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On the Connectedness of the Set of Constrained Equilibria * §

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Abstract

In this paper it is shown that in a general equilibrium model with price rigidities there exists a connected set of constrained equilibria containing both trivial equilibria. The proof of this theorem combines results in the areas of mathematical programming with those in topology. This result is proved without using differentiability assumptions and is also extended to the case with upper semi-continuous demand correspondences. All known existence results for the model discussed follow as easy corollaries from these results.

1 Introduction

In economic models with fixed prices and quantity rationing many existence results have been given in the past. In Drèze (1975) the existence of a constrained equilibrium without rationing on the market of the numeraire commodity was shown. In van der Laan (1980b) it was remarked that there exist two trivial equilibria which also satisfy the conditions for a constrained equilibrium given by Drèze (1975). In one of these trivial equilibria the prices of all commodities are set equal to their lower bound and supply of all commodities is completely rationed, while the demand of every commodity is not rationed. The second trivial equilibrium is obtained by setting all prices equal to their upper bound, rationing the demand of all commodities completely and not rationing the supply of any commodity. Van der Laan (1980b) and, in a somewhat different model, Kurz (1982) showed the existence of non-trivial equilibria without rationing on the demands, while van der Laan (1982) showed the existence of a non-trivial equilibrium without rationing on the demands and without rationing on at least one market. This constrained equilibrium is called a real unemployment equilibrium. Similar results have been obtained in Dehez and Drèze (1984), van der Laan (1984), and Weddepohl (1987), for models with a different set of admissible prices. In van der Laan and Talman (1990) and Herings (1992) more equilibrium existence results are given.

Van der Laan (1982) states a theorem that there is a connected set of constrained equilibria containing both the trivial equilibrium with complete supply rationing and a real unemployment equilibrium. The proof is based on properties of points generated by a simplicial algorithm applied to a model with price rigidities and quantity rationing. However, the proof is not complete since it assumes that a certain sequence of connected 1-manifolds has a subsequence that converges to some connected 1-manifold. This reasoning is not valid in general. Nevertheless, the basic idea of the proof, the use of the properties of the path of points generated by a simplicial algorithm in order to obtain insight into the structure of the set of constrained equilibria, will turn out to be very useful. This idea will be used to show the existence of a connected set of constrained equilibria containing the two trivial equilibria, which is a generalization of the result in van der Laan (1982). No differentiability assumptions will be made on demand functions in order to derive this result. In fact it will be shown that it is possible to obtain the same result in the case with upper semi-continuous excess demand correspondences.

In Section 2 the model and equilibrium concept used is briefly discussed. In Section 3 the algorithm used in van der Laan (1982) is presented. It is applied on a function derived from the excess demand function of an economy with price rigidities and quantity constraints. It is shown that the algorithm generates a path of points connecting the two trivial equilibria. In Section 4 it is shown that the set of constrained equilibria has a component, i.e. a maximally connected subset, which contains the two trivial equilibria. In Section 5 it is shown that the results of Section 4 remain valid if weaker assumptions are made with respect to the economy, guaranteeing only that the excess demand correspondence is upper semi-continuous instead of being a continuous function. In Section 6 it is shown that all the earlier mentioned equilibrium existence results can be proved by a one line argument using the results of Sections 4 and 5. In Section 7 the main conclusions are briefly summarized.

2 The Model

In the following, for $k \in \mathbf{N}$, define $I_k = \{1, \ldots, k\}$, $Q^k = \{x \in \mathbf{R}^k \mid \forall j \in I_k,\ 0 \leq x_j \leq 1\}$, let 0^k be a $k-$dimensional vector of zeros, and let 1^k be a $k-$dimensional vector of ones. Consider an exchange economy with price rigidities and a rationing system, $\mathcal{E} = \left(\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})}, (l, L)\right)$. There are m consumers indexed $i = 1, \ldots, m$ and n commodities indexed $j = 1, \ldots, n$. A consumer is defined by a consumption set X^i, a preference ordering $\succeq^i$ on X^i, and a vector of initial endowments w^i. The set of admissible prices is denoted by $P_{(\underline{p},\bar{p})}$. Here $\underline{p}$ denotes a lower bound and $\bar{p}$ an upper bound on the set of admissible prices. The rationing system (l, L) is a pair of functions, each function having mn components. Function values describe rationing schemes permitted in the economy. For every $i \in I_m$ and for every $j \in I_n$ component $(i-1)n + j$ of l is denoted by l_j^i and component $(i-1)n + j$ of L is denoted by L_j^i. This description of the possible rationing schemes is very general and includes special cases as uniform rationing (Drèze (1975)), rationing determined by initial endowments (Kurz (1982)), rationing determined by market shares (Weddepohl (1983)), rationing determined by priority (Weddepohl (1987)), or no constraints on the admissible rationing schemes (Herings (1992)).

With respect to the economy $\mathcal{E}$ the following assumptions are made:

A1. For every $i \in I_m$, X^i is a convex, closed, non-empty subset of $\mathbf{R}^n$, $X^i \subset \mathbf{R}^n_+$, and $X^i + \mathbf{R}^n_+ \subset X^i$.

A2. For every $i \in I_m$, the preference ordering $\succeq^i$ on X^i is transitive, complete, continuous, weakly monotonic, and convex.

A3. For every $i \in I_m$, the initial endowments w^i are an element of $\mathrm{Int}\,(X^i)$.

A4. The set of admissible prices is equal to

$$P_{(\underline{p},\bar{p})} = \left\{p \in \mathbf{R}^n_+ \,\middle|\, \forall j \in I_n,\ \underline{p}_j \leq p_j \leq \bar{p}_j\right\},$$

for some given $\underline{p}, \bar{p} \in \mathbf{R}^n_+$ such that for every $j \in I_n$, $0 < \underline{p}_j \leq \bar{p}_j$.

A5. The functions $l : Q^n \to -\mathbf{R}_+^{mn}$ and $L : Q^n \to \mathbf{R}_+^{mn}$ specifying the rationing system are continuous on Q^n and satisfy for every $i \in I_m$, $j \in I_n$, $q \in Q^n$,

$$\begin{aligned} l_j^i(q) &= l_j^i(r) \text{ if } r \in Q^n \text{ and } q_j = r_j, \\ l_j^i(q) &= 0 \text{ if } q_j = 0, \text{ and } l_j^i(q) < -w_j^i \text{ if } q_j = 1, \\ L_j^i(q) &= L_j^i(r) \text{ if } r \in Q^n \text{ and } q_j = r_j, \\ L_j^i(q) &> \sum_{h \in I_m \setminus \{i\}} w_j^h \text{ if } q_j = 0, \text{ and } L_j^i(q) = 0 \text{ if } q_j = 1. \end{aligned}$$

A6. The preference ordering $\succeq^i$ on X^i is strongly convex. [1]

A preference relation $\succeq^i$ on X^i is weakly monotonic if $x^i, y^i \in X^i$ and $\forall j \in I_n$, $x_j^i \geq y_j^i$ implies $x^i \succeq^i y^i$. For $x^i, y^i \in X^i$, let $x^i \succ^i y^i$ be defined as $x^i \succeq^i y^i$ and not $y^i \succeq^i x^i$. A preference relation $\succeq^i$ on X^i is convex if $x^i, y^i \in X^i$ and $x^i \succ^i y^i$ implies for every $0 < \lambda < 1$, $\lambda x^i + (1-\lambda) y^i \succ^i y^i$. A preference relation $\succeq^i$ on X^i is strongly convex if $x^i, y^i \in X^i$, $x^i \neq y^i$, and $x^i \succeq^i y^i$ implies for every $0 < \lambda < 1$, $\lambda x^i + (1-\lambda) y^i \succ^i y^i$.

Given $p \in P_{(\underline{p},\bar{p})}$, $\bar{l}^i \in -\mathbf{R}_+^n$, and $\bar{L}^i \in \mathbf{R}_+^n$, the constrained budget set of consumer $i \in I_m$ at price p and rationing scheme $(\bar{l}^i, \bar{L}^i)$ is defined by

$$B^i(\bar{l}^i, \bar{L}^i, p) = \left\{ x^i \in X^i \,\middle|\, p \cdot x^i \leq p \cdot w^i,\ \bar{l}^i \leq x^i - w^i \leq \bar{L}^i \right\},$$

and the constrained demand set of consumer $i \in I_m$ at price p and rationing scheme $(\bar{l}^i, \bar{L}^i)$ is defined by

$$\delta^i(\bar{l}^i, \bar{L}^i, p) = \left\{ x^i \in B^i(\bar{l}^i, \bar{L}^i, p) \,\middle|\, \forall y^i \in B^i(\bar{l}^i, \bar{L}^i, p), x^i \succeq^i y^i \right\}.$$

Definition 2.1 (Constrained Equilibrium)
A constrained equilibrium of the economy $\mathcal{E} = \left(\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})}, (l, L)\right)$ *is an element*

$$\left(x^{*1}, \ldots, x^{*m}, l^{*1}, \ldots, l^{*m}, L^{*1}, \ldots, L^{*m}, p^*\right) \in \prod_{i=1}^m X^i \times \prod_{i=1}^m -\mathbf{R}_+^n \times \prod_{i=1}^m \mathbf{R}_+^n \times P_{(\underline{p},\bar{p})}$$

such that

1. $\forall i \in I_m : x^{*i} \in \delta^i\left(l^{*i}, L^{*i}, p^*\right)$;

2. $\sum_{i=1}^m x^{*i} - \sum_{i=1}^m w^i = 0^n$;

3. $\forall j \in I_n : x_j^{*h} - w_j^h = L_j^{*h}$ *for some* $h \in I_m$ *implies* $x_j^{*i} - w_j^i > l_j^{*i}$, $\forall i \in I_m$, *and* $x_j^{*h} - w_j^h = l_j^{*h}$ *for some* $h \in I_m$ *implies* $x_j^{*i} - w_j^i < L_j^{*i}$, $\forall i \in I_m$;

4. $\forall j \in I_n : p_j^* < \bar{p}_j$ *implies* $L_j^{*i} > x_j^{*i} - w_j^i$, $\forall i \in I_m$, *and* $p_j^* > \underline{p}_j$ *implies* $l_j^{*i} < x_j^{*i} - w_j^i$, $\forall i \in I_m$;

[1] This assumption will be dropped in Sections 5 and 6.

5. $(l^{*1},\ldots,l^{*m}) \in l(Q^n)$ *and* $(L^{*1},\ldots,L^{*m}) \in L(Q^n)$.

For $j \in I_n$, define the component $\hat{p}_j$ of the function $\hat{p} : Q^n \to P_{(\underline{p},\bar{p})}$ by

$$\hat{p}_j(q) = \max\left\{\underline{p}_j, \min\left\{(2-3q_j)\,\underline{p}_j + (3q_j - 1)\,\bar{p}_j, \bar{p}_j\right\}\right\},\ \forall q \in Q^n.$$

Moreover, define the functions $\hat{l} : Q^n \to -\mathbf{R}^{mn}_+$ and $\hat{L} : Q^n \to \mathbf{R}^{mn}_+$ by

$$\begin{aligned}\hat{l}(q) &= l(\min\{1^n, 3q\}),\ \forall q \in Q^n,\\ \hat{L}(q) &= L(\max\{0^n, 3q - 2\,1^n\}),\ \forall q \in Q^n,\end{aligned}$$

where the maximum and the minimum are taken componentwise. The components $\hat{l}^i_j$ of $\hat{l}$ and $\hat{L}^i_j$ of $\hat{L}$ are defined in the same way as the components l^i_j and L^i_j. It can be shown, as in Herings (1992), that there is no loss of generality in describing the prices and rationing schemes simultaneously using the function $(\hat{p} \times \hat{l} \times \hat{L}) : Q^n \to P_{(\underline{p},\bar{p})} \times -\mathbf{R}^{mn}_+ \times \mathbf{R}^{mn}_+$ if one is interested in all the possible constrained equilibrium allocations and prices, or binding rationing constraints. For every $q \in Q^n$ define the set $W(q)$ of vectors in $\mathbf{R}^n$ orthogonal on $\hat{p}(q)$, so $W(q) = \{z \in \mathbf{R}^n \mid \hat{p}(q)\cdot z = 0\}$, and define the total excess demand correspondence $\zeta : Q^n \to \mathbf{R}^n$ by

$$\zeta(q) = \left(\sum_{i=1}^{m} \delta^i\left(\hat{l}^i(q), \hat{L}^i(q), \hat{p}(q)\right) - \sum_{i=1}^{m}\left\{w^i\right\}\right) \cap W(q),\ \forall q \in Q^n.$$

Using the results in Herings (1992) the following theorem can be shown.

Theorem 2.2

Let be given the economy $\mathcal{E} = \left(\{X^i, \succeq^i, w^i\}_{i=1}^m, P_{(\underline{p},\bar{p})}, (l, L)\right)$ *and let the Assumptions A1-A5 be satisfied. Then the correspondence* $\zeta : Q^n \to \mathbf{R}^n$ *satisfies the following conditions*

1. *ζ is a non-empty and convex valued correspondence;*
2. *ζ is an upper semi-continuous correspondence;*
3. $\forall q \in Q^n,\ \forall z \in \zeta(q),\ \forall j \in I_n,\ z_j \geq 0$ *if* $q_j = 0$;
4. $\forall q \in Q^n,\ \forall z \in \zeta(q),\ \forall j \in I_n,\ z_j \leq 0$ *if* $q_j = 1$;
5. $\forall q \in Q^n,\ \forall z \in \zeta(q),\ \hat{p}(q)\cdot z = 0.$

If in addition Assumption A6 is satisfied, then ζ is a continuous function.

If for some $q \in Q^n$, $0 \in \zeta(q)$, then it is easily verified that q induces a constrained equilibrium $(x^{*1},\ldots,x^{*m}, \hat{l}(q), \hat{L}(q), \hat{p}(q))$, where $\forall i \in I_m$, $x^{*i} \in \delta^i(\hat{l}^i(q), \hat{L}^i(q), \hat{p}(q))$. Condition 1 of Definition 2.1 is guaranteed by the definition of the correspondences δ^i and ζ. Condition 2 is satisfied since $0 \in \zeta(q)$. Finally, Conditions 3, 4, and 5 are satisfied by the definition of the function $(\hat{p} \times \hat{l} \times \hat{L})$.

Let $q = 0^n$. By Properties 3 and 5 of Theorem 2.2 it follows immediately that $\zeta(0^n) = 0^n$. Moreover, 0^n induces a constrained equilibrium $(w^1, \ldots, w^m, 0^{mn}, \hat{L}(0^n), \underline{p})$. This is the trivial equilibrium with complete supply rationing.

Let $q = 1^n$. By Properties 4 and 5 of Theorem 2.2 it follows immediately that $\zeta(1^n) = 0^n$. Moreover, 1^n induces a constrained equilibrium $(w^1, \ldots, w^m, \hat{l}(1^n), 0^{mn}, \bar{p})$. This is the trivial equilibrium with complete demand rationing.

3 The Algorithm

In Sections 3 and 4 it will always be assumed that the Assumptions A1-A6 are satisfied for a given economy $\mathcal{E}$. In this section an algorithm is discussed which computes a zero point of ζ. First some preliminaries are given. Let $x^1, \ldots, x^{t+1}$ be $t+1$ affinely independent points in $\mathbf{R}^n$. Then the $t-$simplex denoted by $\sigma(x^1, \ldots, x^{t+1})$ is defined as the convex hull of the set with elements $x^1, \ldots, x^{t+1}$, so

$$\sigma\left(x^1, \ldots, x^{t+1}\right) = co\left(\left\{x^1, \ldots, x^{t+1}\right\}\right).$$

The points $x^1, \ldots, x^{t+1}$ are called the vertices of $\sigma(x^1, \ldots, x^{t+1})$. A $(t-1)-$simplex τ being the convex hull of t vertices of the simplex $\sigma(x^1, \ldots, x^{t+1})$ is called a facet of σ. There is exactly one vertex of σ, say x^k for some $k \in I_{t+1}$, which is not a vertex of a facet τ of σ and therefore τ is called the facet of σ opposite the vertex x^k. Two different simplices σ^1 and σ^2 are called adjacent if one of them is a facet of the other or if both share a common facet.

Definition 3.1 (Triangulation)

Let S be a $t-$dimensional convex subset of $\mathbf{R}^k$. A collection G of $t-$simplices is a simplicial subdivision or triangulation of S if

1. $\cup_{\sigma \in G} \sigma = S$;

2. *The intersection of two simplices in G is either empty or the convex hull of $s \leq t+1$ common vertices;*

3. *If a facet τ of a simplex $\sigma^1 \in G$ lies in the boundary of S then there is no $\sigma^2 \in G$ such that $\sigma^2 \neq \sigma^1$ and τ is a facet of σ^2, and if τ does not lie in the boundary of S then there is exactly one $\sigma^2 \in G$ such that $\sigma^2 \neq \sigma^1$ and τ is a facet of σ^2.*

In this section the set Q^n will be triangulated. It is possible to show that the triangulation of a compact set S contains a finite number of simplices. Hence all triangulations considered in this section contain a finite number of simplices. If G is a triangulation of a compact, convex subset of $\mathbf{R}^k$ then the mesh size of the triangulation G, denoted mesh (G), is given by

$$\text{mesh}(G) = \max\{\|x-y\|_\infty \,|\exists \sigma \in G \text{ such that } x, y \in \sigma\}.$$

For the existence of a triangulation of Q^n with arbitrarily chosen positive mesh size, see van der Laan and Talman (1987).

An essential part of the algorithm is that to each point in Q^n a label in the set I_{n+1} is assigned. For every $q \in Q^n$ let $I(q)$ be defined by

$$I(q) = \left\{ j^* \in I_n \,\middle|\, q_{j^*} \neq 1 \text{ and } \zeta_{j^*}(q) = \max_{j \in I_n} \zeta_j(q) \right\}.$$

Define the labelling function $\phi : Q^n \to I_{n+1}$ by

$$\begin{aligned} \phi(q) &= j^* \text{ if } j^* \in I(q) \text{ and } \forall j \in I(q),\ j^* \leq j, \\ \phi(q) &= n+1 \text{ if } I(q) = \emptyset. \end{aligned} \tag{1}$$

Definition 3.2 (Proper Labelling Function)
The labelling function $\phi : Q^n \to I_{n+1}$ is proper if for every $j \in I_n$ and $x \in Q^n$, $q_j = 1$ implies $\phi(q) \neq j$ and $q_j = 0$ implies $\phi(q) \neq n+1$.

It will be shown that the labelling function ϕ given in (1) is proper, if the Assumptions A1-A6 are satisfied.

The algorithm which will be used to compute an approximation of a set of zero points of the function ζ is identical to the one in van der Laan (1982) which is a special case of the algorithm in Chapter 5 of van der Laan (1980a) and van der Laan and Talman (1981). For the sake of completeness the steps of the algorithm are given below. If the labelling function is proper, then it can be shown (van der Laan (1980a)) that each step described in the algorithm is feasible. Define for $J \subset I_n$ the sets

$$\begin{aligned} A(J) &= \{x \in Q^n \,|\forall j \in I_n \setminus J,\ x_j = 0\}, \\ G(J) &= \{\sigma \cap A(J) \,|\sigma \in G \text{ and } \dim(\sigma \cap A(J)) = |J|\}. \end{aligned}$$

From Theorem 2.3 in Todd (1976) it follows immediately that $G(J)$ is a triangulation of $A(J)$. In the description of the algorithm given below, σ^i will denote a simplex and x^j a vertex generated by the algorithm. J^k is a subset of I_n generated by the algorithm and determines a set $A(J^k)$ and a triangulation $G(J^k)$ in which the algorithm generates simplices. Let a triangulation G of Q^n and a proper labelling function $\phi : Q^n \to I_{n+1}$ be given. Then the algorithm operates as follows.

Algorithm

Step 0. Let $t = 0$, $x^0 = 0^n$, $\sigma^0 = \sigma(x^0)$, $J^0 = \emptyset$, $i = j = k = 0$. Go to Step 1.

Step 1. If $\phi(x^i) = n+1$ then stop. If $\phi(x^i) \notin J^k$ then go to Step 3. Otherwise there is a unique vertex $\bar{x}^i$ of σ^j such that $\bar{x}^i \neq x^i$ and $\phi(\bar{x}^i) = \phi(x^i)$. Go to Step 2.

Step 2. Let τ be the facet of σ^j opposite $\bar{x}^i$. If there exists $h \in J^k$ such that $\tau \subset A(J^k \backslash \{h\})$ then go to Step 4. Otherwise there is a unique point $x^{i+1} \in A(J^k)$ such that $\sigma^{j+1} = co(\tau \cup \{x^{i+1}\})$ is a t–simplex of $G(J^k)$ and $\sigma^{j+1} \neq \sigma^j$. Increase the values of i and j by 1. Go to Step 1.

Step 3. Define $J^{k+1} = J^k \cup \{\phi(x^i)\}$. There is a unique point $x^{i+1} \in A(J^{k+1})$ such that $\sigma^{j+1} = \mathrm{co}(\sigma^j \cup \{x^{i+1}\})$ is a $(t+1)$–simplex of $G(J^{k+1})$. Increase the values of i, j, k, t by 1. Go to Step 1.

Step 4. Let $\bar{\bar{x}}^i$ be the unique vertex of σ^j such that $\phi(\bar{\bar{x}}^i) = h$ and $\bar{\bar{x}}^i \neq \bar{x}^i$. Define $J^{k+1} = J^k \setminus \{h\}$. Define $\sigma^{j+1} = \tau$. Increase the values of j and k by 1 and decrease the value of t by 1. Let $\bar{x}^i$ be the element $\bar{\bar{x}}^i$. Go to Step 2.

In Step 1 the algorithm is initiated. In Step 2 a simplex is replaced by another simplex of the same dimension, which is possible by the properness of the labelling function. In Step 3 a new simplex is generated which contains the old simplex as a facet and in Step 4 the new simplex generated is a facet of the old simplex.

Definition 3.3 (J**–completeness**)
Let $J \subset I_{n+1}$ be given where $|J| = t$. A $(t-1)$–simplex $\sigma(x^1, \ldots, x^t)$ in Q^n is J–complete if $\phi(\{x^1, \ldots, x^t\}) = J$.

The simplices generated have the property that they are an element of $G(J)$ for some $J \subset I_n$ with $t = |J|$. Furthermore, let two adjacent simplices σ^j, σ^{j+1} generated by the algorithm be given and let $\sigma^j \in G(J)$, $\sigma^{j+1} \in G(\bar{J})$. Then $\sigma^j \cap \sigma^{j+1}$ is a $J \cup \bar{J}$–complete simplex in $A(J \cap J)$.

Theorem 3.4
Let G be a triangulation of Q^n and $\phi : Q^n \to I_{n+1}$ a proper labelling function. Then the algorithm terminates in a finite number of steps with an I_{n+1}–complete n–simplex in Q^n.
Proof
A detailed proof is given in Chapter 5 of van der Laan (1980a). There it is shown that Step 2 in the algorithm is feasible due to the proper labelling, and the number of steps of the algorithm is finite due to the fact that cycling is impossible by using the "door-in door-out"

argument of Lemke and Howson (1964) and the finiteness of the number of simplices in the collection $\cup_{J \subset I_n} G(J)$. Hence the algorithm stops in a finite number of steps with a simplex having a vertex with label $n+1$. This simplex has to be $n-$dimensional and $I_{n+1}-$complete since $\phi(q) \neq n+1$ if $q_j = 0$ for some $j \in I_n$.

Q.E.D.

Hence, if the labelling function is proper the algorithm generates a finite sequence of adjacent simplices, say $\sigma^0, \ldots, \sigma^{\bar{j}}$. The dimension of the simplices is varying from $t = 0$ at σ^0 to $t = n$ at $\sigma^{\bar{j}}$.

Theorem 3.5

Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Then the labelling function $\phi : Q^n \to I_{n+1}$ defined in (1) is proper.

Proof

Let $q \in Q^n$ be given and let $q_j = 0$ for some $j \in I_n$. By Property 3 of Theorem 2.2 it holds that $\zeta_j(q) \geq 0$. If for some $k \in I_n$, $q_k = 1$, then by Property 4 of Theorem 2.2 $\zeta_k(q) \leq 0$. Hence $I(q) \neq \emptyset$ and so $\phi(q) \neq n+1$. Let $q \in Q^n$ be given and let $q_j = 1$ for some $j \in I_n$. Then by definition $j \notin I(q)$ and hence $\phi(q) \neq j$.

Q.E.D.

Theorem 3.6

Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Let σ be a simplex generated by the algorithm using a triangulation G of Q^n. Then for every $\varepsilon > 0$ there exists a $\delta > 0$ such that if mesh$(G) < \delta$ *then for all $q \in \sigma$, $\|\zeta(q)\|_\infty < \varepsilon$.*

Proof

Assume that σ is the $j-$th simplex generated by the algorithm and therefore it will be denoted by σ^j. If $j = 0$ then $q = 0^n$ which implies by Property 3 of Theorem 2.2 that $\forall j \in I_n$, $\zeta_j(q) \geq 0$. Hence by Property 5 of Theorem 2.2 and since $\forall j \in I_n$, $\hat{p}_j(q) > 0$, it follows that $\zeta(q) = 0^n$. So consider the case where $j > 0$. By the remarks above Theorem 3.4, $\sigma^{j-1} \cap \sigma^j$ is a $J-$complete simplex in $A(J)$ for some $J \subset I_n$. Hence, for all $q \in \sigma^{j-1} \cap \sigma^j$ it holds that $h \in I_n \setminus J$ implies $q_h = 0$ and therefore by Property 3 of Theorem 2.2 that $\zeta_h(q) \geq 0$. Let $k \in J$ and let x be the vertex of the simplex $\sigma^{j-1} \cap \sigma^j$ such that $\phi(x) = k$. Then $\zeta_k(x) = \max_{h \in I_n} \zeta_h(x) \geq 0$. So for every $k \in I_n$ there is a vertex $x \in \sigma^{j-1} \cap \sigma^j$ such that $\zeta_k(x) \geq 0$. By Theorem 2.2, ζ is a continuous function. Define

$$\bar{\varepsilon} = \frac{\min_{h \in I_n} \{\underline{p}_h\}}{\sum_{h \in I_n} \bar{p}_h} \varepsilon \tag{2}$$

and choose $\delta > 0$ such that $x, y \in Q^n$ and $\|x - y\|_\infty < \delta$ implies $\|\zeta(x) - \zeta(y)\|_\infty < \bar{\varepsilon}$. Let $q \in \sigma^j$. Then $\text{mesh}(G) < \delta$ implies $\forall k \in I_n$, $\zeta_k(q) > -\bar{\varepsilon}$. Using Property 5 of Theorem 2.2, $\hat{p}(q) \cdot \zeta(q) = 0$. Hence

$$\hat{p}_k(q)\,\zeta_k(q) = -\sum_{h \in I_n \setminus \{k\}} \hat{p}_h(q)\,\zeta_h(q) < \bar{\varepsilon} \sum_{h \in I_n \setminus \{k\}} \hat{p}_h(q).$$

So $\zeta_k(q) < \bar{\varepsilon}\frac{\sum_{h \in I_n \setminus \{k\}} \hat{p}_h(q)}{\hat{p}_k(q)} \leq \bar{\varepsilon}\frac{\sum_{h \in I_n \setminus \{k\}} \bar{p}_h}{\underline{p}_k} \leq \varepsilon$.

Q.E.D.

So the total excess demand corresponding with all points in the simplices generated can be made less than an arbitrarily specified $\varepsilon > 0$. Moreover the first simplex generated is $\{0^n\}$. Now it will be shown that 1^n is a vertex of the last simplex generated.

Theorem 3.7
Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Then $\phi(1^n) = n + 1$ and there is no other point $q \in Q^n$ such that $\phi(q) = n + 1$.

Proof
If $q = 1^n$ then $I(q) = \emptyset$ and hence $\phi(q) = n + 1$. Now let $q \in Q^n$ be such that $q \neq 1^n$. Properties 4 and 5 of Theorem 2.2 guarantee that $\zeta_j(q) \geq 0$ for some $j \in I_n$ for which $q_j < 1$ while $q_k = 1$ implies that $\zeta_k(q) \leq 0$. Consequently $I(q) \neq \emptyset$ and $\phi(q) \neq n + 1$.

Q.E.D.

Therefore the algorithm generates a finite sequence of adjacent simplices such that 0^n is a vertex of the first simplex generated and 1^n is a vertex of the last simplex generated. This result combined with Theorem 3.6 will be used extensively in the next section to prove the existence of a connected set containing the two trivial equilibria.

4 The Existence of a Connected Set Containing the Two Trivial Equilibria

In this section the properties of the points in the simplices generated by the algorithm will be used to prove the existence of a connected set C in Q^n such that $0^n, 1^n \in C$ and for each element q in C it holds that $\zeta(q) = 0$. Theorem 4.1 first gives an interesting result for approximate constrained equilibria.

Theorem 4.1
Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Then for every $r \in \mathbf{N}$ there

exists a continuous function $f^r : [0,1] \to Q^n$ *satisfying* $f^r(0) = 0^n$, $f^r(1) = 1^n$, *and* $\forall t \in [0,1]$, $\|\zeta(f^r(t))\|_\infty < \frac{1}{r}$.

Proof

Consider the simplices $\sigma^0, \ldots, \sigma^{\bar{j}}$ generated by the algorithm for a triangulation G with $\mathrm{mesh}(G) < \delta$ where δ is chosen such that Theorem 3.6 holds for $\varepsilon = \frac{1}{r}$. Define for $j \in \{0\} \cup I_{\bar{j}}$ the elements $q^j \in Q^n$ as follows. For $0 \le j \le \bar{j}-1$, q^j is the barycentre of $\sigma^j \cap \sigma^{j+1}$, and $q^{\bar{j}} = 1^n$. Clearly $q^0 = 0^n$. By Theorem 3.7 it holds that $1^n \in \sigma^{\bar{j}}$. Moreover, for every $j \in I_{\bar{j}}$ it holds that q^{j-1} and q^j are elements of σ^j. For $x \in \mathbf{R}$ let $\lfloor x \rfloor$ denote the greatest integer which is less than or equal to x. By the convexity of simplices and Theorem 3.6, it is easily verified that the function $f^r : [0,1] \to Q^n$ defined by

$$\begin{aligned} f^r(t) &= (1 - \bar{j}t + \lfloor \bar{j}t \rfloor)\, q^{\lfloor \bar{j}t \rfloor} + (\bar{j}t - \lfloor \bar{j}t \rfloor)\, q^{\lfloor \bar{j}t \rfloor + 1},\ 0 \le t < 1, \\ f^r(t) &= q^{\bar{j}},\ t = 1, \end{aligned}$$

satisfies all conditions of Theorem 4.1.

Q.E.D.

Define

$$E = \{q^* \in Q^n \mid \zeta(q^*) = 0^n\}.$$

Clearly 0^n and 1^n are elements of E and therefore $E \ne \emptyset$. Moreover E is a closed set by the continuity of the function ζ. As is argued below Theorem 2.2 each element $q \in E$ induces a constrained equilibrium

$$\left(\delta^1(\hat{l}^1(q), \hat{L}^1(q), \hat{p}(q)), \ldots, \delta^m(\hat{l}^m(q), \hat{L}^m(q), \hat{p}(q)), \hat{l}(q), \hat{L}(q), \hat{p}(q)\right).$$

For a non-empty, compact set $S \subset \mathbf{R}^n$ define the function $d_S : \mathbf{R}^n \to \mathbf{R}$ by

$$d_S(x) = \min\{\|x - y\|_\infty \mid y \in S\}.$$

By the theorem of Weierstrass, the function d_S is well-defined. Let S, T be non-empty, compact subsets of $\mathbf{R}^n$. Define $e(S,T)$ by

$$e(S,T) = \min\{\|x - y\|_\infty \mid x \in S,\ y \in T\}.$$

Using the theorem of Weierstrass it follows immediately that $e(S,T)$ is well-defined. Clearly, if S and T are disjoint, non-empty, compact subsets of $\mathbf{R}^n$, then $e(S,T) > 0$.

Proposition 4.2

Let $x, y \in \mathbf{R}^n$ *and a non-empty, compact set* $S \subset \mathbf{R}^n$ *be given. Then* $|d_S(x) - d_S(y)| \le \|x - y\|_\infty$.

Proof

By definition of d_S there exist $\bar{x},\ \bar{y} \in S$ such that $\|x-\bar{x}\|_\infty = d_S(x)$ and $\|y-\bar{y}\|_\infty = d_S(y)$. So it has to be shown that $|\|x-\bar{x}\|_\infty - \|y-\bar{y}\|_\infty| \le \|x-y\|_\infty$ or equivalently

$$-\|x-y\|_\infty \le \|x-\bar{x}\|_\infty - \|y-\bar{y}\|_\infty \text{ and } \|x-\bar{x}\|_\infty - \|y-\bar{y}\|_\infty \le \|x-y\|_\infty.$$

This holds since $\|y-\bar{y}\|_\infty \le \|y-\bar{x}\|_\infty \le \|y-x\|_\infty + \|x-\bar{x}\|_\infty$ and $\|x-\bar{x}\|_\infty \le \|x-\bar{y}\|_\infty \le \|x-y\|_\infty + \|y-\bar{y}\|_\infty$.

Q.E.D.

Proposition 4.3

Let be given a non-empty compact set $S \subset \mathbf{R}^n$. Then the function d_S is continuous. **Proof**

Let be given a sequence $(x^r)_{r\in\mathbf{N}}$ in $\mathbf{R}^n$ such that $x^r \to x$. Clearly by Proposition 4.2 $0 \le |d_S(x^r) - d_S(x)| \le \|x^r - x\|_\infty \to 0$. So $d_S(x^r) \to d_S(x)$.

Q.E.D.

The following theorem shows that the approximate constrained equilibria given by the function f^r are uniformly close to the set of constrained equilibria.

Theorem 4.4

Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$ and for every $r \in \mathbf{N}$ let $f^r : [0,1] \to Q^n$ be a continuous function satisfying the properties given in Theorem 4.1. Then $\forall \varepsilon > 0,\ \exists R \in \mathbf{N}$ such that $\forall r \ge R,\ \forall t \in [0,1],\ d_E(f^r(t)) < \varepsilon$.

Proof

Suppose $\exists \varepsilon > 0$ such that $\forall R \in \mathbf{N},\ \exists r^R \ge R,\ \exists t^R \in [0,1]$ satisfying $d_E(f^{r^R}(t^R)) \ge \varepsilon$. Consider the sequence $(f^{r^R}(t^R))_{R\in\mathbf{N}}$ in Q^n. There exists a convergent subsequence $(f^{r^{R^s}}(t^{R^s}))_{s\in\mathbf{N}}$ with limit, say, $q \in Q^n$. It holds that

$$d_E(q) = d_E\left(\lim_{s\to\infty} f^{r^{R^s}}\left(t^{R^s}\right)\right) = \lim_{s\to\infty} d_E\left(f^{r^{R^s}}\left(t^{R^s}\right)\right) \ge \varepsilon.$$

However,

$$0 \le \|\zeta(q)\|_\infty = \left\|\zeta\left(\lim_{s\to\infty} f^{r^{R^s}}\left(t^{R^s}\right)\right)\right\|_\infty = \left\|\lim_{s\to\infty} \zeta\left(f^{r^{R^s}}\left(t^{R^s}\right)\right)\right\|_\infty \le \lim_{s\to\infty} \frac{1}{r^{R^s}} = 0.$$

So $q \in E$ and therefore $d_E(q) = 0$, which is a contradiction.

Q.E.D.

Before proving the main theorem a few definitions and properties are given first. Loosely speaking, a topological space is connected if it is of one piece. The following formal definitions of connectedness can be found for example in Dugundji (1965).

Definition 4.5 (Connectedness)
A topological space X is connected if it is not the union of two non-empty, disjoint, closed sets.

A subset of a topological space is connected if it becomes a connected space when given the induced topology.

Definition 4.6 (Component)
The component of a point x in a topological space X is the union of all connected subsets of X containing x.

It is easily seen that each component is connected and therefore the component of an element x is the largest connected subset containing x.

Definition 4.7 (Quasicomponent)
The quasicomponent of an element x in a topological space X is the intersection of all subsets of X which are both open and closed and contain x.

Following Dugundji (1965) it is not difficult to show that the collection of all components partitions X. Similarly the collection of all quasicomponents partitions X. In general topological spaces, components and quasicomponents need not coincide. However, it is easily seen that the component of a point is a subset of the quasicomponent of this point. The following example illustrates the fact that it is possible that the component of a point is a proper subset of the quasicomponent of this point. It is a modified version of Example 115 in Steen and Seebach Jr. (1970).

Example 4.8
Let

$$\begin{aligned} R^0 &= \left\{(q_1, q_2) \in Q^2 \,|\, q_1 = 0\right\}, \\ R^1 &= \left\{(q_1, q_2) \in Q^2 \,|\, q_1 = 1\right\}, \\ S^r &= \left\{(q_1, q_2) \in Q^2 \,\middle\|\, \left\|(q_1, q_2) - (\frac{1}{2}, \frac{1}{2})\right\|_\infty = \frac{r}{2(r+1)}\right\}, \ \forall r \in \mathbf{N}. \end{aligned}$$

For every $r \in \mathbf{N}$, S^r is a square with center $\left(\frac{1}{2}, \frac{1}{2}\right)$ and diameter $\frac{r}{r+1}$. Consider the set $T = R^0 \cup R^1 \cup (\cup_{r \in \mathbf{N}} S^r)$. Give T the topology induced by the topology of the Euclidean space $\mathbf{R}^2$. Since $\forall k \in \mathbf{N}$ the set $\cup_{r=1}^k S^r$ is open and closed in T, it holds that the quasicomponent of 0^2 is a subset of $R^0 \cup R^1$. Clearly R^0 is the component of 0^2. Hence R^0 is a subset of the quasicomponent of 0^2. Since every open and closed set in T containing R^0

has to contain R^1 it has to hold that the quasicomponent of 0^2 is $R^0 \cup R^1$.

Example 4.8 makes clear that the quasicomponent of a point is not necessarily connected. Fortunately, the set T of Example 4.8 cannot result as a set of elements $q \in Q^2$ for which $0^2 \in \zeta(q)$, since the set T in Example 4.8 is not closed. The following theorem gives sufficient conditions guaranteeing that the component and the quasicomponent of a point coincide.

Theorem 4.9
Let E be a compact subset of the Euclidean space $\mathbf{R}^n$. Then the component and the quasicomponent of each point of the set E coincide. [2]

Proof
If S is a subset of E then $\mathrm{cl}(S)$ and $\mathrm{bd}(S)$ will denote respectively the closure of S in E and the boundary of S in E. Let an element q of E be given. Clearly the component of q is contained in the quasicomponent of q. Now the converse will be shown. Let C be the quasicomponent of some $q \in E$. Since C is an intersection of sets closed in E it is closed in E itself. It has to be shown that C is connected. Suppose C is not connected. Then there exist two non-empty disjoint sets A and B such that $A \cup B = C$ and A and B are both closed in C, hence in $\mathbf{R}^n$. Without loss of generality it can be assumed that $q \in A$. Clearly, A and B are compact sets. Therefore the set D defined by

$$D = \left\{ q \in E \,\middle|\, d_A(q) < \frac{1}{2} e(A, B) \right\} = d_A^{-1}\left(\left(\leftarrow, \frac{1}{2} e(A, B) \right) \right) \cap E$$

is well-defined. It follows immediately that D is open in E, $A \subset D$, and $B \cap \mathrm{cl}(D) = \emptyset$. Moreover, these three properties imply

$$C \cap \mathrm{bd}(D) = (A \cup B) \cap \mathrm{bd}(D) = \emptyset. \tag{3}$$

Let $\bar{q} \in B$. In the following step of the proof a set which is both open and closed in E and which contains q but which does not contain $\bar{q}$ is constructed. This yields a contradiction since C is the quasicomponent of q and $\bar{q}$ is an element of C. If $\mathrm{bd}(D) = \emptyset$ then the set D satisfies the requirements mentioned above and the proof is finished. So assume $\mathrm{bd}(D) \neq \emptyset$. For every $r \in \mathrm{bd}(D)$ it holds by (3) that $r \notin C$. Hence, since C is the quasicomponent of q, for every $r \in \mathrm{bd}(D)$ there exists a set F^r which is open and closed in E and which is such that $q \in F^r$ and $r \notin F^r$. The collection $\{E \setminus F^r \,|\, r \in \mathrm{bd}(D)\}$ is an open cover of $\mathrm{bd}(D)$ in E and since $\mathrm{bd}(D)$ is compact, there exists a finite subcover, say $\left\{E \setminus F^{r^1}, \ldots, E \setminus F^{r^k}\right\}$ of $\mathrm{bd}(D)$. The set $\cap_{i \in I_k} F^{r^i}$ is open and closed in E and contains the element q. Moreover, $\mathrm{bd}(D) \cap \left(\cap_{i \in I_k} F^{r^i}\right) = \emptyset$. Finally consider the set F defined by

[2] It is possible to verify that the proof given is valid whenever E is a compact metric space.

$F = D \cap \left(\cap_{i \in I_k} F^{r^i}\right)$. Clearly, $q \in F$ and $\bar{q} \notin F$. F is an open set in E as an intersection of finitely many open sets. Furthermore,

$$\mathrm{cl}(F) \subset \mathrm{cl}(D) \cap \left(\cap_{i \in I_k} F^{r^i}\right) = \left(D \cap \left(\cap_{i \in I_k} F^{r^i}\right)\right) \cup \left(\mathrm{bd}(D) \cap \left(\cap_{i \in I_k} F^{r^i}\right)\right) = F.$$

Q.E.D.

The next theorem finally gives the desired result. It states that there is a component, i.e. a maximally connected subset of E, containing 0^n and 1^n.

Theorem 4.10
Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Then the set E contains a component C such that $0^n \in C$ and $1^n \in C$.
Proof
Suppose E does not contain a component such that 0^n and 1^n are in it. Then by Theorem 4.9 the quasicomponent of 0^n does not contain 1^n. Hence there exists a set E^0 which is both open and closed in E, contains 0^n, but does not contain 1^n. Define $E^1 = E \setminus E^0$. Clearly E^1 is both open and closed in E and contains 1^n. Since E is a closed subset of $\mathbf{R}^n$, E^0 and E^1 are compact sets in $\mathbf{R}^n$. Since $E^0 \cap E^1 = \emptyset$ this implies that for some $\varepsilon > 0$ it holds that $e(E^0, E^1) \geq \varepsilon$. By Theorem 4.4 there is an r such that for all $t \in [0,1]$ it holds that $d_E(f^r(t)) < \frac{1}{2}\varepsilon$. It will be shown that for some $t^* \in [0,1]$ it holds that $d_{E^0}(f^r(t^*)) < \frac{1}{2}\varepsilon$ and $d_{E^1}(f^r(t^*)) < \frac{1}{2}\varepsilon$. Define the function $h : [0,1] \to \mathbf{R}$ by

$$h(t) = d_{E^0}(f^r(t)) - d_{E^1}(f^r(t)), \ \forall t \in [0,1].$$

By the continuity of the functions f^r, d_{E^0}, and d_{E^1}, the function h is continuous. Moreover $h(0) < -\varepsilon$ and $h(1) > \varepsilon$. Hence there exists a $t^* \in [0,1]$ such that $h(t^*) = 0$, and therefore

$$d_{E^0}(f^r(t^*)) = d_{E^1}(f^r(t^*)) = d_E(f^r(t^*)) < \frac{1}{2}\varepsilon.$$

Consequently there exists $q^0 \in E^0$ and $q^1 \in E^1$ such that $\|f^r(t^*) - q^0\|_\infty < \frac{1}{2}\varepsilon$ and $\|f^r(t^*) - q^1\|_\infty < \frac{1}{2}\varepsilon$. Hence

$$\varepsilon \leq e\left(E^0, E^1\right) \leq \|q^0 - q^1\|_\infty \leq \|f^r(t^*) - q^0\|_\infty + \|f^r(t^*) - q^1\|_\infty < \varepsilon,$$

which is a contradiction.

Q.E.D.

Corollary 4.11
Let the Assumptions A1-A6 be satisfied for an economy $\mathcal{E}$. Then there exists a connected set of constrained equilibria, containing the trivial equilibria $(w^1, \ldots, w^m, 0^{mn}, \hat{L}(0^n), \underline{p})$ and

$(w^1, \ldots, w^m, \hat{l}(1^n), 0^{mn}, \bar{p})$.

Proof

Define the continuous function $\psi : Q^n \to \mathbf{R}^{3mn+n}$ by

$$\psi(q) = \left(\delta^1\left(\hat{l}^1(q), \hat{L}^1(q), \hat{p}(q)\right), \ldots, \delta^m\left(\hat{l}^m(q), \hat{L}^m(q), \hat{p}(q)\right), \hat{l}(q), \hat{L}(q), \hat{p}(q)\right), \ \forall q \in Q^n.$$

Let C be the component of Theorem 4.10, containing 0^n and 1^n. Since the image of a connected set under a continuous function is a connected set, it follows that $\psi(C)$ is a connected set. Moreover, every element of $\psi(C)$ is a constrained equilibrium, $\psi(0^n) = \left(w^1, \ldots, w^m, 0^{mn}, \hat{L}(0^n), \underline{p}\right)$ and $\psi(1^n) = (w^1, \ldots, w^m, \hat{l}(1^n), 0^{mn}, \bar{p})$.

Q.E.D.

5 The Case of Upper Semi-Continuous Correspondences

In Sections 5 and 6 the Assumptions A1-A5 are made, so Assumption A6 is dropped. Now ζ is an upper semi-continuous correspondence. Let the set E be defined by

$$E = \{q \in Q^n \,|\, 0^n \in \zeta(q)\}.$$

This could also be denoted by $E = \zeta^{-1}(\{0^n\})$, where ζ^{-1} is the strong inverse of the correspondence ζ. So again, E is the set of all elements $q \in Q^n$ inducing a constrained equilibrium. It is easily seen that $0^n, 1^n \in E$ if Assumptions A1-A5 are satisfied for an economy $\mathcal{E}$. In this section it will be shown that Theorem 4.10 and Corollary 4.11 are still true if Assumption A6 is not made. However, it is possible to construct examples which show that Theorem 4.1 need not be true. The way Theorem 4.10 and Corollary 4.11 are shown to be true if Assumption A6 is not made, uses many of the ideas of Sections 3 and 4. Again it is based on a simplicial algorithm which generates a path of points with interesting properties. However, the integer labelling algorithm given in Section 3 does not yield the properties needed if we are working with upper semi-continuous excess demand correspondences instead of continuous excess demand functions. In Herings, Talman, and Yang (1993) an algorithm is introduced which generates a path of points with the properties given in Theorem 5.2. In order to make these properties clear a definition is given first.

Definition 5.1 (Piecewise Linear Approximation)

Let G be a triangulation of Q^n and $\zeta : Q^n \to \mathbf{R}^n$ a (non-empty valued) correspondence. For any point q of Q^n let $z(q)$ be an element of $\zeta(q)$. Then the piecewise linear approximation to ζ with respect to G is given by the continuous function $Z : Q^n \to \mathbf{R}^n$ defined *by* $Z(q) = \sum_{j=1}^{n+1} \lambda_j z(x^j)$, *where* $\sigma(x^1, \ldots, x^{n+1})$ *is a simplex in* G *and* $\forall j \in I_{n+1}$, $\lambda_j \geq 0$, $\sum_{j=1}^{n+1} \lambda_j = 1$, $q = \sum_{j=1}^{n+1} \lambda_j x^j$.

In Herings, Talman, and Yang (1993) the following theorem is shown.

Theorem 5.2
Let G *be a triangulation of* Q^n *and let* $\zeta : Q^n \to \mathbf{R}^n$ *be a correspondence satisfying Properties 1-5 of Theorem 2.2. Let* $Z : Q^n \to \mathbf{R}^n$ *be a piecewise linear approximation of* ζ *with respect to* G. *Then there exists a continuous function* $f : [0,1] \to Q^n$ *satisfying* $f(0) = 0^n$, $f(1) = 1^n$, *and* $\forall t \in [0,1]$, *it holds that for some* $\beta \in \mathbf{R}$ *and* $\mu_j \geq 0$ *for all* $j \in I_n$,

$$\begin{aligned} Z_j(f(t)) &= \beta - \mu_j \text{ if } f_j(t) = 0, \\ Z_j(f(t)) &= \beta \text{ if } 0 < f_j(t) < 1, \\ Z_j(f(t)) &= \beta + \mu_j \text{ if } f_j(t) = 1. \end{aligned}$$

Theorem 5.3
Let the Assumptions A1-A5 be satisfied for an economy $\mathcal{E}$. *Then* E *contains a component* C *such that* $0^n \in C$ *and* $1^n \in C$.

Proof
By Proposition 1 on page 22 of Hildenbrand (1974) the set E is closed. Because it is a subset of Q^n it therefore has to be compact. Suppose Theorem 5.3 is not true. Then by Theorem 4.9 the quasicomponent of 0^n does not contain 1^n. Then, similarly to the proof of Theorem 4.10, there exist two sets E^0 and E^1 which are both closed in E and which are such that $E^0 \cap E^1 = \emptyset$, $E^0 \cup E^1 = E$, $0^n \in E^0$, and $1^n \in E^1$. Hence for some $\varepsilon > 0$, $e(E^0, E^1) \geq \varepsilon$. For every $r \in \mathbf{N}$ let G^r be a triangulation of Q^n with mesh size less than or equal to $\frac{1}{r}$ and denote the piecewise linear approximation of ζ with respect to G^r by Z^r, and the function f given in Theorem 5.2 with respect to G^r by f^r. Define $h^r : [0,1] \to \mathbf{R}$ by

$$h^r(t) = d_{E^0}(f^r(t)) - d_{E^1}(f^r(t)), \ \forall t \in [0,1].$$

Clearly, by the continuity of the functions d_{E^0}, d_{E^1}, f^r, the function h^r is continuous for every $r \in \mathbf{N}$. Moreover, $h^r(0) \leq -\varepsilon$ and $h^r(1) \geq \varepsilon$. Let t^r be such that $h^r(t^r) = 0$. Then

$$d_{E^0}(f^r(t^r)) = d_{E^1}(f^r(t^r)) = d_E(f^r(t^r)) \geq \frac{1}{2}\varepsilon. \tag{4}$$

Let $\lambda_1^r, \ldots, \lambda_{n+1}^r$ be non-negative reals satisfying $\sum_{j=1}^{n+1} \lambda_j^r = 1$, let $x^{1^r}, \ldots, x^{n+1^r}$, be vertices of a simplex $\sigma(x^{1^r}, \ldots, x^{n+1^r})$ of G^r,

$$f^r(t^r) = \sum_{j=1}^{n+1} \lambda_j^r x^{j^r} \text{ and } Z^r(f^r(t^r)) = \sum_{j=1}^{n+1} \lambda_j^r z(x^{j^r}).$$

Consider the sequence

$$\left(\lambda_1^r, \ldots, \lambda_{n+1}^r, z(x^{1^r}), \ldots, z(x^{n+1^r}), f^r(t^r), Z^r(f^r(t^r))\right).$$

This sequence remains in a compact set because for all $q \in Q^n$, $z(q) \in \zeta(q)$ implies $\forall j \in I_n$, $-\sum_{i=1}^m w_j^i \leq z_j(q) \leq \frac{\bar{p} \cdot \sum_{i=1}^m w^i}{\underline{p}_j}$ and, by taking a convergent subsequence, without loss of generality it can be assumed that this sequence converges to an element, say

$$\left(\lambda_1^*, \ldots, \lambda_{n+1}^*, z^{*1}, \ldots, z^{*n+1}, q^*, Z^*\right).$$

Since the mesh size of G^r is less than or equal to $\frac{1}{r}$ it holds that $\forall j \in I_{n+1}$, $x^{j^r} \to q^*$. Since ζ is upper semi-continuous it holds that $\forall j \in I_{n+1}$, $z^{*j} \in \zeta(q^*)$. Since $Z^* = \sum_{j=1}^{n+1} \lambda_j^* z^{*j}$, where $\forall j \in I_{n+1}$, $\lambda_j^* \geq 0$ and $\sum_{j=1}^{n+1} \lambda_j^* = 1$, it holds that $Z^* \in \zeta(q^*)$ because ζ is convex valued according to Property 1 of Theorem 2.2.

If $\forall j \in I_n$, $0 < q_j^* < 1$, then, using Theorem 5.2, it has to hold that $\forall j, k \in I_n$, $Z_j^* = Z_k^*$. By Property 5 of Theorem 2.2 $\hat{p}(q^*) \cdot Z^* = 0$, where $\forall j \in I_n$, $\hat{p}_j(q^*) > 0$. Hence it has to hold that $Z^* = 0^n$.

If $\exists j \in I_n$ such that $q_j^* = 0$, then define $I^0(q^*) = \{j \in I_n \mid q_j^* = 0\}$. If $j \in I^0(q^*)$ and $k \in I^n \setminus I^0(q^*)$ then by Theorem 5.2 and Property 3 of Theorem 2.2 it holds that $0 \leq Z_j^* \leq Z_k^*$. Therefore by Property 5 of Theorem 2.2, $Z^* = 0^n$.

If $\exists j \in I_n$ such that $q_j^* = 1$, then define $I^1(q^*) = \{j \in I_n \mid q_j^* = 1\}$. If $j \in I^1(q^*)$ and $k \in I^n \setminus I^1(q^*)$ then by Theorem 5.2 and Property 4 of Theorem 2.2 it holds that $0 \geq Z_j^* \geq Z_k^*$ which again implies $Z^* = 0^n$.

Hence $q^* \in E$ and $d_E(q^*) = 0$. However, by Proposition 4.3 and (4),

$$d_E(q^*) = d_E\left(\lim_{r\to\infty} f^r(t^r)\right) = \lim_{r\to\infty} d_E\left(f^r(t^r)\right) \geq \frac{1}{2}\varepsilon > 0,$$

which is a contradiction.

Q.E.D.

Theorem 5.4

Let the Assumptions A1-A5 be satisfied for an economy $\mathcal{E}$. Then there exists a connected set of constrained equilibria, containing the trivial equilibria $(w^1, \ldots, w^m, 0^{mn}, \hat{L}(0^n), \underline{p})$ and $(w^1, \ldots, w^m, \hat{l}(1^n), 0^{mn}, \bar{p})$.

Proof

Let C be the component obtained in Theorem 5.3. Define the set $M = \{(x^1, \ldots, x^m) \in \prod_{i=1}^m \mathbb{R}^n \mid \sum_{i=1}^m x^i = \sum_{i=1}^m w^i\}$. Define the non-empty and convex valued upper semi-continuous correspondence $\psi : C \to \mathbb{R}^{3mn+n}$ by

$$\psi(q) = \prod_{i=1}^m \delta^i\left(\hat{l}^i(q), \hat{L}^i(q), \hat{p}(q)\right) \times \left\{\left(\hat{l}(q), \hat{L}(q), \hat{p}(q)\right)\right\} \cap (M \times \mathbb{R}^{2mn+n}), \ \forall q \in C.$$

Theorem 5.4 is shown if it is proved that the set $\psi(C) = \{x \in \mathbf{R}^{3mn+n} \mid \exists q \in C \text{ such that } x \in \psi(q)\}$ is connected, since the set $\psi(C)$ contains only constrained equilibria, $0^n, 1^n \in C$, $\psi(0^n) = \left\{\left(w^1, \ldots, w^m, 0^{mn}, \hat{L}(0^n), \underline{p}\right)\right\}$, and $\psi(1^n) = \left\{(w^1, \ldots, w^m, \hat{l}(1^n), 0^{mn}, \bar{p})\right\}$. Suppose the set $\psi(C)$ is not connected, then it can be partitioned in non-empty, disjoint sets S^1 and S^2, which are both closed in the set $\psi(C)$. Consider the non-empty sets $\psi^{-1}(S^1)$ and $\psi^{-1}(S^2)$. By Proposition 1 on page 22 of Hildenbrand (1974) both $\psi^{-1}(S^1)$ and $\psi^{-1}(S^2)$ are closed in C. Clearly $\psi^{-1}(S^1) \cup \psi^{-1}(S^2) = C$. Hence if it can be shown that $\psi^{-1}(S^1) \cap \psi^{-1}(S^2) = \emptyset$ then a contradiction with the connectedness of C is obtained and Theorem 5.4 has been proved. Suppose $q \in \psi^{-1}(S^1) \cap \psi^{-1}(S^2)$ and $x^1,\ x^2 \in \psi(q)$ where $x^1 \in S^1$ and $x^2 \in S^2$. Since ψ is convex valued it holds that for every $\lambda \in [0,1]$, $(1-\lambda)x^1 + \lambda x^2 \in \psi(q)$. Hence there exists a continuous function $h : [0,1] \to \psi(C)$ such that $h(0) = x^1$ and $h(1) = x^2$. By Theorem 5.3 of Dugundji (1965) this implies that x^2 is contained in the component of x^1, which is a contradiction. Hence $\psi^{-1}(S^1) \cap \psi^{-1}(S^2) = \emptyset$.

6 Equilibrium Existence Results

The following definition gives a slight generalization of the equilibrium concept used in Drèze (1975).

Definition 6.1 (Drèze Equilibrium)
A Drèze equilibrium with respect to commodity $j \in I_n$ of an economy $\mathcal{E}$ is a constrained equilibrium $(x^{*1}, \ldots, x^{*m}, l^{*1}, \ldots, l^{*m}, L^{*1}, \ldots, L^{*m}, p^*)$ of the economy $\mathcal{E}$ satisfying $\forall i \in I_m$, $l_j^{*i} < x_j^{*i} - w_j^i < L_j^{*i}$.

It is easily seen that the existence of a Drèze equilibrium with respect to commodity $j \in I_n$ is equivalent to the set $E \cap \{q \in Q^n \mid \frac{1}{3} \leq q_j \leq \frac{2}{3}\}$ being non-empty. It is also easily observed that the two trivial equilibria do not satisfy the requirements of a Drèze equilibrium with respect to some commodity $j \in I_n$. In van der Laan and Talman (1990) it is shown that for an economy with uniform rationing schemes it holds that

$$\forall j \in I_n,\ \forall \lambda \in [0,1],\ E \cap \{q \in Q^n \mid q_j = \lambda\} \neq \emptyset. \tag{5}$$

In the following a constrained equilibrium induced by an element in the set given in (5) will be called a Drèze equilibrium with respect to the pair (j, λ). For the following definition see van der Laan (1980, 1982) and Kurz (1982).

Definition 6.2 (Supply Constrained Equilibrium)
A real unemployment equilibrium or a supply constrained equilibrium of an economy $\mathcal{E}$ is a constrained equilibrium $(x^{*1}, \ldots, x^{*m}, l^{*1}, \ldots, l^{*m}, L^{*1}, \ldots, L^{*m}, p^*)$ of the economy $\mathcal{E}$

satisfying $\forall i \in I_m$, $\forall j \in I_n$, $x_j^{*i} - w_j^i < L_j^{*i}$ *and* $\exists k \in I_n$ *such that* $\forall i \in I_m$, $l_k^{*i} < x_k^{*i} - w_k^i < L_k^{*i}$.

It is easily seen that the existence of a supply constrained equilibrium is equivalent to the set $E \cap \{q \in Q^n \mid \frac{1}{3} \leq \max_{j \in I_n} q_j \leq \frac{2}{3}\}$ being non-empty. It is also easily observed that the two trivial equilibria do not satisfy the requirements of a supply constrained equilibrium. A demand constrained equilibrium could be defined analogously to a supply constrained equilibrium. In Herings (1992) it is shown that

$$\forall \alpha \in Q^n, \; E \cap \{q \in Q^n \mid \forall j \in I_n, \; q_j \leq \alpha_j, \; \exists k \in I_n, \; q_k = \alpha_k\} \neq \emptyset.$$

Corresponding equilibria are called extended supply constrained equilibria with respect to the vector α in Q^n. Moreover in Herings (1992) it is shown that

$$\forall \beta \in Q^n, \; E \cap \{q \in Q^n \mid \forall j \in I_n, \; q_j \geq \beta_j, \; \exists k \in I_n, \; q_k = \beta_k\} \neq \emptyset.$$

Corresponding equilibria are called extended demand constrained equilibria with respect to the vector β in Q^n. Using Theorem 5.3 the existence of each of the constrained equilibria mentioned above is very easily shown. The existence results follow as easy corollaries to Theorem 6.3.

Theorem 6.3
Let the Assumptions A1-A5 be satisfied for an economy $\mathcal{E}$. Let $g : Q^n \to \mathbf{R}$ be a continuous function satisfying $g(0^n) \leq 0$ and $g(1^n) \geq 0$. Then $g^{-1}(\{0\}) \cap E \neq \emptyset$.

Proof
Let C be the connected component obtained in Theorem 5.3. Since g is continuous it follows that $g(C)$ is a connected subset of $\mathbf{R}$. Since all connected subsets of $\mathbf{R}$ are intervals and since $0^n, 1^n \in C$, $g(0^n) \leq 0$, and $g(1^n) \geq 0$, it holds that $0 \in g(C) \subset g(E)$.

Q.E.D.

Besides the existence of each one of the constrained equilibria defined above, Theorem 6.3 also immediately yields the existence of a component of the set of constrained equilibria, which contains the two trivial equilibria and all the constrained equilibria defined above.

Now $\forall j \in I_n$, $\forall \lambda \in [0, 1]$, the existence of a Drèze equilibrium with respect to the pair (j, λ) follows by Theorem 6.3, where $g : Q^n \to \mathbf{R}$ is defined by

$$g(q) = q_j - \lambda, \; \forall q \in Q^n.$$

Moreover, for each $\alpha \in Q^n$, the existence of an extended supply constrained equilibrium with respect to α follows by Theorem 6.3, where $g : Q^n \to \mathbf{R}$ is defined by

$$g(q) = \max\{q_j - \alpha_j \mid j \in I_n\}, \; \forall q \in Q^n.$$

Finally, for each $\beta \in Q^n$, the existence of an extended demand constrained equilibrium with respect to β follows by Theorem 6.3, where $g : Q^n \rightarrow \mathbf{R}$ is defined by

$$g(q) = \min \{q_j - \beta_j \mid j \in I_n\}, \ \forall q \in Q^n.$$

7 Conclusions

Using concepts from mathematical programming and topology it is possible to show that the set of constrained equilibria of an economy with some rationing system has a component containing the two trivial equilibria. It is shown that this result is true if the demand functions of this economy are continuous, or even if the demand correspondences of this economy are upper semi-continuous. Using these results all known equilibrium existence results for these economies are very easily shown.

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