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Osaka University, Toyonaka, Osaka 560-0043, JAPAN

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Junko Doi† and Kazuo Mino‡

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Abstract

This paper introduces external habit formation into one of the basic models of endogenous growth in which continuing expansion of product variety sustains long-term growth. We assume that households consume a range of final goods and they set a benchmark level of consumption for each good. The benchmark consumption is determined by external habit formation so that there are commodity-specific external effects. Each good is produced by a monopolistically competitive firm and the firm's optimal pricing decision exploits the fact that consumers' demand is subject to the external habit formation. Given those settings, we show that the introduction of consumption externalities may affect the balanced-growth characterization, transitional dynamics as well as policy impacts in fundamental manners.

JEL Classification code: E2, O3, O4

Keywords: consumption externalities, habit formation, monopolistic competition, R&D-based growth model

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†Kyoto Sangyo University, Motoyama, Kamigamo, Kita-ku, Kyoto, 603-8555, Japan (phone: 81-75-705-1768, e-mail: jdoi@cc.kyoto-su.ac.jp)

‡Corresponding Author, Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560 0043, Japan. E-mail: mino@econ.osaka-u.ac.jp.

1 Introduction

In the recent development of growth economics, external effects in production and knowledge creation have played a pivotal role. Seminal contributions to the endogenous growth theory such as Aghion and Howitt (1992), Lucas (1988) and Romer (1986 and 1990) have emphasized the effects of production externalities and knowledge spillovers that may sustain persistent long-term growth. In contrast, the role of externalities in consumption activities has been a minor topic in growth theory. While consumption externalities have been taken seriously by the researchers in the fields of asset pricing, business cycles and fiscal policy (for example, Abel 1990, Galí 1994 and Ljungqvist and Uhlig 2000), growth economists in general have not paid much attention to the presence of external effects in the consumption side of the economy.

The recent contributions by Alonso-Carrera et al. (2004 and 2005), Carroll et al. (1977 and 2000), Alvarez-Cuadrado et al. (2004) and Liu and Turnovsky (2005), however, have kindled a renewed interest in the role of consumption externalities in growing economies. Alonso-Carrera et al. (2004 and 2005), Alvarez-Cuadrado et al. (2004) and Liu and Turnovsky (2005) examine implications of consumption external effects in the standard neoclassical growth models. The main research concern of those authors is to explore the effects of interdependency among consumers on welfare and transitional dynamics of the economy. Carroll et al. (1997 and 2000) examine the roles of external as well as internal habit formation in an endogenous growth model with an Ak technology and analyze how the presence of consumption externalities affects savings and the pattern of growth.[1] Those studies have clearly demonstrated that external effects of consumption may have significant implications for growing economies in both qualitative and quantitative senses.

Departing from the existing studies on the effects of consumption externalities in growing economies, we investigate the role of consumption external effects in the context of a variety-expansion model of growth. The analytical framework of this paper is based on Grossman and Helpman (1991, Chapter 3). In our setting, there are a variety of consumption goods and each commodity is produced by a monopolistically competitive firm. The range of

[1] Harbaugh (1996) also discusses the relation between growth and saving in the presence of consumption externalities by using a two-period model with uncertainty.

consumption goods variety is enhanced by R&D activities.[2] Those assumptions enable us to introduce two distinctive features of consumption external effects that have not been considered in the existing literature assuming a homogenous consumption good and perfect competition. First, we may assume that consumers set a benchmark consumption level for each good. The benchmark level of each consumption good is determined by external (outward-looking) habit formation so that there exist commodity-specific external effects. Second, since each commodity is produced by a monopolist, the firm may exploit the fact that consumer's demand for its own product is affected by the commodity-specific external effect. This means that the firm can internalize the consumption external effect when maximizing its profits. As a result, the firm's marginal cost involves the implicit 'internalization costs' of the consumption external effects, and hence the pricing decision of the firm is affected by the behavior of benchmark level of consumption set by the consumers. The basic idea of this kind of modelling has been proposed by Ravn et al. (2002 and 2006) who examine the effects of commodity-specific consumption externalities in a real business cycle model with monopolistic competition.[3] In this paper, we consider the implications of consumption external effects in an imperfectly competitive, growing economy.

Given the analytical framework described above, we explore the balanced-growth equilibrium and transitional dynamics. We find that the presence of consumption externalities may yield significant effects on the balanced-growth-path characterization as well as on equilibrium dynamics of the model economy. First, if there are negative external effects, that is, each consumer's felicity is negatively related to the benchmark level of average consumption of the economy at large, then there generally exists a unique, feasible balanced-growth path that satisfies saddlepoint stability. In contrast, if the externalities positively affect the individual felicity, the local behavior of the economy around the balanced-growth equilibrium exhibits either saddlepoint stability or local indeterminacy. Additionally, if both cases establish saddle stability, the behaviors of key variables such as the rate of technical change may be different depending on whether external effects are negative or positive. Second, the

[2] See Gancia and Zilbotti (2005) for a detailed survey on this class of models.

[3] Since Ravan et al. (2002 and 2006) explore real business cycles in the context of a stochastic dynamic general equilibrium framework, their discussion riles on a model calibration. In contrast, we use a simpler deterministic, continuous-time model of growth, which enables us to study the behavior of the model economy analytically.

policy implications obtained in our model can be quite different from those established in the original Grossman and Helpman model. For example, in our framework a policy that stimulates R&D investment does not necessarily promote long-term growth. In addition, due to the presence of consumption external effects, the level of R&D spending determined in the competitive equilibrium may not be lower than its optimal level that attains the efficient resource allocation.

The reminder of the paper is organized as follows. Section 2 constructs the analytical framework. Section 3 derives a complete dynamic system. Section 4 examines the balanced-growth equilibrium and investigates equilibrium dynamics out of the steady state. Section 5 considers the effects of R&D subsidy and the socially optimal level R&D spending. A brief conclusion is given in Section 6.

2 The Model

2.1 Households

There is a continuum of identical households whose number is normalized to one. The representative household consumes a variety of consumption goods, ranging from index 0 to n. We assume that the consumer's felicity depends not only on her own consumption of each good but also on the benchmark level of consumption that is determined by outward-looking habit formation. The instantaneous sub-utility of the household is given by

$$C = \left(\int_0^n \left(c_i s_i^{-\theta} \right)^{\frac{\alpha-1}{\alpha}} di \right)^{\frac{\alpha}{\alpha-1}}, \quad \theta < 1, \quad \alpha > 1, \tag{1}$$

where c_i is consumption of good $i \in [0, n]$ and α denotes the elasticity of substitution between consumption goods. Here, s_i is the household's benchmark level of consumption of good i, which represents a commodity-specific external habit formation. More specifically, we assume that s_i is a weighted sum of the average consumption of good i up to the present period:

$$s_i(t) = \beta \int_{-\infty}^{t} e^{-\beta(\tau - t)} \bar{c}_i(\tau)\, d\tau, \quad \beta > 0,$$

where $\bar{c}_i(\tau)$ denotes the average consumption of good i in the economy at large. This gives the following dynamic equation of s_i:

$$\dot{s}_i = \beta \left(\bar{c}_i - s_i \right), \quad \beta > 0. \tag{2}$$

Note that the instantaneous utility of consumption of good i can be written as

$$c_i s_i^{-\theta} = c_i^{1-\theta}\left(\frac{c_i}{s_i}\right)^{\theta}; \quad \theta \neq 0, \quad \theta < 1.$$

This shows that the felicity obtained by consuming the i-th good depends on the relative consumption, c_i/s_i, as well as on the absolute level of consumption, c_i. While it is usually assumed that θ has a positive value, in this paper we do not specify the sign of θ. If θ is positive, a rise in the benchmark consumption, s_i, negatively affects the felicity of consumer. Namely, each consumer's preference exhibits jealousy as to consumption of others. In contrast, if θ is negative, then the felicity of consumer increases with the benchmark consumption. In this case consumers' preferences show admiration for consumption of other members in the society.[4] It is also to be noted that if $\beta = +\infty$, then $s_i = \bar{c}_i$ so that the external effects are only intratemporal. In addition, if $\theta = 0$, then each consumer's preference becomes the standard one in which her felicity depends on the absolute levels of private consumption alone.

Given (1), the households maximizes a discounted sum of subutilities

$$U = \int_0^{\infty} e^{-\rho t} \log C dt, \quad \rho > 0,$$

subject to the flow budget constraint:

$$\dot{a} = ar + wN - \int_0^n c_i p_i di, \tag{3}$$

where a denotes the asset holding of the household, w is the real wage rate and p_i denotes the price of consumption good i. We assume that in each moment the representative household supplies N units of labor inelastically. Notice that the habit formation is external to an individual household, so that when deciding her optimal plan, the household takes the future sequence of benchmark consumption, $\{s(t)\}_{t=0}^{\infty}$, as given.

Denoting $\hat{c}_i = c_i s_i^{-\theta}$ and $\hat{p}_i = p_i s_i^{\theta}$, we first consider the following cost minimization problem:

$$\min \int_0^n \hat{c}_i \hat{p}_i di$$

subject to

$$\left(\int_0^n \hat{c}_i^{\frac{\alpha-1}{\alpha}} di\right)^{\frac{\alpha}{\alpha-1}} = C.$$

[4]See Dupor and Lin (2003) for a useful taxonomy as to formulating consumption externalities.

Solving this problem gives the demand function such that

$$\hat{c}_i = \left(\frac{\hat{P}}{\hat{p}_i}\right)^{\alpha} C,$$

where

$$\hat{P} \equiv \left(\int_0^n \hat{p}_i^{1-\alpha} di\right)^{\frac{1}{1-\alpha}} \tag{4}$$

denotes a price index of the subutility (aggregate consumption), C. The demand equation of good i is thus given by

$$c_i = s_i^{\theta(1-\alpha)} \left(\frac{\hat{P}}{p_i}\right)^{\alpha} C. \tag{5}$$

This equation states that given prices and the composite consumption, C, the demand for good i decreases with s_i if $\theta > 0$. When $\theta < 0$ (so that each consumer has admiration for other consumers), a higher s_i increases c_i.

Now define

$$E \equiv \min \int_0^n c_i p_i di = \hat{P}^{\alpha} C \int_0^n s_i^{\theta(1-\alpha)} p_i^{1-\alpha} di = \hat{P} C.$$

This yields

$$C = E/\hat{P}. \tag{6}$$

As a result, the optimization problem can be expressed as

$$\max \int_0^{\infty} e^{-\rho t} \left[\log E - \log \hat{P}\right] dt$$

subject to

$$\dot{a} = ra + wN - E.$$

The optimization conditions for this problem gives the Euler equation,

$$\frac{\dot{E}}{E} = r - \rho, \tag{7}$$

together with the transversality condition:

$$\lim_{t \to \infty} (a/E)\, e^{-\rho t} = 0.$$

As for choosing a numeraire, we follow Grossman and Helpman (1991). We normalize prices so that nominal spending, E, remains constant over time. Thus, by setting $E = 1$ for all $t \geq 0$, from (7) the real interest rate, r, equals the time discount rate in every moment:

$$r = \rho. \tag{8}$$

2.2 Producers

Each consumption good is produced by a monopolistically competitive firm. The profits of the firm producing consumption good i are given by

$$\pi_i = p_i c_i - wbc_i, \quad b > 0.$$

The firm produces by using labor alone and the production function of good i is assumed to be $c_i = (1/b)\, l_i$, where l_i is labor devoted to production of the i-th good. From the demand function (5), π_i becomes

$$\pi_i = s_i^{\theta(1-\alpha)} \hat{P}^{\alpha} C \left[p_i^{1-\alpha} - wbp_i^{-\alpha} \right]. \tag{9}$$

Following Ravn et al. (2006), we assume that the firm exploits the fact that consumers' demand behavior is affected by the benchmark consumption level, s_i, and that s_i changes according to (2). This means that the firm maximizes a discounted sum of its profits over an infinite-time horizon subject to (2). The optimization behavior of the firm is thus formulated as follows:

$$\max \int_0^{\infty} \exp\left(- \int_0^t r(\xi)\, d\xi \right) \pi_i(t)\, dt$$

subject to (9) and

$$\dot{s}_i = \beta \left[s_i^{\theta(1-\alpha)} \hat{P}^{\alpha} C p_i^{-\alpha} - s_i \right], \tag{10}$$

where $s_i(0)$ is given. In this problem, the firm's control and state variables are p_i and s_i, respectively.

To derive the optimization conditions, let us set up the following Hamiltonian function:

$$H_i = s_i^{\theta(1-\alpha)} \hat{P}^{\alpha} C \left[p_i^{1-\alpha} - wbp_i^{-\alpha} \right] + \lambda_i \beta \left[s_i^{\theta(1-\alpha)} \hat{P}^{\alpha} C p_i^{-\alpha} - s_i \right],$$

where λ_i is the shadow value of the benchmark consumption level, s_i. Maximizing the Hamiltonian function with respect p_i, we obtain

$$(1-\alpha)\, p_i^{-\alpha} + bw\alpha p_i^{-\alpha-1} - \lambda_i \beta \alpha p_i^{-a-1} = 0.$$

This yields the optimal pricing formula in such a way that

$$p_i = \frac{\alpha}{\alpha - 1} (bw - \beta \lambda_i). \tag{11}$$

Equation (11) means that the price of good i equals the marginal cost of labor input, bw, plus the shadow cost of habit formation, $-\beta\lambda_i$, multiplied by a coefficient, $\alpha/(\alpha-1)$. Since production needs labor alone and λ_i is an implicit cost for the firm, in the conventional expression (11) may be written as

$$p_i = \frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda_i}{bw}\right)bw.$$

Therefore, if we take into account of the explicit labor costs, bw, alone, the markup ratio is represented by $\frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda_i}{bw}\right)$ so that it changes with the relative costs, $(-\beta\lambda_i)/bw$. In the standard formulation without consumption externalities, the markup formula is given by $p_i = \alpha wb/(\alpha-1)$, which has a constant markup rate, $\alpha/(\alpha-1)$. The endogenous markup ratio in our setting will be one of the sources that make the analytical results diverge from those obtained in the original Grossman and Helpman model.

The shadow value λ_i changes according to

$$\begin{aligned}\dot{\lambda}_i &= r\lambda_i - \frac{\partial H_i}{\partial s_i}\\ &= (r+\beta)\lambda_i - \theta(1-\alpha)s_i^{\theta(1-\alpha)-1}\hat{P}^{\alpha}C\left[p_i^{1-\alpha} - bwp_i^{-\alpha} + \lambda_i\beta p_i^{-a}\right]. \qquad (12)\end{aligned}$$

The solution of (12) is expressed as

$$\begin{aligned}\lambda(t) &= \int_t^{\infty}\left\{\exp\left(-\int_t^{\tau}(r(\xi)+\rho)d\xi\right)\theta(1-\alpha)s_i(\tau)^{\theta(1-\alpha)-1}\hat{P}^{\alpha}(\tau)C(\tau)\right.\\ &\quad \left.\times\left[p_i(\tau)^{1-\alpha} - bw(\tau)p_i(\tau)^{-\alpha} + \lambda\beta p_i(\tau)^{-a}\right]\right\}d\tau.\end{aligned}$$

Since $\alpha > 1$, the sign of $\theta(1-\alpha)s_i^{\theta(1-\alpha)-1}\hat{P}^{\alpha}C\left[p_i^{1-\alpha} - bwp_i^{-\alpha} + \lambda\beta p_i^{-a}\right]$ is negative (resp. positive) if $\theta > 0$ (resp. $\theta < 0$).[5] Hence, $\lambda(t) \leq 0$ for all $t \geq 0$ if $\theta > 0$, while $\lambda(t) \geq 0$ for all $t \geq 0$ if $\theta < 0$. If the firm i sells an additional unit of product, then an increase in consumption of good i raises the benchmark consumption, s_i, which represents the weighted average of

[5] Note that from the first-order condition

$$(1-\alpha)p_i^{-\alpha} + bw\alpha p_i^{-\alpha-1} - \lambda_i\beta\alpha p_i^{-a-1} = 0.$$

Hence, we see that

$$\begin{aligned}&p_i^{1-\alpha} - bwp_i^{-\alpha} + \lambda\beta p_i^{-a}\\ =\ &(1/\alpha)p_i\left(\alpha p_i^{-\alpha} - \alpha bwp_i^{-\alpha-1} + \lambda_i\alpha\beta p_i^{-a-1}\right) = p_i^{1-\alpha}/\alpha > 0.\end{aligned}$$

past consumption of good i in the economy at large. When $\theta > 0$, such an increase in s_i will lower the future consumption demand for good i. Therefore, an increment in production of good i yields two types of additional costs: the marginal cost of labor employment, bw, and the marginal penalty cost, $-\beta\lambda_i$, that counts the expected reduction of future consumption demand for good i due to the marginal increase in s_i. In contrast, when $\theta < 0$, a rise in s_i has a positive impact on the future consumption demand. Thus the marginal cost of production equals the marginal labor cost minus the marginal benefit of production expansion, $\beta\lambda_i\,(> 0)$.

Notice that from (11) we obtain $p_i - bw + \beta\lambda = p_i/\alpha$. Thus (12) is written as

$$\dot{\lambda}_i = (r + \beta)\,\lambda_i - \theta\left(\frac{1}{\alpha} - 1\right) s_i^{\theta(1-\alpha)-1}\hat{P}^{\alpha} C p_i^{1-\alpha}. \tag{13}$$

2.3 R&D

The research and development sector is assumed to be competitive. R&D activities enhance variety of consumption goods by using labor. The production function of the R&D firm is given by

$$\dot{n} = \delta L_R n, \quad \delta > 0, \tag{14}$$

where L_R denote labor input for R&D activities.[6] Denoting the patent price by v, we see that the zero-excess-profit condition for the R&D sector, i.e. $v\dot{n} - wL_R = 0$, gives

$$w = \delta n v. \tag{15}$$

We assume that the patent length is infinite. Since in the monopolistically competitive final good markets the zero-excess-profit condition holds, the patent price paid by the monopolist is equal to the discounted present value of its profits:

$$v\,(t) = \int_t^{\infty} \exp\left(-\int_t^{\tau} r\,(s)\,ds\right) \pi_i\,(\tau)\,d\tau,$$

where r is the real interest rate. The above condition yields

$$\frac{\dot{v}}{v} = r - \frac{\pi_i}{v}. \tag{16}$$

[6] As usual, we assume that n in the right-hand side of (14) represents external spilliovers of the existing knowledge.

2.4 Labor Market Equilibrium Condition

We have assumed that each household supplies a fixed amount of labor, N, in each moment. Since the number of households is normalized to one, N also expresses the aggregate labor supply. Thus the full employment condition for labor labor is

$$L_R + L_f = N, \tag{17}$$

where L_f is the total labor used for consumption goods production:

$$L_f = \int_0^n l_i di.$$

3 Dynamic System

3.1 Symmetric Equilibrium

In order to make our model analytically tractable, we focus on the symmetric equilibrium in which the following conditions are fulfilled:

$$c_i = c, \quad p_i = p, \quad s_i = s, \ \lambda_i = \lambda \ \text{ for all } i \in [0, n] \,. \tag{18}$$

In our product-variety expansion model of growth, we assume that in each moment newly invented goods are introduced into the market. Suppose good i is created at period $t - \xi \ (\xi > 0)$. Then the level of s at period t is

$$s_i(t) = \beta \int_{t-\xi}^{t} e^{-\beta\tau} \bar{c}_i(\tau) \, d\tau + s_i(t - \xi),$$

where the initial value of $s_i(t-\xi)$ is exogenously given. Thus if we set $s_i(t-\xi) = 0$, consumption goods have a vintage structure, so that the symmetric equilibrium will not hold. To avoid analytical complexity, we assume that in the symmetric equilibrium the following holds:

$$s_i(t-\xi) = s(t-\xi), \quad \text{for all } i \in [0, n(t-\xi)] \text{ and for all } \xi \geq 0.$$

In words, in the symmetric equilibrium each consumer sets the same amount of benchmark consumption for every good, regardless of its timing of introduction into the market. In the Dixit-Stiglittz type of variety-expansion model, the key to the consumer utility is the number of goods available rather than the character of each good, because in the symmetric equilibrium the difference in character of each good disappears. Our assumption of symmetric treatment of the benchmark consumption is, therefore, plausible one.

If we assume that $s_i = s$, it also holds that $\lambda_i = \lambda$ for all $i \in [0, n]$. Hence, the prices are the same for all goods, $p_i = \alpha bw / [bw - \beta\lambda] = p$. Additionally, due to the normalization of the number of households, in equilibrium the instantaneous level of average consumption satisfies that $\bar{c}_i = c$ for all $i \in [0, n]$. As a consequence, the dynamic equation of benchmark consumption becomes

$$\dot{s} = \beta (c - s). \tag{19}$$

Using the symmetric conditions, we find that (4) and (5) respectively yield:

$$\hat{P} = n^{\frac{1}{\alpha-1}} \hat{p} = n^{\frac{1}{1-\alpha}} s^{\theta} p, \tag{20}$$

$$C = n^{\frac{\alpha}{\alpha-1}} \hat{c} = n^{\frac{\alpha}{\alpha-1}} s^{-\theta} c. \tag{21}$$

Due to the normalization, $\hat{P}C = E = 1$, we obtain

$$\hat{P}C = pnc = 1. \tag{22}$$

Thus it holds that

$$\hat{P}^{\alpha} C p^{1-\alpha} = \hat{P}C \left(\frac{\hat{P}}{p}\right)^{\alpha-1} = \left(\frac{\hat{P}}{p}\right)^{\alpha-1},$$

which gives

$$\hat{P}^{\alpha} C p^{1-\alpha} = \left(n^{\frac{1}{1-\alpha}} s^{\theta}\right)^{\alpha-1} = n^{-1} s^{\theta(\alpha-1)}.$$

Hence , (13) is written as

$$\dot{\lambda} = (r + \beta) \lambda - \theta \left(\frac{1}{\alpha} - 1\right) \frac{1}{sn}. \tag{23}$$

In the symmetric equilibrium the markup formula (11) is written as

$$p = \frac{\alpha}{\alpha - 1} (bw - \beta\lambda). \tag{24}$$

This gives the profits of consumption good producers:

$$\begin{aligned} \pi &= s^{\theta(1-\alpha)} \hat{P}^{\alpha} C \left[p^{1-\alpha} - wbp^{-\alpha}\right] \\ &= \frac{1}{n}\left[1 - \frac{wb(\alpha - 1)}{\alpha (bw - \beta\lambda)}\right]. \end{aligned} \tag{25}$$

3.2 A Complete Dynamic System

The production function of each firm is given by $bc = l$ so that $bcn = L_f$. Therefore, (22) yields

$$L_f = \frac{b}{p}.$$

Namely, because of the nomalization, $E = 1$, the aggregate labor employment for final goods production decreases with the consumption good price, p. Hence, using (24), we obtain

$$L_f = \frac{b(\alpha - 1)}{\alpha (bw - \beta\lambda)}. \tag{26}$$

Combining (14), (17) and (26), we can derive:

$$\frac{\dot{n}}{n} = \delta (N - L_f) = \delta \left[N - \frac{b(\alpha - 1)}{\alpha (bw - \beta\lambda)} \right]. \tag{27}$$

From (2) the dynamic behavior of s is shown by

$$\dot{s} = \beta \left[\frac{1}{np} - s \right] = \beta \left[\frac{wb(\alpha - 1)}{n\alpha (bw - \beta\lambda)} - s \right]. \tag{28}$$

Now denote $sn \equiv x$. Then (19), (27) and $\frac{\dot{x}}{x} = \frac{\dot{s}}{s} + \frac{\dot{n}}{n}$ give

$$\frac{\dot{x}}{x} = \beta \left[\frac{wb(\alpha - 1)}{x\alpha (bw - \beta\lambda)} - 1 \right] + \delta \left[N - \frac{b(\alpha - 1)}{\alpha (bw - \beta\lambda)} \right]. \tag{29}$$

Substituting (8) and (25) into (16) yields

$$\frac{\dot{v}}{v} = \rho - \frac{\delta}{w} + \frac{\delta b(\alpha - 1)}{\alpha (bw - \beta\lambda)}. \tag{30}$$

Hence, in view of (15) and (27), we obtain the following:

$$\frac{\dot{w}}{w} = \frac{\dot{n}}{n} + \frac{\dot{v}}{v} = \rho + \delta N - \frac{\delta}{w}. \tag{31}$$

Finally, from (23) the implicit price of the benchmark consumption changes according to

$$\dot{\lambda} = (\rho + \beta)\lambda - \theta \left(\frac{1}{\alpha} - 1 \right) \frac{1}{x}. \tag{32}$$

To sum up, we have derived a complete dynamic system consisting of (29), (31) and (32) that describe the dynamic motions of, $x\,(= ns)$, $w\,(= \delta vn)$ and λ. In words, our derived system depicts the behaviors of the aggregate level of benchmark consumption $x\,(= ns)$, the aggregate value of knowledge $(vn = w/\delta)$ and the shadow value of the benchmark consumption, λ.

4 Balanced-Growth and Equilibrium Dynamics

4.1 Existence of the Balanced-Growth Equilibrium

In the balanced-growth equilibrium, x, w and λ stay constant over time. Hence, it holds that

$$\frac{\dot{s}}{s} = \frac{\dot{c}}{c} = \frac{\dot{v}}{v} = -\frac{\dot{n}}{n} = -\delta\left[N - \frac{b(\alpha-1)}{\alpha(bw^* - \beta\lambda^*)}\right] < 0,$$

where w^* and λ^* denotes the steady-state values of of w and λ, respectively. Because of normalization, in the balanced-growth equilibrium where n grows at a constant rate, s, c and v contract at the rate of $-\dot{n}/n$. In addition, p and L_f stay constant on the balanced-growth path. Note that in the symmetric equilibrium we have

$$C = n^{\frac{\alpha}{\alpha-1}} s^{-\theta} c, \quad \hat{P} = n^{\frac{1}{1-\alpha}} s^{\theta} p.$$

Thus we see that the aggregate consumption changes according to

$$\frac{\dot{C}}{C} = \left[\frac{1}{\alpha-1} + \theta\right]\left(\frac{\dot{n}}{n}\right),$$

and that the rate of change in price index is given by

$$\frac{d\hat{P}/dt}{\hat{P}} = -\left[\frac{1}{\alpha-1} + \theta\right]\left(\frac{\dot{n}}{n}\right).$$

Since we have set $E = 1$, the instantaneous utility equals $-\log \hat{P}$. Therefore, welfare expansion requires that

$$-\frac{1}{\alpha-1} < \theta.$$

When $\theta > 0$, this condition is always satisfied. In what follows, we assume that this condition holds for the case of $\theta < 0$ as well.

Condition $\dot{w} = 0$ gives the steady-state level of the real wage rate:

$$w^* = \frac{\delta}{\delta N + \rho}. \tag{33}$$

This shows that the real wage on the balanced-growth path increases with the R&D efficiency, δ, and decreases with the total labor supply, N. Remember that from (15) the real wage rate satisfies $w = \delta v n$. Thus (33) indicates that the steady-state value of total net wealth, vn, is $1/(\delta N + \rho)$.

The steady-state values of x and λ are obtained by setting $\dot{\lambda} = 0$ and $\dot{x} = 0$ in (29) and (32), respectively. Those conditions are given by

$$\beta \left[\frac{wb(\alpha - 1)}{x\alpha(bw - \beta\lambda)} - 1 \right] + \delta \left[N - \frac{b(\alpha - 1)}{\alpha(bw - \beta\lambda)} \right] = 0, \tag{34}$$

$$x = \frac{\theta(1 - \alpha)}{(\rho + \beta)\alpha\lambda}. \tag{35}$$

Substituting the steady-state level of real wage w^* given by (33) into (34) and using (35), we find that the steady-state values of x and λ are respectively given by

$$x^* = \frac{(\alpha - 1)\beta[\delta b(\rho + \beta) + \theta(\delta N - \beta)(\delta N + \rho)]}{(\rho + \beta)\delta b[(\alpha - 1)(\delta N + \rho) - (\delta N - \beta)\alpha]}, \tag{36}$$

$$\lambda^* = \frac{\theta\delta b[\alpha(\delta N - \beta) - (\alpha - 1)(\delta N + \rho)]}{\alpha\beta[\delta b(\rho + \beta) + \theta(\delta N - \beta)(\delta N + \rho)]}. \tag{37}$$

In order to define a feasible steady state, the parameter values should satisfy certain conditions. First, suppose that $\theta > 0$. Since $\lambda^* < 0$ in this case[7], we should assume the following conditions:

$$\alpha(\delta N - \beta) - (\alpha - 1)(\delta N + \rho) < 0, \tag{38}$$

$$\delta b(\rho + \beta) + \theta(\delta N - \beta)(\delta N + \rho) > 0. \tag{39}$$

Notice that under (38) and (39), λ^* has a positive value for $\theta < 0$, and hence (38) and (39) ensure the feasibility conditions for the case of $\theta < 0$ as well.

The balanced-growth rate is given by

$$g = \frac{\dot{n}}{n} = \delta\left(N - L_f^*\right),$$

where L_f^* denotes the steady-state value of labor devoted to consumption goods production. Since $L_f^* = b(\alpha - 1)/\alpha(bw^* - \beta\lambda^*)$, from (33) and (37) we obtain

$$L_f^* = \frac{(\alpha - 1)(\delta N + \rho)[\delta b(\rho + \beta) + \theta(\delta N - \beta)(\delta N + \rho)]}{\delta\left[\alpha\delta b(\rho + \beta) + \theta(\alpha - 1)(\delta N + \rho)^2\right]}. \tag{40}$$

The above demonstrates that, if $\theta > 0$, the steady-state value of L_f is positive under (39). When $\theta < 0$, we should assume that

$$\alpha\delta b(\rho + \beta) + \theta(\alpha - 1)(\delta N + \rho)^2 > 0 \tag{41}$$

[7] If $\theta > 0$ and $\lambda^* < 0$, then $x^* > 0$ from (35).

to make L_f^* positive. In addition, it is easy to confirm that $N > L_f^*$ if (38), (39) and (41) are fulfilled. Hence, L_f^* satisfies the feasibility condition such that $0 < L_f^* < N$.

Summing up the above discussion, we have shown:

Proposition 1 *In the case of negative consumption externalities $(\theta > 0)$, the economy has a unique, feasible balanced-growth path, if the parameter values satisfy*

$$-\frac{\delta b(\beta+\rho)}{\theta(\delta N+\rho)} < \delta N - \beta < \frac{(\alpha-1)(\delta N+\rho)}{\alpha}.$$

In the case of positive consumption externalities $(\theta < 0)$, the presence of a unique and feasible balanced-growth path is ensured if

$$\delta N - \beta < \min\left\{\frac{(\alpha-1)(\delta N+\rho)}{\alpha},\ -\frac{\delta b(\rho+\beta)}{\theta(\delta N+\rho)^2}\right\}$$
$$\textit{and}\ \ \alpha\delta b(\rho+\beta) + \theta(\alpha-1)(\delta N+\rho)^2 > 0.$$

4.2 The Long-Run Growth Rate

By use of (33) and (37), we may express the balanced-growth rate as a function of given parameters:

$$g = \delta N - \frac{(\alpha-1)(\delta N+\rho)\left[\delta b(\rho+\beta) + \theta(\delta N-\beta)(\delta N+\rho)\right]}{\alpha\delta b(\rho+\beta) + \theta(\alpha-1)(\delta N+\rho)^2}. \tag{42}$$

This demonstrates that the balanced-growth rate depends on all the parameters involved in the model. If there is no consumption external effect, i.e. $\theta = 0$, the balanced-growth rate determined by (42) is reduced to

$$\hat{g} = \delta N - \frac{(\alpha-1)(\delta N+\rho)}{\alpha} = \frac{\delta N}{\alpha} - \rho. \tag{43}$$

As (43) shows, the balanced-growth rate in the standard model with product-variety expansion increases with the labor supply, N, and the efficiency of R&D, δ, while it decrease with the elasticity of substitution among consumption goods, α, and the time discount rate, ρ. Intuitive implications of these comparative statics results in the standard setting have been well understood: see, for example, Chapter 3 in Grossman and Helpman (1991). In contrast to these simple results in the standard model, (42) shows that the effects of parameter changes on the long-term growth rate are rather complex in the presence of external habit formation.

First, compare the balanced-growth rate given by (42) with that determined by (43) . To do so, it is helpful to remember that the balanced-growth rate in the presence of consumption externalities is written as

$$g = \delta \left(N - L_f^* \right) = \delta \left[N - \frac{\alpha - 1}{\alpha w^* - (\alpha \beta / b) \lambda^*} \right],$$

while in the absence of external effect it is expressed as

$$\hat{g} = \delta \left(N - \frac{\alpha - 1}{\alpha w^*} \right),$$

where $w^* = \delta / (\delta N + \rho)$. Since the steady-state level of real wage is independent of θ, if there are negative consumption externalities ($\theta > 0$) so that λ^* has a negative value, then the aggregate employment for final goods production, L_f, is smaller than that in the absence of consumption externalities ($\theta = 0$) . As a result, other things being equal, the balanced-growth equilibrium with negative consumption externalities may attain a higher growth rate than that realized in the standard setting without consumption externalities. In contrast, if there are positive consumption external effects ($\theta < 0$) , the steady-state rate of L_f is larger than that in the case of $\theta = 0$. Hence, the introduction of positive consumption external effects has a negative impact on long-term growth.

Economic interpretations of these results are rather obvious. For example, consider the case of negative consumption externalities. If there are negative externalities in consumption, a higher growth of consumption demand will enhance the social level of stock of habits, which in turn depresses the future consumption demand and thus future profits of firms. Since each firm correctly anticipates such an effect of social habit formation on the consumers' decision, it has an incentive to set a higher price in order to slow down the growth of habit accumulation. Consequently, in the symmetric equilibrium the aggregate consumption demand will decline and thus the total labor devoted to final goods production decreases. This means that labor will shift from the production activities to R&D sector, which accelerates the long-term growth. In the case of positive consumption externalities, the exposition given above is completely reversed. We have thus shown:

Proposition 2 *Other things being equal, the economy with negative consumption externalities attains a higher balanced-growth rate than the economy without externalities. In contrast,* *if there are positive consumption externalities, the balanced-growth rate is lower than that sustained by the economy without externalities.*

It is worth emphasizing that in our setting the growth effects of changes in other parameters would be also different from those obtained in the standard modelling. For instance, consider the effect of a change in the level of labor supply, N. As was shown by (43), an increase in N raises the balanced-growth rate, if $\theta = 0$. In this case a higher N depresses the steady-state level of real wage, w^*, which has a negative impact on growth. At the same time, a rise in labor supply stimulates technical progress, because it allows the R&D sector to employ a larger amount of labor. In the standard model without consumption externalities, the latter effect dominates the former, so that a higher labor supply accelerates growth. That is, there exists a scale effect in the presence of knowledge externalities in the R&D sector. If there are consumption externalities, there is an additional effect generated by a change in N: an increase in N may change the value of λ^* (see equation (37)). More precisely, it is seen that when $\theta > 0$, the effect of a raise in N on the magnitude of λ^* is ambiguous. If $\theta < 0$, then an increase in N raises λ^*. This produces an additional increase in $L_f^* \left(= b\left(\alpha - 1\right)/\alpha\left(bw^* - \beta\lambda^*\right)\right)$. If this increase in L_f^* is large enough to hold $dL_f^*/dN > 1$, then a larger labor supply lowers the balanced-growth rate: we may have an anti-scale effect even though there are knowledge spillovers in the R&D sector.

Similarly, if $\theta \neq 0$, the relationship between the time discount rate and the balanced-growth rate should be reconsidered. In the absence of consumption externalities, a higher ρ decreases the steady-state rate of real wage, w^*, which increases the labor input for final goods production. Hence, the balanced growth rate will decline. If $\theta > 0$, (37) states that an increase in ρ lowers the absolute value of λ^*. Therefore, we obtain $dL_f^*/d\rho > 0$, so that the balanced growth rate decreases. However, if $\theta < 0$, it is seen that we cannot determine the sign of $dL_f^*/d\rho$ without imposing further constraints on the magnitudes of the parameter values.

4.3 Equilibrium Dynamics

To examine the stability property of our dynamic system, it is to be noted that the dynamic behavior of w given by (33) is independent of other variables and it is completely unstable.

Since the initial value of w is not specified in the perfect-foresight equilibrium, the unusable behavior of w means that it always holds that $w = w^*$. Consequently, we may focus on the dynamics of x and λ under the fixed level of $w = w^*$. Keeping in mind that the predetermined variable in our system is $x\,(= sn)$ alone, we see that there is a unique stable converging path around the balanced-growth equilibrium if the dynamic system consisting of (29) and (32) exhibits a saddle-point property.

Linearizing (29) and (32) around the steady state where $\dot{x} = \dot{\lambda} = 0$, we obtain:

$$\begin{pmatrix} \dot{x} \\ \dot{\lambda} \end{pmatrix} = J \begin{pmatrix} x - x^* \\ \lambda - \lambda^* \end{pmatrix},$$

where

$$J \equiv \frac{\alpha\beta b\,(\alpha - 1)}{(x\alpha\,(bw^* - \beta\lambda))^2} \begin{pmatrix} -\,(bw^* - \beta\lambda)\,w^* & (\beta w^* - \delta x)x \\ \theta\left(\frac{1}{\alpha} - 1\right)\frac{1}{x^2} & \rho + \beta \end{pmatrix}.$$

The sign of the determinant of the coefficient matrix J satisfies the following:

$$\begin{aligned} \text{sign } \det J &= \text{sign } \left\{[-\,(bw^* - \beta\lambda)\,w^*]\,(\rho + \beta) - \theta\left(\frac{1}{\alpha} - 1\right)\frac{1}{x}(\beta w^* - \delta x)\right\} \\ &= \text{sign } \left\{-\delta\left[\frac{\alpha\,(\rho + \beta)\,b\delta + \theta(\delta N + \rho)^2\,(\alpha - 1)}{(\delta N + \rho)^2\alpha}\right]\right\}. \end{aligned}$$

Hence, if $\theta > 0$, then $\det J < 0$ so that the linearized dynamic system exhibits a saddle-point property. Additionally, it is easy to see that if (41) holds, the system with a negative θ also satisfies the saddlepoint stability. We have thus shown the following:

Proposition 3 *When* (38) , (39) *and* (41) *are held, the balanced-growth path satisfies local saddle-point stability.*

Figures 1 (a), (b) and (c) depict typical phase diagrams of the dynamic system. Figures 1 (a) and (b) assume that conditions (38), (39) and (41) are fulfilled. Moreover, it is assumed that $\delta N - \beta > 0$. In figure 1 (a) where $\theta > 0$, the converging saddle paths have positive slopes. If the initial position of $x\,(= sn)$ is smaller than its steady-state value, x^*, then the initial value selected on the saddle path is also smaller than λ^*. On the transitional process in which both x and λ continue increasing, the absolute value of λ diminishes. Thus in the transition, the price of consumption good decreases and the labor input for final goods production rises, which means that the rate of technical progress, $\dot{n}/n$, declines during the transition toward the balanced-growth equilibrium. Figure 1 (b) shows the case of saddle stability under $\theta < 0$. In this situation the converging paths are negatively sloped. Since $\lambda > 0$ for the case of positive θ, we again find that if the initial level of x is lower than its steady state value, the consumption good price increases and the rate of technical change declines in the transition process.

It is worth pointing out that in the case of $\theta < 0$, we may have an alternative possibility. Now let us assume that θ is negative and it has a large absolute value. Then both (39) and (41) may be violated and the following conditions hold:

$$\delta b\,(\rho+\beta)+\theta(\delta N-\beta)\,(\delta N+\rho)<0, \tag{44}$$

$$\alpha\delta b\,(\rho+\beta)+\theta\,(\alpha-1)\,(\delta N+\rho)^2<0. \tag{45}$$

Observe that if (38) is satisfied, λ^*, x^* and L_f^* have positive values under (44) and (45). Furthermore, we see that

$$\begin{aligned}\text{sign}\,\{\text{trace}J\} &= \text{sign}\,\{-\left(bw^*-\beta\lambda\right)w^*+(\rho+\beta)\}\\ &= \text{sign}\,\Big\{-\frac{b\delta^2[\alpha\,(\rho+\beta)\,b\delta+\theta(\delta N+\rho)^2]}{(\delta N+\rho)^2\alpha[(\rho+\beta)\,b\delta+\theta(\delta N+\rho)(\delta N-\beta)]}\\ &\quad+(\delta N+\rho)^2\alpha\{(\rho+\beta)\,b\delta+\theta(\delta N+\rho)(\delta N-\beta)\Big\}.\end{aligned}$$

Consequently, given (44) and (45), we find that $\det J > 0$ and trace $J < 0$. If this is the case, the linearized dynamic system is asymptotically stable around the balanced-growth path. Since the initial value of λ is not predetermined, in this case there is a continuum of converging paths around the balanced-growth equilibrium and, therefore, local indeterminacy emerges. Noting that (44) needs $\delta N > \beta$, we may state:

Proposition 4 *Suppose that $\theta < 0$ and $\beta < \delta N$. Then the balanced-growth path may exhibits local indeterminacy, if*

$$\theta<\min\left\{-\frac{\delta b\,(\rho+\beta)}{(\delta N-\beta)\,(\delta N+\rho)},\ -\frac{\alpha\delta b\,(\rho+\beta)}{(\alpha-1)\,(\delta N+\rho)^2}\right\}.$$

Figure 1 (c) depicts the situation to which Proposition 4 can apply. In this case the steady state is a sink, which yields a continuum of converging equilibria at least around the balanced-growth path.

5 Discussion

5.1 R&D Subsidy

In the original Grossman and Helpman model, any policy that promotes R&D activities has a clear implication. Since the market economy fails to internalize the knowledge externalities in the R&D sector, the resource allocation to the R&D activities in the competitive equilibrium is too low to attain the social optimum. The government's R&D subsidy to the research firms, therefore, enhances growth and welfare, as long as it is financed by non-distortionary taxation. Such an unambiguous policy implication may not hold in our model. To see this, consider a simple R&D subsidy scheme in which a portion of labor costs of the R&D firms is subsidized at a rate of $\phi \in (0,1)$. We assume that the government finances the R&D subsidies by levying a lump-sum tax on the households' income. Then profits of the R&D firms is $v\dot{n} - (1-\phi)\, wL_R$, so that the zero-excess-profit condition for the R&D firms is given by

$$(1-\phi)\, w = \delta n v.$$

Using this relation, we see that the arbitrage condition (30) becomes

$$\frac{\dot{v}}{v} = \rho b - \frac{\delta}{(1-\phi)\, w} + \frac{\delta b\,(\alpha-1)}{\alpha\,(1-\phi)\,(bw-\beta\lambda)}. \tag{46}$$

Since the growth rate of variety of goods is not directly affected by the introduction of the R&D subsidy, we still have

$$\frac{\dot{n}}{n} = \delta\,(N - L_f) = \delta\left[N - \frac{b\,(\alpha-1)}{\alpha\,(bw-\beta\lambda)}\right].$$

Combining (46) with the above, we find that the dynamic equation of the real wage rate is:

$$\frac{\dot{w}}{w} = \frac{\dot{v}}{v} + \frac{\dot{n}}{n} = \rho + \delta N - \frac{\delta}{(1-\phi)\, w} + \frac{\phi\delta b\,(\alpha-1)}{\alpha\,(1-\phi)\,(bw-\beta\lambda)} \tag{47}$$

Notice that if $\phi = 0$, then (46) is reduced to (31), and hence the dynamic behavior of real wage rate is independent of the other state variables, x and λ. However, as shown above, if $\phi \neq 0$, the behavior of w depends on x as well as on λ. The steady-state conditions are the following:

$$\beta\left(\frac{wL_f}{x} - 1\right) + \delta\,(N - L_f) = 0 \Longleftrightarrow \dot{x} = 0, \tag{48}$$

$$(\rho+\beta)\lambda-\theta\left(\frac{1}{\alpha}-1\right)\frac{1}{x}=0 \Longleftrightarrow \dot{\lambda}=0, \tag{49}$$

$$\rho+\delta N-\frac{\delta}{(1-\phi)w}+\frac{\phi\delta L_f}{1-\phi}=0 \Longleftrightarrow \dot{w}=0. \tag{50}$$

Keeping in mind that $L_f=b(\alpha-1)/\alpha(bw-\beta\lambda)$ and using (48) and (49), we obtain the following equation:

$$L_f=\frac{(\alpha-1)\left[\alpha b(\rho+\beta)w+\delta N\theta\right]}{\alpha^2 b(\rho+\beta)w^2+\delta(\alpha-1)\theta}. \tag{51}$$

Equations (50) and (51) determine the steady-state values of w and L_f.

First, consider the standard setting where there are no consumption externalities $(\theta=0)$. In this case, the real wage behaves according to

$$\frac{\dot{w}}{w}=\rho+\delta N-\frac{\delta}{(1-\phi)w}+\frac{\phi\delta(\alpha-1)}{\alpha(1-\phi)bw}, \tag{52}$$

implying that the steady-state level of real wage rate is

$$w^*=\frac{\delta}{\delta N+\rho}\left[1+\frac{\phi}{\alpha\delta(1-\phi)}\right].$$

Since w^* increases with ϕ and since the balanced-growth rate in the absence of consumption externalities is given by

$$g=\delta\left(N-\frac{\alpha-1}{\alpha w^*}\right),$$

a rise in the rate of R&D subsidy, ϕ, increases the long-run growth rate by reducing the labor allocation to final goods production.

In order to examine the growth effect of a change in ϕ in the presence of external habit formation, let us rewrite (50) as

$$L_f=\frac{1}{\phi}\left(\frac{1}{w}-\frac{\delta N+\rho}{\delta}\right)+\frac{\delta N+\rho}{\delta}. \tag{53}$$

Then the steady-state levels of w and L_f are determined at the intersection of the graphs of (51) and (53). As an example, consider the case of negative externalities $(\theta>0)$. As shown by Figure 2, when $\theta>0$, the graph of (51) has an intercept of N on the L_f axis and it has a negative slope for $L_f\leq N$.[8] Moreover, $\lim_{w\to\infty}L_f=0$. The graph of (53) also has a negative

[8] From (51) we obtain

$$\frac{dL_f}{dw}=\frac{\alpha b(\alpha-1)(\rho+\beta)\left[\delta\theta(\alpha-1)-2\left(\alpha b^2(\rho+\beta)w+\delta\alpha N\theta\right)w\right]}{\left[\alpha^2 b(\rho+\beta)w^2+\delta(\alpha-1)\theta\right]^2}.$$

Thus the graph of (51) has a positive slope for $0<w<\hat{w}$, where $dL_f/dw=0$ at $w=\hat{w}$. However, we see that $L_f>N$ for $0<w<\hat{w}$ and that $dL_f/dw<0$ for the feasible region in which L_f is strictly less than N.

slope and $\lim_{w\to\infty} L_f = -\left(\frac{1}{\phi}-1\right)\left(\frac{\delta N+\rho}{\theta}\right) < 0$. While the shapes of these graphs suggest that there may exist multiple steady states, we restrict our attention to the case where the steady state is uniquely given. Notice that if ϕ increases, the graph of (53) shifts downward for $w \leq \delta/(\delta N+\rho)$, while it shifts upward for $w > \delta/(\delta N+\rho)$. Thus we can confirm that a rise in the subsidy rate increases the balanced-growth rate if the real wage rate in the initial steady state satisfies $w^* > \delta/(\delta N+\rho)$: see Figure 2 (a). In this case, a higher ϕ lowers L_f by increasing the price level p, so that the labor allocation to the R&D sector increases to enhance long-term growth. In contrast, as depicted by Figure 2 (b), when the initial level of real wage is less than $\delta/(\delta N+\rho)$, a higher ϕ increases L_f, and hence a rise in R&D subsidy fails to stimulate long-run growth.

The Intuition behind the above-mentioned results is the following. Remember again that the steady-state level of employment in the final good sector is written as

$$L_f^* = \frac{b}{p^*} = \frac{1}{\frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda^*}{bw^*}\right)w^*}.$$

In the model without externalities, a rise in the real wage w^* caused by a higher R&D subsidy rate directly increases the prices of final goods, because $\lambda^* = 0$ in the above equation. Hence, a higher ϕ reduces L_f so that the balanced-growth rate increases. In the presence of consumption externalities, an increase in ϕ yields the indirect as well as direct effects on the equilibrium price level p^*. First, a rise in subsidy to the R&D sector increases the labor demand of the R&D firms, and hence, other things being equal, the real wage rate tends to rise, which increases the equilibrium price p^*. At the same time, a higher p^* reduces consumption demand so that the external habit formation will be slow down. This lowers the implicit 'internalization costs' for the firm, i.e. the absolute value of λ^*, which depresses the mark-up rate, $\frac{\alpha}{\alpha-1}\left(1-\frac{\beta\lambda^*}{bw^*}\right)$.[9] If this reduction in the mark-up rate dominates the initial increase in the real wage rate, the equilibrium price p^* may fall down so that L_f^* increases. As a consequence, a higher ϕ lowers the real wage and raises L_f^*, which depresses the balanced-growth rate. Our graphical analysis indicates that such a conclusion tends to hold if the initial level of w^* is less than $\delta/(\delta N+\rho)$. In contrast, if the initial w^* exceeds $\delta/(\delta N+\rho)$, then a decrease in the mark-up rate cannot cancel the direct effect of a rise in w^*, and therefore in

[9]Remember that we are concerned with the case of negative externalities ($\theta > 0$) so that λ has a negative value.

the new steady state p increases to lower L_f^*.

5.2 The Optimal Level of R&D

As emphasized above, in the standard R&D based growth model with product-variety expansion, the labor allocation devoted to R&D activities is too small to attain the efficient allocation. Since in our model externalities present both in production and consumption sides, we may not establish such a straightforward result. To confirm this, let us derive the social optimal allocation in the product-variety expansion model of growth without any distortion. This is examined by solving the following planning problem. If we focus on the symmetric equilibrium, the objective function for the planner is given by

$$U = \int_0^\infty e^{-\rho t} \log C dt = \int_0^\infty e^{-\rho t} \left[\frac{1}{\alpha - 1} \log n - \theta \log s + \log L_f - \log b \right] dt.$$

In the above, we use $C = n^{\frac{\alpha}{\alpha-1}} s^{-\theta} c$ and $nc = L_f/b$. We assume that the planner maximizes U by controlling labor allocation to production, L_f, subject to

$$\dot{n} = \delta n \left(N - L_f \right), \tag{54}$$

$$\dot{s} = \beta \left(\frac{L_f}{bn} - s \right), \tag{55}$$

and the initial values of n and s.

The Hamilton function for this problem can be set as

$$H = \frac{1}{\alpha - 1} \log n - \theta \log s + \log L_f + \mu \delta n \left(N - L_f \right) + \eta \beta \left(\frac{L_f}{bn} - s \right),$$

where μ and η respectively denote the shadow values of n and s. The necessary conditions for an optimal involve the following:

$$\frac{1}{L_f} = \mu \delta n - \eta \beta \frac{1}{bn}, \tag{56}$$

$$\dot{\mu} = \rho \mu - \mu \delta \left(N - L_f \right) - \eta \beta \frac{L_f}{bn^2} - \frac{1}{\left(\alpha - 1 \right) n}, \tag{57}$$

$$\dot{\eta} = \rho \eta + \frac{\theta}{s} + \beta \eta, \tag{58}$$

together with the dynamic equations of n and s.

If consumption externalities do not exist $(\theta = 0)$, then the planning problem is simply given by

$$\max \int_0^\infty e^{-\rho t} \left[\frac{1}{\alpha - 1} \log n + \log L_f - \log b \right] dt$$

subject to (54) alone. It is a simple exercise to show that in this problem the optimal balanced-growth rate is

$$g^* = \frac{\delta N}{\alpha} - \left(1 - \frac{1}{\alpha}\right) \rho,$$

which is unambiguously higher than the balanced-growth rate in the competitive equilibrium given by (43). Hence, the balanced-growth rate in the decentralized economy (so that the labor allocation to R&D activities) is too low to attain the social optimum.

If $\theta \neq 0$, on the balanced-growth path it holds that

$$g = \frac{\dot{\eta}}{\eta} = \frac{\dot{n}}{n} = -\frac{\dot{s}}{s} = -\frac{\dot{\mu}}{\mu} \tag{59}$$

and L_f stays constant over time. From (54) through (58), together with (59), we obtain the following equations:

$$\rho = \left[\frac{\beta}{b} \left(\frac{\eta}{n} \right) + \frac{1}{\alpha - 1} \right] \frac{1}{n\mu},$$

$$\delta (N - L_f) = \beta \left(1 - \frac{L_f}{bns} \right),$$

$$\delta (N - L_f) = \rho + \beta + \frac{\theta}{s\eta},$$

$$\frac{1}{L_f} = \delta \mu n - \frac{\beta}{b} \left(\frac{\eta}{n} \right).$$

After some manipulation, we find that the above set of equations can be summarized as a single equation such that

$$\frac{\delta}{(\rho - \delta)(\alpha - 1)} L_f = \frac{\rho b}{\beta (\rho - \delta)} + \frac{\theta [\beta - \delta (N - L_f)]}{\delta (N - L_f) - (\rho + \beta)}. \tag{60}$$

A positive solution of this equation gives the steady-state level of L_f in the socially optimum balanced-growth path in which every distortion is internalized. Suppose that (60) has a unique solution in between 0 and N. We can confirm that the steady-state value of L_f determined by (60) would be larger than L_f^* given by (37). Thus the competitive level of R&D is not necessarily smaller than the optimal level of R&D that realizes the social optimum. This finding as well as one shown in the previous subsection, indicate that we need careful consideration as to the policy recommendation in the R&D-based growth model if the consumers' preferences involve external habit formation.

6 Conclusion

This paper has introduced commodity-specific external effects into one of the standard models of endogenous growth in which continuing growth is sustained by expansion of product variety. We have shown that the presence of consumption externalities may significantly affect both the balanced-growth equilibrium and transitional dynamics of the economy. In addition, the scale effect, the effect of R&D subsidy and the characterization of efficient growth in our setting would be fundamentally different form those obtained in the standard model without consumption externalities. Obviously, unlike production externalities, the presence of consumption externalities cannot be the main engine of growth. Our study have, however, demonstrated that they may yield significant implications for growing economies in both positive and normative senses.

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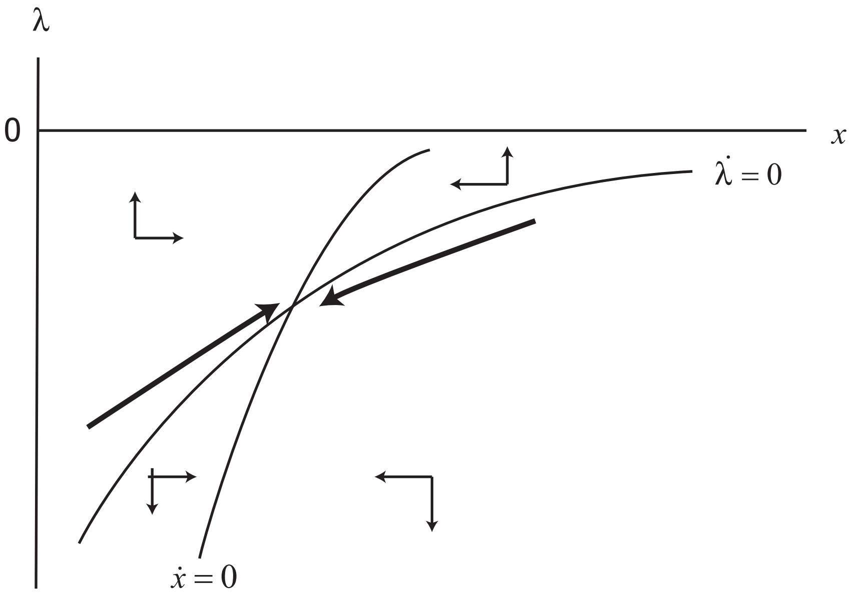

Figure 1 (a): $\theta > 0$

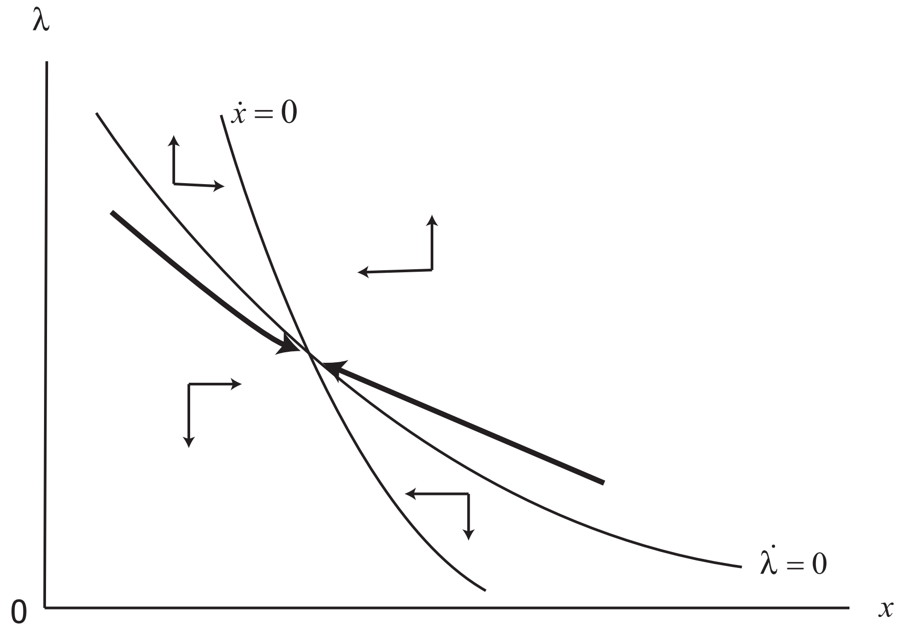

Figure 1 (b) : $\theta < 0$

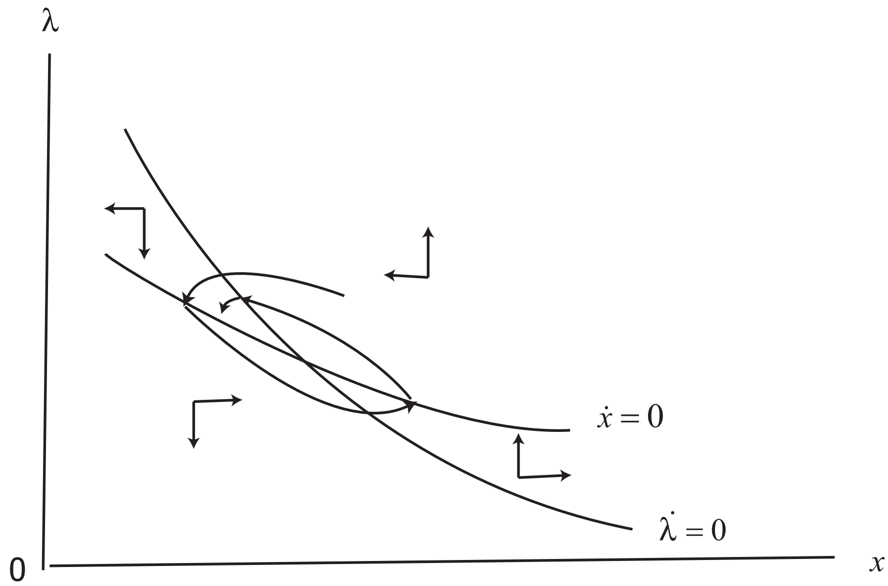

Figure 1 (c) : $\theta < 0$

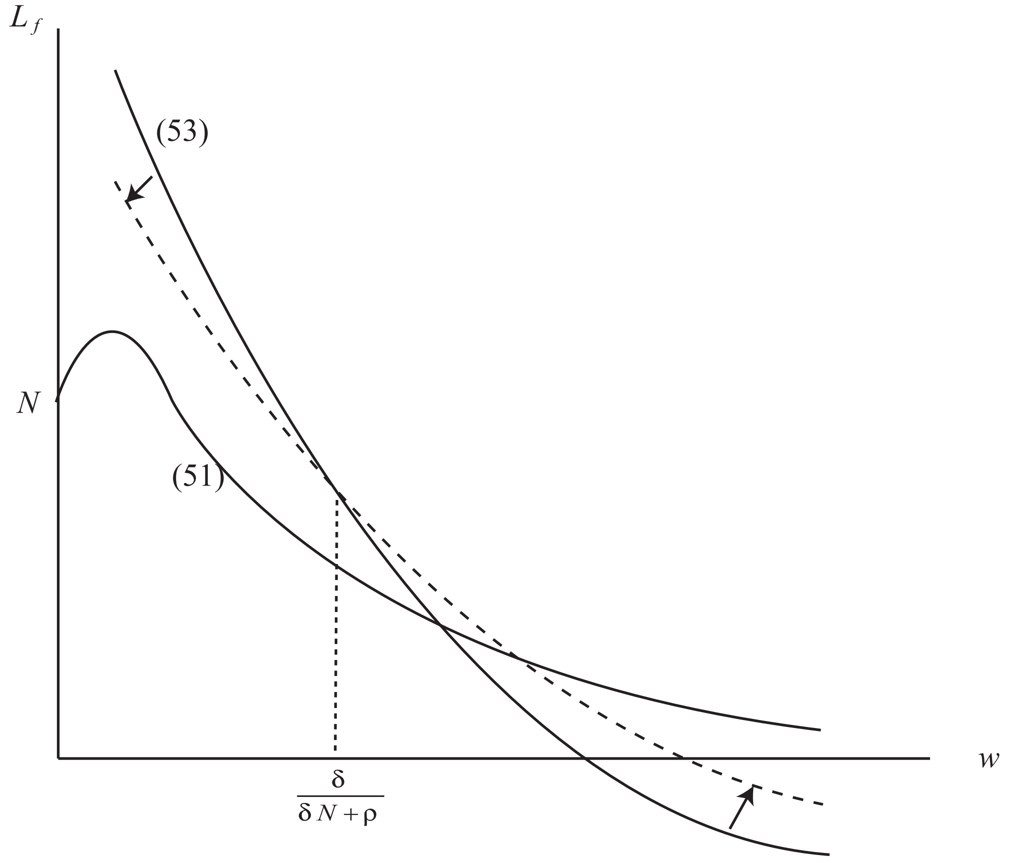

Figure 2 (a)

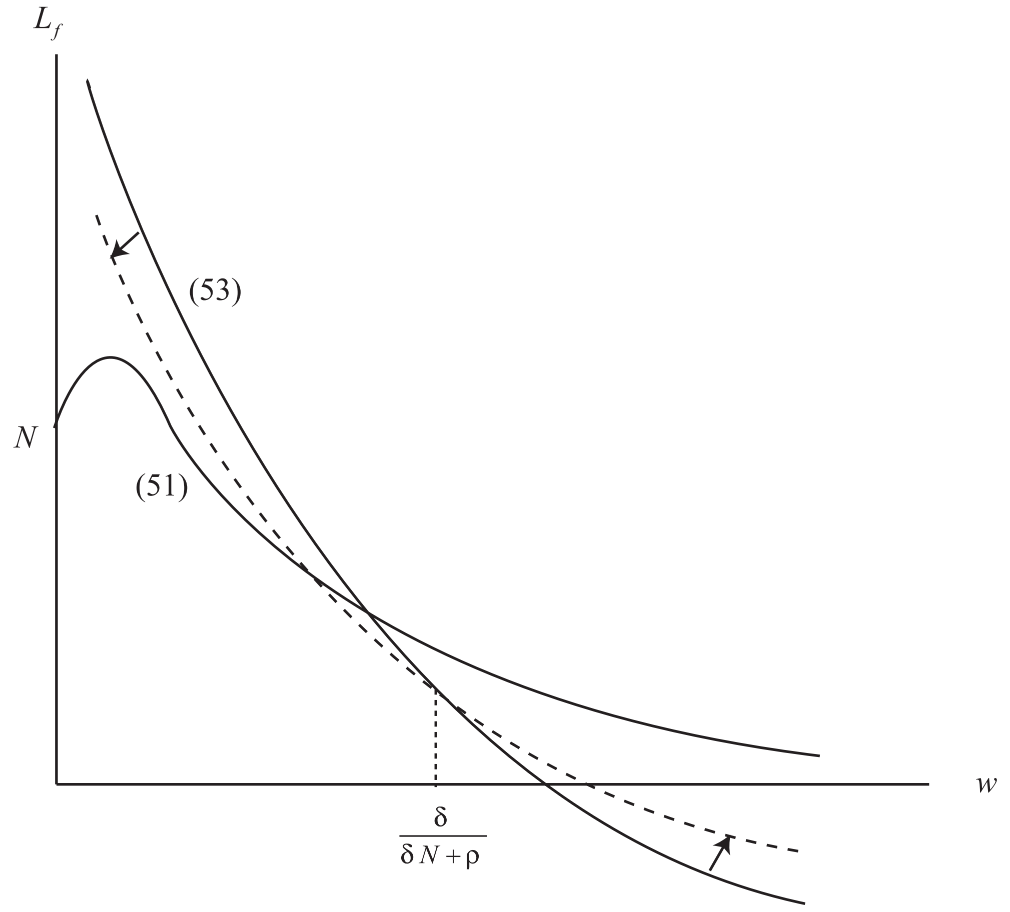

Figure 2 (b)