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Abstract

A Grossman–Helpman–Romer-type endogenous-growth model is developed in this study. This model has two countries in which there are knowledge spillovers that are partially local. Owing to these spillovers, innovation cost in a particular country decreases as the number of firms locating in both that country and the other country increases. If international knowledge spillovers are symmetric, innovation cost is lower in the country that has the larger market. However, if a small-market country can absorb the international knowledge spillovers better than a large-market country, the innovation cost may be lower in the small-market country. When the innovation cost is lower in the country that has a large market, the growth rate increases with agglomeration, which is generated by a reduction in the transportation costs. However, when the innovation cost is lower in the country that has a small market, the growth rate decreases with the reduction in the transportation costs.

Key words: knowledge spillovers, growth rate, transportation costs, market scale

JEL classification: F43: O30: R12

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1 Introduction

In this paper, a Grossman–Helpman–Romer-type endogenous-growth model is developed that has two countries. The activity of the innovation sector results in an increased variety of differentiated goods. In our model, there are asymmetric knowledge spillovers that are partially local between the two countries. Given these knowledge spillovers, the innovation cost in a particular country decreases as number of firms locating in both that country and the other country increases. We assume that intra-national spillovers are complete but international spillovers are incomplete. In addition, we assume that international knowledge spillovers between the two countries are not symmetric.¹

Coe and Helpman (1995) and Eaton and Kortum (1996) produced empirical studies about international knowledge spillovers in OECD countries and found that these spillovers were not symmetric. Coe and Helpman (1995) showed that, in developed nations, intra-national knowledge spillovers played an important role in economic growth, although international knowledge spillovers had only small effects on economic growth. They also found that, in developing countries, international knowledge spillovers from developed countries played an important role in economic growth, as did intra-national knowledge spillovers. These results suggest that international knowledge spillovers between developed and developing countries are asymmetric. Thus, we think it important to construct a model in which there are asymmetric international knowledge spillovers. In this context, we interpret asymmetry in international knowledge spillovers as the difference between developed and developing countries in their ability to absorb international knowledge spillovers.²

Our model is closely related to those of Martin (1999) and Martin and Ottaviano (1999), who constructed models in which international knowledge spillovers are symmetric. When the shipping of differentiated goods incurs transportation costs, more firms locate in the large-market country than in the small-market country to economize on transportation costs. In addition,

¹Baldwin et al. (2001) and Fujita and Thisse (2003) assumed that knowledge spillovers are partially local. However, in their models, knowledge spillovers between two countries are symmetric.

²Asymmetries in knowledge spillovers may be due to language. For example, Coe and Helpman (1995) found that, while Japan absorbed international spillovers from the U.S., there were no international knowledge spillovers from Japan to the U.S. The explanation may be that most Japanese recognized English as the international language and tried to learn it, but Americans did not learn Japanese. Consequently, Japan can absorb international knowledge spillovers from the U.S., but there are no international knowledge spillovers from Japan to the U.S.

if international knowledge spillovers are symmetric, the innovation cost is always lower in the country that has the larger income, since the country that has the larger income has a larger market than the other country. This is because the larger the number of firms locating in a particular country, the lower the innovation cost in that country due to partially local knowledge spillovers. Thus, when international knowledge spillovers are symmetric, the growth rate increases monotonically with agglomeration in the large-income country.

However, this result may not be valid if international knowledge spillovers are asymmetric. If the small-market country can absorb international knowledge spillovers better than the large-market country, the innovation costs may be lower in the small-market country. In this case, the growth rate decreases as the share of differentiated firms in the large-market country increases, since the innovation cost in the small-market country increases as the share of differentiated firms in the large-market country increases. In Martin (1999) and Martin and Ottaviano (1999), owing to symmetric international spillovers, there is always a trade-off between international income equality and a high growth rate. This trade-off arises in our model for some of the parameter values even if international spillovers are not symmetric. However, in our model, we show that there is a situation in which greater income *equality* raises the growth rate. Hence, although our model is closely related to those of Martin (1999) and Martin and Ottaviano (1999), it has richer implications.³

Given this framework, our model yields two patterns relating to the innovation sector's location.⁴ In one pattern, the innovation sector locates in the large-income country. In the other pattern, the innovation sector locates in the small-income country. Which location pattern emerges depends on the ability to absorb international knowledge spillovers, international income inequality, and transportation costs. In the former case, the growth rate increases as the transportation costs decrease and the international income inequality increases. In the latter case, the growth rate decreases as the transportation costs decrease and the international income inequality decreases. It must be remembered that, as the transportation costs decrease,

³Martin (1999) focused on the relationship between intra-national transportation costs and international transportation costs. However, in his model, there is always a trade-off between international income equality and the growth rate.

⁴Barro and Sala-i-Martin (1997) explained the movement of innovation activity between countries by another mechanism. Their model showed that this movement occurs because agents in a particular country who imitate others (by copying their innovations) find that innovation is preferable to imitation when some parameters change owing to, for example, changes in government policy.

agglomeration in the large-income country increases. Therefore, there is a critical value of the transportation costs. Below this critical value, the former pattern applies. Above the critical value, the economy follows the latter pattern. In the economy, there are two cases, one in which only the former pattern emerges, and another one, in which both location patterns occur simultaneously. When both location patterns occur, the effect of reduced transportation costs on the growth rate changes at the critical value.

Many papers have shown that trade liberalization increases growth rates. However, there are cases in which, in the process of economic development, the regulation of trade has increased the growth and industrialization under certain conditions. For example, Komiya et al. (1984) have argued that the Japanese government regulated trade for protection and to expand domestic industry in the high-growth period between 1950 and 1970. By doing so, Japan became industrialized and achieved a high growth rate. In addition, Bernstein and Mohnen (1994), Coe and Helpman (1995), and Eaton and Kortum (1996) reported that, from the 1960s to the 1980s, there were substantial international knowledge spillovers from the U.S. to Japan. Between the 1950s and the 1980s, the Japanese government is perceived to have substantially regulated international trade. That is, transportation costs were high, and Japan could easily absorb international spillovers. We suggest that our model can help to explain why high transportation costs and the ability to absorb international knowledge spillovers led to high growth between the 1950s and the 1970s.

However, since the 1980s, international trade has been liberalized in Japan. Moreover, Japan has not intensively innovated. Coe and Helpman (1995) found that the elasticity of total factor productivity with respect to foreign R&D in Japan fell from 0.037 to 0.027 between 1980 and 1990. Bernstein and Mohnen (1994) recently showed that intensive R&D was not taking place in Japan. This suggests that, after the 1980s, the transportation costs fell and the ability to absorb international knowledge spillovers declined. Our model suggests that, when the transportation costs are low, it is desirable for the innovation sector to be located in a technologically advanced country. In fact, since the 1980s, by comparison with the U.S., Japan's R&D performance has been relatively poor.

This paper is organized as follows. In the next section, we present the basic model and distinguish between two patterns relating to the innovation sector's location. In section 3, we examine the steady state and the growth rate in the context of these two patterns. In section 4, we present the results from the model and their interpretation. Section 5 is the conclusion.

2 The model

2.1 The economy

A basic setting in our model follows Martin and Ottaviano (1999). Consider two countries, called Home and Foreign. Each country is endowed with a fixed amount of labor, L , which is immobile between countries. We normalize so that $L = 1$.⁵ The countries are symmetric except for the amount of initial patent holding and the ability to absorb international knowledge spillovers. Suppose that Home and Foreign initially own H_0 and H_0^* with $H_0 > H_0^*$. Given symmetry, we consider only Home (with Foreign's variables labeled *).

Consumer preferences are as follows:

$$U = \int_0^\infty \log[D(t)^\alpha Y(t)^{1-\alpha}] e^{-\rho t} dt, \quad (1)$$

where Y is the consumption of a homogeneous good, ρ is the rate of time preference, and $\alpha \in (0, 1)$ is the share of expenditure devoted to the composite good, $D(t)$. $D(t)$ is given by

$$D(t) = \left[\int_{i=0}^{N(t)} D_i(t)^{1-\frac{1}{\sigma}} di \right]^{\frac{1}{1-\frac{1}{\sigma}}}, \sigma > 1, \quad (2)$$

where D_i is the consumption of differentiated goods indexed by i , N is the total number of varieties available in the economy, and σ is the elasticity of substitution between varieties.

The value of expenditure E is

$$E = \int_0^n p_i D_i di + \int_{j=n}^N \tau p_j^* D_j dj + p_Y Y, \quad (3)$$

where p_Y is the price of the homogeneous good, p_i is the price of the i th variety, and n is the number of varieties produced in Home and $N = n + n^*$. The international shipping of differentiated goods incurs transportation costs taking the form of iceberg costs, as in Samuelson (1954). If a quantity of differentiated goods is shipped, then a $1/\tau$ ($\tau > 1$) proportion of this quantity reaches the customer.⁶ There is no transportation cost for the homogeneous good.

⁵With this normalization, our model lacks a scale effect. However, our focus is not on the scale effect on growth, but on geography and growth. The geography of firms and the innovation sector is independent of the scale effect.

⁶These costs are interpreted as transportation costs that are affected by trade policy, such as tariffs and quotas.

On the supply side, the homogeneous good is produced using only labor with constant returns to scale technology. For simplicity, one unit of labor is required to produce one unit of the homogenous good. We assume that, since the demand for this good is sufficiently large, it is produced in both countries in equilibrium. Because there are no transportation costs for the homogenous good, the nominal wage is equal between the two countries. Solving the optimization problem shows that $p_Y = w$.

For all varieties, differentiated goods are produced in a monopolistically competitive sector with increasing returns to scale technology. If there is no cost of discrimination, each firm produces its own variety. We assume that a patent is required for starting production and β units of labor are used to produce one unit of these goods. Once the patent is made, differentiated goods firms have monopoly power over the variety produced and can freely relocate production facilities across countries. Under this assumption, optimal pricing implies $p_i = p_i^* = w\beta\sigma/(\sigma - 1)$. The operating profit is

$$\pi = px - w\beta x = \frac{w\beta x}{\sigma - 1}, \quad (4)$$

where x is the size of the firm.

In contrast to firms, workers are immobile. Thus, their incomes are geographically fixed. Solving the first-order condition, we find the demand for differentiated goods:

$$D_i = \frac{\sigma - 1}{w\beta\sigma} \frac{\alpha E}{n + \tau^{1-\sigma}n^*}, \quad D_j = \frac{\sigma - 1}{w\beta\sigma} \frac{\tau^{-\sigma}\alpha E}{n + \tau^{1-\sigma}n^*}.$$

Suppose that there is perfect global financial market, which yields the interest rate, r . Suppose the patent lives infinitely. Therefore, firms that have a patent have a perpetual monopoly power. The value of differentiated firms is the present discounted value of all future operating profits. Thus, the value of the firms, v , is

$$v(t) = \int_t^\infty e^{-[R(s)-R(t)]} \frac{w\beta x(s)}{\sigma - 1}, \quad (5)$$

where $R(t) = \int_0^t r(\tau)d\tau$. By differentiating with respect to t , we find the no-arbitrage condition:

$$\dot{v}(t) = r(t)v(t) - \pi(t). \quad (6)$$

Solving the consumer's intertemporal optimization problem yields $\dot{E}/E = \dot{E}^*/E^* = r - \rho$. As in Grossman and Helpman (1991), our economy does not have a monetary instrument, so there is nothing to pin down the price level. Therefore, we can normalize aggregate income at any time; i.e., $E + E^* = 1$. On this basis, the solution to the optimization problem derives that $r = \rho$.

2.2 Equilibrium

Owing to the free movement of patents, operating profits are equal between countries. Thus, from (4), $x = x^*$.

There is an equilibrium condition for the market clearing of goods, which incorporates transportation costs. Using the consumer's demand for differentiated goods, the market-clearing conditions are as follows:

$$\begin{aligned} x &= \frac{\alpha(\sigma - 1)}{\beta\sigma} \left(\frac{E}{n + \delta n^*} + \frac{E^* \delta}{n\delta + n^*} \right), \\ x^* &= \frac{\alpha(\sigma - 1)}{\beta\sigma} \left(\frac{E\delta}{n + \delta n^*} + \frac{E^*}{\delta n + n^*} \right). \end{aligned}$$

For simplicity, we define the parameter δ as $\delta \equiv \tau^{1-\sigma}$. It is clear that $0 < \delta < 1$ and a higher δ implies lower transportation costs since $\tau > 1$ and $\sigma > 1$. In addition, we denote the share of differentiated goods firms in Home as γ .

Given the equilibrium condition, $x = x^*$, the equilibrium share of differentiated goods firms in Home is

$$\gamma = \frac{\theta_E}{1 - \delta} - \frac{(1 - \theta_E)\delta}{1 - \delta}, \quad (7)$$

where $\theta_E = E/(E + E^*)$, which is the share of total expenditure of Home. This equation implies that a higher share of expenditure leads to greater agglomeration in Home, owing to increasing returns to scale in the differentiated goods sector. When the transportation costs are low, i.e. δ is high, the firms tend to locate in Home since the firms desire shipping to Foreign while locating in Home.

From $x = x^*$, the size of the firms, wherever they and the innovation sector locate, is

$$x = x^* = \alpha \frac{\sigma - 1}{w\beta\sigma} \frac{E + E^*}{N}. \quad (8)$$

It must be recalled that one patent is needed to produce a new differentiated good. Thus, the number of varieties and the number of firms in the economy are equal to the number of patents in the economy; hence, $n + n^* = H + H^*$ at each point of the time.

3 The steady state and the equilibrium growth rate

Consider the innovation sector, which produces a patent by using only labor. This sector operates as described in Grossman and Helpman (1991). Suppose that the number of labor inputs required to produce the patent is $\eta/(\epsilon n + \epsilon^* n^*)$. The parameter η represents the constant labor productivity implied per patent produced. The parameters $0 \leq \epsilon^* \leq 1$ and $0 \leq \epsilon \leq 1$ represent the degree of knowledge spillovers from Foreign to Home and from Home to Foreign, respectively. We interpret ϵ (ϵ^*) as the ability of Foreign (Home) to absorb international knowledge spillovers. Note that the innovation cost is a decreasing function of the number of firms located in own and the other countries.⁷ We refer to knowledge spillovers within a country as “local knowledge spillovers,” as do Martin and Ottaviano (1999), and assume that local knowledge spillovers are perfect. However, it is assumed that the effect of the other country’s diversity on the innovation cost may be not perfect: ($0 \leq \epsilon \leq 1$ and $0 \leq \epsilon^* \leq 1$).⁸ Owing to free entry and zero profits in the innovation sector, $v_H = w\eta/(n + \epsilon^* n^*)$ when innovation takes place in Home. Similarly, $v_F = w\eta/(n^* + \epsilon n)$ when the innovation takes place in Foreign. The terms v_H and v_F denote the value of a patent when the innovation sector is located in Home and Foreign, respectively.

Note that γ is constant in equilibrium. Therefore, in the steady state, n , n^* , and N grow at the same constant rate, $g = \dot{N}/N$. The other equilibrium conditions, which ensure that v_H and v_F decrease at the same rate as the growth rate of N , are as follows:

$$g = \frac{\dot{N}}{N} = -\frac{\dot{v}_H}{v_H} = -\frac{\dot{v}_F}{v_F}.$$

3.1 Location of the innovation sector

The innovation sector operates in the country with the lowest innovation cost. From the setting of the innovation cost, the following patterns of the location of the innovation sector are identified.

- (Pattern 1) The innovation sector operates in Home ($n + \epsilon^* n^* > \epsilon n + n^*$).

⁷Martin (1999) refers to this type of knowledge spillover as the Jacobs type of knowledge spillover. The author suggested that the source of these benefits is, for example, direct observation of the production process: researchers observe the production process and find new goods easier to invent.

⁸The production costs for the patent in Home and Foreign are $w\eta/(n + \epsilon^* n^*)$ and $w\eta/(n^* + \epsilon n)$, respectively.

- (Pattern 2) The innovation activity operates in Foreign ($n + \epsilon^* n^* < \epsilon n + n^*$).

When the location pattern is in pattern 1, the share of differentiated firms in Home is as follows:

$$\gamma > \hat{\gamma} \equiv \frac{1 - \epsilon^*}{2 - \epsilon - \epsilon^*}. \quad (9)$$

This discussion implies the property of the economy described by the following lemma.

Lemma 1 *When $\gamma > \hat{\gamma}$ ($\gamma < \hat{\gamma}$), pattern 1 (pattern 2) applies.*

Next, we consider income. Given $v = \pi/(\rho + g)$, (4) and (8), the equilibrium aggregate income in each country is

$$E = w + \frac{2\alpha\rho wh}{(\rho + g)\sigma - \alpha\rho}, \quad E^* = w + \frac{2\alpha\rho w(1 - h)}{(\rho + g)\sigma - \alpha\rho}, \quad (10)$$

where $h = H/N = H/(H + H^*)$ and H is the number of patents in Home. That is, h is the share of patents owned by the households in Home. In our model, this share is constant because patents are free to move between the two countries.⁹ Thus, H and N grow at the same rate. Next, we determine the consumer's share of income in Home, θ_E .

From (10)

$$\theta_E = \frac{\sigma(\rho + g) + \alpha\rho(2h - 1)}{2\sigma(\rho + g)}. \quad (11)$$

Equation (11) shows that international income inequality, θ_E , is a decreasing function of the growth rate. This is because a higher growth rate implies a higher rate of entry of firms, which lowers both the monopoly profits and the value of patents. The decrease in income from patent holdings is greater in Home than in Foreign because Home owns more patents. Thus, international income inequality decreases as the growth rate increases.

It is found that $\theta_E > 1/2$ provided that $h > 1/2$. In other words, when Home has more patents than Foreign, the aggregate income is higher in Home than in Foreign. (7) should be kept in mind. From assumption $h > 1/2$, it is found that $\gamma > 1/2$ since, if $h > 1/2$, $\theta_E > 1/2$. From (9), it is clear that $\hat{\gamma} \leq 1/2$ when $\epsilon^* \geq \epsilon$. This generates the property of the economy described in the next lemma.

⁹Baldwin (1999) and Baldwin et al. (2001) constructed models in which h is an endogenous variable. In these models, assets are not free to move internationally, and this assumption generates the cumulative causation mechanism for agglomeration. In our model, the focus is not on the cumulative causation mechanism but on the effect of partially local knowledge spillovers on the relationship between geography and growth.

Lemma 2 *With the assumption $h > 1/2$, pattern 2 cannot arise if $\epsilon^* \geq \epsilon$.*

This lemma states that, if Foreign is less able to absorb international knowledge spillovers than Home, pattern 2 cannot arise.¹⁰

Wherever the innovation sector is located, Home's nominal GDP, defined as the sum of value added in the three sectors, is $GDP_H = w + \gamma\beta xN/(\sigma - 1)$. Home's nominal GDP exceeds Foreign's nominal GDP, which is $GDP_F = w + (1 - \gamma)\beta xN/(\sigma - 1)$, because $\gamma > 1/2$. Each country's GDP is an increasing function of its share of firms producing differentiated goods.

3.2 The steady state and the equilibrium growth rate under pattern 1.

When $\gamma > \hat{\gamma}$, this pattern emerges. Since labor is employed in three sectors, when the innovation sector is in Home, the labor-market clearing condition is

$$E + E^* = \frac{2\sigma w}{\sigma - \alpha} - \frac{\sigma w}{(\sigma - \alpha)} \frac{\eta}{\gamma + \epsilon^*(1 - \gamma)} g_H, \quad (12)$$

where g_H is the growth rate when the innovation sector operates in Home. It must be remembered that the total expenditure is constant due to normalization.

Given free entry in the innovation sector, $r = \rho$, (12) and (6), the growth rate is

$$g_H = \frac{2\alpha[\gamma + \epsilon^*(1 - \gamma)]}{\sigma\eta} - \frac{\sigma - \alpha}{\sigma} \rho. \quad (13)$$

Note that the increased agglomeration in Home raises the equilibrium growth rate when the innovation activity takes place in Home. Since agglomeration in Home reduces the innovation cost in Home, agglomeration in Home has a positive effect on the growth rate. From (7) and (13), the equilibrium growth rate as a function of the share of Home's total income is

$$g_H = \frac{2\alpha}{\sigma\eta} \left[\frac{1 - \epsilon^*}{1 - \delta} [(1 + \delta)\theta_E - \delta] + \epsilon^* \right] - \frac{\sigma - \alpha}{\sigma} \rho. \quad (14)$$

Substituting (13) into (11) yields the following relationship between international income inequality and the location of firms:

$$\theta_{EH} = \frac{(1 - \epsilon^*)\gamma + \epsilon^* + \eta\rho h}{2(1 - \epsilon^*)\gamma + 2\epsilon^* + \eta\rho}, \quad (15)$$

¹⁰In Martin and Ottaviano (1999), $\epsilon = \epsilon^* = 1$ or $\epsilon = \epsilon^* = 0$. Hence, in their model, the innovation sector always locates in Home. However, in our model, the case in which innovation activity is conducted in Foreign may emerge when $\epsilon < \epsilon^*$.

where θ_{EH} is the consumer's share of income in Home when the innovation sector is in Home. It is clear that the consumer's share of income in Home decreases as the number of differentiated firms in Home increases. This is because agglomeration in Home reduces the innovation cost in Home and raises the growth rate, which reduces the monopoly power of existing firms. Since Home owns more patents, the fall in patent income is greater in Home. Therefore, international income inequality is reduced when the growth rate increases. This is known as the competition effect: see Martin (1999).

The value of γ that satisfies both (7) and (15) is the steady-state solution of the model. Combining (7) and (15) and imposing $\gamma > 0$ yields the equilibrium value of γ .¹¹ We denote γ_H as the equilibrium value of γ when the innovation sector is located in Home.

$$\gamma_H = \frac{(1 - 3\epsilon^* - \eta\rho) + \sqrt{\Delta}}{4(1 - \epsilon^*)}, \quad (16)$$

where

$$\Delta = (1 - 3\epsilon^* - \eta\rho)^2 + 8(1 - \epsilon^*) \left[\epsilon^* + \frac{\eta\rho}{1 - \delta}(h - (1 - h)\delta) \right].$$

The equilibrium growth rate can be computed from (13).

It is clear that $\partial\gamma_H/\partial h > 0$ from (16), and, thus, $\partial g_H/\partial h > 0$ from (14). In addition, $\partial\gamma_H/\partial\delta > 0$, and, thus, $\partial g_H/\partial\delta > 0$.

Let us define $\bar{\delta}$ such that $\gamma_H = 1$ if $\delta \geq \bar{\delta}$. From (16), we find that

$$\bar{\delta} = \frac{1 + \eta\rho(1 - h)}{1 + \eta\rho h}. \quad (17)$$

It is obvious that $\bar{\delta} < 1$ and $\partial\bar{\delta}/\partial h < 0$ since $h > 1/2$. From (14), if $\delta \geq \bar{\delta}$, then g_H is constant at the maximum growth rate, $g_H = (2\alpha/\sigma\eta) - (\sigma - \alpha)\rho/\sigma$, among all potential growth rates. These results are described in the next lemma.

Lemma 3 • *When $\gamma_H \geq \hat{\gamma}$ and $\delta < \bar{\delta}$, the equilibrium growth rate, g_H , is an increasing function of the share of patent holdings in Home, h , and a decreasing function of the transportation costs τ .*

- *When $\delta \geq \bar{\delta}$, g_H is constant at the maximum growth rate, $g_H = (2\alpha/\sigma\eta) - (\sigma - \alpha)\rho/\sigma$, among all potential growth rates.*

¹¹The equation that we find by combining (7) and (15) is

$$2(1 - \delta)(1 - \epsilon^*)\gamma^2 + (1 - \delta)(3\epsilon^* - 1 + \eta\rho)\gamma - \{(1 - \delta)\epsilon^* + \eta\rho(h - (1 - h)\delta)\} = 0.$$

In this location pattern, the effect of the share of patent holdings in Home on the equilibrium growth is positive. However, the effect of the high inequality of patent holding on international income inequality is ambiguous. This is because there are two effects. First, the high h simply indicates the high international income inequality. Second, there is the competition effect.

Decreasing transportation costs raises the equilibrium growth rate. The decline in the transportation costs induces agglomeration in Home, which results in a reduction in the innovation costs in Home because the strength of the degree of spillovers from local knowledge is greater than that from international knowledge. The increase in the growth rate reduces international income inequality because of the competition effect. Hence, the decrease in the transportation costs raises the equilibrium growth rate and reduces international income inequality.

The effect of an increase in ϵ^* is ambiguous. We show that $\partial\gamma_H/\partial\epsilon^* < 0$ from $1/2 < \gamma_H \leq 1$. Thus, it is found that the increase in ϵ^* reduces the growth rate. However, it is found that an increase in ϵ^* increases the growth rate for a given γ_H from (14). The increase in the growth rate reduces international income inequality, which reduces the share of firms because of the competition effect. These effects reduce the equilibrium growth rate. The increase in ϵ^* reduces international income inequality, but the effect on the growth rate is ambiguous.

The stability properties of the equilibrium are analyzed following Grossman and Helpman (1991). First, we define the variable $W \equiv 1/w$. That is, W is the inverse of the wage rate. In addition, $g = \dot{N}/N$ represents the instantaneous rate of innovation in the economy.

(6) is rewritten as a function of the wage rate as follows:

$$g_H = \frac{\alpha[\gamma_H + \epsilon^*(1 - \gamma_H)]}{\sigma\eta w} - \rho. \quad (18)$$

Given the definition of W , (12) is rewritten as

$$W = \frac{2\sigma}{\sigma - \alpha} - \frac{\sigma}{(\sigma - \alpha)} \frac{\eta}{\gamma_H + \epsilon^*(1 - \gamma_H)} g_H. \quad (19)$$

Note that (19) must be satisfied at all times because this equation expresses the resource constraint. The higher the innovation rate, the greater the employment in the innovation sector, which lowers employment in the differentiated goods sector and, therefore, the supply of differentiated goods. This increases the price of these goods as well as the wage. This is why the curve representing (19) slopes downward.

The steady-state properties of the model are illustrated in Figure 1. (20) and (19) intersect at point M. At this point, dynamic forces in the economy

do not affect W and g . In other words, if the economy reaches point M, innovation continues at a constant rate, and the division of resources among the innovation sector, the homogenous goods sector, and the differentiated goods sector remains constant. The wage rate does not change. The upward-sloping curve represents the combinations of g and W .

$$W = \frac{\sigma\eta}{\alpha[\gamma_H + \epsilon^*(1 - \gamma_H)]}(g_H + \rho). \quad (20)$$

At all points above the line representing (20), an increase in the number of varieties reduces the wage rate, so W rises. Below this line, the opposite applies. These movements follow (19).

As in Grossman and Helpman (1991), expectations can be fulfilled only if the economy jumps immediately to this steady state. Expectations that differ from those implied by point M cannot be fulfilled. In the long run, initial beliefs that contradict those implied by point M lead to either $g = 0$ or $W = 0$. In the first case, N eventually stops growing at a finite value, while W grows infinitely. The finite value of N could only arise were w to approach zero. However, this value of N requires a strictly positive wage rate, not one of zero. Since the wage rate remains positive, this path requires unfulfilled expectations. In the second case, g reaches its maximum value, while the value of W falls arbitrarily close to zero. Owing to an unbounded and continual increase in the number of varieties, the wage rate eventually falls to zero. Hence, beliefs are clearly contradicted. The only remaining possibility is for the economy to start at point M and stay there forever.

Suppose that the economy starts with the innovation cost being lower in Home. If the transportation costs decrease or the share of patents in Home increases, the share of differentiated firms rises because the demand for differentiated goods increases. This lowers the innovation cost in Home further. The lower innovation cost raises the growth rate. This movement continues until profits in the innovation sector are zero and firm profits in the differentiated goods sector are equal in both countries. Stability analysis indicates that this process is instantaneous.

3.3 The steady state and the equilibrium growth rate under pattern 2.

As in subsection 3.2, we examine the steady state and the growth rate.

The labor-market clearing condition is

$$E + E^* = \frac{2\sigma w}{\sigma - \alpha} - \frac{\sigma w}{(\sigma - \alpha)} \frac{\eta}{(1 - \gamma) + \epsilon\gamma} g_F, \quad (21)$$

where g_F is the growth rate when the innovation sector operates in Foreign. The equilibrium growth rate is described as a function of γ as follows:

$$g_F = \frac{2\alpha[\epsilon\gamma + (1 - \gamma)]}{\sigma\eta} - \frac{\sigma - \alpha}{\sigma}\rho. \quad (22)$$

Unlike (13), (22) shows that the equilibrium growth rate is a decreasing function of γ . This is because the progress of agglomeration in Home increases the innovation cost in Foreign. Thus, the growth rate decreases as the progress of agglomeration in Home.

Substituting (7) into (22) yields the equilibrium growth rate described as a function of θ_E as follows:

$$g_F = \frac{2\alpha}{\sigma\eta} \left[\frac{\epsilon - 1}{1 - \delta} [(1 + \delta)\theta_E - \delta] + 1 \right] - \frac{\sigma - \alpha}{\sigma}\rho. \quad (23)$$

Combining (22) and (11) shows the following relationship between international income inequality and the location of firms:

$$\theta_{EF} = \frac{(\epsilon - 1)\gamma + 1 + \eta\rho h}{2(\epsilon - 1)\gamma + 2 + \eta\rho}, \quad (24)$$

where θ_{EF} is the household's share of income in Home when the innovation sector operates in Foreign. Compared with (15), θ_{EF} is an increasing function of γ . This is because agglomeration in Home raises the innovation cost in Foreign, thereby reducing the equilibrium growth rate, and, thus, reinforces the monopoly power of existing firms. Since Home owns more patents, the income increases more in Home than in Foreign. Hence, the income gap widens.

The value of γ that satisfies (7) and (15) is the steady-state solution of the model. The equilibrium share of firms in Home, γ_F , is as follows:¹²

$$\gamma_F = \frac{(3 - \epsilon + \eta\rho) \pm \sqrt{(3 - \epsilon + \eta\rho)^2 - 8(1 - \epsilon)\left\{1 + \frac{\eta\rho}{1 - \delta}(h - (1 - h)\delta)\right\}}}{4(1 - \epsilon)}.$$

For the existence of a solution for γ , we impose the following condition:

$$(3 - \epsilon + \eta\rho)^2 - 8(1 - \epsilon) \left[1 + \frac{\eta\rho}{1 - \delta}(h - (1 - h)\delta) \right] > 0.$$

¹²The equation that we find by combining (7) and (15) is

$$2(1 - \delta)(1 - \epsilon)\gamma^2 + (1 - \delta)(3 - \epsilon + \eta\rho)\gamma - \{(1 - \delta)\epsilon + \eta\rho(h - (1 - h)\delta)\} = 0.$$

(7) shows that, if $h = 1/2$, $\gamma = 1/2$. Therefore, we find that the value of γ_F is

$$\gamma_F = \frac{(3 - \epsilon + \eta\rho) - \sqrt{\Phi}}{4(1 - \epsilon)}, \quad (25)$$

where

$$\Phi = (3 - \epsilon + \eta\rho)^2 - 8(1 - \epsilon) \left[1 + \frac{\eta\rho}{1 - \delta}(h - (1 - h)\delta) \right].$$

We calculate the equilibrium growth rate by substituting (25) into (22).

It is shown that $\partial\gamma_F/\partial\epsilon < 0$. Thus, $\partial g_F/\partial\epsilon > 0$ from (22). In addition, from (25), $\partial\gamma_F/\partial\delta > 0$, and $\partial\gamma_F/\partial h > 0$. Therefore, $\partial g_F/\partial\delta < 0$, and $\partial g_F/\partial h < 0$. We summarize these results in the next lemma.

Lemma 4 *If $\gamma_F < \hat{\gamma}$, the equilibrium growth rate is an increasing function of the transportation costs, the ability of Foreign to absorb international knowledge spillovers, and a decreasing function of Home's share of patents.*

In this location pattern, the equilibrium growth rate rises if the share of patent holdings in Home decreases. In addition, the increase in the equilibrium growth rate reduces the international income inequality. This is because there are two effects. First, the small h simply implies low international income inequality. A small h raises the share of firms locating in Foreign and thus reduces the innovation cost in Foreign. Second, there is the competition effect. Hence, there is no trade-off between international income equality and the growth rate.¹³

Lower transportation costs reduce the equilibrium growth rate. The decline in the transportation costs increases the agglomeration in Home. Given increasing returns to scale, firms in Foreign relocate to Home due to a reduction in the transportation costs. This relocation of firms raises the innovation cost in Foreign, and the growth rate declines. The fall in the growth rate raises international income inequality because the lower growth rate weakens the competition effect. Hence, lower transportation costs reduce the equilibrium growth rate and raise international income inequality.

An increase in ϵ raises the equilibrium growth rate and reduces international income inequality. The increase in ϵ directly increases the growth rate for a given γ_F from (22). The increase in the growth rate reduces Home's share of income and raises Foreign's share of income because of the competition effect. Thus, the increase in ϵ raises Foreign's share of income. This

¹³In Martin (1999), there is a trade-off between international income equality and the growth rate. This is because, in his model, innovation activities locate in the country with the larger market.

effect reinforces the direct effect of ϵ on the growth rate and raises the equilibrium growth rate. The increase in ϵ reduces international income inequality and raises the equilibrium growth rate.

The stability properties are investigated as in subsection 3.2.¹⁴

Here, we investigate the factors that determine the location of the innovation sector. As we explained earlier, if $\gamma < \hat{\gamma}$, the innovation sector is in Home and, if $\gamma > \hat{\gamma}$, the innovation sector operates in Foreign. From (7), it is clear that the lower the transportation costs are, the higher the share of firms that locate in Home is. Thus, there is a critical value of δ that represents the level of the transportation costs. We define $\hat{\delta}$ as the value that leads to $\gamma = \hat{\gamma}$. We find the value of $\hat{\delta}$ using the equation to derive the steady-state γ and the critical value $\hat{\gamma}$:¹⁵

$$\hat{\delta} = \frac{(\epsilon - \epsilon^*)(1 - \epsilon\epsilon^*) - (2 - \epsilon - \epsilon^*)\{(1 - \epsilon^*) - (2 - \epsilon - \epsilon^*)h\}\eta\rho}{(\epsilon - \epsilon^*)(1 - \epsilon\epsilon^*) - (2 - \epsilon - \epsilon^*)\{(1 - \epsilon^*) - (2 - \epsilon - \epsilon^*)(1 - h)\}\eta\rho}. \quad (26)$$

In this context, $\partial\gamma_H/\partial\delta > 0$ and $1 > \hat{\gamma} = (1 - \epsilon^*)/(2 - \epsilon - \epsilon^*)$. Hence, $\bar{\delta} > \hat{\delta}$. It must be recalled that $\bar{\delta}$ is the value of δ that implies $\gamma = 1$. In addition, we find that, when $\gamma = \hat{\gamma}$, $g_H = g_F$ and $\theta_{EH} = \theta_{EF}$.¹⁶

From the definition, the economy has three cases. If $\hat{\delta}$ is not positive, the innovation sector is always in Home regardless of the level of transportation costs. If $\hat{\delta} \geq 1$, the innovation sector is always in Foreign regardless of the level of the transportation costs. If $0 < \hat{\delta} < 1$, both location patterns of the innovation sector exist in the economy, and which location pattern emerges depends on the level of the transportation costs.

For simplicity, we focus on the following two cases: the case in which $\epsilon^* = 1$, $0 \leq \epsilon \leq 1$ and that in which $\epsilon^* = 0$, $0 \leq \epsilon \leq 1$.¹⁷

¹⁴We derive a labor-market clearing condition and a no-arbitrage condition. These are

$$W = \frac{2\sigma}{\sigma - \alpha} - \frac{\sigma}{(\sigma - \alpha)} \frac{\eta}{\epsilon\gamma_F + (1 - \gamma_F)} g_F,$$

$$W = \frac{\sigma\eta}{\alpha[\epsilon\gamma_F + (1 - \gamma_F)]} (g_F + \rho).$$

¹⁵Substituting (25) into $\gamma_F = \hat{\gamma}$ yields the same value of $\hat{\delta}$.

¹⁶We calculate the growth rate and the share of expenditure of Home. These are, respectively,

$$g_H = g_F = \frac{2\alpha(1 - \epsilon\epsilon^*)}{\sigma\eta(2 - \epsilon - \epsilon^*)} - \frac{\sigma - \alpha}{\sigma} \rho,$$

$$\theta_{EH} = \theta_{EF} = \frac{(1 - \epsilon\epsilon^*) + \eta\rho h(2 - \epsilon - \epsilon^*)}{2(1 - \epsilon\epsilon^*) + \eta\rho(2 - \epsilon - \epsilon^*)}.$$

¹⁷If we analyze the cases of $0 < \epsilon^* < 1$ and $0 < \epsilon < 1$, the results are qualitatively the

When $\epsilon^*=1$ and $0 \leq \epsilon \leq 1$, it is clear that $\epsilon^* \geq \epsilon$. Thus, in this case, the location of the innovation sector is described by pattern 1 from Lemma 2. The location of pattern 1 was analyzed in subsection 3.1.¹⁸

Hence, we focus on the cases of $\epsilon^* = 0$ and $0 \leq \epsilon \leq 1$. Substituting $\epsilon^* = 0$ into (26) yields the following value of $\hat{\delta}$:

$$\hat{\delta} = \frac{\epsilon + \eta\rho(2 - \epsilon)\{1 - (2 - \epsilon)h\}}{\epsilon + \eta\rho(2 - \epsilon)\{1 - (2 - \epsilon)(1 - h)\}}. \quad (27)$$

Using $\hat{\delta}$, we restate the relationship between the location of the innovation sector and the transportation costs from (7) and Lemma 1. When $\delta > \hat{\delta}$, the innovation sector is in Home (pattern 1). When $\delta < \hat{\delta}$, the innovation sector is in Foreign (pattern 2).

From (27), we find that $h > 1/2$ implies that $\hat{\delta} < 1$. Thus, we find that, in the economy, cases such as $\hat{\delta} \leq 0$ and $0 < \hat{\delta} < 1$ emerge. We denote the former and the latter as case A and case B, respectively.¹⁹ Now, we have to investigate the sign of $\hat{\delta}$. Since the denominator of $\hat{\delta}$ is positive provided that $0 \leq \epsilon \leq 1$ and $h > 1/2$, the sign of $\hat{\delta}$ is determined by the sign of the numerator of $\hat{\delta}$. Here, we have to examine how the sign of the numerator of $\hat{\delta}$ is determined.

When the numerator of $\hat{\delta}$ is not positive, the following relationship holds:

$$h \geq \hat{h} \equiv \frac{1}{2 - \epsilon} + \frac{\epsilon}{\eta\rho(2 - \epsilon)^2}. \quad (28)$$

From this equation, we find that, when $\epsilon = 0$, $\hat{h} = 1/2$, and, when $\epsilon = 1$, $\hat{h} > 1$ and \hat{h} is an increasing function of ϵ . Next, let us define $\bar{\epsilon}$ as the value associated with $\hat{h} = 1$. We derive the value of $\bar{\epsilon}$ as follows:

$$\bar{\epsilon} \equiv \frac{3\eta\rho + 1 + \sqrt{(3\eta\rho + 1)^2 - 8(\eta\rho)^2}}{2\eta\rho}. \quad (29)$$

same as those in our model. If $\epsilon^* \geq \epsilon$, the innovation sector's location is always pattern 1. If $\epsilon > \epsilon^*$, the relationship between the growth rate and the transportation costs may not be monotonic, as shown later.

¹⁸When $\epsilon = \epsilon^* = 0$, $\hat{\delta} = -1$. Therefore, only pattern 1 arises in the economy. This case corresponds to the case of local spillovers in Martin and Ottaviano (1999). In addition, when $\epsilon = \epsilon^* = 1$, the growth rate is independent of the transportation costs and reaches its maximum value,

$$g_H = g_F = \frac{2\alpha}{\sigma\eta} - \frac{\sigma - \alpha}{\sigma}\rho.$$

This is consistent with the case of global spillovers in Martin and Ottaviano (1999).

¹⁹In case A, only pattern 1 emerges in the economy. In case B, the economy has both location pattern.

It is clear that, if $\epsilon > \bar{\epsilon}$, then $\hat{h} > 1$ because h increases with ϵ . Then, we define $\hat{\epsilon}$ as the value associated with $\hat{\delta} = 0$. Deriving $\hat{\epsilon}$, it is shown that, from $0 \leq \epsilon \leq 1$,

$$\hat{\epsilon} = \frac{(1 - \eta\rho + 4\eta\rho h) - \sqrt{(1 - \eta\rho + 4\eta\rho h)^2 - 8(\eta\rho)^2 h(2h - 1)}}{2\eta\rho h}. \quad (30)$$

We find that $0 < \hat{\epsilon} < 1$. Here, we show from (27) that, when $\epsilon = 0$, $\hat{\delta} = -1$ and, when $\epsilon = 1$, $0 < \hat{\delta} < 1$. In addition, $\partial\hat{\delta}/\partial\epsilon > 0$. Thus, it is found that, if $\epsilon > \hat{\epsilon}$, then $0 < \hat{\delta} < 1$.

In short, (28) implies that, if Home's share of patent holdings is high, the economy is in case A. This is because there is high-income inequality due to high inequality. However, a higher ϵ directly reduces the innovation cost in Foreign. Therefore, the tendency that the innovation sector is in Foreign is strong. (29) implies that, for any value of $1/2 < h \leq 1$, case A is not observed, provided $\epsilon > \bar{\epsilon}$. From (30), it is shown that, if $\epsilon > \hat{\epsilon}$, the economy is in case B due to that high ϵ leads to a reduction in the innovation cost in Foreign. We show that, if ϵ is sufficiently low and if the share of patent holding in Home is high, the economy is in case A.

Consider the relationship between $\hat{\delta}$ and h . It is clear that $\partial\hat{\delta}/\partial h < 0$ and, when $h = 1/2$, $\hat{\delta} = 1$. This implies that, if h is high, higher transportation costs are necessary for a decline in the innovation cost in Foreign. This is because, when the transportation costs are high, differentiated firms relocate in Foreign, which leads to a reduction in the innovation cost in Foreign. Thus, it is possible for the innovation sector to locate in Foreign due to an increase in the transportation costs. In addition, it can be shown that an increase in ϵ raises $\hat{\delta}$ from (27). In particular, if $\epsilon > \bar{\epsilon}$, the economy is in case B for any values of $1/2 < h \leq 1$.

Given Lemmas 1 through 4 and the discussion in this section, we present the following proposition, which summarizes the main results of our model.

Proposition 1 *With the assumption $h > 1/2$, the following conditions are satisfied:*

- *The economy is in case A when $h \geq \hat{h}$, when $\epsilon^* \geq \epsilon$, or when $\epsilon^* = 0$ and $\epsilon \leq \hat{\epsilon}$. In case A economy, the equilibrium growth rate is g_H , and g_H is an increasing function of Home's share of the patent and a decreasing function of the transportation costs.*
- *The economy is in case B when $\epsilon > \bar{\epsilon}$ or when $\epsilon > \hat{\epsilon}$. In case B economy, the growth rate is not monotonic.*

- When the innovation sector is in Home, the equilibrium growth rate is g_H , and g_H is an increasing function of Home's share of patents and a decreasing function of the transportation costs.
 - When the innovation sector is in Foreign, the equilibrium growth rate is g_F , and g_F is an increasing function of the transportation costs, the ability of Foreign to absorb international knowledge spillovers, and a decreasing function of Home's share of patent holdings.
- When $\delta \geq \bar{\delta}$, g_H is constant at the maximum growth rate, $g_H = (2\alpha/\sigma\eta) - (\sigma - \alpha)\rho/\sigma$, among all potential growth rates.

Here, we investigate the relationship between the equilibrium growth rate and the transportation costs. If case A is observed, the equilibrium growth rate monotonically increases with the transportation costs. This case is illustrated in Figure 2.

However, if case B is observed, the relationship between the equilibrium growth rate and the transportation costs is not monotonic. It must be recalled that, when $\hat{\delta} < \delta$, the innovation sector is in Home, and, when $\hat{\delta} > \delta$, the innovation sector is in Foreign. The lower the transportation costs are, the higher the equilibrium growth rate is, provided that $\hat{\delta} < \delta$. However, lower transportation costs reduce the equilibrium growth rate if $\hat{\delta} > \delta$. This is because lower transportation costs increase the agglomeration of firms in Home. The agglomeration has different effects on the innovation cost in each country. Thus, the effect of the transportation costs on the equilibrium growth rate changes at $\delta = \hat{\delta}$. This relationship is illustrated in Figure 3.

Next, we show the effects of a change in ϵ on the equilibrium growth rate. It must be recalled that Lemma 4 and $\partial\hat{\delta}/\partial\epsilon > 0$. This implies that, if the ability of Foreign to absorb international knowledge spillovers improves, it is easier for the innovation sector to locate in Foreign. As Lemma 4 shows, an increase in ϵ raises the equilibrium growth rate provided that $\delta < \hat{\delta}$. However, it is clear from (13) and (14) that, although ϵ varies, the equilibrium growth rate is constant if the innovation sector is in Home. Moreover, when $\delta = \hat{\delta}$, $g_H = g_F$.²⁰ This discussion reveals the effect of an increase in ϵ on the relationship between the equilibrium growth rate and the transportation

²⁰We find that, when $\gamma = \hat{\gamma}(\delta = \hat{\delta})$,

$$g_H = g_F = \frac{2\alpha(1 - \epsilon\epsilon^*)}{\sigma\eta(2 - \epsilon - \epsilon^*)} - \frac{\sigma - \alpha}{\sigma}\rho$$

costs when the economy is observed in case B. This is illustrated in Figure 4.

Next, consider the effects of a change in h on $\hat{\delta}$, $\bar{\delta}$, and the equilibrium growth rate. From (39) and (21), we show that, if Home's share of patent holdings increases, it is easier for the innovation sector to locate in Home, and for the economy to achieve its maximum growth rate. As Lemma 4 shows, in pattern 2, the equilibrium growth rate, g_F , is a decreasing function of h . Due to these effects, the equilibrium growth rate falls when the location of the innovation sector is in pattern 2, g_F . On the contrary, the equilibrium growth rate rises when the location of the innovation sector is g_H , as shown in Lemma 3. The effect of an increase in h is illustrated in Figure 5.

Next, we discuss some implications for welfare in the two countries. Price indices for differentiated goods in Home and Foreign are as follows:

$$G_H = N^{\frac{1}{\sigma-1}} \frac{\sigma\beta w}{\sigma-1} (\gamma + (1-\gamma)\delta)^{\frac{1}{\sigma-1}}, \quad G_F = N^{\frac{1}{\sigma-1}} \frac{\sigma\beta w}{\sigma-1} (\gamma\delta + (1-\gamma))^{\frac{1}{\sigma-1}}, \quad (31)$$

where G_H and G_F represent the price indices for Home and Foreign, respectively. It is clear that G_H is a decreasing function of γ and that G_F is an increasing function of γ . Thus, with respect to the price index, agglomeration in Home improves welfare in Home and reduces welfare in Foreign. This is because shipping differentiated goods incurs transportation costs in our model. It must be noted that, if there are no transportation costs, both G_H and G_F are independent of γ . Given this discussion, we refer to these effects of the transportation costs as "transportation cost effects."

Clearly, welfare in Home improves as agglomeration increases in Home provided that $\delta \geq \hat{\delta}$ because of the high growth rate and the low price index in Home. In particular, when $\delta \geq \bar{\delta}$, the economy achieves its maximum growth rate and $\gamma = 1$, and welfare in Home is maximized. However, when $\delta < \hat{\delta}$, the effect of agglomeration in Home on welfare in Home is ambiguous. The reason is as follows. The equilibrium growth rate decreases as agglomeration in Home increases, which lowers Home's welfare. However, increased transportation costs raise Home's welfare as agglomeration increases in Home.

Welfare in Foreign decreases as agglomeration in Home increases provided that $\delta < \hat{\delta}$. In this range, the equilibrium growth rate decreases with increased agglomeration in Home. When $\delta \geq \hat{\delta}$, the equilibrium growth rate increases with increased agglomeration in Home. This raises welfare in Foreign. However, higher transportation costs reduce Foreign's welfare as agglomeration in Home increases.

Our results have important implications, for example, for trade policy. When the ability of a small-market country to absorb international knowl-

edge spillovers is high and there is little international income inequality, the economy has two location patterns of the innovation sector, and the growth rate does not monotonically increase as the transportation costs decline. In this case, if this economy initially has high transportation costs, the innovation sector locates in that country. As the transportation costs decrease, the growth rate decreases. When the transportation costs continue to decrease and are below some critical level, the innovation sector relocates from the small-market country to a large-market country. Once the innovation sector relocates to a large-market country as the agglomeration of firms producing differentiated goods increases in this country, the growth rate increases as the transportation costs decrease.²¹

Considering a real economy, Komiya et al. (1984) argued that the Japanese government regulated trade to protect and expand the domestic industry. By doing so, Japan accomplished industrialization and a high growth rate between the 1950s and the 1970s. In addition, Bernstein and Mohnen (1994), Coe and Helpman (1995), and Eaton and Kortum (1996) reported that, between the 1960s and the 1980s, there were substantial international knowledge spillovers from the U.S. to Japan. Our results suggest that, at that time, the Japanese economy was described by pattern 2 and that highly regulated trade and the Japan's ability to absorb international knowledge spillovers led to technological development in Japan and contributed to its high economic growth between the 1950s and the 1970s.

However, since the 1980s, international trade has been liberalized in Japan and has not intensively innovated.²² Our results suggest that, after the 1980s, the economy was in pattern 1 and that low transportation costs and Japan's reduced ability to absorb international knowledge spillovers led the innovation sector to locate in the U.S. From our result, in that pattern, as the transportation costs fall, it is more favorable for the innovation sector to operate in a country that has more advanced technology (the U.S.). In fact, after the 1980s, relative to the U.S., Japan's R&D sector has not performed well.

²¹These results explain only one of the many mechanisms affecting the relationship between the transportation costs and the growth rate. For example, Yamamoto (2003) has considered other mechanisms.

²²Coe and Helpman (1995) found that the elasticity of total factor productivity with respect to foreign R&D in Japan decreased from 0.037 to 0.027 between 1980 and 1990. Bernstein and Mohnen (1994) found that, recently, intensive R&D has not been taking place in Japan.

4 Conclusions

In this paper, we have constructed a model in which knowledge spillovers are partially local and international knowledge spillovers are asymmetric. Partially local and asymmetric international spillovers generate particular patterns of growth and agglomeration in the economy.

The effect on the growth of agglomeration has been analyzed using partially local knowledge spillovers with agglomeration: agglomeration in a country reduces the innovation cost in that country and stimulates growth. With agglomeration, the interaction between the transportation costs and the extent of international knowledge spillovers leads the innovation sector to relocate. The effect of a reduction in the transportation costs on the growth rate is not monotonous due to this relocation. In our model, in the context of a growing economy, two key factors are the ability of the country to absorb international knowledge spillovers and the level of the transportation costs. By focusing on these two points, our model analysis of growth due to agglomeration has provided numerous implications.

In the modern world, local knowledge spillovers play an important role, and the development of telecommunications technology has facilitated international knowledge spillovers. Thus, an investigation of the influence of international knowledge spillovers on the economy is worthwhile. Our study has contributed to this objective.

References

- [1] Baldwin, R. E., 1999, Agglomeration and endogenous capital. *European Economic Review* 43, 253-280.
- [2] Baldwin, R. E., P. Martin, and G. I. P. Ottaviano, 2001, Global income divergence, trade, and industrialization: The geography of the growth take-offs. *Journal of Economic Growth* 6, 5-37.
- [3] Barro, R., and X. Sala-i-Martin, 1997, Technological diffusion, convergence, and growth. *Journal of Economic Growth* 2, 1-26.
- [4] Bernstein, J. I., and P. Mohnen, 1994, International R&D spillovers between U.S. and Japanese R&D intensive sectors. NBER Working Paper #4682.
- [5] Coe, D. T., and E. Helpman, 1995, International R&D spillovers. *European Economics Review* 39, 859-887.

- [6] Eaton, J., and S. Kortum, 1996, Trade in ideas: Patenting and productivity in the OECD. *Journal of International Economics* 40, 251–278.
- [7] Fujita, M., and J.-F. Thisse, 2003, Does geographical agglomeration foster economic growth, and who gains and loses from it? *Japanese Economic Review* 54, 121–145.
- [8] Grossman, G., and E. Helpman, 1991, *Innovation and growth in the world economy* (MIT Press, Cambridge, MA).
- [9] Komiya, R., M. Okuno, and K. Suzumura, 1984, *Nihonnosangyou-seisaku, (Industrial policies in Japan)* (University of Tokyo Press, Tokyo). in Japanese
- [10] Martin, P., 1999, Public policies, regional inequalities, and growth. *Journal of Public Economics* 73, 85–105.
- [11] Martin, P., and G. I. P. Ottaviano, 1999, Growing locations: Industry location in a model of endogenous growth. *European Economic Review* 43, 281–302.
- [12] Romer, P., 1990, Endogenous technical change. *Journal of Political Economy* 98, 71–102.
- [13] Samuelson, P., 1954, The transfer problem and transport costs, : Analysis of effects of trade impediments. *Economic Journal* 64, 264–289.
- [14] Yamamoto, K., 2003, Agglomeration and growth with innovation in the intermediate goods sector. *Regional Science and Urban Economics* 33, 335–360.

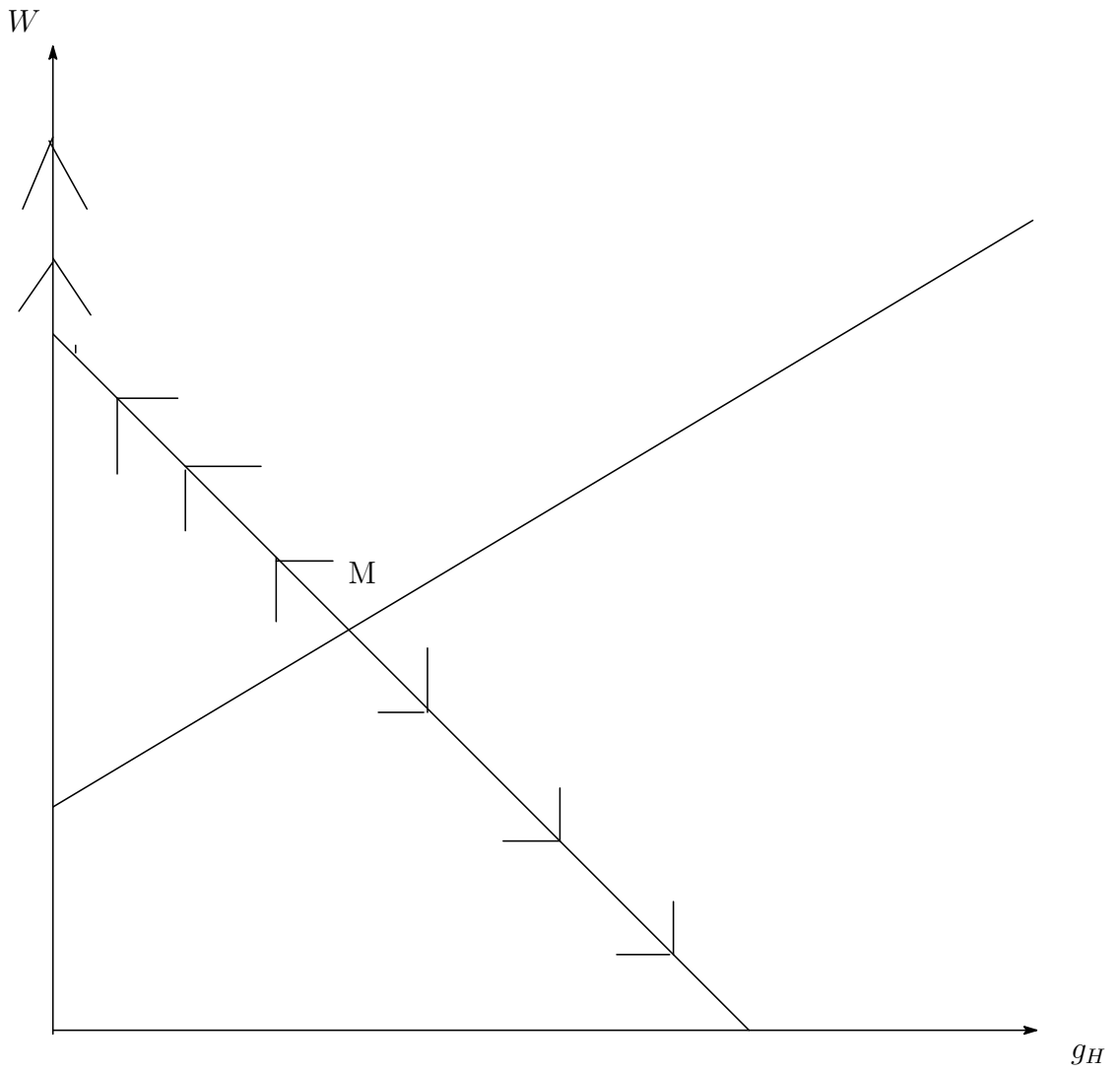


Figure 1:

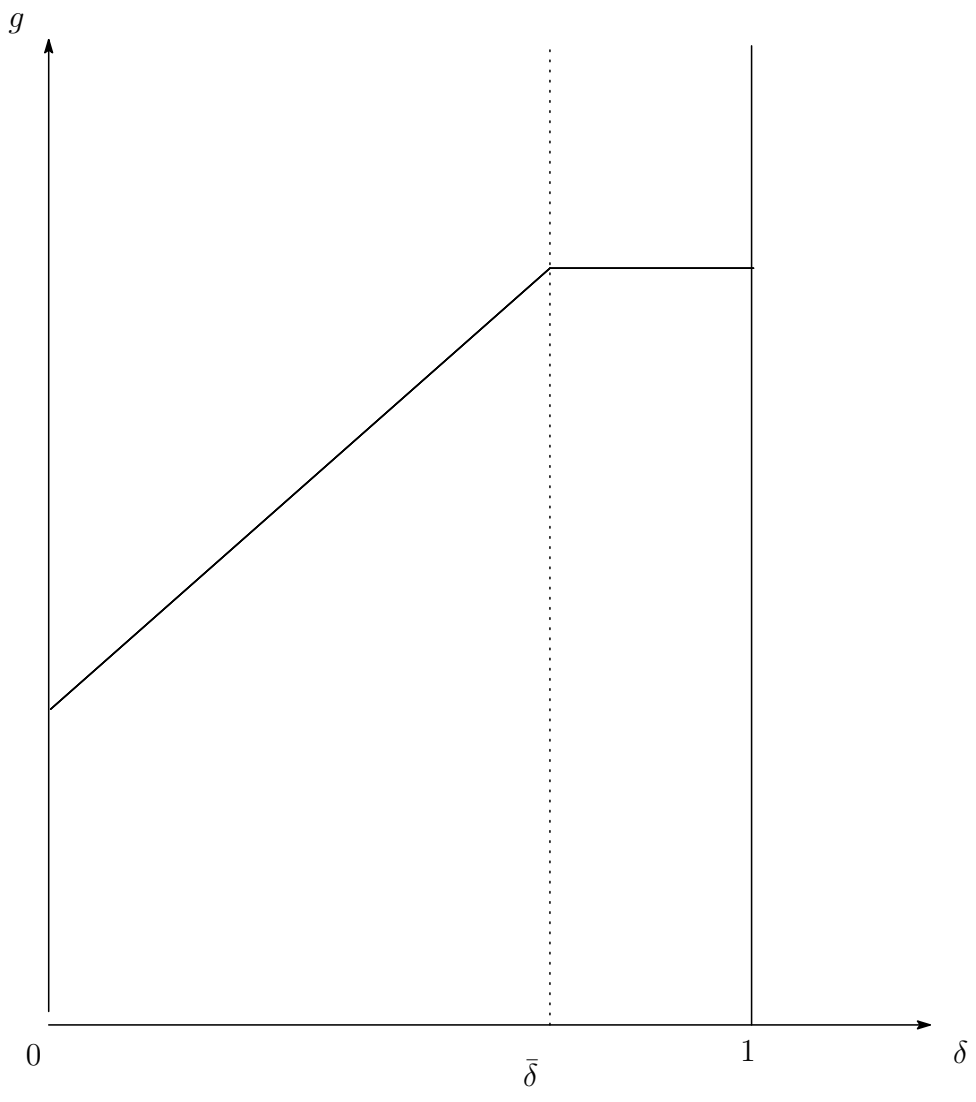


Figure 2:

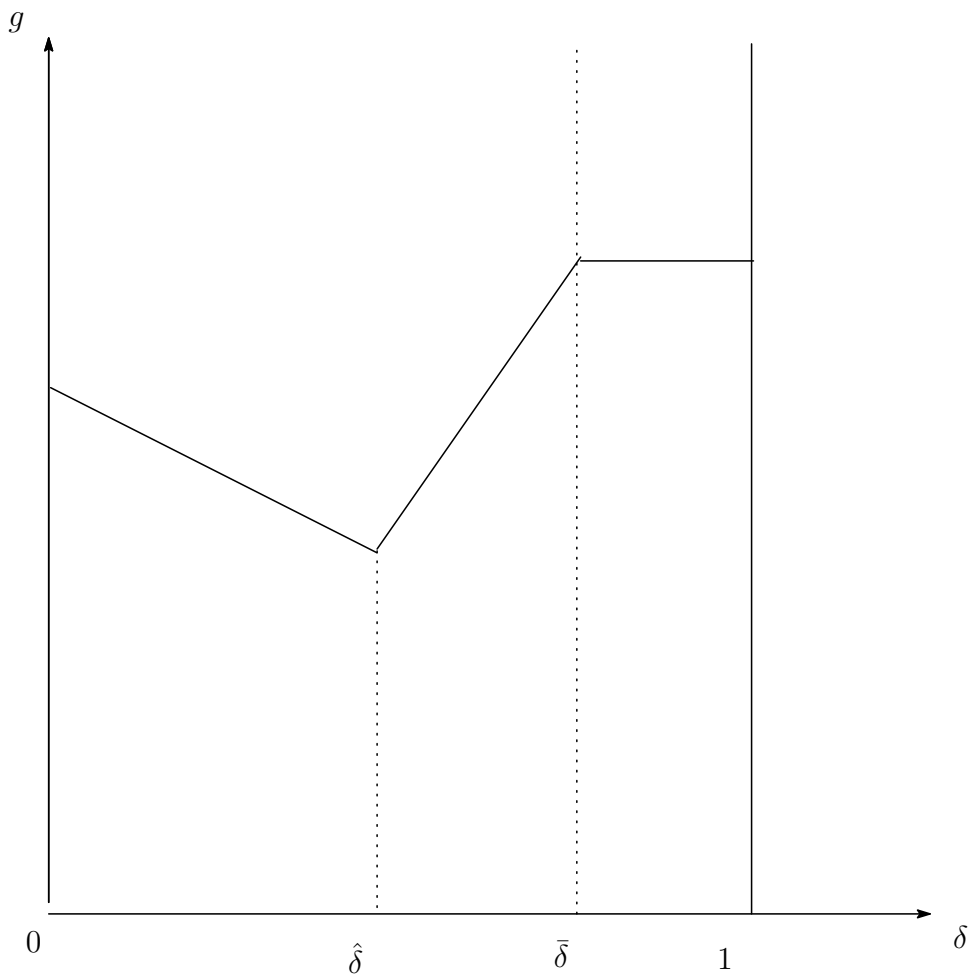


Figure 3:

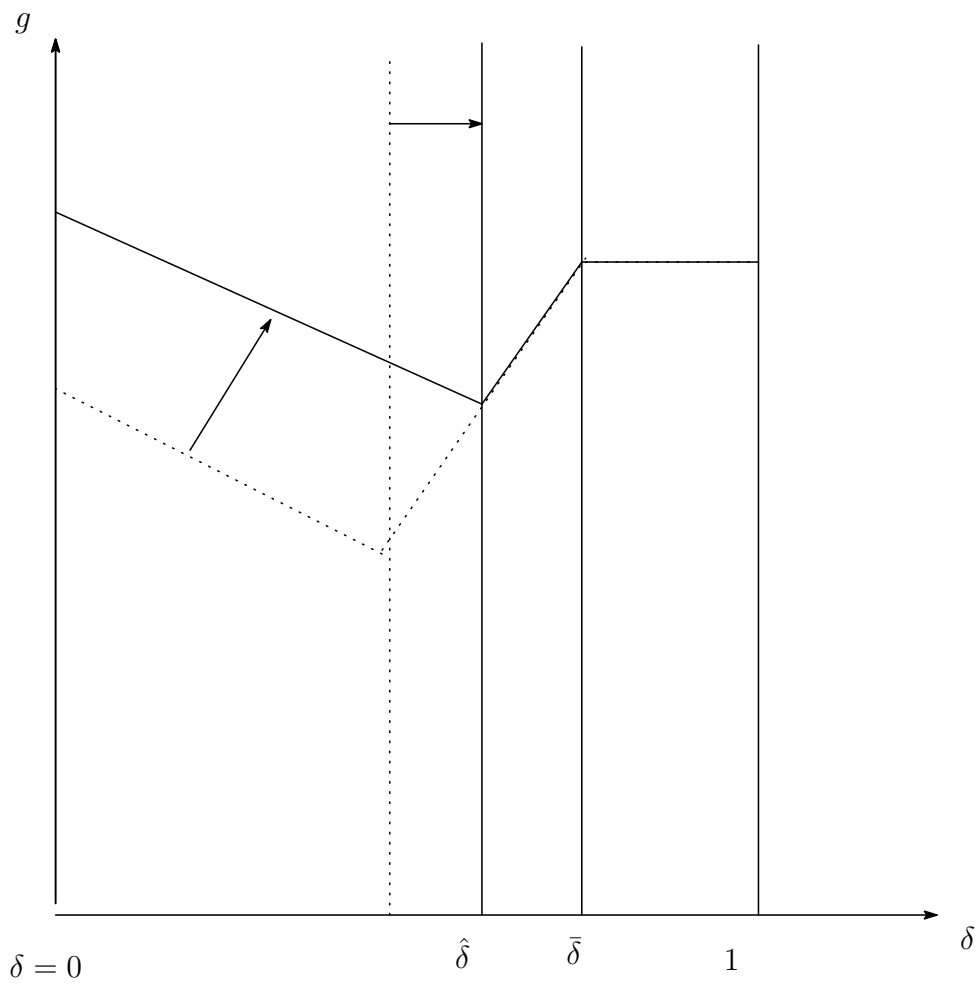


Figure 4:

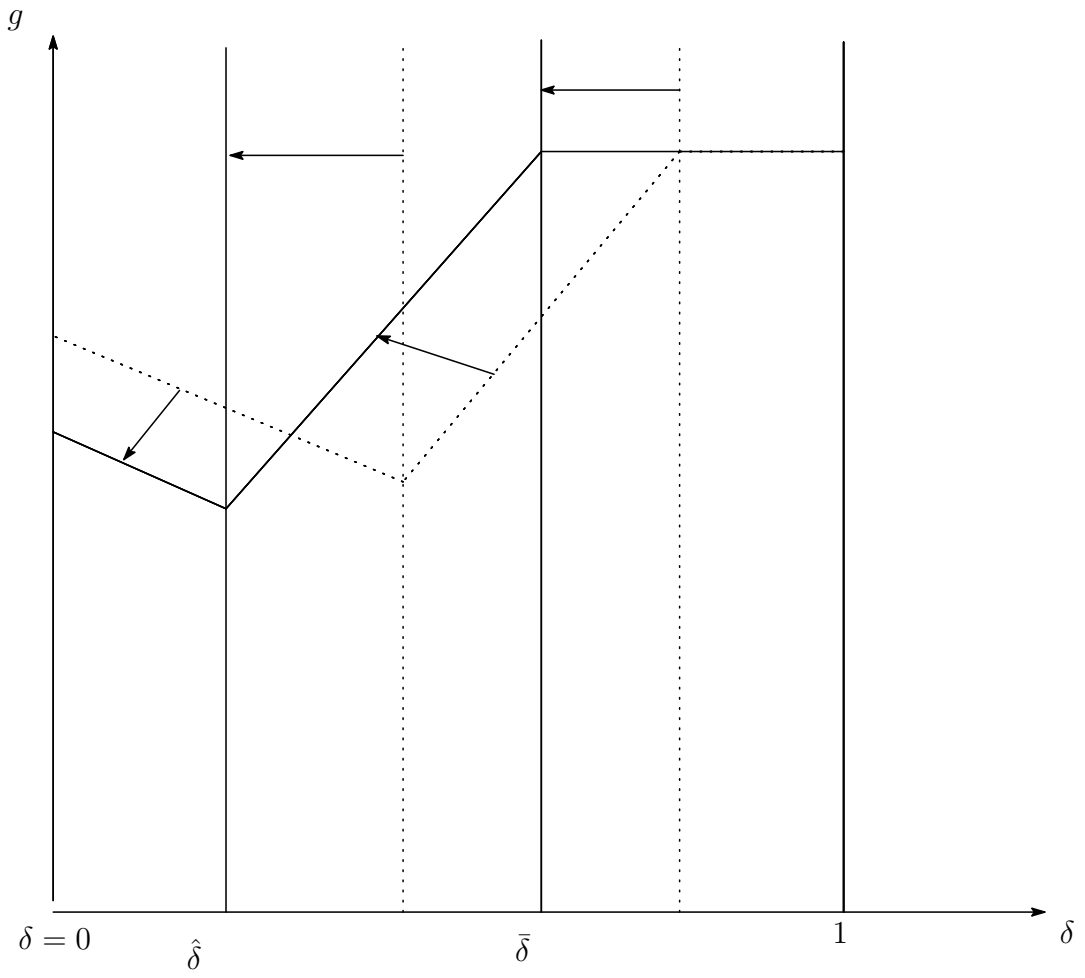


Figure 5: