibrium s^* to be commitment robust if, whenever players are sure that they will play s^* if they come to the second period, there is a perfect equilibrium of the first stage game that induces the outcome s^*. Hence, the definition forces players to play s^* in the second period, and second period mistakes are excluded. We then show that an equilibrium s^* of a 2-person normal form game is commitment robust if no player can improve his payoff by acting as a Stackelberg leader. Hence, if s^* is a pure equilibrium that is commitment robust, then s^* must also be the outcome of a subgame perfect equilibrium of the two games in which one player acts as the Stackelberg leader.

It has to be admitted that the combination of perfection and truncation consistency as employed in our definition of commitment robustness is somewhat artificial: What is the rationale of allowing mistakes in the first period, but eliminating them in the second period? Hence, the question remains whether one can give a more convincing argument to eliminate the equilibrium outcome "1,1" in the game of Table 1b, that is, whether there exist solution concepts that force players to coordinate on "2,2" in that game. It turns out that the answer is in the affirmative for a variety of set-valued solution concepts that have an evolutionary flavor, i.e. these concepts seem to correspond more closely to the interpretation of an equilibrium as a fixed point of an unspecified dynamic process, than to the interpretation of an equilibrium as a self-enforcing agreement. Specifically, the concepts of persistent equilibria (Kalai and Samet (1984)), curb-equilibria and curb*-equilibria (Basu and Weibull (1991)) force players to coordinate on (T, L). Let us illustrate this result for curb-equilibria, i.e. for equilibria that belong to minimal sets of mixed strategy pairs that are closed with respect to taking best responses. The game of Table 1b has two sets that are closed under best responses, viz. the entire strategy set and the set of strategies in which zero probability is assigned to B^1 and R^1. Obviously, only the latter set is minimal. Furthermore, each equilibrium in this set induces the outcome "2,2", hence, each curb equilibrium of the game of Table 1b produces the commitment robust equilibrium of Table 1a. In Section 7 we show that the above argument generalizes to any game that admits a unique and pure commitment robust equilibrium.

In particular, for each common interest game we, hence, have a "justification" for playing the Pareto-dominant equilibrium.

3 Preliminaries

We start our analysis from a given finite 2-person normal form game $g = (A_1, A_2, u_1, u_2)$. We write $S_i = \Delta(A_i)$ for the set of mixed strategies of player i in g and we denote a generic mixed strategy by s_i. We write $C(s_i) = \{a_i \in A_i : s_i(a_i) > 0\}$. The set of mixed strategy pairs is $S = S_1 \times S_2$, and, for $s \in S$, $C(s) = C(s_1) \times C(s_2)$. Throughout the paper we assume the mild regularity condition that, whenever a player i is indifferent between the strategy pairs a and a' in $A = A_1 \times A_2$, the same holds true for player j, hence[1]

$$\text{if } u_i(a) = u_i(a'), \text{ then } u_j(a) = u_j(a'). \tag{3.1}$$

For $s_i \in S_i$ and $a_i \in A_i$, we define

$$\begin{aligned} u_j(s_i) &= \max\{u_j(s_i, a_j) : a_j \in A_j\} && (3.2)\\ B(s_i) &= \{a_j \in A_j : u_j(s_i, a_j) = u_j(s_i)\} && (3.3)\\ u_i(a_i) &= u_i(a_i, a_j) \text{ where } a_j \in B(a_i) && (3.4)\\ \bar{u}_i &= \max_{a_i \in A_i} u_i(a_i) && (3.5) \end{aligned}$$

Hence, $u_i(a_i)$ is player i's payoff when he commits to a_i and $\bar{u}_i$ is this player's payoff when he acts as the Stackelberg leader. These concepts are meaningfully defined because of the regularity assumption (3.1): Although player j may have multiple best responses

[1] Whenever i and j occur in this paper, then $i, j \in \{1, 2\}$. If i and j occur in combination, it is understood that $i \neq j$.

against a_i, player i does not care which best reply is taken. We say that player i has an incentive to move first at s if $u_i(s) < \bar{u}_i$, that is if this player would be better off if he would be allowed to act as a Stackelberg leader.

Note that any pure strategy Nash equilibrium s satisfies $u_i(s) \leq \bar{u}_i$ for $i = 1, 2$. Hence, if no player i has an incentive to move first at such s, then $u_i(s) = \bar{u}_i$ for $i = 1, 2$ and s is a subgame perfect equilibrium outcome in each of the two games where one of the players is forced to move first. Conversely, if s is pure and a subgame perfect equilibrium outcome of each of these two games, then s is a Nash equilibrium of g and no player has an incentive to move first at s. Obviously, any such equilibrium must Pareto-dominate any other pure Nash equilibrium and we have proved

Corollary 1 *No player has an incentive to move first at the pure strategy equilibrium s if and only if s is an SPE outcome of the two games in which one of the players is forced to move first. A pure equilibrium at which no player has an incentive to move first Pareto-dominates any other pure Nash equilibrium, hence, two pure equilibria at which no player has an incentive to move first are payoff equivalent.*

This corollary is no longer correct for mixed equilibria. For example, the zero-sum game of matching pennies has a unique (symmetric) mixed strategy equilibrium, it is not an SPE outcome of the game where i has to move first: Although player i has no incentive to move first (he is sure to lose this game if he is forced to disclose his action before j moves), a pure equilibrium at which no player has an incentive to move first may also be Pareto dominated by a mixed equilibrium: Add to Matching Pennies a third strategy for each player such that if both play this strategy both lose half a penny, but both players lose one penny if only one player chooses this strategy. In each Stackelberg outcome, both players choose the third strategy, but the resulting equilibrium is Pareto dominated by the original mixed equilibrium of Matching Pennies.

It is easy to construct an example of a game that does not have an equilibrium at which no player has an incentive to move first. In fact, most games will not have such an equilibrium, although the set of games having such an equilibrium certainly is not of measure zero: All games in the neighborhood of the coordination game from Table 1a have this property. Two classes of games that admit equilibria at which no player has an incentive to move first are strictly competitive games (Friedman (1983)) and common interest games (Aumann and Sorin (1989)). The game g is said to be strictly competitive if for all s, s' and $i \neq j$, if $u_i(s) \geq u_i(s')$, then $u_j(s) \leq u_j(s')$. The game g is a game with common interest if there exists s such that $u_i(s) \geq u_i(s')$ for all i and s'.

Lemma 1 *a) If g is strictly competitive and s is a Nash equilibrium of g, then no player has an incentive to move first at s.*
b) If g has common interest and s is a Pareto dominant Nash equilibrium of g, then no player has an incentive to move first at s.

Proof. The proof of b) is trivial, so we only prove a). Assume s is an equilibrium of a strictly competitive game g and $u_1(s) < \bar{u}_1$. Let $\bar{u}_1 = u_1(\bar{a}_1) = u_1(\bar{a}_1, \bar{a}_2)$. Then $u_2(\bar{a}_1, a_2) \leq u_2(\bar{a}_1, a_2)$ for all a_2, hence, $u_1(\bar{a}_1, a_2) \geq u_1(\bar{a}_1, \bar{a}_2) > u_1(s)$ for all a_2. But then by playing $\bar{a}_1$ player 1 could guarantee himself more than the equilibrium payoff $u_1(s)$, which is impossible. □

Sometimes we will make use of stronger regularity conditions than (3.1). One such condition is that different outcomes are associated with different payoffs, i.e.

$$\text{if } a \neq a', \text{ then } u_i(a) \neq u_i(a') \qquad (i \in \{1,2\}). \tag{3.6}$$

Obviously, (3.6) is satisfied generically. The final regularity condition that we will use has first been introduced in Lemke and Howson (1964) and is also satisfied generically. For $s_j \in S_j$, define the matrix u_{is_j} as the restriction of u_i to $B(s_j) \times C(s_j)$. The regularity condition that Lemke and Howson impose is

$$\text{rank } (u_{is_j}) \geq |B(s_j)| \text{ for all } j \in \{1,2\} \text{ and all } s_j \in S_j \tag{3.7}$$

where $|B(s_j)|$ denotes the cardinality of the set $B(s_j)$. Note that (3.7) does not imply (3.1) but that, for a game satisfying (3.7), the expressions (3.4) and (3.5) are still well-defined since $|B(a_j)| = 1$ for each a_j. Also note that the class of 2-person games satisfying both (3.6) and (3.7) is still generic. The following Lemma lists some properties of games that satisfy (3.7).

Lemma 2 *Let g satisfy (3.7) and let s be an equilibrium of g that is not pure. Then*

(i) $|C(s_1)| = |C(s_2)| = |B(s_1)| = |B(s_2)|$

(ii) If s' is an equilibrium and $C(s') = C(s)$, then $s' = s$.

(iii) For each $a_i \in C(s_i)$, there exists $a_j \in C(s_j)$ such that $a_j \notin B(a_i)$.

(iv) For each $a_i \in C(s_i)$, there exists $a_j \in C(s_j)$ such that $a_i \notin B(a_j)$.

Proof.

(i) Since s is an equilibrium, we have $C(s_i) \subset B(s_j)$ for $i, j \in \{1,2\}$. Since the rank of a matrix cannot exceed the number of columns, (3.7) implies $|B(s_j)| \leq |C(s_j)|$ for all s_j. Combining these observations yields $|C(s_1)| \leq |B(s_2)| \leq |C(s_2)| \leq |B(s_1)| \leq |C(s_1)|$, hence, all inequalities must be equalities.

(ii) Assume s and s' are equilibria with $C(s) = C(s')$. If $u_i(s) = 0$, then the columns of the matrix u_{is_j} are dependent, hence, (3.7) is violated. So assume $u_i(s) \neq 0$. Consider the vector v with a_j–th coordinate equal to $v(a_j) = u_i(s')s_j(a_j) - u_i(s)s'_j(a_j)$. Then, if we premultiply v by u_{is_j} we get zero, hence by (3.7) v must be the zero vector. Therefore, $u_i(s')s_j = u_i(s)s'_j$ and since both s_j and s'_j are probability vectors $u_i(s') = u_i(s)$. But then $s_j = s'_j$. A similar argument implies that $s_i = s'_i$.

(iii) This follows immediately from the fact that $|B(a_i)| = 1$ and $|C(s_j)| \geq 2$.

(iv) If $a_i \in B(a_j)$ for all $a_j \in C(s_j)$, then $\{a_i\} = B(a_j)$ for all such a_j, hence $\{a_i\} = B(s_j)$ contradicting (i). □

4 The Action Commitment Game

To formally consider the questions of which outcomes will result when the timing of the moves is endogenous and of which equilibria of the original game are still viable in this context, we use the 2-stage game of action commitment that has been introduced in Hamilton and Slutsky (1990). In this game, which will be referred to as γ^2, each player can choose between committing to an action in period 1 or to wait till period 2. Formally, the rules are as follows

Stage 1: Simultaneously the players choose to commit to actions in A_1 and A_2, respectively, or to wait till stage 2.

Stage 2: Each player i who did not yet choose an action in A_i is informed about what action his opponent j took in stage 1. After having received this information, the player is required to choose an action in A_i with players moving simultaneously if both still have to make a choice.

Payoffs: The players' actions in γ^2 lead to a unique outcome in A. If $a \in A$ results, then player i's payoff in γ^2 is $u_i(a)$. Hence, there is no discounting, moving late does not entail any cost.

We will use σ_i to denote a strategy of player i in γ^2. Player i's payoff resulting from $\sigma = (\sigma_1, \sigma_2)$ is denoted by $U_i(\sigma)$. We will be interested in SPE of γ^2. There are three types of subgames in γ^2: a) the original game, b) the stage-2 subgame in which both players still have to move and c) for each i and a_i, a subgame in which player i already has committed to a_i in stage 1 but in which player j still has to move. Assumption (3.1) implies that these latter subgames are completely determined as far as the payoffs are concerned: The assumption of persistent rationality embodied in the SPE concept implies that player j will choose an element in $B(a_i)$ and that the payoffs will be $(u_i(a_i), u_j(a_i))$. We will write γ^{2r} for the extensive form game that results from γ^2 if we replace each subgame $a_i \in A_1 \cup A_2$ with the terminal payoff $(u_1(a_i), u_2(a_i))$. Note that each SPE of γ^{2r} can be seen as a representative of an equivalence class of SPE of γ^2. Because of this one-to-one correspondence, we will henceforth restrict attention to γ^{2r}.[2]

A behavior strategy σ_i of player i in γ^{2r} is a rule that tells this player which action from $A_i \cup \{w_i\}$ to choose in stage 1 and what to do in stage 2 in the information set $w = (w_1, w_2)$ that corresponds to the case where both players waited. We write $\sigma_i = (\sigma_i^1, \sigma_i^2)$ where σ_i^1 denotes the randomization at time 1 and σ_i^2 is the mixed action in information set w. For the case where $\sigma_i^1(w_i) < 1$, i.e. player i moves with positive probability in the first period, it will also be convenient to write

$$\omega_i = \sigma_i^1(w_i), \; s_i^1 = (1-\omega_i)^{-1}\sigma_i^1, \text{ and } s_i^2 = \sigma_i^2 \tag{4.1}$$

hence, s_i^1 is the mixed action that player i plays if he moves in period 1. In this case we will also write $\sigma_i = (\omega_i, s_i^1, s_i^2)$. The normal form of γ^{2r} will be denoted as g^{2r}. In this normal form several pure strategies appear as equivalent rows, resp. columns, because of the fact that the second stage matters only if w_i was chosen in stage 1 that is, any two pure strategies $\bar{\sigma}_i = (0, \bar{a}_i^1, \bar{a}_i^2)$ and $\tilde{\sigma}_i = (0, \tilde{a}_i^1, \tilde{a}_i^2)$ with $\bar{a}_i^1 = \tilde{a}_i^1$ are equivalent. Taking a representative from each such equivalence class of pure strategies, we see that a pure strategy σ_i is equivalent to a rule that assigns to exactly one time point $t \in \{1,2\}$ an action in A_i. If $a_i \in A_i$, then we will write a_i^1 for the pure strategy "commit to a_i in period 1" while a_i^2 denotes the strategy "wait till period 2 and best respond if the opponent moves in period 1, otherwise play a_i". Hence, the set of pure strategies of player i in g^{2r} is $\{a_i^t : a_i \in A_i, t \in \{1,2\}\}$.

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[2]In the appendix we show that most of our results do not depend on this, and how the other results can be preserved by a slight adjustment.

For a game g satisfying (3.6) or (3.7) we define

$$\delta_i(a) = \begin{cases} 1 & \text{if } a_i \in B(a_j) \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

Note that, for a_j given, $\delta_i(a)$ is equal to 1 for exactly one a_i. If σ is played in γ^{2r}, then the outcome $a \in A$ results with probability

$$p^\sigma(a) = \sigma^1(a) + (1-\omega_1)\omega_2\sigma_1^1(a_1)\delta_2(a) + \omega_1(1-\omega_2)\sigma_2^1(a_2)\delta_1(a) + \omega_1\omega_2\sigma^2(a), \tag{4.3}$$

where we have used $\sigma^t(a)$ as a shorthand notation for $\sigma_1^t(a_1)\sigma_2^t(a_2)$. The probability distribution p^σ on A defined by (4.3) is called the outcome of σ and σ is said to induce the outcome s if $p^\sigma = s$. If g does not satisfy (3.6), then one has some freedom to specify the behavior in stage-2 subgames where already one of the players has moved, hence, each strategy pair σ in g^{2r} induces a set of outcomes, which are all payoff equivalent. In this case we will say that σ induces s if there exist $\delta = (\delta_1, \delta_2)$ with $\delta_i : A_j \to S_i$ and $\delta_i(a_j) \in \Delta B(a_j)$ such that, for p^σ as defined in (4.3), we have $p^\sigma = s$.

5 Nonrobustness of Mixed Strategy Equilibria

The relevance of mixed strategy equilibria has frequently been questioned. In particular, the traditional, naive[3], interpretation according to which players consciously randomize their actions is intuitively not very appealing. The reason a player must randomize in equilibrium is only to keep the others from deviating; the player himself will not lose if he deviates and instead uses any of the pure strategies in the support of the mixed strategy. Furthermore, unlike a pure equilibrium, a mixed strategy equilibrium typically

[3]We do not consider here more sophisticated interpretations like mixed strategies as beliefs or as population frequencies.

is not ex post self-enforcing in that, given the realizations of the actions, some player would like to deviate. Aspects of timing only compound these difficulties and as a result it is unlikely that a mixed strategy equilibrium is viable in the action commitment game. Namely, consider an equilibrium s^* of a game g satisfying (3.7) in which at least one of the players randomizes. Then, according to Lemma 2, each player randomizes and there is positive probability that the realized actions do not constitute an equilibrium ex post. Hence, if player j plays s_j^* in period 1 of γ^{2r}, then player i strictly prefers waiting to moving in the first period, and, consequently, both players playing s^* in the first period is not a Nash equilibrium of γ^{2r}. Obviously, we also have that both players waiting till the second period of γ^{2r} and then playing s^* is an equilibrium only if no player has an incentive to move first at s^*. In the next proposition we show that the latter is actually a necessary and sufficient condition for the existence of a Nash equilibrium of γ^{2r} with outcome s^*. Hence, only a very limited set of mixed strategy equilibria is viable when the timing of the moves is endogenous and the following Proposition can be viewed as a foundation and formalization of the critique of mixed strategy Nash equilibria in the applied literature (cf. Bester (1992)).

Proposition 1 *Let g be a game that satisfies the regularity condition (3.7) and let s^* be an equilibrium of g that is not in pure strategies. Then there exists a Nash equilibrium σ of γ^{2r} with outcome s^* if and only if no player has an incentive to move first at s^*.*

Proof.

Sufficiency. No player has an incentive to move first at s^* if and only if $u_i(s^*) \geq \bar{u}_i$ for $i = 1, 2$. In this case the strategy pair $\sigma = (\sigma_1, \sigma_2)$ with $\sigma_i^1 = w_i$, $\sigma_i^2 = s_i^*$ is a Nash equilibrium (in fact a subgame perfect equilibrium) of γ^{2r} with outcome s^*.

Necessity. Assume $u_i(s^*) < \bar{u}_i$ for some i and that σ is a Nash equilibrium of γ^{2r} with outcome s^*. We will derive a contradiction. Without loss of generality we may assume $u_1(s^*) < \bar{u}_1$. The proof is divided into a number of steps.

Step 1. $\omega_i = \sigma_i^1(w_i) < 1$ for $i = 1, 2$, hence, each player moves with positive probability in the first period.

Clearly, we must have $\sigma_2^1(w_2) < 1$, for otherwise player 1 can guarantee the payoff $\bar{u}_1$ by committing in the first period, hence $U_1(\sigma) \geq \bar{u}_1$ and σ cannot have the outcome s^*. Suppose that player 1 waits till the second period, i.e. $\sigma_1^1(w_1) = 1$. Let a^* be such that $\sigma_2^1(a_2^*) > 0$ and $a_1^* \in C(s_1^*) \backslash B(a_2^*)$. Then the outcome a^* can only be obtained if it is played in the second period, hence, $p^\sigma(a^*) \leq \omega_2 s^*(a^*)$, a contradiction.

Step 2. $\omega_i = \sigma_i^1(w_i) > 0$ for $i = 1, 2$, hence, with positive probability each player waits till the second period of γ^{2r} in σ.

Namely, assume $\sigma_i^1(w_i) = 0$ for some i. Then $\sigma_i^1(a_i) = s_i^*(a_i)$ for all a_i and i chooses at least two strategies with positive probability (Lemma 2). Furthermore, since there does not exist $a_j \in A_j$ with $a_j \in B(a_i)$ for all $a_i \in C(s_i^*)$, we have $U_j(\sigma_i, a_j^2) > U_j(\sigma_i, a_j^1)$ for all a_j, hence, $\sigma_j^1(w_j) = 1$. This contradicts step 1.

From the steps 1 and 2 we can conclude that the stage-2 subgame (w_1, w_2) is reached with positive probability if σ is played in γ^{2r}. Since σ is a Nash equilibrium of γ^{2r}, σ^2 must be a Nash equilibrium of g. Furthermore, obviously $C(\sigma^2) \subset C(s^*)$. We will distinguish two cases:

Case A $C(\sigma^2) = C(s^*)$, hence (by Lemma 2) $\sigma^2 = s^*$.

Case B $C(\sigma^2) \neq C(s^*)$, hence (by Lemma 2), there exists for each player i a strategy a_i^* such that $a_i^* \in C(s^*) \backslash C(\sigma_i^2)$.

We first continue the proof for case A.

Step A3. For each player i: If $\sigma_i^1(a_i) > 0$, then $u_i(a_i) > u_i(s^*)$, hence, a player commits himself only to strategies that yield more than the equilibrium payoff.

Suppose, to the contrary, that there exists some $\bar{a}_i$ such that $\sigma_i^1(\bar{a}_i) > 0$ but $u_i(\bar{a}_i) \leq u_i(s^*)$. Since player i must be indifferent between $\bar{a}_i^1$ and a_i^2 for any $a_i \in C(s^*)$, we must have

$$\sum_{a_j} \sigma_j^1(a_j)u_i(\bar{a}_i, a_j) + \omega_j u_i(\bar{a}_i) = \sum_{a_j} \sigma_j^1(a_j)u_i(a_j) + \omega_j u_i(s^*)$$

hence

$$\begin{aligned} u_i(\bar{a}_i) &= u_i(s^*) \quad \text{and} \\ u_i(\bar{a}_i, a_j) &= u_i(a_j) \quad \text{for all } a_j \in C(\sigma_j^1). \end{aligned}$$

In particular, $\bar{a}_i$ must be a best response against any pure action that player j plays with positive probability in the first period. From Lemma 2 we can conclude that there exists $a_j^* \in C(s_j^*)$ such that $\bar{a}_i \notin B(a_j^*)$ and $\sigma_j^1(a_j^*) = 0$. We may assume that $a_j^* \in B(\bar{a}_i)$, since otherwise by (4.3)

$$p^\sigma(\bar{a}_i, a_j^*) = \omega_1\omega_2 s^*(\bar{a}_i, a_j^*) < s^*(\bar{a}_i, a_j^*).$$

Applying Lemma 2 once more we see that we can find a_i^* such that $a_j^* \notin B(a_i^*)$. Then, for $a^* = (a_1^*, a_2^*)$,

$$p^\sigma(a^*) = \omega_1\omega_2 s^*(a^*) < s^*(a^*),$$

another contradiction.

Step A4. Conclusion of the proof for case A.

We have seen up to now that each player must move in both periods with positive probability and that for each player i and for each action a_i with $\sigma_i^1(a_i) > 0$ we have $u_i(a_i) > u_i(s^*)$. Since $U_i(\sigma) = u_i(s^*)$ and since waiting is a best response for each player we have

$$\sum_{a_j} \sigma_j^1(a_j)u_i(a_j) + \omega_j u_i(s^*) = u_i(s^*)$$

so that there must exist a_j^* with $\sigma_j^1(a_j^*) > 0$ and $u_i(a_j^*) \leq u_i(s^*)$. Hence, $\sigma_j^1(a_j^*) > 0$

and $u_i(a_i, a_j^*) \le u_i(s^*)$ for all $a_i \in A_i$, and, therefore

$$\sigma_j^1(a_j^*) > 0 \quad \text{and} \quad a_j^* \notin B(a_i) \quad \text{for all } a_i \text{ with } \sigma_i^1(a_i) > 0.$$

Consider $a^* = (a_1^*, a_2^*)$. Then $p^\sigma(a^*) = \sigma^1(a^*) + \omega_1\omega_2 s^*(a^*)$, hence, since $p^\sigma = s^*$, we must have

$$\sigma^1(a^*) = (1 - \omega_1\omega_2)s^*(a^*). \tag{5.1}$$

On the other hand, for each player i, we have by summing up (4.3) over all a_j, $p^\sigma(a_i^*) = \sigma_i^1(a_i^*) + \omega_1\omega_2 s_i^*(a_i^*)$, hence

$$\sigma^1(a^*) = (1 - \omega_1\omega_2)^2 s^*(a^*). \tag{5.2}$$

Combining (5.1) with (5.2) and using $a^* \in C(s^*)$ yields $\omega_1\omega_2 \in \{0,1\}$. But this contradicts the steps 1 and 2 and completes the proof for Case A.

We now continue with the proof of case B. Let a^* be such that, for each player i, $a_i^* \in C(s_i^*) \backslash C(\sigma_i^2)$.

Step B3. For all players i, j and all actions, if $a_j \in C(s_j^*)$ and $a_j \notin B(a_i^*)$, then $a_i^* \in B(a_j)$.

Assume there exists $\bar{a}_j \in C(s_j^*)$ such that $\bar{a}_j \notin B(a_i^*)$ and $a_i^* \notin B(\bar{a}_j)$. In σ the outcome $(a_i^*, \bar{a}_j)$ can occur only if both players play it in the first period, hence

$$s^*(a_i^*, \bar{a}_j) = \sigma^1(a_i^*, \bar{a}_j) \tag{5.3}$$

By summing up (4.3) over all a_l it follows that, for $k \neq l$, $s_k^*(a_k) = p^\sigma(a_k) \geq \sigma_k^1(a_k)$. Combining this with (5.3) and (4.3) it follows that

$$\sigma_i^1(a_i^*) = s_i^*(a_i^*) \qquad \text{and} \qquad a_i^* \notin B(a_j) \text{ if } \sigma_j^1(a_j) > 0.$$

This implies that any outcome (a_i^*, a_j) with $a_j \notin B(a_i^*)$ can occur only in the first period, hence,

$$s^*(a_i^*, a_j) = \sigma^1(a_i^*, a_j) \qquad \text{for all } a_j \notin B(a_i^*),$$

and hence, $s_j^*(a_j) = \sigma^1(a_j)$ for $a_j \notin B(a_i^*)$. This implies that if player j acts in period two, then he plays the unique $\bar{a}_j \in B(a_i^*)$ for sure. But this implies (by Lemma 2(i)) that $\bar{a}_j \in B(a_i)$ for all $a_i \in C(s_i^*)$. This contradicts Lemma 2(iv).

Step B4. Conclusion of the proof for case B.

Consider the pair $a^* = (a_1^*, a_2^*)$. Without loss of generality, step B3 allows us to assume that $a_1^* \in B(a_2^*)$. By Lemma 2 we have

$$\text{if } a_1 \in C(s_1^*),\ a_1 \neq a_1^*, \text{ then } a_1 \notin B(a_2^*).$$

By step B3, therefore $a_2^* \in B(a_1)$ for all such a_1. But then we have that $a_2^* \in B(\sigma_1^2)$ and $a_2^* \notin C(\sigma_2^2)$. By Lemma 2 this is impossible since σ^2 is an equilibrium of g and g satisfies the regularity condition (3.7). □

6 Robustness of Pure Strategy Equilibria

In this section we show that for pure strategy equilibria of g the situation is fundamentally different from that for mixed equilibria: Each pure equilibrium is a subgame perfect equilibrium outcome of the action commitment game, and each pure equilibrium that satisfies a mild regularity condition corresponds to a perfect equilibrium of that game. Specifically, if $\bar{a}$ is a pure equilibrium of g and if my opponent j decides to commit to $\bar{a}_j$ in γ^{2r} whenever he has to move I can do no better but to commit to $\bar{a}_i$ as well.

Proposition 2 *If a is a pure strategy equilibrium of g, then the strategy profile $\sigma = (\sigma_1, \sigma_2)$ with $\sigma_i^t = a_i \quad (i, t \in \{1,2\})$, i.e. each player i chooses a_i whenever he has to move is a subgame perfect equilibrium of γ^{2r}.*

Proof. Trivial. □

To eliminate pure equilibria of g we, hence, have to use equilibrium refinements. As most of these refinements have been defined in the strategic form of the game, we take a normal form perspective in the rest of this paper. The next proposition shows that all pure equilibria that satisfy a mild regularity requirement satisfy the weakest of these more refined criteria, i.e. they are perfect (undominated) equilibria.

Proposition 3 *If g satisfies (3.6) and $\bar{a}$ is a pure equilibrium of g with $u_i(\bar{a}) > \min_{a_j} u_i(\bar{a}_i, a_j)$ for $i = 1, 2$, then there exists a perfect equilibrium of g^{2r} with outcome $\bar{a}$.*

Proof. The strategy profile $\sigma = (\sigma_1, \sigma_2)$ defined by $\sigma_i = \bar{a}_i^1$ (i.e. each player i commits to $\bar{a}_i$ in period 1) is a Nash equilibrium of g^{2r}. We will show that σ_i is an undominated strategy in g^{2r}. Since $\bar{a}_i$ is undominated in g and g satisfies (3.6), $\bar{a}_i$ can be dominated in g^{2r} only if it is dominated by a strategy in which player i moves only in period 2. If s_i^2 (with meaning "play the mixed action s_i in period 2") is such a dominating strategy we must have $u_i(s_i, a_j) \geq u_i(a_i) = u_i(a)$, but then s_i must be a best response against $\bar{a}_j$ in g. The regularity condition (3.6) implies that the best response is unique, hence, $s_i = \bar{a}_i$. Now, since $u_i(\bar{a}) > \min_{a_j} u_i(\bar{a}_1, a_j)$ we have $u_i(\bar{a}_i) > u_i(\bar{a}_i, a_j)$ for some a_j, hence $U_i(\bar{a}_i^1, a_j^2) > U_i(\bar{a}_i^2, a_j^2)$ for some a_j. But this implies that $\bar{a}_i^1$ is not dominated by $\bar{a}_i^2$ in g^{2r}, hence $(\bar{a}_1^1, \bar{a}_2^1)$ is perfect. □

Let us remark that the result does not hold for games that satisfy just the weaker condition (3.1). One obvious reason for why the proposition might fail is that $\bar{a}$ might not be a perfect equilibrium of g. However, it can fail even for perfect equilibria. Consider the game g from Table 2a. In this game, B_i is a weakly dominated strategy for player i, hence, a perfect equilibrium of g must be a Nash equilibrium of the 3×3 game that results if B_1 and B_2 are eliminated. However, in this 3×3 game C_i is strictly dominated and in the 2×2 game that results after C_1 and C_2 have been eliminated D_i is strictly dominated, so that (A_1, A_2) is the unique perfect equilibrium of g. The game g^{2r} is given in Table 2b.

	A_2	B_2	C_2	D_2
A_1	1 1	1 1	3 -3	0 0
B_1	1 1	1 1	2 2	0 0
C_1	-3 3	2 2	0 0	-1 8
D_1	0 0	0 0	8 -1	-2 -2

Table 2a: Game g.

	A_2^1	B_2^1	C_2^1	D_2^1	A_2^2	B_2^2	C_2^2	D_2^2
A_1^1	1 1	1 1	3 -3	0 0	1 1	1 1	1 1	1 1
B_1^1	1 1	1 1	2 2	0 0	2 2	2 2	2 2	2 2
C_1^1	-3 3	2 2	0 0	-1 8	-1 8	-1 8	-1 8	-1 8
D_1^1	0 0	0 0	8 -1	-2 -2	0 0	0 0	0 0	0 0
A_1^2	1 1	2 2	8 -1	0 0	1 1	1 1	3 -3	0 0
B_1^2	1 1	2 2	8 -1	0 0	1 1	1 1	2 2	0 0
C_1^2	1 1	2 2	8 -1	0 0	-3 3	2 2	0 0	-1 8
D_1^2	1 1	2 2	8 -1	0 0	0 0	0 0	8 -1	-2 -2

Table 2b: Game g^{2r}.

The important observation is that A_i^1 is weakly dominated by $\frac{1}{2}B_i^1 + \frac{1}{2}A_i^2$. Furthermore, C_i^1, D_i^1 and B_i^2 are all weakly dominated by A_i^2. Hence, any perfect equilibrium of g^{2r} has to be a Nash equilibrium of the game in which player i is restricted to choose from the set $\{B_i^1, A_i^2, C_i^2, D_i^2\}$. Now it is easily seen that any Nash equilibrium of this 4×4 game either results in the payoff (2,2) or is the strategy pair in which each player i chooses $\frac{1}{2}B_i^1 + \frac{1}{2}A_i^2$. Hence, each perfect equilibrium of g^{2r} yields each player a payoff of at least $\frac{3}{2}$ and there is no perfect equilibrium of g^{2r} with outcome (A_1, A_2). It is also easy to construct an example showing that the condition $u_i(\bar{a}) > \min_{a_j} u_i(\bar{s}_i, a_j)$ in Proposition 3 is essential. Let g be as in Table 3a. Then g satisfies (3.6) and A is a perfect equilibrium of g. In g^{2r}, however, A_i^1 is dominated by A_i^2, hence, each player can guarantee the payoff 4 by committing to B_i. Every perfect equilibrium of g^{2r} results in the outcome (B_1, B_2).

	A_2		B_2	
A_1	2	2	3	0
B_1	0	3	4	4

Table 3a: Game g.

	A_2^1		B_2^1		A_2^2		B_2^2	
A_1^1	2	2	3	0	2	2	2	2
B_1^1	0	3	4	4	4	4	4	4
A_1^2	2	2	4	4	2	2	3	0
B_1^2	2	2	4	4	0	3	4	4

Table 3b: Game g^{2r}.

At this stage it is appropriate to compare our work with Hamilton and Slutsky (1990). Hamilton and Slutsky consider the game g^{2r} where g is the standard quantity setting Cournot duopoly game. In their Theorem VIII they claim

> "the two Stackelberg equilibria are the only pure strategy equilibria in undominated strategies. Playing the Cournot equilibrium strategy at the first turn is dominated by waiting to play after one's rival."

Although the standard Cournot duopoly game does not fit our context, discretized versions of this game do and the Cournot equilibrium satisfies the conditions from Proposition 3. Hence, it follows that the above claim is wrong. Playing the Cournot equilibrium strategy at the first stage would indeed be dominated if players could be sure that they would continue with the Cournot equilibrium in the second stage in case both players still have to move, but, if mistakes are possible, players cannot be sure of this. The total quantity in the second period might be above the Cournot quantity, if only by mistake, and in this case it pays to commit to the Cournot quantity if the opponent does the same. It is the impression of the present authors that, although not stating it explicitly, Hamilton and Slutsky actually had this truncated game in mind when making the above claim. (See also their companion paper Hamilton and Slutsky (1993) in which in the proof of Theorem III they display the payoff matrix of the truncated game rather than of the full game g^{2r}. In the next section we will analyze this truncated game and show that only equilibria in which no player has an incentive to move first are perfect in it. Note that in the economic context studied by Hamilton and Slutsky, there is some justification for studying the truncated game rather than the full game g^{2r}. It is well-known that the 2-person quantity setting duopoly game is dominance solvable, i.e. by iterative elimination of weakly dominated strategies, the game can be reduced to the Cournot quantities. This implies that any quantity choice in the second period that is different from the Cournot quantity is iteratively dominated in g^{2r} and, hence, can be eliminated. Hence, if g is the Cournot duopoly game, then by iterative elimination of weakly dominated strategies the game g^{2r} can be reduced to the truncated game and in the latter the above claim of Hamilton and Slutsky is correct. (Recall, however, that the example in Section 2 has shown that the full game cannot always be reduced this way.) Finally, it is interesting to note that, again for the special case of Cournot duopoly, whether the equilibrium $\bar{a}$ from Proposition 3 is proper or not depends on the descritization. Let q^* be the Cournot quantity and let q^- (resp. q^+) be the largest (smallest) quantity in the grid that is less (more) than q^*. (Assume that q^* is in the grid as well.) Then committing to q^* is a proper equilibrium in g^{2r} if and only if $u_i(q^-, q^*) < u_i(q^+, q^*)$.

7 Commitment Robust Equilibria

The SPE that was described in proposition 3 is fragile in the sense that, if a player is sure that a will be played in period 2, then he has nothing to lose from waiting. However, if player j waits and $u_i(a) < \bar{u}_i$, then player i has an incentive to commit. To put this more formally, we have that committing to a in period 1 is not a perfect (i.e. undominated) equilibrium of the game in which players are forced to play a in the second period of γ^{2r}. This section is devoted to proving that only equilibria in which no player has an incentive to move first can be perfect in this game.

We introduce some notation first. If $s^* \in S$, we will write $\gamma^{2r}(s^*)$ for the extensive form game that results from γ^{2r} if we replace the subgame w at stage 2 in which both players still have to move by an endpoint with payoff vector $(u_1(s^*), u_2(s^*))$. Hence, we assume that players will play s^* if both still have to move. (To have an SPE of γ^{2r} we must of course require that s^* is a Nash equilibrium of g.) The associated normal form game is denoted by $g^{2r}(s^*)$. In this game, the set of pure strategies of player i is $S_i \cup \{w_i\}$ and payoffs are determined by g if no player waits, by (3.2) and (3.4) if only j waits, and they are equal to $(u_1(s^*), u_2(s^*))$ if both players wait.

Our definition of commitment robustness formalizes the idea that "sensible behavior" in the first stage of γ^{2r} should produce the outcome s if players foresee that in the second stage play will continue with s.

Definition 1 *An equilibrium s^* of g is commitment robust (is a CRE) if there exists a perfect equilibrium σ of $g^{2r}(s^*)$ with outcome s^*.*

Proposition 4 *An equilibrium s^* of g is commitment robust if and only if no player has an incentive to move first at s^*, i.e. $u_i(s^*) \geq \bar{u}_i$ for $i = 1, 2$.*

REMARK Note that if s is mixed and g satisfies the regularity requirement (3.7), then this statement follows from Proposition 1. The proof of this Proposition is straightforward if s^* is pure (the steps 1 – 4 below). In the case s^* is mixed, the proof will rely on the proof of Proposition 1 (see step 7).

Proof.

Sufficiency: If $u_i(s^*) \geq \bar{u}_i$ for $i = 1, 2$, then waiting is a dominant strategy for each player in $g^{2r}(s^*)$, hence (w_1, w_2) is a perfect equilibrium and this equilibrium results in the outcome s^*.

Necessity: Assume $u_i(s^*) < \bar{u}_i$ for some i. We have to show that there does not exist a perfect equilibrium of $g^{2r}(s^*)$ with outcome s^*. We assume that such an equilibrium, σ, exists and derive a contradiction. The proof is divided in a number of steps.

Step 1. We may assume $u_i(s^*) < \bar{u}_i$ for $i = 1, 2$.

Namely, assume $u_i(s^*) < \bar{u}_i$ but $u_j(s^*) \geq u_j$. Then in $g^{2r}(s^*)$ we have that w_j is a dominant strategy for j. If $\sigma_j(w_j) = 1$, then player i can guarantee $\bar{u}_i > u_i(s^*)$ by committing to a Stackelberg leader strategy. Hence, σ cannot result in outcome s^*. If $\sigma_j(w_j) < 1$, then player j must have a dominant strategy $\bar{a}_j$ in the game g, hence $u_j(a_i, \bar{a}_j) = u_j(a_i)$ for all a_i, and, therefore, $u_i(a_i, \bar{a}_j) = u_i(a_i)$ because of Assumption (3.1). Hence, all of j's dominant strategies are equivalent for i and i's best response to σ_j yields $\bar{u}_i > u_i(s^*)$. Consequently, there cannot be a perfect equilibrium with outcome s^*.

Step 2. $\sigma_i(w_i) < 1$ for $i = 1, 2$.

The argument is the same as in step 1. If player j waits for sure then i can commit himself and thereby guarantee more than $u_i(s^*)$.

Step 3. If $\sigma_i(a_i) > 0$, then $u_i(a_i) \geq u_i(s^*)$ with the inequality being strict if a_i is not a dominant strategy in g.

This follows from the observation that, if the condition is not satisfied, a_i is dominated in $g^{2r}(s^*)$ by w_i.

Step 4. s_i^* cannot be pure for $i = 1, 2$.

Assume s_i^* is pure, $s_i^* = a_i^*$. By step 2, player i must play a_i^* with positive probability in the first period. However, $u_i(a_i^*) = u_i(a^*)$, so that, by step 3, a_i^* must be a dominant strategy in g. Then $\bar{u}_j = \max_{a_j} u_j(a_i^*, a_j) = u_j(s^*)$, contradicting step 1.

Step 5. We must have $\sigma_i(w_i) > 0$ for all i.

Assume $\sigma_i(w_i) = 0$, hence, player i moves for sure in period 1. Assume there exists $a_i \in C(s_i^*), a_j, a_j' \in C(s_j^*)$ such that $u_j(a_i, a_j) > u_j(a_i, a_j')$. Then, since $u_j(s_i^*, a_j) = u_j(s_i^*, a_j')$, there must exist $a_i' \in C(s_i^*)$ with $u_j(a_i', a_j') > u_j(a_i', a_j)$. This implies that w_j is a strictly better response in $g^{2r}(s^*)$ against s_i^* than any $a_j \in C(s_j^*)$, hence $\sigma_j(w_j) = 1$, contradicting step 2. Consequently, we must have $u_j(a_i, a_j) = u_j(a_i, a_j')$ for all $a_i \in C(s_i^*), a_j, a_j' \in C(s_j^*)$. Assumption (3.1) then implies $u_i(a_i, a_j) = u_i(a_i, a_j')$ for all such a_i, a_j, a_j'. Hence, since s^* is an equilibrium $u_i(a_i, a_j) = u_i(a_i', a_j)$ for all $a_i, a_i' \in C(s_i^*), a_j \in C(s_j^*)$. Using (3.1) once more we see that both players' payoffs must be constant and equal to $u(s^*)$ on $C(s^*)$. If there exists $a_i \in C(s_i^*)$ with $u_j(a_i) > u_j(s^*)$, then player j's unique best response to s^* is to wait and σ cannot result in s^*, hence, we must have $u_j(a_i) = u_j(s^*)$ for all $a_i \in C(s_i^*)$. By step 3, all a_i with $s_i^*(a_i) > 0$ must therefore be dominant strategies in g. But then, for any such a_i we have, $\bar{u}_j = \max_{a_j} u_j(a_i, a_j) = u_j(s^*)$, contradicting step 1.

Step 6. If $a_i \in C(s_i^*)$ is a dominant strategy in g, then $\sigma_i(a_i) = 0$.

Assume a_i is dominant in g, $s_i^*(a_i) > 0$ and $\sigma_i(a_i) > 0$. Then $u_i(a_i) \leq u_i(s^*)$ by step 5 and $u_i(a_i) \geq u_i(s^*)$ by step 3. Exactly as in the proof of the previous step one can show that both players' payoffs must be constant on $C(s^*)$. The contradiction is obtained in exactly the same way as before.

Step 7. Conclusion of the proof.

We have seen up to now that each player must move in both periods with positive probability, that in the first period a player cannot choose a strategy that is dominant in g, and that for each strategy a_i with $\sigma_i(a_i) > 0$ we have $u_i(a_i) > u_i(s^*)$. We can now mimic the proof of step A4 of Proposition 1 (in which no use was made of the regularity requirement (3.7)) to derive a contradiction. □

To conclude this section, we will compare our concept of commitment robustness with that proposed in Rosenthal (1991). Rosenthal defines an equilibrium s to be commitment robust if s is an SPE outcome of each of the two games in which one of the players moves first. Hence, Proposition 4 seems to establish the equivalence of Rosenthal's concept to ours. However, whereas we assume that only commitments to pure actions are possible, Rosenthal assumes that a player can commit to a mixed strategy and that the mixture can be communicated. Hence, Rosenthal works with what Schelling (1960, p. 185) calls "fractional commitments" and Schelling already points out that these may be more efficient than pure ones. (Compare our discussion of the extended matching pennies game in Section 3 in which a mixed strategy equilibrium Pareto dominates the pure strategy equilibrium.) Of course, having the opportunity to commit to a mixed action can never be worse than having the opportunity to commit to a pure action and, hence, any equilibrium that is commitment robust according to Rosenthal, which will be called an RCRE, is also a CRE. Of course, frequently there will exist no CRE and it is easy to construct a game that admits a CRE but no RCRE. To conclude this section we derive an alternative characterization of RCRE for a game satisfying (3.7).

Proposition 5 *For a game g and a player i define u_i^+ by means of*

$$u_i^+ = \max_{s_i \in S_i} \max_{a_j \in B(s_i)} u_i(s_i, a_j)$$

Let s^* be a Nash equilibrium of g. If $u_i(s^*) \geq u_i^+$ for $i = 1, 2$, then s^* is an RCRE. If g satisfies (3.7), then s^* is an RCRE if and only if $u_i(s^*) \geq u_i^+$ for $i = 1, 2$.

Proof. Consider the game γ^{ij} in which player i is allowed to act as a Stackelberg leader, with this player's mixed action being revealed to player j. Because of the bilinearity of

u_i, player i's payoff if he chooses s_i and j best responds, is at most $\max_{a_j \in B(s_i)} u_i(s_i, a_j)$. Hence, each SPE yields player i at most u_i^+. If $u_i(s^*) = u_i^+$, then any strategy pair σ with $\sigma_i = s_i^*$ and σ_j with $\sigma_j(s_i^*) = s_j^*$ and $\sigma_j(s_i) \in B(s_i)$ for all s_i is an SPE of γ^{ij}. This proves the first part of the proposition.

To prove the second part, let $s_i^+ \in S_i$ and $a_j^+ \in B(s_i^+)$ be such that $u_i(s_i^+, a_j^+) = u_i^+$. Because of (3.7), there exists $x \in \mathbf{R}^{A_i}$ with $x(a_i) = 0$ if $a_i \notin C(s_i^+)$ such that $u_j(x, a_j^+) > 0$ and $u_j(x, a_j) = 0$ for all $a_j \in B(s_i^+) \backslash \{a_j^+\}$. (Here $u_j(x, a_j)$ is shorthand notation for $\sum_{a_i} x(a_i) u_j(a_i, a_j)$.) Write $t = \sum_{a_i} x(a_i)$ and $s_i(\varepsilon, t) = (1 + \varepsilon t)^{-1}(s_i^+ + \varepsilon x)$. Then $s_i(\varepsilon, t) \in S_i$ if ε is sufficiently small, $B(s_i(\varepsilon, t)) = \{a_i^+\}$ and $s_i(\varepsilon, t) \to s_i^+$ as $\varepsilon \to 0$. Hence, if s is an SPE outcome of γ^{ij} we must have $u_i(s) \geq u_i(s_i(\varepsilon, t), a_j^+)$, and, therefore, $u_i(s) \geq u_i(s_i^+, a_j^+) = u_i^+$. This completes the proof of the second part of the proposition.
□

8 Endogenous Timing and Coordination

In this section we return to the untruncated game from section 6 and we investigate whether, by employing more refined equilibrium notions, we can obtain the conclusion that only commitment robust equilibria are viable when the order of the moves is endogenous. Specifically, we address the question of whether in a game that has a unique and pure CRE players will automatically coordinate on this CRE if the timing of the moves is endogenous. We will show that some set-valued "evolutionary" concepts, viz. the notions of persistent equilibria (Kalai and Samet (1984)) and of curb and curb* equilibria (Basu and Weibull (1991)) do indeed allow this conclusion. We first formally define these concepts. Write $\mathcal{B}(s_i)$ for the set of all mixed best replies against s_i hence $\mathcal{B}(s_i) = \Delta B(s_i)$ and $\mathcal{B}(s) = \mathcal{B}(s_2) \times \mathcal{B}(s_1)$. For a set $\tilde{S}$ of mixed strategy pairs, write $\mathcal{B}(\tilde{S}) = \cup_{s \in \tilde{S}} \mathcal{B}(s)$. Similarly, define $\mathcal{B}^*(\tilde{S})$ for the set of all undominated best replies against $\tilde{S}$, i.e. best replies that are undominated strategies.

Definition 2 *Let g be a 2-person normal form game*

(i) A retract is a set $R = R_1 \times R_2$ *where* R_i *is a nonempty, closed and convex subset of* S_i.

(ii) Given a certain property (X) *of retracts, an* X*-retract is a retract* R *that is minimal with respect to this property, i.e. there does not exist a retract* R' *with* $R' \subset R$, $R' \neq R$ *having property* (X).

(iii) An X*-equilibrium is an equilibrium that belongs to an* X*-retract.*

(iv) Persistent retracts, curb retracts and curb retracts are defined, respectively, by the properties* (P), (C) *and* (C^*):

$$\mathcal{B}(s) \cap R \neq \emptyset \text{ for all } s \text{ in some open neighborhood } \mathcal{O} \text{ of } R \qquad (P)$$

$$\mathcal{B}(R) \subset R \qquad (C)$$

$$\mathcal{B}^*(R) \subset R \qquad (C^*)$$

We will make use of the following proposition which proof can, for example, be found in Balkenborg (1992).

Proposition 6 *(i) Every curb retract contains a curb* retract and every curb* retract contains a persistent retract.*

(ii) Every game has at least one curb (resp. curb, resp. persistent) equilibrium.*

(iii) If R *is a curb retract (resp. a curb* retract), then* $R_i = \Delta(A_i')$ *for some* $A_i' \subset A_i$.

(iv) Different curb retracts are disjoint and so are different curb retracts.*

(v) If no player i has equivalent strategies in g (i.e. there do not exist $a_i, a_i' \in A_i$ *with* $a_i \neq a_i'$ *and* $u_i(a_i, a_j)$ *for all* a_j*), then the properties (iii) and (iv) also hold for persistent retracts, i.e. each persistent retract is a convex hull of pure strategies and different persistent retracts are disjoint.*

The main result of this paper is

Proposition 7 *Assume g satisfies (3.6) and $\bar{a}$ is a pure CRE of g. Then*

(i) Any curb (resp. curb, resp. persistent) equilibrium of g^{2r} yields each player i a payoff of at least $u_i(\bar{a})$.*

(ii) If $\bar{a}$ is the unique CRE of g, then each curb (resp. curb. resp. persistent) equilibrium of g^{2r} results in the outcome $\bar{a}$.*

Proof. In this proof, let "x" stand for "curb", "curb*" or "persistent". Note that since g satisfies (3.6), no player i has equivalent strategies in g^{2r}, hence x-retracts are convex hulls of pure strategies and different x-retracts are disjoint (Proposition 6). Let the retract $\bar{R}$ be defined by

$$\bar{A}_i = \{\bar{a}_i^1\} \cup \{a_i^2 : \ a_i \in A_i\}, \ \bar{R}_i = \Delta \bar{A}_i, \ \bar{R} = \bar{R}_1 \times \bar{R}_2.$$

Note that $\bar{R}$ satisfies the properties $(C), (C^*)$ and (P). We will show that any x-retract is contained in $\bar{R}$.

The proof is easy in case some player i has a dominant strategy $\tilde{a}_i$ in g. Then $\tilde{a}_i = \bar{a}_i$ and since $\bar{a}$ is a CRE we have that for all $a_i \neq \bar{a}_i$, a_i^t is dominated by $\bar{a}_i^t$ $(t = 1, 2)$. This implies that if R is a persistent retract, then

$$R \subset \Delta(\{\bar{a}_i^1, \bar{a}_i^2\}) \times \Delta(\bar{A}_j) \subset \bar{R}.$$

Since any x-retract contains a persistent retract, and since $\bar{R}$ has properties (C) and (C^*), it follows that any x-retract is contained in $\bar{R}$.

Now assume that no player has a dominant strategy in g. Then it is easily seen that $\bar{a}_i^2$ is an undominated strategy for each player i. Furthermore, for each j there exists some $\tilde{a}_j$ such that $\bar{a}_i \notin B(\tilde{a}_j)$. We will show that, if $a_j \neq \bar{a}_j$, then committing to a_j cannot belong to any x-retract. Note that if R is an x-retract and $a_j^1 \in R_j$, then $\bar{a}_i^2 \in R_i$.

Namely, $\bar{a}_i^2$ is an undominated best response against a_j^1 (hence $\bar{a}_i^2 \in R_i$ if x stands for curb or curb*, and $\bar{a}_i^2$ is the unique best response against $(1-2\varepsilon)a_j^1 + \varepsilon\tilde{a}_j^1 + \varepsilon\bar{a}_j^2$ (so that $a_i^2 \in R_i$ if x stands for persistent). Now, since a_j^2 is the unique best response against $(1-\varepsilon)\bar{a}_i^2 + \varepsilon\tilde{a}_i^1$ it follows that $(\bar{a}_1^2, \bar{a}_2^2) \in R$ whenever $a_i^1 \in R_i$ for some $a_i \neq \bar{a}_i$. Since $\bar{R}$ satisfies the properties $(C), (C^*)$ and (P), we have that any x-retract that contains $(\bar{a}_1^2, \bar{a}_2^2)$ is contained in $\bar{R}$ and there does not exist an x-retract containing some a_i^1 with $a_i \neq \bar{a}_i$. Hence, any x-retract is contained in $\bar{R}$.

Now note that, if players are restricted to choose strategies from $\bar{R}$, then each player i can guarantee the payoff $u_i(\bar{a})$ by playing $\bar{a}_i^1$. Consequently, if σ is an x-equilibrium of g^{2r}, then $C(\sigma) \subset \bar{R}$ and $u_i(\sigma) \geq u_i(\bar{a})$ for $i = 1, 2$, which proves the first part of the proposition.

Now assume that there exists an x-equilibrium σ that results in an outcome different from $\bar{a}$. Then for each player i we must have $\sigma_i(\bar{a}_i^1) < 1$. Define the mixed strategy s_i of player i in g by

$$s_i(a_i) = (1 - \sigma_i(\bar{a}_i^1))^{-1}\sigma_i(a_i^2) \qquad (a_i \in A_i)$$

Since σ is an equilibrium of g^{2r} we have that s is an equilibrium of g and, furthermore $u_i(s) \geq \bar{u}_i$ for $i = 1, 2$, since each player i can guarantee $\bar{u}_i$ by playing a_i^1. Hence, s is a CRE of g. This completes the proof. □

Note that in the proof we did not use the full power of the regularity condition (3.6). We used that $\bar{a}$ is a strict equilibrium, i.e. $\bar{a}_i$ is the unique best response against $\bar{a}_j$, and that, for each player i, $\bar{a}_i$ is the unique Stackelberg leader strategy. The game from Table 4 may show that the latter assumption is essential. Note that this game satisfies (3.1). (T, L) is a CRE in this game, however, the unique curb retract of g^{2r} is the entire game, so that, in particular (B^1, R^1) is a curb equilibrium.

	L		M		R	
T	3	3	0	0	0	0
M	2	2	3	3	0	0
B	0	0	4	1	2	2

Table 4

9 Conclusion

We have addressed the question of whether only equilibria at which no player has an incentive to move first are viable when the order of the moves is endogenously determined and players have the opportunity to commit themselves. We have seen that in order to answer this question in the affirmative one needs quite strong equilibrium concepts, but that, if one is willing to accept such concepts, one can indeed conclude that, with endogenous timing, players will indeed coordinate on the commitment robust equilibrium whenever the latter is pure and unique. We have restricted ourselves in this paper to 2-person games and we have only allowed one point in time at which a player can commit himself. It is important to investigate the extent to which our results depend on these assumptions. One can easily define the game γ^t in which there are $t-1$ periods in which a player has the opportunity to commit. (Obviously $\gamma^1 = g$.) The reader can verify that our main results remain valid in this extended context. Hence, mixed equilibria are typically not viable and any curb (resp. curb*, resp. persistent) equilibrium of the game in which the players have $t-1$ opportunities to commit themselves results in the commitment robust equilibrium. We have not investigated whether our results extend to games with more than two players, although we expect they do. It is clear, however, that in some cases (as in the proof of Proposition 1) different techniques are needed.

It has to be admitted that the class of games with a commitment robust equilibrium, i.e. the class for which we were able to determine the outcome with endogenous timing in this paper is quite limited. In a companion paper (Van Damme and Hurkens (1993)) we address the question of which outcomes can be expected for some economic games without a commitment robust equilibrium. It turns out that in this case the length of the timing game plays an important role. To illustrate the phenomena that arise, consider the game g from Table 5a in which $x > 0$.

	L		R	
T	2	1	0	0
B	3	0	1	x

Table 5a

	L		R		W	
T	2	1	0	0	2	1
B	3	0	1	x	1	x
W	3	0	1	x	y	z

Table 5b

In game g, B is a dominant strategy, hence, $s = (B, R)$ is the unique Nash equilibrium. If we substitute $(y, z) = (1, x)$ in Table 5b, then we obtain the game $g^{2\tau}(s)$. In this game, (T, W) is the unique perfect equilibrium. Only player 1 has an incentive to commit himself and indeed this player commits. However, now assume that $x > 1$ and consider γ^3, i.e. there are two time periods at which players can commit. Player 1 has no incentive to commit himself right away: he is sure of getting his Stackelberg payoff if he waits one more period. Player 2 foresees that, if he does not commit right away, player 1 will do so in the next instance. Hence, if he does not commit, his payoff is 1, while committing to R yields $x > 1$. Formally, if players foresee that as of stage 2 (T, W) will be played, they will analyze the stage 1 game by substituting $(y, z) = (2, 1)$ in Table 5b and in this game the unique perfect equilibrium is (W, R). We come to the conclusion that the predicted outcome is very sensitive with respect to how many times one has the opportunity to commit themselves. If t is odd, the predicted outcome of γ^t is (B, R), while it is (T, L) if t is even. Note that these problems do not arise in case one of the Stackelberg equilibria Pareto-dominates the other. If $x < 1$, then for the sequence $\{s^t\}_t$ which is recursively defined by (i) s^1 is a Nash equilibrium of g and, (ii) for each $t \geq 1$, s^{t+1} is a perfect equilibrium of $g^{2\tau}(s^t)$, then s^t is (T, L) if $t \geq 3$, hence, players coordinate on the Pareto dominant Stackelberg equilibrium. More delicate issues are left for our companion paper.

Appendix

In this appendix we show that the truncation we use throughout the paper —namely to substitute the SPE payoffs in subgames where one player has committed and one player has waited— is innocuous for most results obtained in this paper. We show how the propositions can be adjusted.

Let g^2 denote the reduced normal form of γ^2. Notice that g^{2r} is obtained from g^2 by elimination of weakly dominated "waiting"-strategies.

Proposition 1 is not correct when γ^{2r} is replaced by γ^2, as can easily be seen from the coordination game from Table 1a. Let s^* denote the mixed equilibrium of this game. Consider the following strategy for player i:
Wait until the second period;
If the opponent has also waited, play s_i^* in period 2;
If the opponent has committed to a_j, then respond with $a_i \notin B(a_j)$.
These strategies constitute a Nash equilibrium of γ^2 that induces the outcome s^*.

However, Proposition 1 can be adjusted by replacing γ^{2r} by γ^2 and by replacing "Nash" by "subgame perfect". The proof remains the same.

In Proposition 2 (resp. Proposition 3) γ^{2r} (resp. g^{2r}) can be replaced by γ^2 (resp. g^2). The proofs remain the same. Propositions 4, 5 and 6 are not affected.

Proposition 7 is not correct for "curb" if g^{2r} is replaced by g^2. However, it remains correct for "curb*" and "persistent". This can be proved as follows:

Let "x" stand for "curb*" or "persistent". First, remark that x-retracts do not contain pure weakly dominated strategies. Hence, the strategies that are contained in an x-retract of g^2 are not eliminated and are, hence, also strategies in g^{2r}. In particular, we can use the same notation $a_i^!$ as before.

Furthermore, we use the following result from Balkenborg (1992) (Corollary 6.2.2, page 80):

If Γ and Γ' are two games and Γ' is obtained from Γ by deleting pure weakly dominated strategies, then any persistent retract of Γ contains a persistent retract of Γ'.

Now it follows from the above and the proof of Proposition 7 that, if R is a persistent retract of g^2 then $R \cap \bar{R} \neq \emptyset$, where $\bar{R}$ is as defined in the proof of Proposition 7. Since $\bar{R}$ has properties (C^*) and (P) in g^2, it follows that every x-retract of g^2 is contained in $\bar{R}$. The remainder of the proof is exactly the same as in the proof of Proposition 7.

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