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The Voluntary Provision of Public Goods When Individuals are Heterogeneous with Respect to Income and Preferences and Equity May Matter

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1. Introduction

When a group of individuals must make voluntary contributions towards the provision of a public good, the frame of the model that is conjectured to characterize the preferences of these individuals is important for identifying the outcome of a voluntary contribution mechanism. Two models of individual behaviour are examined in this note. One model is the conventional model introduced by Bergstrom, Blum and Varian (1986) and the other is the Bergstrom et al. model augmented to reflect the equity theory of Walster, Walster and Berscheid (1978).[1]

The environments described below are developed for implementation in a controlled laboratory environment. In all cases subjects will know their own endowments, payoff tables, the total endowment of the group and when the session will end. The environment was introduced by Chan, Mestelman, Moir and Muller (1999).

Each individual i has an endowment of w_i tokens. The lab dollar payoff to individual i , u_i, is derived from the function

$$u_i = x_i + \alpha_i\, G + x_i\, G \qquad \textbf{(1)}$$

where x_i is the allocation to the private good, $G = \Sigma\, g_i$, is the aggregate allocation to the public good, $g_i = w_i - x_i$ is the individual's allocation to the public good and α_i is a parameter which

[1] The augmented model is comparable to, but more general than, the individual utility function introduced by Fehr and Schmidt (1999) for individuals with inequality aversion. One difference is that Fehr and Schmidt (1999) identify payoffs as the object of inequality aversion and we have used contributions to the public good relative to endowments. Payoffs relative to endowments would yield the same results as relative contributions.

characterizes individual preferences for the public good.

In the baseline homogeneity treatment, all agents have the same endowment and preference parameters. These agents are identified as S-type individuals. Heterogeneity is introduced by making one agent (the D-type agent) different from the other two (the S-type agents). The D-type agent will have either a larger endowment than the others, a stronger preference for the public good or both. There are two levels of heterogeneity in endowments: same endowment with $w_i = 20$ for all i , and different endowment with $w_1 = w_2 = 18$, $w_3 = 24$, and two levels of heterogeneity in preferences: same preferences with $\alpha_i = 9$ for all i , and different preferences with $\alpha_1 = \alpha_2 = 6$, $\alpha_3 = 15$. In all treatments, the group endowment, W, is 60 tokens per period and the aggregate preference parameter $\alpha = \Sigma\, \alpha_i$ is 27.

2. The Conventional Model

Following Bergstrom et al. (1986), in a non-cooperative environment the best response function for individual i given payoff function **(1)** is

$$g_i = \max\,(0, (\, w_i - G_{-i} + \alpha_i - 1)/2) \qquad \textbf{(2)}$$

which is constrained to be non-negative. Assuming the constraint is not binding on any subject, setting $n = 3$, recognizing that $G_{-i} = G - g_i$ and summing over i we obtain

$$G = (W + \alpha - 3)/4 \qquad \textbf{(3)}$$

Aggregate contributions in equilibrium depend only on the aggregate group endowment, W, and the aggregate preference parameter, α. Given our experimental parameterization this is 21 tokens in all conditions. Using the equations from **(2)** for each of the three subjects, the individual Nash equilibrium contributions may be calculated for each type of subject in each of the four treatments. These equilibria are reported in Table 1 along with corresponding distribution of payoffs. The group optimum contribution is 43 tokens. Any combination of contributions totalling 43 will yield the same aggregate optimal payoff, but none of these combinations will be a Nash equilibrium.

Note that when groups are heterogeneous in only one dimension, endowments or preferences, the predicted measure of equity is intermediate between the case of homogeneous groups and groups with heterogeneity in both dimensions. Heterogeneity is predicted to lead to increasing inequality of contributions in the equilibrium state. However, when a conventional Nash equilibrium is realized, payoffs are equalized (or nearly equalized) across all agents regardless of the heterogeneity condition. These predictions are included in Table 1.

3. The Equity Theory Model

Chan et al. (1997) introduced a version of equity theory as presented by Walster et al. (1978) into the conventional public good model by augmenting the payoff function **(1)** by adding the term $\Pi_i = - f_i (w_i , s_i)$ where $s_i = (g_i / w_i) - ((g_1 + g_2 + g_3)/ (w_1 + w_2 + w_3))$ and $\partial f_i / \partial s_i$ <=> 0 if s_i <=> 0. s_i is the difference between the share of a group's endowment invested in Market 2 and the share of endowment invested by individual *i* in Market 2. The resulting Nash equilibria, given that the same functional form of $f_i (w_i , s_i)$ applies to each subject in a group, lead to the predictions regarding voluntary contributions when the augmented payoff function is relevant that people with lower (higher) endowments, lower (higher) preferences for the public good or both will voluntary contribute more (less) when they realize a Nash equilibrium than they would have contributed if the equity theory term $f_i (w_i , s_i)$ did not augment the conventional payoff function.

In this formulation of an alternative model, the average contribution by the group is the measure of the equitable contribution. Deviations from this value will induce psychic loses, perhaps due to feelings of guilt or spite, and will result in total payoffs lower than the induced payoff reflects. The actual payoff function will then be the payoff function **(1)** augmented by $- f_i (w_i , s_i)$.

4. Derivation of Predictions for Homogeneity and Heterogeneity Cases when Equity Theory Considerations are included in the Model

In all cases, the augmented payoff function is

$$u_i = x_i + \alpha_i G + x_i G - f_i (w_i , s_i) \qquad \textbf{(4)}$$

where

$$s_i = (g_i /w_i) - (G/W) \text{ and } \partial f_i/\partial s_i \leq 0 \text{ if } s_i \leq 0 \text{ or } \partial f_i/\partial s_i \geq 0 \text{ if } s_i \geq 0 \qquad \textbf{(5)}$$

and the budget constraint for each individual is $g_i + x_i = w_i$, where g_i, x_i,and w_i are the contribution to the public good, the expenditure on the private good and the endowment of individual i. G is the total group contribution to the public good, W is the total group endowment and α_i is individual i's preference weight. The greater this value, the greater the individual's preference for the public good.

4.1. Case 1: Homogeneous Groups

In this Case, $\alpha_i = \alpha$ for all individuals and $w_i = w$ for all individuals. From the identity of the endowments and payoff functions, the assumption that in the equilibrium state all individuals (1, 2 and 3) will make the same voluntary contributions, and from **(5)**, in an equilibrium state $s_i = 0$ for all individuals. Therefore, the constrained maximization of (**4**) results in the same voluntary contributions for each individual as in the conventional model because in equilibrium $f_i(w_i, 0) = 0$.

4.2.1 Case 2: Groups with Income Heterogeneity (the proofs for this Case are reproduced from Chan et al., 1997)

In this Case $\alpha_i = \alpha$ for all individuals and $w_1 = w_2 < w_3$. From the symmetry of the endowments and payoff functions and identity of preferences, the assumption that in the equilibrium state low income individuals (1 and 2) will make the same voluntary contributions, and from **(5)**, in an equilibrium state

$$s_1 = ((g_1 / w_1) - (g_3 / w_3))(w_3 / W) = - s_3(w_3 / 2w_1) \tag{6}$$

Lemma 1: $- sign\ \partial f_1/\partial s_1 = sign\ \partial f_3/\partial s_3$

Proof: From **(6)**, $s_3 > 0 \leftrightarrow s_1 < 0$. Hence, from the properties of $f_i\,(\cdot)$, $\partial f_3/\partial s_3 > 0 \leftrightarrow \partial f_1/\partial s_1 < 0$.

The first order condition for the maximization of **(4)** by individual i , under zero conjectural variations is

$$\alpha_i - 1 + w_i - \Sigma\, g_{j\neq i} - 2g_i - (\partial f_i/\partial s_i)(W - w_i)/w_i W = 0 \tag{7}$$

Once again, from the symmetry of the endowments and payoff functions, in an equilibrium state individuals with the same endowments will make the same voluntary contributions. From these assumptions follow

$$\alpha - 1 + w_1 - g_3 - 3g_1 - (\partial f_1/\partial s_1)(W - w_1)/w_1 W = 0 \tag{8}$$

$$\alpha - 1 + w_3 - 2g_1 - 2g_3 - (\partial f_3/\partial s_3)(W - w_3)/w_3 W = 0 \tag{9}$$

which may be solved to find expressions for voluntary contributions by individuals 1, 2 and 3 as a function of the $f_i(\cdot)$ component of the augmented payoff function and the predicted voluntary contribution under the induced payoff function. These expressions are

$$g_1 = g_1^o + 0.25[\, - 2(\partial f_1/\partial s_1)(W - w_1)/w_1 W + (\partial f_3/\partial s_3)(W - w_3)/w_3 W] \tag{10}$$

$$g_3 = g_3^o + 0.25[2(\partial f_1/\partial s_1)(W - w_1)/w_1 W - 3(\partial f_3/\partial s_3)(W - w_3)/w_3 W] \tag{11}$$

where $g_i^o = (\alpha - 1 + 4w_i - W)/4$ is the Nash equilibrium voluntary contribution of the ith subject based on the induced payoff function.

Lemma 2: $(g_3^o / w_3) > (g_1^o / w_1)$ if $w_3 > w_1$ and $\alpha_1 = \alpha_3 = \alpha$

Proof: For corner solution cases $g_1^o = 0$ and $g_3^o > 0$ the result is obvious. For interior solution cases, substitute $g_i^o = (\alpha - 1 + 4w_i - W)/4$ (the Nash equilibrium voluntary contribution) into the inequality $(g_3^o / w_3) > (g_1^o / w_1)$. By rearranging the terms in the inequality the result follows directly.

4.2.2 Comparison of Conventional and Equity Theory Contributions for Case 2

From Lemma 1, $\Omega = -\gamma(\partial f_1/\partial s_1) + \beta(\partial f_3/\partial s_3)$, where γ and $\beta > 0$, can be less than, greater than or equal to zero.

1. Suppose $\Omega \leq 0$, then **(10)** $\rightarrow g_1 \leq g_1^o \rightarrow (g_1 / w_1) \leq (g_1^o / w_1)$ and **(11)** $\rightarrow (g_3 / w_3) \geq (g_3^o / w_3)$. Therefore $(g_3 / w_3) > (g_1 / w_1)$, and **(6)** $\rightarrow \partial f_1/\partial s_1 < 0$ and $\partial f_3/\partial s_3 > 0$. This contradicts supposition 1.

2. Suppose $\Omega > 0$, then **(10)** $\rightarrow g_1 > g_1^o \rightarrow (g_1 / w_1) > (g_1^o / w_1)$ and **(11)** $\rightarrow (g_3 / w_3) < (g_3^o / w_3)$. Therefore, either $(g_3 / w_3) > (g_1 / w_1)$ or $(g_3 / w_3) < (g_1 / w_1)$. If $(g_3 / w_3) < (g_1 / w_1)$ then **(6)** $\rightarrow \partial f_1/\partial s_1 > 0$ and $\partial f_3/\partial s_3 < 0$. This contradicts supposition 2. However, if $(g_3 / w_3) > (g_1 / w_1)$ then **(6)** $\rightarrow \partial f_1/\partial s_1 < 0$ and $\partial f_3/\partial s_3 > 0$. This does not contradict supposition 2. Therefore $(g_3 / w_3) > (g_1 / w_1)$.

This last result demonstrates that the term in the square brackets in **(10)** must be positive while the term in the square brackets in **(11)** must be negative. The equilibrium contribution of low (high) endowment individuals will be greater (less) than the predicted conventional (without the equity consideration) induced payoff contribution.

4.3.1 Case 3: Groups with Preference Heterogeneity

In this case, $\alpha_1 = \alpha_2 < \alpha_3$ and $w_i = w$ for all individuals. From the identity of the endowments and the asymmetry of the payoff functions, the assumption that in the equilibrium state low preference individuals (1 and 2) will make the same voluntary contributions, and from **(5)**, in an equilibrium state

$$s_1 = ((g_1 /w) - (g_3 /w))(w /W) = - s_3 /2 \qquad \textbf{(12)}$$

Lemma 3: $- \, sign\ \partial f_1/\partial s_1 = sign\ \partial f_3/\partial s_3$

Proof: From (12), $s_3 > 0 \leftrightarrow s_1 < 0$. Hence, from the properties of $f_i\,(\cdot)$, $\partial f_3/\partial s_3 > 0 \leftrightarrow \partial f_1/\partial s_1 < 0$.

The first order condition for the maximization of **(4)** by individual *i*, under zero conjectural variations, is given by equation **(7)**. From the identity of the endowments and asymmetry of the payoff functions, in an equilibrium state individuals with the same preferences will make the same voluntary contributions. From these assumptions follow

$$\alpha_1 - 1 + w - g_3 - 3g_1 - (\partial f_1/\partial s_1)(W - w)/wW = 0 \tag{13}$$

$$\alpha_3 - 1 + w - 2g_1 - 2g_3 - (\partial f_3/\partial s_3)(W - w)/wW = 0 \tag{14}$$

which may be solved to find expressions for voluntary contributions by individuals 1, 2 and 3 as a function of the $f_i(\cdot)$ component of the augmented payoff function and the predicted voluntary contribution under the induced payoff function. These expressions are

$$g_1 = g_1^o + 0.25[\,-2(\partial f_1/\partial s_1)(W - w)/wW + (\partial f_3/\partial s_3)(W - w)/wW] \tag{15}$$

$$g_3 = g_3^o + 0.25[2(\partial f_1/\partial s_1)(W - w)/wW - 3(\partial f_3/\partial s_3)(W - w)/wW] \tag{16}$$

where $g_1^o = (2\alpha_1 - \alpha_3 - 1 + 4w - W)/4$ and $g_3^o = (3\alpha_3 - 2\alpha_1 - 1 + 4w - W)/4$ are the Nash equilibrium voluntary contributions of the low preference and high preference subjects based on the induced payoff function.

Lemma 4: $(g_3^o / w) > (g_1^o / w)$ if $w_3 = w_1 = w$ and $\alpha_1 < \alpha_3$

Proof: For corner solution cases $g_1^o = 0$ and $g_3^o > 0$ the result is obvious. For interior solution cases, substitute $g_1^o = (2\alpha_1 - \alpha_3 - 1 + 4w - W)/4$ and $g_3^o = (3\alpha_3 - 2\alpha_1 - 1 + 4w - W)/4$ (the Nash equilibrium voluntary contributions) into the inequality $(g_3^o / w) > (g_1^o / w)$. By rearranging the terms in the inequality the result follows directly.

4.3.2 Comparison of Conventional and Equity Theory Contributions for Case 3

From Lemma 3, $\Omega = -\gamma(\partial f_1/\partial s_1) + \beta(\partial f_3/\partial s_3)$, where γ and $\beta > 0$, can be less than, greater than or equal to zero.

3. <u>Suppose</u> $\Omega \leq 0$, then **(15)** $\rightarrow g_1 \leq g_1^o \rightarrow (g_1 / w) \leq (g_1^o / w)$ and **(16)** $\rightarrow (g_3 / w) \geq (g_3^o / w)$. Therefore $(g_3 / w) > (g_1 / w)$, and **(A9)** $\rightarrow \partial f_1/\partial s_1 < 0$ and $\partial f_3/\partial s_3 > 0$. This contradicts supposition 3.

4. <u>Suppose</u> $\Omega > 0$, then **(15)** $\rightarrow g_1 > g_1^o \rightarrow (g_1 / w) > (g_1^o / w)$ and **(16)** $\rightarrow (g_3 / w) < (g_3^o / w)$. Therefore, either $(g_3 / w) > (g_1 / w)$ or $(g_3 / w) < (g_1 / w)$. If $(g_3 / w) < (g_1 / w)$ then **(12)** $\rightarrow \partial f_1/\partial s_1 > 0$ and $\partial f_3/\partial s_3 < 0$. This contradicts supposition 4. However, if $(g_3 / w) > (g_1 / w)$ then **(12)** $\rightarrow \partial f_1/\partial s_1 < 0$ and $\partial f_3/\partial s_3 > 0$. This does not contradict supposition 4. Therefore $(g_3 / w) > (g_1 / w)$.

This last result demonstrates that the term in the square brackets in **(15)** must be positive while the term in the square brackets in **(16)** must be negative. The equilibrium contribution of low (high) preference individuals will be greater (less) than the predicted conventional (without the equity consideration) induced payoff contribution for these individuals.

4.4 Case 4: Groups with Income and Preference Heterogeneity

In this Case, $\alpha_1 = \alpha_2 < \alpha_3$ and $w_1 = w_2 < w_3$. This combines Cases 2 and 3. The result that the equilibrium contribution of the low (high) preference and low (high) endowment individuals will be greater (less) than the predicted conventional (without equity consideration) induced payoff contributions for these individuals follows directly from the proofs provided for Cases 2 and 3.

5. Summary

When a voluntary contribution environment such as the one described above is characterized by heterogeneous agents

1) individuals with higher incomes will contribute more than individuals with lower incomes,

2) individuals with greater preference for the public good will contribute more than individuals with lesser preference,
3) if equity in contributions relative to income matters, then individuals with greater incomes or greater preferences will contribute less than they would have contributed if equity in contributions did not matter, and
4) if equity in contributions relative to income matters, then individuals with lower incomes or lesser preferences will contribute more than they would have contributed if equity in contributions did not matter.

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Table 1. Experimental Design: Parameterization and Nash Equilibria by Treatment (from Chan et al. 1999)

	Same Preferences (α_i = 9 for i= 1, 2, 3)		Different Preferences (α_i = 6 for i = 1 and 2, α_3 = 15)	
	Same Endowments (w_i = 20 for i = 1, 2, 3)	Different Endowments (w_i = 18 for i = 1 and 2, w_3 = 24)	Same Endowments (w_i = 20 for i = 1, 2, 3)	Different Endowments (w_i = 18 for i = 1 and 2, w_3 = 24)
Contributions				
Individual Nash	{7, 7, 7}	{5, 5, 11}	{4, 4, 13}	{2, 2, 17}
Group Nash	21	21	21	21
Group Optimum	43	43	43	43
Payoffs				
Individual Nash	{475, 475, 475}	{475, 475, 475}	{478, 478, 469}	{478, 478, 469}
Group Nash	1425	1425	1425	1425
Group Optimum	1909	1909	1909	1909

Notes: The parameters identified above are the subject's preference parameter for the public good, α_i, and the subject's endowment for each decision round, w_i. If the value of either of these parameters increases, then the subject's return to public good consumption or the subject's endowment in each decision round increases. In heterogeneous environments, individuals 1 and 2 are the S-type individuals and individual 3 is the D-type individual.