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Abstract

This paper explores a two-country model of capital accumulation with country-specific production externalities. The main concern of our discussion is to investigate equilibrium determinacy (aggregate stability) conditions in a financially integrated world economy. We show that the well-established equilibrium determinacy conditions for the case of small-open economy are still valid if heterogeneity between two countries is small enough. As the technological difference between the countries increases, the equilibrium determinacy conditions may diverge from those for the small country setting.

Keywords: financial integration, two-country model, equilibrium determinacy, social constant returns

JEL classification: F43, O41

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1 Introduction

Does financial integration enhance economic volatility? With regard to this question, the equilibrium business cycle theory based on indeterminacy and sunspots has claimed that economic fluctuations caused by extrinsic uncertainty tend to be more prominent in financially open economies than in closed economies. Weder (2001), for example, inspects a small-open economy version of the two-sector model studied by Benhabib and Farmer (1996) and shows that the model needs a lower degree of external increasing returns to yield indeterminacy of equilibrium than the closed-economy counterpart. In a similar vain, Aguiar-Conraria, and Wen (2005), Lahiri (2001) and Meng and Velasco (2003 and 2004) demonstrate that perfect capital mobility may enhance the possibility of equilibrium indeterminacy for small-open economies.[1] The main reason for this results is that in the small-open economies with capital mobility the interest rate is fixed in the world financial market, so that consumption behaves as if the utility function were linear. Since a high elasticity of intertemporal substitutability in consumption generally serves as a source of indeterminacy, a small-open economy tends to be more volatile than a closed economy with the same technologies and preferences.

In this paper we explore whether volatility of small-open economies emphasized in the existing studies may hold in the world economy model as well. We construct a two-country model with commodity trade and financial interactions. Our main concern is to compare the indeterminacy conditions for the small-open economy with those for the world economy consisting of large countries. Since the world economy is a closed economy with heterogeneous agents (countries), the present paper may be considered a study on the relationship between heterogeneity and indeterminacy.[2] The analytical framework of this paper is a two-goods, two-country model of capital accumulation in which there are sector as well as country-specific production externalities. Following many of the foregoing studies, we assume that technology of each production sector satisfies social constant returns to scale, while the private technol-

[1] Meng and Velasco (2003 and 2004) study two-sector dependent economy models with sector-specific externalities in which investment goods are not traded. Lahiri (2001) examines a endogenously growing small-open economy with capital mobility.

[2] In a different context, Ghiglino and Olszak-Duquenne (2005) also consider the relation between heterogeneity of agents and equilibrium indeterminacy in a closed economy model of economic growth with production externalities.

ogy exhibits decreasing returns. Given this setting, the small-open economy yields a clear conclusion about aggregate stability: its equilibrium path around the steady-state is locally intermediate, if and only if the technology of investment good sector is more capital intensive than the consumption good sector from the social perspective, but it is less capital intensive from the private perspective. If both sectors have the same factor intensity ranking from private as well as social perspectives, then the competitive equilibrium is uniquely determined. We re-examine these stability/instability conditions in the context of a financially integrated world economy.

Our main finding is that the well-known stability conditions for the small-open economy mentioned above are still valid in the general equilibrium model of world economy, if the fundamentals of both countries are close to each other. More specifically, if two counties are symmetric in the sense that they share the same technology and preference structures, the conditions for equilibrium determinacy/indeterminacy are exactly the same as those for the case of small open economy. When each country has different total factor productivities, so that the steady-state equilibrium is not identical for each country, then the stability conditions under which the small-open economy exhibits local indeterminacy also produce multiple converging equilibria in the world economy model. If the two countries have different degree of production externalities, then the equilibrium determinacy conditions for the world economy may diverge from those established in the small-open economy. These results demonstrate that heterogeneity of economies would be a relevant determinant of equilibrium determinacy in a financially integrated world.

It is to be noted that the topic of this paper is closely related to a seminal contribution by Nishimura and Shimomura (2002b) who study equilibrium indeterminacy of a two-country world economy with the same type of production technologies assumed in this paper. Nishimura and Shimomura (2002b) utilize a dynamic Heckscher-Ohlin model where both consumption and investment goods are freely traded but there is no financial transactions between the two countries. Additionally, following the standard formulation, they assume that the both countries have the same production technologies. In their model, there is a continuum of steady state distribution of capital and which steady state is realized depends on expectations of agents if the equilibrium path is indeterminate.[3] In this paper we assume

[3]Sim and Ho (2008) point out that if one of the two countries has no production externalities in the

that investment goods are not traded but international lending and borrowing are allowed. As a result, the steady-state distribution of capital between two countries is uniquely determined. As mentioned above, we also consider the case in which both countries have different production technologies and explore how the asymmetry of technologies affects the stability conditions. The present paper, therefore, studies the equilibrium determinacy issue in the open-macroeconomics context rather than in the traditional trade theory context.[4]

The rest of the paper is organized as follows. Section 2 constructs the base model and summarizes the main results of the foregoing studies on the case of small-open economy. Section 3 extends the base model to the general equilibrium model of the world economy. Section 4 examines equilibrium determinacy in the symmetric as well as in the asymmetric steady-state equilibrium. Section 5 makes remarks on our main discussion and Section 6 concludes.

2 The Base Model

Before examining the world economy model, we construct the base model and summarize the main findings in small-open economy models studied by Meng and Velasco (2003 and 2004) and Weder (2001). This review is useful to capture the similarity and difference between the stability conditions for the small-country setting and the world economy model.

2.1 Households

Consider an open economy that is financially integrated with the rest of the world. The economy produces investment and consumption goods. While consumption goods are internationally traded, the investment goods are not tradable. We assume that while the domestic

Nishimura-Shimomura model, then indeterminacy may not hold in the world economy even if the autarkic equilibrium of the other country is indeterminate. The assumption of symmetric technology, thus, plays a relevant role in equilibrium determinacy of the global economy. For further development of the study by Nishimura and Shimomura (2002b), see Chiglino (2007) and Nishimura, et al. (2006 and 2008).

[4] In their related studies, Nishimura and Shimomura (2002a) examine equilibrium determinacy of a small-country version of dynamic Heckscher-Ohlin model. Moreover, Nishimura and Shimomura (2006) examine the possibility of equilibrium indeterminacy in a dynamic Heckscher-Ohlin model of a world economy in the absence of production externalities. Indeterminacy in their model comes from the fact that intertemporal trade between the home and foreign countries is prohibited.

households cannot directly own the capital stocks in foreign countries, they can freely lend to or borrow from the foreign households in the international financial market.[5]

There is a continuum of identical households with a unit mass. Each household inelastically supplies a unit of labor in each moment. The objective of the representative household is to maximize a discounted sum of utility

$$U = \int_0^\infty e^{-\rho t} \frac{c^{1-\sigma}}{1-\sigma} dt, \quad \sigma > 0, \quad \rho > 0$$

subject to the following constraints:

$$\dot{b} = Rb + rk + w - c - pv, \tag{1}$$

$$\dot{k} = v - \delta k, \tag{2}$$

where c is consumption, b stock of foreign bonds, R interest rate, r real rate of return to capital (in terms of the consumption good), k capital stock, w real wage, p price of investment good (in terms of the consumption good), v real investment, and δ denotes the depreciation rate of capital. The value of foreign bonds is also evaluated by the consumption goods. Condition (2) comes from our assumption that investment goods are not internationally traded. The optimization behavior are also subject to the initial holdings of capital and foreign bonds as well as the non-Ponzi game condition:

$$\lim_{t \to \infty} b(t) \exp\left(-\int_0^t R(s)\, ds \right) = 0. \tag{3}$$

The Hamiltonian function for the household's optimization problem is given by

$$H = \frac{c^{1-\sigma}}{1-\sigma} + \phi (Rb + rk + w - c - pv) + q (v - \delta k),$$

where ϕ and q respectively denote implicit prices of foreign bond and capital. The necessary conditions for an interior optimum are:

$$\max_c H \Rightarrow c^{-\sigma} = \phi, \tag{4}$$

[5] Although the same assumption has been frequently used in open-macroecomic models (e.g. Turnovsky and Sen 1995 and Chapter 7 in Turnovsky 1997), it is obviously restrictive to assume that investment goods are not internationally traded. For example, Bems (2008) shows that about 40% of investment goods are tradables in developed as well as developing countries. It is, however, plausible to consider that transportation costs of capital goods are much higher than transaction costs of financial assets. We assume that costs for transporting capital goods are prohibitively higher than international lending and borrowing.

$$\max_{v} H \Rightarrow \phi p = q, \tag{5}$$

$$\dot{\phi} = \phi\left(\rho - R\right), \tag{6}$$

$$\dot{q} = q\left(\rho + \delta\right) - \phi r, \tag{7}$$

together with (1), (2), the transversality conditions, $\lim_{t\to\infty} b\phi e^{-\rho t} = 0$ and $\lim_{t\to\infty} ke^{-\rho}q = 0$, and the initial conditions on b and k.

Equation (5) yields $\dot{\phi}/\phi + \dot{p}/p = \dot{q}/q$, so that from (6) and (7) we obtain

$$R = \frac{r}{p} - \delta + \frac{\dot{p}}{p}. \tag{8}$$

That is, the optimal portfolio choice requires that the interest rate on foreign bonds, R, equals the net rate of return to real asset, $r/p - \delta$, plus the capital gain, $\dot{p}/p$.

2.2 Production

The production functions of the investment goods ($i = 1$) and the consumption goods ($i = 2$) are specified by

$$\begin{gathered} y_i = A_i k_i^{a_i} l_i^{b_i} \bar{k}_i^{\alpha_i - a_i} \bar{l}^{\beta_i - b_i},\ i = 1, 2 \\ 0 < a_i, b_i < 1, \quad a_i < \alpha_i, \quad b_i < \beta_i, \quad \alpha_i + \beta_i = 1, \end{gathered} \tag{9}$$

where y_i is output, k_i capital and l_i is labor input of the i-th sector. Moreover, $\bar{k}_i$ and $\bar{l}_i$ denote sector-specific external effects generated by the average levels of capital and labor employed in sector i. We assume that every firm is identical, so that in equilibrium it holds that $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$. As a consequence, the social level of production function is

$$y_i = A_i k_i^{1-\alpha_{ii}} l_i^{1-\alpha_i}, \quad 0 < \alpha_i < 1, \quad i = 1, 2.$$

Therefore, the production technology of each sector satisfies socially constant returns to scale, but it exhibits privately decreasing returns.[6]

Each production sector is assumed to be perfectly competitive so that the rate of return to capital and the real wage rate respectively equal the private marginal productivity of

[6] As far as the small-country models are concerned, the Cobb-Douglas specification of production is not necessary to obtain our main conclusion. As shown by Mino (2001), we obtain the same conclusion when the production function is specified as

$$y_i = f^i\left(k_i, l_i\right)\phi^i\left(\bar{k}_i, \bar{l}_i\right),$$

where functions $f^i(.)$ and $\phi^i(.)$ are homogenous of degree α and $1 - \alpha$, respectively.

capital and labor. Thus, considering that $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$, we obtain the following profit maximization conditions of the firms:

$$r = pa_1 \frac{y_1}{k_1} = a_2 \frac{y_2}{k_2}, \qquad w = pb_1 \frac{y_1}{l_1} = b_2 \frac{y_2}{l_2}. \tag{10}$$

The full-employment conditions of capital and labor are:

$$k_1 + k_2 = k, \qquad l_1 + l_2 = 1. \tag{11}$$

From (10) we can derive the relation between factor intensity of each sector and the relative factor price in the following manner:

$$\frac{k_i}{l_i} = \frac{b_i}{a_i}\omega, \quad i = 1, 2, \tag{12}$$

where $\omega = w/r$. Hence, the relative price p can be expressed as

$$p = \frac{A_2 a_2 \left(b_2/a_2\right)^{\alpha_2 - 1} \left(k_2/l_2\right)^{\alpha_2 - 1}}{A_1 a_1 \left(b_1/a_1\right)^{\alpha_1 - 1} \left(k_1/l_1\right)^{\alpha_1 - 1}} = \frac{A_2 a_2^{\alpha_2} b_2^{1-\alpha_2}}{A_1 a_1^{\alpha_1} b_1^{1-\alpha_1}} \omega^{\alpha_2 - \alpha_1} \tag{13}$$

and the real rate of return to capital is written as

$$\frac{r}{p} = a_1 A_1 \left(\frac{k_1}{l_1}\right)^{\alpha_1 - 1} = A_1 a_1^{\alpha_1} b_1^{1-\alpha_1} \omega^{\alpha_1 - 1} = A_1 \tilde{r}\left(\omega\right), \tag{14}$$

where

$$\tilde{r}\left(\omega\right) \equiv b_1 \left(\frac{a_1}{b_1}\right)^{\alpha_1} \omega^{\alpha_1 - 1}.$$

The full-employment conditions in (11) yield

$$l_1 = \frac{k - (k_2/l_2)}{(k_1/l_1) - (k_2/l_2)}, \qquad l_2 = \frac{(k_1/l_1) - k}{(k_1/l_1) - (k_2/l_2)}.$$

Using (10), we see that the supply functions of each production sector is written as

$$\begin{aligned} y_1 &= l_1 A_1 \left(\tfrac{k_1}{l_1}\right)^{\alpha_1} = \tfrac{k - (a_2/b_2)\omega}{\Delta_p} A_1 \left(\tfrac{a_1}{b_1}\right)^{\alpha_1} \omega^{\alpha_1 - 1} = A_1 \left(\tfrac{b_2 k - a_2 \omega}{b_1 b_2 \Delta_p}\right) \tilde{r}(\omega) \\ &\equiv A_1 y^1\left(k, \omega\right), \end{aligned} \tag{15a}$$

$$\begin{aligned} y_2 &= l_2 A_2 \left(\tfrac{k_2}{l_2}\right)^{\alpha_2} = \tfrac{(a_1/b_1)\omega - k}{\Delta_p} A_2 \left(\tfrac{a_2}{b_2}\right)^{\alpha_2} \omega^{\alpha_2 - 1} = \tfrac{A_1}{A_2} \left(\tfrac{a_1 \omega - b_1 k}{b_1 b_2 \Delta_p}\right) \tilde{r}(\omega) p\left(\omega\right) \\ &\equiv A_2 y^2\left(k, \omega\right), \end{aligned} \tag{15b}$$

where Δ_p denotes the gap of factor intensities between two sectors from the private perspective:

$$\Delta_p \equiv \frac{b_1}{a_1} - \frac{b_2}{a_2}. \tag{16}$$

The partial derivative of supply functions are given by the following:

$$y_k^1(k,\omega) = \frac{b_1}{b_1 b_2 \Delta_p}\tilde{r}(\omega), \tag{17a}$$

$$y_\omega^1(k,\omega) = \frac{1}{b_1 b_2 \Delta_p}\left\{(-a_2)\left[\omega\tilde{r}'(\omega) + r(\omega)\right] + b_2 k\tilde{r}'(\omega)\right\}, \tag{17b}$$

$$y_k^2(k,\omega) = \left(\frac{A_1}{A_2}p\right)\left(\frac{-b_1}{b_1 b_2 \Delta_p}\right)\tilde{r}(\omega), \tag{17c}$$

$$y_\omega^2(k,\omega) = \left(\frac{A_1}{A_2}p\right)\frac{\tilde{r}(\omega)}{\omega b_1 b_2 \Delta_p}\left[\alpha_1 a_1 \omega + (1-\alpha_1) b_1 k\right]. \tag{17d}$$

where

$$\tilde{r}'(\omega) = (\alpha_1 - 1) a_1^{\alpha_1} b_1^{1-\alpha_1}\omega^{\alpha_1 - 2} < 0,$$

$$\omega\hat{r}'(\omega) + r(\omega) = \alpha_1 a_1^{\alpha_1} b_1^{1-\alpha_1}\omega^{\alpha_1 - 1} > 0.$$

Using the notation given above, we can confirm that the signs of partial derivatives of $y^i(k,\omega)$ functions satisfy the following:

$$\begin{aligned} \text{sign } y_k^1(k,\omega) &= \text{sign } \Delta_p, \\ \text{sign } y_\omega^1(k,\omega) &= - \text{ sign } \Delta_p, \\ \text{sign } y_k^2(k,\omega) &= - \text{ sign } \Delta_p, \\ \text{sign } y_\omega^2(k,\omega) &= \text{sign } \Delta_p. \end{aligned} \tag{18}$$

2.3 Dynamics of a Small-Open Economy

The dynamic equation of capital, k, is given by

$$\dot{k} = A_1 y^1(k,\omega) - \delta k. \tag{19}$$

The relative factor price changes according to

$$\frac{\dot{\omega}}{\omega} = \frac{1}{\alpha_2 - \alpha_1}\frac{\dot{p}}{p} = \frac{1}{\Delta_s}\left[A_1\tilde{r}(\omega) - R - \delta\right], \tag{20}$$

where

$$\Delta_s = \alpha_1 - \alpha_2 \tag{21}$$

measures the relative factor intensity from the social perspective.

If the home country is a small-open economy, the interest rate R is determined in the world financial market, and hence the time discount rate ρ must equal the given R to sustain a competitive equilibrium satisfying the non-Ponzi-game and feasibility conditions. In such a simple situation, it is easy to see that the steady-state equilibrium is uniquely given. Let us denote the steady-state values of k and ω by $\hat{k}$ and $\hat{\omega}$, respectively. Then the approximated dynamic system linearized at the steady state is given by

$$\begin{bmatrix} \dot{k} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} A_1 y_k^1\left(\hat{k},\hat{\omega}\right) - \delta & A_1 y_\omega^1\left(\hat{k},\hat{\omega}\right) \\ 0 & \dfrac{\hat{\omega}}{\Delta_s} A_1 \tilde{r}'\left(\hat{\omega}\right) \end{bmatrix} \begin{bmatrix} k - \hat{k} \\ \omega - \hat{\omega} \end{bmatrix},$$

where, in view of the steady-state conditions, it holds that

$$\begin{aligned} A_1 y_k^1\left(\hat{k},\hat{\omega}\right) - \delta &= \frac{1}{\Delta_p a_1} A_1 \tilde{r}\left(\omega\right) - \delta = \frac{\rho+\delta}{a_1 \Delta_p} - \delta \\ &= \frac{a_1\rho + [a_2\left(1-b_1\right) + b_2 a_1]\delta}{\Delta_p}. \end{aligned}$$

The above relation means that

$$\text{sign } \left[A_1 y_k^1\left(\hat{k},\hat{\omega}\right) - \delta\right] = \text{sign } \Delta_p. \tag{22}$$

Inspecting the coefficient matrix of the linearized system given above, we see that the steady state satisfies saddle-point stability if and only if

$$\Delta_p \Delta_s > 0.$$

If $\Delta_p < 0$ and $\Delta_s > 0$, then the trace of the coefficient matrix has a negative value, while its determinant is positive, implying that the steady state is a sink.[7] Thus the steady state equilibrium of an small-open economy exhibits local indeterminacy if and only if

$$\Delta_p < 0 \quad \text{and} \quad \Delta_s > 0.$$

Consequently, the aggregate stability of the small-open economy may be summarized as the following proposition:[8]

[7] Conversely, if $\Delta_s < 0$ and $\Delta_p > 0$, the steady-state equilibrium is totally unstable.

[8] See Meng and Velasco (2003 and 2004) and Weder (2001) for the details. See also Lahiri (2001), Yong and Meng (2004) and Zhang (2008) for further discussions on equilibrium determinacy in small-open economies.

Proposition 1 *The steady-state equilibrium of a small-open economy is locally indeterminate, if and only if the capital good sector's technology is more capital intensive than that of the consumption good sector from the social perspective, but it is less capital intensive from the private perspective. If the private and social factor intensity rankings are the same, then the dynamics of small open economy satisfies determinacy of equilibrium.*

Intuitive implication of the above proposition is as follows. Suppose that the economy initially stays at the steady-state equilibrium. As an example of expectation driven fluctuations, assume that the agents anticipate an expansion of production of investment goods. Then the price of investment goods (in terms of consumption goods), p, is expected to fall. If the social technology of investment good sector is more capital intensive than that of consumption good sector ($\Delta_s > 0$), then the Stolper-Samuelson condition (equation (13)) states that a decrease in the relative price is associated with an increase in the factor-price ratio, $\omega\left(= w/r\right)$. Therefore, if the private technology of investment good sector is also more capital intensive ($\Delta_p > 0$), then a rise in the factor price ratio depresses the supply of investment goods: see (18). Therefore, the initial expectation about an enhancement of investment good demand cannot be self-fulfilled, implying that the equilibrium path is uniquely determined and that extrinsic uncertainty cannot produce economic fluctuations. However, if $\Delta_s > 0$ and $\Delta_p < 0$, a fall in the investment good price lowers the factor price ratio and, hence, the investment good production will increase: see again (18). As a result, the initial expectations can be self-fulfilled, which indicates that expectations-driven fluctuations may be observable.[9]

3 The World Economy

We now introduce a foreign country who has the same market structure as that of the home country. We still assume that consumption goods are tradable but investment goods are not traded. In addition, it is assumed that residents in each country cannot directly own

[9] Using the same type of production function, we may confirm that Proposition 1 also holds in a two-sector, closed-economy model of exogenous growth with a linear utility function: see Benhabib et al. (2000). It can be also shown that the same result holds for the two-sector endogenous growth model of closed economies with physical and human capital: see Benhabib et al. (2000), Bond et al. (1996), Mino (2001) and Bond and Driskill (2006). The mathematical structure of those models is essentially the same.

capital stock in the other country. However, they can freely lend to and borrow from the households in the other country by transacting in the international bond market. Since we will assume that both home and foreign countries produce homogenous consumption goods, the intratemporal trade of consumption goods means intertemporal lending and borrowing between the two countries.

3.1 Equilibrium Conditions and the Dynamic System

The production technologies of the foreign country also have the same Cobb-Douglas forms except for the total factor productivity:

$$y_i^* = A_i^* k_i^{*a_i} l_i^{*bi} \bar{k}_i^{\alpha_i - a_i} \bar{l}_i^{\beta_i - b_i}, \quad i = 1, 2,$$

where star superscripts denote foreign variables. We also assume that the representative household in the foreign country has the same preference structure as that of the households in the home country. The foreign households maximize

$$U^* = \int_0^\infty e^{-\rho t} \frac{c^{*1-\sigma}}{1-\sigma} dt, \quad \sigma > 0, \quad \rho > 0$$

subject to the following constraints:

$$\dot{b}^* = Rb^* + r^* k^* + w^* - c^* - p^* v^*,$$

$$\dot{k}^* = v^* - \delta k^*,$$

together with the initial conditions and the non-Ponzi-game condition.

Since the production structures are the same as those of home country (except for the total factor productivity), the dynamic behaviors of k^* and $\omega^* \, (= w^*/r^*)$ are respectively given by

$$\dot{k}^* = A_1^* y^1 \left(k^*, \omega^*\right) - \delta k^*, \tag{23}$$

$$\dot{\omega}^* = \frac{\omega^*}{\Delta_s} \left[A_1^* \tilde{r} \left(\omega^*\right) - R - \delta\right], \tag{24}$$

where the forms of $y^i \left(k^*, \omega^*\right)$ $(i = 1, 2)$ and $\hat{r} \left(\omega^*\right)$ functions are the same as those of the home country.

To complete the two-country world model, we should endogenize the world interest rate, R. Since the consumption good is tradeable, its world-wide market equilibrium condition is

$$c + c^* = y_2 + y_2^*. \tag{25}$$

Note that factor income distribution in the competitive equilibrium in each country means that $rk + w = py_1 + y_2$ and $r^*k^* + w^* = p^*y_1^* + y_2^*$. Therefore, the flow budget constraint for the households in each country is respectively written as

$$\dot{b} = Rb + y_2 - c,$$

$$\dot{b}^* = Rb^* + y_2^* - c^*,$$

each of which describes the current account in the home and foreign countries. The world market equilibrium condition (25) yields the market clearing condition for the bond market:

$$b + b^* = 0. \tag{26}$$

Condition (4) gives $c = \phi^{-1/\sigma}$. The shadow value of asset of the foreign country, ϕ^*, also follows $\dot{\phi}^*/\phi^* = \rho - R = \dot{\phi}/\phi$, which means that the relative value of ϕ^*/ϕ stays constant over time. As a result, letting $\phi^*/\phi = m$ and using (4), we express the equilibrium condition for consumption goods as:

$$\phi^{-1/\sigma} + (m\phi)^{-1/\sigma} = A_2 y^2 (k, \omega) + A_2^* y^2 (k^*, \omega^*) . \tag{27}$$

Keeping in mind that m is constant over time, we take the time derivative of both sides of (27) to obtain:

$$\begin{gathered} \tfrac{1}{\sigma} \left[\phi^{-1/\sigma} + (m\phi)^{-1/\sigma} \right] (R - \rho) \\ = A_2 y_k^2 (k, \omega) \left[A_1 y^1 (k, \omega) - \delta k \right] + A_2 y_\omega^2 (k, \omega) \tfrac{\omega}{\Delta_s} \left[A_1 \tilde{r} (\omega) - R - \delta \right] \\ + A_2^* y_{k^*}^2 (k^*, \omega^*) \left[A_1^* y^1 (k^*, \omega^*) - \delta k^* \right] \\ + A_2 y_{\omega^*}^2 (k^*, \omega^*) \tfrac{\omega^*}{\Delta_s} \left[A_1^* \tilde{r} (\omega^*) - R - \delta \right] . \end{gathered} \tag{28}$$

In deriving the above, we use (19), (20), (23) and (24). Plugging (27) into the left-hand-side of the above equation, we find that R depends on k, k^*, ω and ω^*. As we see in the next subsection, in the steady state equilibrium of the world economy, the interest rate equals to the time discount rate $(R = \rho)$. Out of the steady state, the above equation states that the interest rate in the world financial market is expressed as a function of k, ω, k^* and ω^* :

$$R = R (k, \omega, k^*, \omega^*) . \tag{29}$$

It is to be noted that function $R(.)$ does not contain m. We characterize this function in the next subsection.

In sum, a complete dynamic system of the financially integrated world economy is given by the following system:

$$\dot{k} = A_1 y^1(k, \omega) - \delta k, \tag{30}$$

$$\dot{\omega} = \frac{\omega}{\Delta_s}\left[A_1 \tilde{r}(\omega) - R(k, \omega, k^*, \omega^*) - \delta\right], \tag{31}$$

$$\dot{k}^* = A_1^* y^1(k^*, \omega^*) - \delta k^*, \tag{32}$$

$$\dot{\omega}^* = \frac{\omega^*}{\Delta_s}\left[A_1^* \tilde{r}(\omega^*) - R(k, \omega, k^*, \omega^*) - \delta\right]. \tag{33}$$

Note again that $m\,(= \phi^*/\phi)$ stays constant over time and it does not directly affects the dynamic behaviors of k, k^*, ω and ω^*. This means that the choice of m only affect the relative consumption levels both in the transition and in the steady state.

3.2 The Steady-State Equilibrium

In the steady-state equilibrium, where $\dot{k} = \dot{k}^* = \dot{\omega} = \dot{\omega}^* = 0$, the following conditions are satisfied:

$$A_1 y^1(k, \omega) - \delta k = 0,$$

$$A_1 \tilde{r}(\omega) - R(k, \omega, k^*, \omega^*) - \delta = 0,$$

$$A_1^* y^1(k^*, \omega^*) - \delta k^* = 0,$$

$$A_1^* \tilde{r}(\omega^*) - R(k, \omega, k^*, \omega^*) - \delta = 0.$$

Notice that in the steady state (29) becomes

$$R = \rho, \tag{34}$$

which pins down the long-run rate of world interest rate. As a result, the second and fourth steady conditions shown above respectively become:

$$A_1 \tilde{r}(\hat{\omega}) = \rho + \delta, \tag{35}$$

$$A_1^* \tilde{r}(\hat{\omega}^*) = \rho + \delta, \tag{36}$$

where $\hat{\omega}$ and $\hat{\omega}^*$ denote the steady-state values of factor price ratios. Since $\tilde{r}(.)$ function is monotonically decreasing, (35) and (36) respectively determines unique levels of ω and ω^* in the steady state. More specifically, the steady-sate values of factor price ratios in both countries satisfy

$$\frac{\hat{\omega}^*}{\hat{\omega}} = \left(\frac{A_1}{A_1^*}\right)^{1-\alpha_1}.$$

Once the steady-state values of $\hat{\omega}$ and $\hat{\omega}^*$ are given, the long-run levels of capital, $\hat{k}$ and $\hat{k}^*$, are also uniquely determined by $A_1 y^1(k,\omega) - \delta k = 0$ and $A_1^* y^1(k^*,\omega^*) - \delta k^* = 0$. Thus we obtain:

$$\frac{\hat{k} - (a_2/b_2)\,\hat{\omega}}{\Delta_p} A_1 \left(\frac{a_1}{b_1}\right)^{\alpha_1} \hat{\omega}^{\alpha_1 - 1} = \delta \hat{k}, \tag{37}$$

$$\frac{\hat{k}^* - (a_2/b_2)\,\hat{\omega}^*}{\Delta_p} A_1^* \left(\frac{a_1}{b_1}\right)^{\alpha_1} \hat{\omega}^{*\alpha_1 - 1} = \delta \hat{k}^*. \tag{38}$$

We assume that there exist feasible levels of $\hat{k}$ and $\hat{k}^*$ that fulfill the two equations shown above. Consequently, the steady-state values of the state variables of the world economy, $\left(\hat{k}, \hat{\omega}, \hat{k}^*, \hat{\omega}^*\right)$, are uniquely given.[10]

Finally, the equilibrium level of the shadow value of asset in the home country determined by

$$\left(1 + m^{-1/\sigma}\right) \hat{\lambda}^{-1/\sigma} = A_2 y^2\left(\hat{k}, \hat{\omega}\right) + A_2^* y^2\left(\hat{k}^*, \hat{\omega}^*\right). \tag{39}$$

where $\hat{\lambda}$ is the steady-state level of the implicit price of asset holding. Thus the long-run level of consumption of each country respectively given by

$$\hat{c} = \hat{\lambda}^{-1/\sigma}, \quad \hat{c}^* = \left(m\hat{\lambda}\right)^{-1/\sigma}. \tag{40}$$

It is worth noting that the steady-state levels of capital and factor price ratios are independent of the choice of m, but the relative consumption levels of both countries in the steady state depends on m. It is obvious that both countries have the identical technologies and thus $A_i = A_i^*$ $(i = 1, 2)$, then it holds that $\hat{k} = \hat{k}^*$ and $\hat{\omega} = \hat{\omega}^*$.

[10] In view of (28), we observe that the steady state conditions, $\dot{k} = \omega = \dot{k}^* = \dot{\omega}^* = 0$, guarantee that $R\left(\hat{k}, \hat{\omega}, \hat{k}^*, \hat{\omega}^*\right) = \rho$ holds as well.

4 Equilibrium Determinacy of the World Economy

4.1 A Linearized System

In this section we analyze the world economy dynamics around the steady-state equilibrium. Conducting a linear approximation of the dynamic system consisting of (30), (31), (32) and (33), we see that the coefficient matrix of the linearized system is given by

$$J_s = \begin{bmatrix} A_1 y_k^1 - \delta & A_1 y_\omega^1 & 0 & 0 \\ -\frac{\hat{\omega}}{\Delta_s} R_k & \frac{\hat{\omega}}{\Delta_s}\left(A_1 \tilde{r}' - R_\omega\right) & -\frac{\hat{\omega}}{\Delta_s} R_{k^*} & -\frac{\hat{\omega}}{\Delta_s} R_{\omega^*} \\ 0 & 0 & A_1^* y_{k^*}^1 - \delta & A_1^* y_{\omega^*}^1 \\ -\frac{\hat{\omega}^*}{\Delta_s} R_k & -\frac{\hat{\omega}^*}{\Delta_s} R_\omega & -\frac{\hat{\omega}^*}{\Delta_s} R_{k^*} & \frac{\hat{\omega}^*}{\Delta_s}\left(A_1^* \tilde{r}'^* - R_{\omega^*}\right) \end{bmatrix}, \tag{41}$$

where all the elements are evaluated at the steady state.[11]

The above matrix involves the partial derivatives of $R(.)$ function evaluated at the steady state. To inspect the sign of those partial derivatives, the following fact will be useful. First, using the steady state conditions, we find:

$$R_k = \frac{A_2}{\Phi} y_k^2\left(\hat{k}, \hat{\omega}\right)\left[A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) - \delta\right], \tag{42a}$$

$$R_\omega = \frac{A_1 A_2}{\Phi}\left[y_k^2\left(\hat{k}, \hat{\omega}\right) y_\omega^1\left(\hat{k}, \hat{\omega}\right) + \frac{\hat{\omega}}{\Delta_s} y_\omega^2\left(\hat{k}, \hat{\omega}\right) \tilde{r}'(\hat{\omega})\right], \tag{42b}$$

$$R_{k^*} = \frac{A_2^*}{\Phi} y_{k^*}^2\left(\hat{k}^*, \hat{\omega}^*\right)\left[A_1^* y_{k^*}^1\left(\hat{k}^*, \hat{\omega}^*\right) - \delta\right], \tag{42c}$$

$$R_{\omega^*} = \frac{A_1^* A_2^*}{\Phi}\left[y_{k^*}^2\left(\hat{k}^*, \hat{\omega}^*\right) y_{\omega^*}^1\left(\hat{k}^*, \hat{\omega}^*\right) + \frac{\hat{\omega}^*}{\Delta_s} y_{\omega^*}^2\left(\hat{k}^*, \hat{\omega}^*\right) \tilde{r}'\left(\hat{\omega}^*\right)\right], \tag{42d}$$

where

$$\hat{C} \equiv \hat{c} + \hat{c}^* > 0,$$

$$\Phi \equiv \tfrac{1}{\sigma}\hat{C} + \tfrac{\hat{\omega}}{\Delta_s} A_2 y_\omega^2\left(\hat{k}, \hat{\omega}\right) + \tfrac{\hat{\omega}^*}{\Delta_s} A_2^* y_{\omega^*}^2\left(\hat{k}^*, \hat{\omega}^*\right).$$

Here, $\hat{C}$ is the world demand on the consumption good, while Φ represents the sensitivity of excess demand for consumption goods with respective to a change in the nominal interest rate, R. From the steady-state conditions and the sign conditions in (18), we confirm the following:

[11] Here, $\tilde{r}' = \tilde{r}'(\hat{\omega})$ and $\tilde{r}'^* = \tilde{r}'(\hat{\omega}^*)$.

Lemma 1 *(i) If* $\Delta_p \Delta_s > 0$, *then* $\Phi > 0$, *and (ii) if* $\Delta_p < 0$ and $\Delta_s > 0$, *then* $\Phi < 0$.

Proof. See Appendix 1. ■

The above lemma indicates the following facts about the signs of partial derivatives of $R(.)$ function:

$$\begin{gathered} \text{sign } R_k = \text{sign } R_{k^*} = - \text{ sign } \Phi, \\ \text{sign } R_\omega = \text{sign } R_{\omega^*} = \text{sign } \Phi \text{ if } \Delta_p \Delta_s < 0. \end{gathered} \tag{43}$$

These results will be useful for analyzing the stability condition shown in the next subsection.

4.2 Symmetric Steady State

We first focus on the symmetric steady state under the assumption that $A_i = A_i^* \ (i = 1, 2)$. In this case heterogeneity between the home and foreign countries is represented by differences in the initial distributions of capital and asset holdings. Note again that the steady-state values of capital stocks and the factor price ratios are respectively identical in both countries, so that $\hat{k} = \hat{k}^*$ and $\hat{\omega} = \hat{\omega}^*$. This means that we can set $y_k^1 = y_{k^*}^1$, $y_\omega^1 = y_{\omega^*}^1$, $\tilde{r}' = \tilde{r}^{*\prime}$ and

$$\begin{aligned} R_k\left(\hat{k}, \hat{\omega}, \hat{k}, \hat{\omega}\right) &= R_{k^*}\left(\hat{k}, \hat{\omega}, \hat{k}, \hat{\omega}\right), \\ R_\omega\left(\hat{k}, \hat{\omega}, \hat{k}, \hat{\omega}\right) &= R_{\omega^*}\left(\hat{k}, \hat{\omega}, \hat{k}, \hat{\omega}\right). \end{aligned}$$

in matrix J displayed above.

Remember that (35) and (36) yield:

$$\begin{aligned} A_1 \ y_k^1\left(\hat{k}, \hat{\omega}\right) &= \frac{1}{b_1 \Delta_p} A_1 \tilde{r}(\omega) = \frac{1}{b_1 \Delta_p}(\rho + \delta) \\ &= \frac{1}{b_1 \Delta_p} A_1^* \tilde{r}(\omega^*) = A_1^* y_{k^*}^1\left(\hat{k}^*, \hat{\omega}^*\right). \end{aligned}$$

Similarly, it can be shown that

$$y_\omega^1\left(\hat{k}, \hat{\omega}\right) = y_{\omega^*}^1\left(\hat{k}^*, \hat{\omega}^*\right).$$

Let us write the characteristic equation of the linearized as $\Psi_s(\lambda) \equiv \det[\lambda I - J] = 0$. Then by use of the symmetric property conditions, we verify that the characteristic equation of the coefficient matrix J can be expressed in the following manner:[12]

$$\begin{aligned}\Psi_s(\lambda) &\equiv \det\begin{bmatrix}\lambda-\lambda_1^s & -A_1 y_\omega^1 & 0 & 0\\ \frac{\hat{\omega}}{\Delta_s}R_k & \lambda-\frac{\hat{\omega}}{\Delta_s}\left(A_1\tilde{r}'-R_\omega\right) & \frac{\hat{\omega}}{\Delta_s}R_k & \frac{\hat{\omega}}{\Delta_s}R_\omega\\ 0 & 0 & \lambda-\lambda_1^s & -A_1 y_\omega^1\\ \frac{\hat{\omega}}{\Delta_s}R_k & \frac{\hat{\omega}}{\Delta_s}R_\omega & \frac{\hat{\omega}}{\Delta_s}R_k & \lambda-\frac{\hat{\omega}}{\Delta_s}\left(A_1\tilde{r}'-R_\omega\right)\end{bmatrix}\\ &= \left(\lambda-\lambda_1^s\right)^2\left(\lambda-\lambda_2^s\right)^2=0,\end{aligned}$$

where

$$\lambda_1^s \equiv A_1 y_k^1\left(\hat{k},\hat{\omega}\right)-\delta=\tfrac{1}{b_1\Delta_p}(\rho+\delta)-\delta,$$

$$\lambda_2^s \equiv \tfrac{\hat{\omega}}{\Delta_s}A_1\tilde{r}'(\hat{\omega}).$$

As a consequence, the linearized dynamic system evaluated at the symmetric steady state has the same characteristic roots as those of the dynamic system that depicts the local behavior of the corresponding small-open economy. To sum up, we have shown:

Proposition 2 *In the symmetric steady state of the world economy, the equilibrium path converging to the steady state is locally intermediate, if and only if the investment good sector uses a more capital intensive technology than the consumption good sector from the social perspective, while it uses a less capital intensive technology from the private perspective. Conversely, if the small open economy holds determinacy conditions, i.e.* $\Delta_s\Delta_p > 0$, *then the world economy is locally determinate as well.*

It is worth pointing out that although in the symmetric steady state capital stocks as well as factor price ratios are the same for both countries, it does not mean that neither international trade nor financial interactions is taking place in the steady-state equilibrium. To see this, suppose that at the outset households in each country have no foreign assets, that is, $b_0 = b_0^* = 0$. Then, due to the no-Ponzi-game conditions given by (3), the intertemporal budget constraint for the home and foreign households are respectively given by the following:

$$k_0+\int_0^\infty \exp\left(-\int_0^t R_s ds\right)w_t dt=\int_0^\infty \exp\left(-\int_0^t R_s ds\right)c_t dt,$$

$$k_0^*+\int_0^\infty \exp\left(-\int_0^t R_s ds\right)w_t^* dt=\int_0^\infty \exp\left(-\int_0^t R_s ds\right)c_t^* dt.$$

[12] See Appendix 2.

Hence, using $c_t^* = (m)^{-1/\sigma} c_t$, we obtain

$$m = \left[\frac{k_0 + \int_0^\infty \exp\left(-\int_0^t R_s ds\right) w_t dt}{k_0^* + \int_0^\infty \exp\left(-\int_0^t R_s ds\right) w_t^* dt} \right]^\sigma . \tag{44}$$

Remember that from (10) and (12) the real wage rates are determined by $w_t = a_2^{\alpha_2} b_2^{1-\alpha_2} \omega_t^{\alpha_2}$ and $w_t^* = a_2^{\alpha_2} b_2^{1-\alpha_2} \omega_t^{*\alpha_2}$. If the steady state holds local determinacy, under a given set of initial capital holdings, (k_0, k_0^*), the paths of factor price ratios, $\{\omega_t, \omega_t^*\}_{t=0}^\infty$, are uniquely determined. This means that the value of m depends on the initial levels of capital stocks in both countries.[13]

For example, assume that the utility functions are logarithmic so that $\sigma = 1$. Since in the symmetric steady state both the home and foreign countries produce the same level of consumption goods, (44) indicates that if $m > 1$ (so $c_t > c_t^*$ for all $t \geq 0$), then the home country imports consumption goods from the foreign country, and hence its asset position is negative in the steady state. The steady-state expressions of the flow budget constraints for the households in both countries are:

$$\begin{aligned} \hat{b} &= \frac{1}{\rho} \left[\hat{c} - y_2\left(\hat{k}, \hat{\omega}\right) \right] < 0, \\ \hat{b}^* &= \frac{1}{\rho} \left[\frac{\hat{c}}{m} - y_2\left(\hat{k}, \hat{\omega}\right) \right] > 0. \end{aligned}$$

Conversely, when $m < 1$, the home country is a creditor and the foreign country is a debtor in the long-run equilibrium.

If $\Delta_p < 0$ and $\Delta_s > 0$ so that there is a continuum of converging paths towards the symmetric steady state, then the initial levels of ω and ω^* may not be uniquely determined under a given set of (k_0, k_0^*). This means that the discounted sum of human wealth, $\int_0^\infty \exp\left(-\int_0^t R_s ds\right) w_t$ and $\int_0^\infty \exp\left(-\int_0^t R_s ds\right) w_t^* dt$ cannot be uniquely given. Thus the choice of m is also affected by expectations formation, which implies that the steady-state levels of consumption and the asset position of each country also depends on expectations of agents in the world economy. This also shows that even if the world economy converges to the steady state, the long-term income distribution between the home and countries is not uniquely determined either. Since the steady-state levels of per-capita income of the home

[13] If $b_0 = -b_0^* \neq 0$, then the initial asset positions also affects the magnitude of m.

and foreign countries are respectively given by

$$\begin{aligned}\hat{c} + \hat{p}\hat{v} &= R\hat{b} + \hat{r}\hat{k} + \hat{w}, \\ \hat{c}^* + \hat{p}^*\hat{v}^* &= -R\hat{b} + \hat{r}^*\hat{k}^* + \hat{w}^*.\end{aligned}$$

Hence, noting that $R = \rho = r/p - \delta = \hat{r}/\hat{p}^* - \delta$ in the steady state, we see that the long-run level of relative income between two countries is expressed as:

$$\frac{\hat{c} + \hat{p}\hat{v}}{\hat{c} + \hat{p}\hat{v}} = \frac{(\rho + \delta)\hat{p}^*(\hat{\omega}^*)\hat{k} + \rho\hat{b} + \hat{w}^*\left(\hat{k}^*\right)}{(\rho + \delta)\hat{p}(\omega^*)\hat{k} - \rho\hat{b} + \hat{w}\left(\hat{k}\right)}.$$

As noted above, while the steady-state values of $\hat{\omega}$, $\hat{\omega}^*, \hat{k}$ and $\hat{k}^*$ are uniquely given, the long-run level of asset position $\hat{b}$ depends on expectations. Hence, the per-capita income of each country may not converge to the same level in the long run, even if both countries are endowed with the same production technologies and preferences.

4.3 Asymmetric Steady State

Next, we examine the case of asymmetric steady state with $A_1 \neq A_1^*$ and $A_2 \neq A_2^*$. In this case, the coefficient matrix of the linearized dynamic system is given by

$$J_a = \begin{bmatrix} A_1 y_k^1 - \delta & A_1 y_\omega^1 & 0 & 0 \\ -\frac{\hat{\omega}}{\Delta_s} R_k & \frac{\hat{\omega}}{\Delta_s}\left(A_1\tilde{r}' - R_\omega\right) & -\frac{\hat{\omega}}{\Delta_s} R_{k^*} & -\frac{\hat{\omega}}{\Delta_s} R_{\omega^*} \\ 0 & 0 & A_1^* y_{k^*}^1 - \delta & A_1^* y_{\omega^*}^1 \\ -\frac{\hat{\omega}^*}{\Delta_s} R_k & -\frac{\hat{\omega}^*}{\Delta_s} R_\omega & -\frac{\hat{\omega}^*}{\Delta_s} R_{\kappa^*} & \frac{\hat{\omega}^*}{\Delta_s}\left(A_1^*\tilde{r}^{*\prime} - R_{\omega^*}\right) \end{bmatrix}.$$

We see that from (35) and (36) the steady-state conditions yield:

$$A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) = \frac{A_1}{\Delta_p}\left(\frac{a_1}{b_1}\right)^{\alpha_1}\hat{\omega}^{\alpha_1 - 1} = \frac{A_1}{b_1\Delta_p}\tilde{r}(\omega) = \frac{1}{b_1\Delta_p}(\rho + \delta),$$

$$A_1^* y_{k^*}^1\left(\hat{k}^*, \hat{\omega}^*\right) = \frac{A_1^*}{\Delta_p}\left(\frac{a_1}{b_1}\right)^{\alpha_1}\hat{\omega}^{*\alpha_1 - 1} = \frac{A_1^*}{b_1\Delta_p}\tilde{r}(\omega^*) = \frac{1}{b_1\Delta_p}(\rho + \delta).$$

The above equations show that $A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) = A_1^* y_k^1\left(\hat{k}, \hat{\omega}\right)$ still holds even in the asymmetric steady state. Similarly, it is also satisfied that

$$\frac{\hat{\omega}}{\Delta_s} A_1 \hat{r}'(\hat{\omega}) = \frac{(\alpha_1 - 1)}{\Delta_s} A_1 \tilde{r}(\hat{\omega}) = \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s},$$

$$\frac{\hat{\omega}^*}{\Delta_s} A_1^* \hat{r}'\left(\hat{\omega}^*\right) = \frac{(\alpha_1 - 1)}{\Delta_s} A_1^* \tilde{r}\left(\hat{\omega}^*\right) = \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s}.$$

Using these facts and denoting $\lambda_1^s = A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) - \delta = \frac{1}{b_1 \Delta_p}(\rho + \delta) - \delta$ and $\lambda_2^s = \frac{\hat{\omega}}{\Delta_s} A_1 \hat{r}'(\hat{\omega})$, we can express the characteristic equation of J_a as follows:

$$\Psi_a(\lambda) = \det \begin{bmatrix} \lambda - \lambda_1^s & -A_1 y_\omega^1 & 0 & 0 \\ \frac{\hat{\omega}}{\Delta_s} R_k & \lambda - \lambda_2^s + \frac{\hat{\omega}}{\Delta_s} R_\omega & \frac{\hat{\omega}}{\Delta_s} R_{k^*} & \frac{\hat{\omega}}{\Delta_s} R_{\omega^*} \\ 0 & 0 & \lambda - \lambda_1^s & -A_1^* y_{\omega^*}^1 \\ \frac{\hat{\omega}^*}{\Delta_s} R_k & \frac{\hat{\omega}^*}{\Delta_s} R_\omega & \frac{\hat{\omega}^*}{\Delta_s} R_{k^*} & \lambda - \lambda_2^s + \frac{\hat{\omega}^*}{\Delta_s} R_{\omega^*} \end{bmatrix} = 0.$$

In the above, we should note:

$$\text{sign } \lambda_1^s = \text{sign } \Delta_p, \qquad \text{sign } \lambda_2^s = - \text{ sign } \Delta_s. \tag{45}$$

As shown in Appendix 3, we can verify that

$$\Psi_a(\lambda) = (\lambda - \lambda_1^s)(\lambda - \lambda_2^s)\,\eta(\lambda) = 0,$$

where

$$\begin{aligned} \eta(\lambda) = \lambda^2 + \left[\frac{\hat{\omega}^*}{\Delta_s}\left(R_{\omega^*} + \left(\frac{\hat{\omega}}{\hat{\omega}^*}\right) R_\omega\right) - (\lambda_1^s + \lambda_2^s)\right]\lambda + \lambda_1^s \lambda_2^s \\ + \frac{\hat{\omega}}{\Delta_s}\left[A_1^* y_{\omega^*}^1 R_{k^*} + \frac{\hat{\omega}}{\hat{\omega}^*} A_1 y_\omega^1 R_k\right] - \lambda_1^s \frac{\hat{\omega}^*}{\Delta_s}\left[R_{\omega^*} + \left(\frac{\hat{\omega}}{\hat{\omega}^*}\right) R_\omega\right]. \end{aligned} \tag{46}$$

Since λ_1^s and λ_2^s are the characteristic roots for the model with symmetric steady state, if the symmetric steady state is determinate, it holds that $\lambda_1^s \lambda_2^s < 0$. In the case of indeterminate steady state, both λ_1^s and λ_2^s have negative real parts so that $\lambda_1^s \lambda_2^s > 0$. Those facts imply that the determinacy/intermediacy conditions depend on the signs of roots contained in (46). We have thus found:

Lemma 2 *Suppose that the steady state of the small open economy exhibits local intermediacy. Then the steady state of the world economy with heterogenous total factor productivities is locally determinate if and only if $\eta(\lambda) = 0$ has two roots with positive real parts. Conversely, suppose that the steady state of the small country satisfies local determinacy. Then the steady state of the world economy is locally indeterminate if $\eta(\lambda) = 0$ has two roots with negative real part.*

Let us denote the roots of (46) by λ_1^a and λ_2^a. Then we obtain:

$$\lambda_1^a + \lambda_2^a = \lambda_1^s + \lambda_2^s - \frac{\hat{\omega}^*}{\Delta_s}\left(R_{\omega^*} + \left(\frac{\hat{\omega}}{\hat{\omega}^*}\right) R_\omega\right), \tag{47}$$

$$\lambda_1^a \lambda_2^a = \lambda_1^s \lambda_2^s + \frac{\hat{\omega}}{\Delta_s}\left[A_1^* y_{\omega^*}^1 R_{k^*} + \frac{\hat{\omega}}{\hat{\omega}^*} A_1 y_\omega^1 R_k\right] - \lambda_1^s \frac{\hat{\omega}^*}{\Delta_s}\left[R_{\omega^*} + \left(\frac{\hat{\omega}}{\hat{\omega}^*}\right) R_\omega\right]. \tag{48}$$

Here, the following lemma is useful for the subsequent discussion:

Lemma 3 *(i) If* $\Phi > 0$, *then sign* $\lambda_1^a + \lambda_2^a < 0$, *and (ii) sign* $[\lambda_1^a \lambda_2^a] =$ *sign* $[\Phi]$ *sign* $[\lambda_1^s \lambda_2^s]$.

Proof. See Appendix 4. ■

Relying on the results obtained so far, we find the following:

Proposition 3 *Even if each country has different total factor productivities, the set of sufficient conditions for determinacy (or indeterminacy) of the equilibrium path of the world economy are the same as these for the case of small-open economy.*

Proof. First, suppose that the small-open economy exhibits local indeterminacy so that the symmetric world steady state is locally indeterminate as well. If this is the case, $\lambda_1^s = A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) - \delta < 0$ and $\lambda_2^s = \frac{\hat{\omega}}{\Delta_s} A_1 \tilde{r}'(\hat{\omega}) < 0$. Since the indeterminacy conditions for the world economy with the symmetric steady state are $\Delta_s > 0$ and $\Delta_p < 0$, Lemma 3 (i) states that if $\Phi > 0$, then $\lambda_1^a + \lambda_2^a < 0$. Additionally, Lemma 3 (ii) shows that if $\Phi > 0$, then $\lambda_1^a \lambda_2^a > 0$. In this case both λ_1^a and λ_2^a have negative real parts. As a result, all the roots of $\Psi_a(\lambda) = 0$ have negative real parts. If $\Phi < 0$, from (ii) of Lemma 3 $\lambda_1^a \lambda_2^a < 0$, implying that $\Psi_a(\lambda) = 0$ have one positive root and three roots with negative real parts. Hence, in view of Lemma 2, the steady-state equilibrium is locally indeterminate, regardless of the sign of Φ. Second, let us assume that the local behavior of small-open economy holds equilibrium determinacy under $\Delta_p \Delta_s > 0$. Since, as shown by Lemma 1, it holds that $\Phi > 0$ in the case of $\Delta_p \Delta_s > 0$. Hence, by Lemma 3 (ii) we see that $\lambda_1^a \lambda_2^a < 0$, so $\eta(\lambda) = 0$ has one negative root, implying that the characteristic equation, $\Psi_a(\lambda) = 0$, has two stables roots. Thus the world economy also holds local determinacy around the steady-state equilibrium. ■

This proposition demonstrates that if the home and foreign countries differ in their total factor productivities as well as on asset and capital holdings, then the stability conditions for the small-open economy model are still valid for the world economy model with two large countries. Only difference from the case of symmetric steady state is that the sable manifold around the steady state would be three dimensional rather than four dimensional in the asymmetric steady state that satisfy the indeterminacy conditions, i.e. $\Delta_p > 0$ and $\Delta_s < 0$.[14]. Therefore, dynamic behavior of each variables would be more restricted in the asymmetric case than in the symmetric steady state, if the indeterminacy conditions hold. Otherwise, the conditions for aggregate stability/instability of the world economy are essentially the same as those for the small-open economy in our setting. In this sense, the relation between volatility and financial integration is the same for a small county as well as for a large country.

4.4 Further Heterogeneity

In order to consider robustness of our results obtained so far, let us introduce further heterogeneity in each country. Assume that in addition to the difference in total factor productivity, each county has different degree of external effects. Hence, the magnitudes of a_i and b_i are the same for both countries, but the values of α_i may be different from α_i^*. In other words, the private factor intensity ranking expressed by Δ_p is common for both countries, while the social factor intensity ranking in the foreign country, Δ_s^*, is not equal to that of the home country, Δ_s. If this is the case, the coefficient matrix of the linearized dynamic system is written as

$$J_{aa} = \begin{bmatrix} A_1 y_k^1 - \delta & A_1 y_\omega^1 & 0 & 0 \\ -\frac{\hat{\omega}}{\Delta_s} R_k & \frac{\hat{\omega}}{\Delta_s}\left(A_1 \tilde{r}' - R_\omega\right) & -\frac{\hat{\omega}}{\Delta_s} R_{k^*} & -\frac{\hat{\omega}}{\Delta_s} R_{\omega^*} \\ 0 & 0 & A_1^* y_{k^*}^1 - \delta & A_1^* y_{\omega^*}^1 \\ -\frac{\hat{\omega}^*}{\Delta_s^*} R_k & -\frac{\hat{\omega}^*}{\Delta_s^*} R_\omega & -\frac{\hat{\omega}^*}{\Delta_s^*} R_{\kappa^*} & \frac{\hat{\omega}^*}{\Delta_s^*}\left(A_1^* \tilde{r}^{*\prime} - R_{\omega^*}\right) \end{bmatrix},$$

where the magnitudes of R_ω and R_{ω^*} also depend on a new parameter Δ_s^*. Note that we have assumed that $\Delta_p = \Delta_p^*$ and thus

$$A_1 y_k^1\left(\hat{k}, \hat{\omega}\right) - \delta = \frac{1}{b_1 \Delta_p}(\rho + \delta) - \delta = A_1^* y_{k^*}^1\left(\hat{k}^*, \hat{\omega}^*\right) - \delta.$$

[14] As mentioned in Proof for Proposition 2, the number of stable root is three if $\Phi < 0$.

As a result, the characteristic equation is expressed as

$$\Psi_{aa}(\lambda) = \det \begin{bmatrix} \lambda - \lambda_1^s & -A_1 y_\omega^1 & 0 & 0 \\ \frac{\hat{\omega}}{\Delta_s} R_k & \lambda - \lambda_2^s + \frac{\hat{\omega}}{\Delta_s} R_\omega & \frac{\hat{\omega}}{\Delta_s} R_{k^*} & \frac{\hat{\omega}}{\Delta_s} R_{\omega^*} \\ 0 & 0 & \lambda - \lambda_1^s & -A_1^* y_{\omega^*}^1 \\ \frac{\hat{\omega}^*}{\Delta_s^*} R_k & \frac{\hat{\omega}^*}{\Delta_s^*} R_\omega & \frac{\hat{\omega}^*}{\Delta_s^*} R_{k^*} & \lambda - \lambda_2^{*s} + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*} \end{bmatrix} = 0.$$

Here, all the elements are evaluated at the asymmetric steady state. As before, λ_i^s and λ_2^{s*} denote the characteristic roots for the dynamics if the home and foreign countries were small-open economies. They are specified as

$$\lambda_1^s = A_1 y_k^1 \left(\hat{k}, \hat{\omega}\right) - \delta = \frac{1}{b_1 \Delta_p} (\rho + \delta) - \delta = \lambda_1^{*s}, \tag{49}$$

$$\lambda_2^s = \frac{\hat{\omega}}{\Delta_s} A_1 \tilde{r}'(\hat{\omega}) = \frac{(\alpha_1 - 1)}{\Delta_s} A_1 \tilde{r}'(\hat{\omega}) = \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s}, \tag{50}$$

$$\lambda_2^{*s} = \frac{\hat{\omega}^*}{\Delta_s} A_1^* \tilde{r}'(\hat{\omega}^*) = \frac{(\alpha_1 - 1)}{\Delta_s^*} A_1^* \tilde{r}'(\hat{\omega}^*) = \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s^*}. \tag{51}$$

The partial derivatives of $R(.)$ function are now replaced with:

$$R_k = \frac{A_2}{\Phi_a} y_k^2 \left(\hat{k}, \hat{\omega}\right) \left[A_1 y_k^1 \left(\hat{k}, \hat{\omega}\right) - \delta\right] = \frac{A_2}{\Phi_a} y_k^2 \left(\hat{k}, \hat{\omega}\right) \lambda_1^s, \tag{52a}$$

$$R_\omega = \frac{A_1 A_2}{\Phi_a} \left[y_k^2 \left(\hat{k}, \hat{\omega}\right) y_\omega^1 \left(\hat{k}, \hat{\omega}\right) + \frac{\hat{\omega}}{\Delta_s} y_\omega^2 \left(\hat{k}, \hat{\omega}\right) \tilde{r}'(\hat{\omega})\right], \tag{52b}$$

$$R_{k^*} = \frac{A_2^*}{\Phi_a} y_{k^*}^2 \left(\hat{k}^*, \hat{\omega}^*\right) \left[A_1^* y_{k^*}^1 \left(\hat{k}^*, \hat{\omega}^*\right) - \delta\right] = \frac{A_2^*}{\Phi_a} y_{k^*}^2 \left(\hat{k}^*, \hat{\omega}^*\right) \lambda_1^s, \tag{52c}$$

$$R_{\omega^*} = \frac{A_1^* A_2^*}{\Phi_a} \left[y_{k^*}^2 \left(\hat{k}^*, \hat{\omega}^*\right) y_{\omega^*}^1 \left(\hat{k}^*, \hat{\omega}^*\right) + \frac{\hat{\omega}^*}{\Delta_s^*} y_{\omega^*}^2 \left(\hat{k}^*, \hat{\omega}^*\right) \tilde{r}'(\hat{\omega}^*)\right], \tag{52d}$$

where

$$\hat{C} \equiv \hat{c} + \hat{c}^* > 0,$$

$$\Phi_a \equiv \frac{1}{\sigma} \hat{C} + \frac{\hat{\omega}}{\Delta_s} A_2 y_\omega^2 \left(\hat{k}, \hat{\omega}\right) + \frac{\hat{\omega}^*}{\Delta_s^*} A_2^* y_{\omega^*}^2 \left(\hat{k}^*, \hat{\omega}^*\right).$$

Notice that since $\alpha_1 \neq \alpha_1^*$, it does not hold that $A_1 \hat{r}'(\hat{\omega}) = A_1^* \hat{r}'(\hat{\omega}^*)$.

It is easy to see that the characteristic equation of J_{aa} is rewritten as

$$\Psi_{aa}(\lambda) = (\lambda - \lambda_1^s)\, \xi(\lambda) = 0,$$

where

$$\xi(\lambda) \equiv (\lambda - \lambda_2^s) A_1 y_\omega^1 \frac{\hat{\omega}}{\Delta_s} R_{k^*} + (\lambda - \lambda_1^s)(\lambda - \lambda_2^{*s}) \frac{\hat{\omega}}{\Delta_s} R_\omega + (\lambda - \lambda_1^s) A_1^* y_{\omega^*}^1 \frac{\hat{\omega}^*}{\Delta_s^*} R_{k^*} + (\lambda - \lambda_1^s)(\lambda - \lambda_2^s)\left[\lambda - \lambda_2^{s*} + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*}\right].$$

This equation is re-expressed as

$$\xi(\lambda) \equiv \lambda^3 + \psi_2 \lambda^2 + \psi_1 \lambda + \psi_0, \tag{53}$$

where

$$\psi_2 = -(\lambda_1^s + \lambda_2^s + \lambda_2^{s*}) + \frac{\hat{\omega}}{\Delta_s} R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*},$$

$$\psi_1 = \lambda_1^s \lambda_2^s + \lambda_1^s \lambda_2^{s*} + \lambda_2^s \lambda_2^{s*} - \left[\frac{\hat{\omega}}{\Delta_s} \lambda_2^s R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} \lambda_2^{s*} R_{\omega^*}\right] - \left[\frac{\hat{\omega}}{\Delta_s}(A_1 y_\omega^1 R_k - \lambda_1^s R_\omega) + \frac{\hat{\omega}^*}{\Delta_s^*}(A_1^* y_{\omega*}^1 R_{k^*} - \lambda_1^s R_{\omega^*})\right],$$

$$\psi_0 = -\lambda_1^s \lambda_2^s \lambda_2^{s*} - \left[\frac{\hat{\omega}}{\Delta_s} \lambda_2^{s*}(A_1 y_\omega^1 R_k - \lambda_1^s R_\omega) + \frac{\hat{\omega}^*}{\Delta_s^*} \lambda_2^s (A_1^* y_{\omega*}^1 R_{k^*} - \lambda_1^s R_{\omega^*})\right].$$

Thus letting λ_i^{aa} $(i = 1, 2, 3)$ be the roots of $\xi(\lambda) = 0$, we obtain the following relations:

$$\lambda_1^{aa} + \lambda_2^{aa} + \lambda_3^{aa} = \lambda_1^s + \lambda_2^s + \lambda_2^{s*} - \left(\frac{\hat{\omega}}{\Delta_s} R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*}\right), \tag{54}$$

$$\lambda_1^{aa}\lambda_2^{aa} + \lambda_2^{aa}\lambda_3^{aa} + \lambda_3^{aa}\lambda_1^{aa} = \lambda_1^s \lambda_2^s + \lambda_1^s \lambda_2^{s*} + \lambda_2^s \lambda_2^{s*} - \left[\frac{\hat{\omega}}{\Delta_s} \lambda_2^s R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} \lambda_2^{s*} R_{\omega^*}\right] + \left[\frac{\hat{\omega}}{\Delta_s}(A_1 y_\omega^1 R_k - \lambda_1^s R_\omega) + \frac{\hat{\omega}^*}{\Delta_s^*}(A_1^* y_{\omega*}^1 R_{k^*} - \lambda_1^s R_{\omega^*})\right], \tag{55}$$

$$\lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} = \lambda_1^s \lambda_2^s \lambda_2^{s*} + \left[\frac{\hat{\omega}}{\Delta_s} \lambda_2^{s*}(A_1 y_\omega^1 R_k - \lambda_1^s R_\omega) + \frac{\hat{\omega}^*}{\Delta_s^*} \lambda_2^s (A_1^* y_{\omega*}^1 R_{k^*} - \lambda_1^s R_{\omega^*})\right]. \tag{56}$$

Notice that since the sign conditions for y_k^i and y_ω^i $(i = 1, 2)$ depend only on the sign of $\Delta_p \left(= \Delta_p^*\right)$, the signs of R_j $(j = k, \omega, k^*, \omega^*)$ are the same as before if Δ_s and Δ_s^* have the same sign.

For inspecting the characteristic roots of $\xi(\lambda) = 0$, it is useful to notice the following:

Lemma 4 (i) *If* $\Delta_p \Delta_s > 0$ *and* $\Delta_p \Delta_s^* > 0$, *then* $\Phi_a > 0$, and: *(ii) if* $\Delta_p < 0$, $\Delta_s > 0$, $\Delta_s^* > 0$, *then* $\Phi_a < 0$.

Proof. See Appendix 5.

Lemma 5 *It holds that*

$$\text{sign } \lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} = \text{ sign } \lambda_1^{s}\lambda_2^{s}\lambda_2^{s*} \text{ if } \Phi_a > 0,$$

$$\text{sign } \lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} = -\text{sign } \lambda_1^{s}\lambda_2^{s}\lambda_2^{s*} \text{ if } \Phi_a < 0.$$

Proof. See Appendix 6.

Lemmas 4 and 5 immediately yield the following outcome:

Proposition 4 *Suppose that the home and foreign countries have different degrees of external effects in production. Then (i) if both home and foreign countries hold indeterminacy conditions for the case of small-open economy, the steady state of the world economy cannot be determinate, and; (ii) if both counties hold determinacy of equilibrium as small countries, the steady state of the world economy can be indeterminate.*

Proof. First, assume that both countries satisfy local indeterminacy condition in the case of small country model, so that $\Delta_p \left(= \Delta_p^*\right) < 0$, $\Delta_s > 0$ and $\Delta_s^* > 0$. Given those conditions, (ii) in Lemma 4 and Lemma 5 show that $\lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} > 0$ because $\lambda_1^s < 0$, $\lambda_2^s < 0$ and $\lambda_2^{s*} < 0$. This means that either $\xi(\lambda) = 0$ have two roots with negative real parts and one negative root or it has three roots with positive real parts. The former means that $\Psi_{aa}(\lambda) = (\lambda - \lambda_1^s)\,\xi(\lambda) = 0$ has three stable roots, while the later shows that it has three unstable roots. Therefore, when each country exhibits indeterminacy as a small country, the world economy cannot establish equilibrium determinacy around the steady state. Conversely, if each country satisfies the determinacy condition, i.e. $\Delta_p\Delta_s > 0$ *and* $\Delta_p\Delta_s^* > 0$, then (i) in Lemma 4 and Lemma 5 states that $\lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} < 0$. Thus $\xi(\lambda) = 0$ has three stable roots or it has one stable and two unstable roots. Therefore, if the former case holds, the steady-state equilibrium of the world economy is indeterminate. ■

The first part of Proposition 4 demonstrates that the local indeterminacy condition for a small country is still valid for the two-large country world, even though the forms of social production functions are different from each other in home and foreign countries. This result does not exclude the possibility that the steady state of the world economy is unstable, that is, $\lambda_1^{aa} > 0$, $\lambda_2^{aa} >$, and $\lambda_3^{aa} > 0$, so that there is only one stable root, λ_1^s. Such a conclusion does not hold if

$$\lambda_1^{aa} + \lambda_2^{aa} + \lambda_3^{aa} = \lambda_1^s + \lambda_2^s + \lambda_2^{s*} - \left(\frac{\hat{\omega}}{\Delta_s} R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*} \right) < 0.$$

We can confirm that if $\Delta_p < 0,\ \Delta_s > 0$ and $\Delta_s^* > 0,$ then $R_\omega + \frac{\hat{\omega}^*}{\Delta_s^*} R_{\omega^*} < 0.$[15] Note that

$$\begin{aligned} & \lambda_1^s + \lambda_2^s + \lambda_2^{s*} \\ = \ & \frac{\rho + \delta}{b_1 \Delta_p} - \delta + \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s} + \frac{(\alpha_1 - 1)(\rho + \delta)}{\Delta_s^*}. \end{aligned} \tag{57}$$

Consequently, when the absolute values of stable roots, $\lambda_1^s,\ \lambda_2^s$ and λ_2^{s*} are sufficiently large, the above inequality can be satisfied. If this is the case $\Psi_{aa}(\lambda) = 0$ has three stable roots so that the steady-state equilibrium of the world economy is not totally unstable in the sense that we can always find a converging path near the steady.

The second part of Proposition 4 indicates that even though both countries have the same factor-intensity rankings from the private as well as social perspectives, the financially integrated world could exhibit local indeterminacy. To derive a sufficient condition for this conclusion, let us assume that $\Delta_p < 0$, $\Delta_s < 0$ and $\Delta_s^* < 0$. Now consider the following sequence:

$$1,\ \psi_2,\ \ \psi_1 - \frac{\psi_0}{\psi_2},\ \psi_0.$$

The Routh theorem states that if the above sequence does not change signs, i.e. $\psi_2 > 0$, $\ \psi_1 - \psi_0/\psi_2 > 0$ and $\psi_0 > 0$, then $\xi(\lambda) = 0$ has no root with positive real part, which means that all the characteristic roots of $\Psi_{aa}(\lambda) = 0$ are stable ones. Since we find that our specification of the factor ranking conditions cannot exclude this possibility, the financial integration may enhance aggregate instability.

Finally, assume that the home country holds indeterminacy conditions, while the steady state of the foreign country is locally determinate, that is, $\Delta_p \left(= \Delta_p^*\right) < 0$, $\Delta_s > 0$ and $\Delta_s^* < 0$. Inspecting the roots of $\xi(\lambda) = 0$, we find that it is now both of the following could hold:

$$\lambda_1^{aa} + \lambda_2^{aa} + \lambda_3^{aa} > 0 \ \text{ and } \lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} < 0,$$

$$\lambda_1^{aa} + \lambda_2^{aa} + \lambda_3^{aa} < 0 \text{ and } \lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} > 0.$$

[15] See proof of Lemma 4 in Appendix 5.

In the former, $\xi(\lambda) = 0$ involves one negative root and two roots with positive real parts, which shows that the characteristic equation has two roots with negative real parts. If the latter set of inequalities are satisfied, then $\xi(\lambda) = 0$ has one positive and three roots with negative real parts. As a consequence, the steady state of the world economy is either locally determinate or indeterminate, depending on the parameter magnitudes involved in the model.

This finding implies that volatility of one country may rely on whether it participates the international financial market as a small country or as a large country. More specifically, when a country that satisfies local determinacy condition opens up international trade, volatility may not be enhanced if that country is small enough not to affect the world interest rate. However, if the country is large enough to affect the world interest rate, then financial integration may destabilize the economy depending on the factor-ranking conditions of the foreign countries.

5 Remarks

In this section, we make two remarks on our central findings in the previous section.

5.1 Reinterpretation of the World Economy Model

So far, we have mainly focused on the comparison of equilibrium determinacy conditions for the world economy consisting of two large countries with those for a small-open economy. In what follows, let us briefly consider the differences between the world economy model and a model of single country. Since the world economy is a closed economy with multiple countries, it would be insightful to examine the similarity and difference between the global economy model and a model of closed single country. To see this, it is convenient to restate the market equilibrium in terms of a pseudo-planning program in which planner maximizes the representative families welfare subject to the resource constraints without internalizing external effects of production.

When we treat a single country model, the optimization problem for the planner may be expressed in the following manner:

$$\max_{v,s} \int_0^\infty e^{-\rho t} \frac{\left[A_2\left((1-v)\,k\right)^{\alpha_2}(1-s)^{b_2}\,\bar{X}_2\right]^{1-\sigma}}{1-\sigma} dt$$

subject to

$$\begin{aligned}
\dot{k} &= A_1 (vk)^{a_1} (s)^{b_1} \bar{X}_1 - \delta k, \\
0 &\leq v,\ s \leq 1, \\
k_0 &= \text{given.}
\end{aligned}$$

In the above, external effects are represented by

$$\bar{X}_1 = \left(\overline{vk}\right)^{\alpha_1 - a_1} (\bar{s})^{1-\alpha_1-b_1}, \quad \bar{X}_2 = \left(\overline{(1-v)\,k}\right)^{\alpha_2 - a_2} \left(\overline{1-s}\right)^{1-\alpha_2-b_2} \tag{58}$$

In this problem the control variables are the factor allocation rates, v and s. In this environment, the planner solves the problem under given expected sequences of external effects, $\left\{\bar{X}_{1t},\ \bar{X}_{2t}\right\}_{t=0}^{\infty}$. It is easy to see that unless $\sigma = 0$, so that the instantaneous utility function is linear in consumption, the indeterminacy condition for the small-open economy is neither necessary nor sufficient for equilibrium indeterminacy for the closed economy. When the instantaneous felicity is linear in consumption so that intertemporal consumption smoothing is perfect, the production possibility frontier between consumption and investment is linear as well. Therefore, the optimal allocation of production factors between the investment and consumption good sectors is determined by the production technology alone in the presence of social constant returns to scale.[16] In this case the determinacy/indeterminacy conditions are thus the same as those for the small-open economy.

In the world-economy model, a pseudo-planning problem whose solution mimics the market equilibrium can be set as

$$\max \int_0^{\infty} e^{-\rho t} \left(\mu \frac{c^{1-\sigma} - 1}{1-\sigma} + \mu^* \frac{c^{*1-\sigma} - 1}{1-\sigma} \right) dt$$

subject to

$$\dot{k} = A_1 (vk)^{a_1} (s)^{b_1} \bar{X}_1 - \delta k,$$

[16] If $\sigma = 0$, the Hamiltonian function for the planner's optimization problem is

$$\begin{aligned}
H_p &= A_2 \left((1-v)\,k\right)^{\alpha_2} (1-s)^{b_2} \bar{X}_2 + p \left[A_1 (vk)^{a_1} (s)^{b_1} \bar{X}_1 - \delta k \right] \\
&= c + p\dot{k},
\end{aligned}$$

where p denotes the shadow value of capital. Thus the optimal selections of v and s simply means maximization of current level of national income (in terms of the consumption goods).

$$\dot{k}^* = A_1^* \left(v^* k^*\right)^{a_1^*} \left(s^*\right)^{b_1^*} \bar{X}_1^* - \delta k^*,$$

$$c + c^* = A_2 \left(\left(1-v\right) k\right)^{a_2} \left(1-s\right)^{b_2} \bar{X}_2 + A_2^* \left(\left(1-v^*\right) k^*\right)^{a_2} \left(1-s^*\right)^{b_2} \bar{X}_2^*,$$

together with the given initial stocks of capital, k_0 and k_0^*. Here, $\bar{X}_1$ and $\bar{X}_2$ are defined by (58) and the external effects in the foreign country are:

$$\bar{X}_1^* = \left(\overline{v^* k^*}\right)^{\alpha_1^* - a_1} \left(\bar{s}^*\right)^{1-\alpha_1^*-b_1}, \qquad \bar{X}_2^* = \left(\overline{\left(1-v^*\right) k^*}\right)^{\alpha_2^* - a_2} \left(\overline{1-s^*}\right)^{1-\alpha_2^*-b_2}.$$

In this problem the control variables for the planner are v, s, c, v^*, s^* and c^*. Setting up the Hamiltonian function such that

$$\begin{aligned} H_w &= \mu \frac{c^{1-\sigma}-1}{1-\sigma} + \mu^* \frac{c^{*1-\sigma}-1}{1-\sigma} \\ &\quad + \psi \left[A_1 \left(vk\right)^{a_1} \left(s\right)^{b_1} \bar{X}_1 - \delta k\right] + \psi^* \left[A_1^* \left(v^* k^*\right)^{a_1^*} \left(s^*\right)^{b_1^*} \bar{X}_1^* - \delta k^*\right] \\ &\quad + \xi \left[A_2 \left(\left(1-v\right) k\right)^{a_2} \left(1-s\right)^{b_2} \bar{X}_2 + A_2^* \left(\left(1-v^*\right) k^*\right)^{a_2} \left(1-s^*\right)^{b_2} \bar{X}_2^* - c - c^*\right]. \end{aligned}$$

We can confirm that if we set $\xi/\psi = p$, $\xi/\psi^* = p^*$ and $\mu^*/\mu = \phi^*/\phi = m$, then the first-order conditions for this planning problem are identical to the equilibrium conditions for the decentralized economy examined in the previous section. Additionally, it can be shown that the no-Ponzi game conditions (3) corresponds to the transversality conditions for the above problem.

With regard to selecting the optimal level of consumption, the key difference between the closed economy model and the world economy setting is that the choice of factor allocation, v, v^*, s and s^*, does not directly restrict the optimal level of consumption in the global economy. In other words, while the total consumption, $c + c^*$, is determined by the world supply of consumption goods, $y_2 + y_2^*$, the intratemporal allocation of labor and capital between the two production sectors in each country is not directly restricted by the preference structure. As a result, although the world economy is a closed economy, its dynamic behavior is not diverge from that of a single, closed economy.

5.2 Alternative Trade Structure

Following the foregoing literature, we have assumed that investment goods are not internationally traded. To see how essential this assumption is, consider an alternative trade regime in which consumption goods are non-tradables, while investment goods are freely traded. We assume that both countries produce a homogenous investment good. In this setting, the optimization behavior of the representative family in the home country is given by

$$\max_{v,s} \int_0^\infty e^{-\rho t} \frac{\left[A_2\left((1-v)\,k\right)^{b_2}(1-s)^{b_2}\,\bar{X}_2\right]^{1-\sigma}}{1-\sigma} dt$$

subject to

$$\dot{b} = Rb + A_1\,(vk)^{a_1}\,(s)^{b_1}\,\bar{X}_1 - v,$$

$$\dot{k} = v - \delta k,$$

with given k_0, b_0 and $\left\{\bar{X}_{1t}, \bar{X}_{2t},\right\}_{t=0}^{\infty}$. It is to be noted that the indeterminacy conditions used so far ($\Delta_p < 0$ and $\Delta_s > 0$) are not effective for this open-economy model unless $\sigma = 0$.

When consumption goods are not traded, the corresponding pseudo-planning problem for the world economy may be set as follows:

$$\max \int_0^\infty e^{-\rho t} \left(\mu \frac{\left[A_2\,(v_2 k)^{a_2}\,(1-s)^{b_2}\,\bar{X}_2\right]^{1-\sigma}}{1-\sigma} + \mu^* \frac{\left[A_2^*\,(v_2^* k)^{a_2}\,(1-s^*)^{b_2}\,X_2^*\right]^{1-\sigma}}{1-\sigma} \right) dt$$

subject to

$$\dot{k} = A_1\,(v_1 k)^{a_1}\,s^{b_1}\bar{X}_1 + A_1^*\,(v_1^* k)^{a_1}\,s^{*b_1}X_1^*,$$

$$v_1 + v_2 + v_1^* + v_2^* = 1,$$

$$v_i,\ v_i^*,\ s,\ s^* \in [0,1]\,,\quad i = 1,2.$$

together with the given expected sequences of external effects, $\left\{\bar{X}_{1t}, \bar{X}_{2t}, \bar{X}_{1t}^*, \bar{X}_{2t}^*\right\}_{t=0}^{\infty}$ and the initial level of capital stock in the world, k_0. In this problem, the control variables are factor allocation rates, v_i, v_i^*, s, s^*. Since the capital goods are traded, the aggregate capital stock in the world is intrateporally allocated across the production sectors in both countries. Again, the optimal allocation of production factors directly affects the level of felicity of the planner. Hence, unless $\sigma = 0$, the indeterminacy conditions in our model may not be applicable for the global economy model with non-traded consumption goods.

Although our model and the model discussed in this subsection impose extreme assumptions on trade structure, they demonstrate that the relation financial integration and aggregate stability would be sensitive not only to production technologies and preference structure but also to trade patterns of commodities.[17]

6 Conclusion

This paper has examined equilibrium determinacy of the financially integrated two-country world with sector as well as country-specific production externalities. Following the foregoing studies, we have assumed that technology of each productions sector satisfies social constant returns to scale, while the private technology exhibits decreasing returns. Our main finding is that the stability conditions for the small-open economy model are valid in the general equilibrium model of world economy as well, if the fundamentals of both countries are close to each other. In particular, we have confirmed that if two counties are symmetric in the sense that they have the same technology and preference structures, then the conditions for equilibrium determinacy are exactly the same as those for the case of small open economy. When each country has a specific value of total factor productivity so that the steady state equilibrium is not symmetric for each country, the stability conditions mentioned above still produce the same conclusion in the case of world economy. If the two countries have different levels of external effects or if capital depreciation rates are not the same for both countries, then determinacy conditions for the world economy may diverge from those established in the small-open economy. Our study has demonstrated that heterogeneity of economies would be a relevant determinant of uniqueness (or multiplicity) of equilibrium.

This paper has treated a two country, two-sector neoclassical growth model with social constant returns and perfect international financial market. We have restricted our attention to the presence of heterogeneity between two countries generated by differences in asset holdings and production technologies. Other factors of heterogeneity such as differences in preferences and policy parameters may produce alternative conclusions about stability.[18] The

[17] Mino (2007) and (2008) examine equilibrium determinacy of two-country models in which there are aggregate increasing returns to scale in production sectors and endogenous labor-leisure choice. The main focus of those studies is to show how the preference structure affects equilibrium determinacy of the world economy with international lending and borrowing. With regard to alternative formulation of macroeconomic open economy models, see Obstfeld and Rogoff (1996) and Schmitt-Grohé and Uribe (2002).

[18] Bosi and Seemuller (2008), Ghiglino and Olszak-Duquenne (2005) and Ghiglino (2007) investigate the role

results may be also affected if endogenous growth is possible (as assumed by the two-country model studied by Famer and Lahiri 2006). In addition, if the international financial market is incomplete, we may obtain different outcomes.[19] Considering our toping in those more general environments may deserve further investigation.

Appendices

Appendix 1. *Proof of Lemma 1:*

(i) From (18), we know sign$[y_\omega^2]$ =sign$[\Delta_p]$. Similarly, sign$[y_{\omega^*}^2]$ =sign$[\Delta_p]$ can be derived. Hence $\Phi > 0$ must be the case when $\Delta_p \Delta_s > 0$.

(ii) Using the world-wide market-clearing condition in (25), we can rearrange Φ as

$$\Phi = \left[\frac{1}{\sigma} A_2 y^2 (k, \omega) + \frac{\omega}{\Delta_s} A_2 y_\omega^2 (k, \omega) \right] + \left[\frac{1}{\sigma} A_2^* y^2 (k^*, \omega^*) + \frac{\omega^*}{\Delta_s} A_2^* y_\omega^2 (k^*, \omega^*) \right]$$

Notice that, from (17c)

$$y_\omega^2 (k, \omega) = \frac{1}{\omega} \left(\frac{A_1}{A_2} \right) \frac{p(\omega)\hat{r}(\omega)}{b_1 b_2 \Delta_p} \left[\alpha_2 a_1 \omega + (1 - \alpha_2) b_1 k \right].$$

Therefore, we obtain:

$$\begin{aligned} & \left[\frac{1}{\sigma} A_2 y^2 (k, \omega) + \frac{\omega}{\Delta_s} A_2 y_\omega^2 (k, \omega) \right] \\ = & \frac{A_1 p(\omega)\hat{r}(\omega)}{b_1 b_2 \Delta_p} \left[\left(\frac{\alpha_2}{\Delta_s} + \frac{1}{\sigma} \right) a_1 \omega + \left(\frac{1}{\Delta_s} - \frac{\alpha_2}{\Delta_s} - \frac{1}{\sigma} \right) b_1 k \right]. \end{aligned}$$

If $\Delta_p < 0$ and $\Delta_s > 0$, then a sufficient condition for $\Phi < 0$ is $\frac{1}{\Delta_s} - \frac{\alpha_2}{\Delta_s} - \frac{1}{\sigma} > 0$, that is, $\sigma > \Delta_s/(1 - \alpha_2)$.

Appendix 2. *Derivation of* $\Psi_s(\lambda)$:

We find that the characteristic equation, $\Psi_s(\lambda) = 0$, can be exoressed in the following manner:

of heterogenous preferences in equilibrium determinacy issue in closed economy models.

[19] In the standard real business cycle literature assuming the presence of technological disturbances, Baxter and Cucini (1995) and Heathcote and Perri (2002) reveal that the financial market structure may play a pivotal role in determining international business cycles.

$$
\begin{aligned}
\Psi_s(\lambda) &= \det\begin{bmatrix}
\lambda-(A_1y_k^1-\delta) & -A_1y_\omega^1 & 0 & 0 \\
0 & \lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r' & 0 & \lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r' \\
0 & 0 & \lambda-(A_1y_k^1-\delta) & -A_1y_\omega^1 \\
\frac{\hat\omega}{\Delta_s}R_1 & \frac{\hat\omega}{\Delta_s}R_2 & \frac{\hat\omega}{\Delta_s}R_1 & \lambda-\frac{\hat\omega}{\Delta_s}(A_1\hat r-R_2)
\end{bmatrix} \\
&= \left(\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r\right)[\lambda-(A_1y_k^1-\delta)]\left[-A_1y_\omega^1\frac{\hat\omega}{\Delta_s}R_1-\frac{\hat\omega}{\Delta_s}R_2\left(\lambda-(A_1y_k^1-\delta)\right)\right] \\
&\quad +A_1y_\omega^1\left[\lambda-(A_1y_k^1-\delta)\right]\left[\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r'\right]\frac{\hat\omega}{\Delta_s}R_1 \\
&\quad +\left[\lambda-\frac{\hat\omega}{\Delta_s}(A_1\hat r-R_2)\right]\left[\lambda-(A_1y_k^1-\delta)\right]\left[\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r'\right]\left[\lambda-(A_1y_k^1-\delta)\right] \\
&= \left(\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r'\right)[\lambda-(A_1y_k^1-\delta)]\chi(\lambda),
\end{aligned}
$$

where

$$
\begin{aligned}
\chi(\lambda) &= -A_1y_\omega^1\frac{\hat\omega}{\Delta_s}R_1-\frac{\hat\omega}{\Delta_s}R_2\left(\lambda-(A_1y_k^1-\delta)\right)+A_1y_\omega^1\frac{\hat\omega}{\Delta_s}R_1 \\
&\quad +\left[\lambda-\frac{\hat\omega}{\Delta_s}\left(A_1\hat r'-R_2\right)\right]\left[\lambda-(A_1y_k^1-\delta)\right] \\
&= \left[\lambda-(A_1y_k^1-\delta)\right]\left[\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r'\right].
\end{aligned}
$$

Consequently, we obtain:

$$
\Psi_s(\lambda)=\left(\lambda-\frac{\hat\omega}{\Delta_s}A_1\hat r'\right)^2[\lambda-(A_1y_k^1-\delta)]^2=0.
$$

Appendix 3. *Derivation of* $\Psi_a(\lambda)$:

The characteristic equation, $\Psi_a(\lambda)=0$, can be derived in the follwoing way:

$$
\begin{aligned}
\Psi_a(\lambda) &= \det \begin{bmatrix} \lambda-\lambda_1^s & -A_1 y_\omega^1 & 0 & 0 \\ 0 & \lambda-\lambda_2^s & 0 & -\frac{\hat{\omega}}{\hat{\omega}^*}(\lambda-\lambda_2^s) \\ 0 & 0 & \lambda-\lambda_1^s & -A_1^* y_{\omega^*}^1 \\ \frac{\hat{\omega}^*}{\Delta_s}R_k & \frac{\hat{\omega}^*}{\Delta_s}R_\omega & \frac{\hat{\omega}^*}{\Delta_s}R_{k^*} & \lambda-\lambda_2^s+\frac{\hat{\omega}^*}{\Delta_s}R_{\omega^*} \end{bmatrix} \\
&= \frac{\hat{\omega}}{\hat{\omega}^*}(\lambda-\lambda_2^s)(\lambda-\lambda_1^s)A_1 y_\omega^1 \frac{\hat{\omega}}{\Delta_s}R_1 + \frac{\hat{\omega}}{\hat{\omega}^*}(\lambda-\lambda_1^s)^2(\lambda-\lambda_2^s)\frac{\hat{\omega}}{\Delta_s}R_\omega \\
&\quad + A_1^* y_{\omega^*}^1 \frac{\hat{\omega}^*}{\Delta_s}R_{k^*}(\lambda-\lambda_1^s)(\lambda-\lambda_2^s) + \left[\lambda-\lambda_2^s+\frac{\hat{\omega}^*}{\Delta_s}R_{\omega^*}\right](\lambda-\lambda_1^s)^2(\lambda-\lambda_2^s) \\
&= (\lambda-\lambda_1^s)(\lambda-\lambda_2^s)\phi(\lambda),
\end{aligned}
$$

where

$$
\begin{aligned}
\phi(\lambda) &= \lambda^2 + \left[\frac{\hat{\omega}^*}{\Delta_s}\left(R_{\omega^*}+\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)^2 R_\omega\right) - (\lambda_1^s+\lambda_2^s)\right]\lambda + \lambda_1^s\lambda_2^s \\
&\quad + \frac{\hat{\omega}^*}{\Delta_s}\left[A_1^* y_{\omega^*}^1 R_{k^*} + \frac{\hat{\omega}}{\hat{\omega}^*}A_1 y_\omega^1 R_k\right] - \lambda_1^s \frac{\hat{\omega}^*}{\Delta_s}\left[R_{\omega^*}+\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)^2 R_\omega\right].
\end{aligned}
$$

Appendix 4. *Proof of Lemma 3.*

Using $(52a)$, $(52b)$, $(52c)$ and $(52d)$,we obtain the following:

$$
\begin{aligned}
\lambda_1^a\lambda_2^a &= \lambda_1^s\lambda_2^s + \frac{\hat{\omega}^*}{\Delta_s}\left[A_1^* y_{\omega^*}^1 R_{k^*} + \frac{\hat{\omega}}{\hat{\omega}^*}A_1 y_\omega^1 R_k\right] - \lambda_1^s\frac{\hat{\omega}^*}{\Delta_s}\left[R_{\omega^*}+\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)^2 R_\omega\right] \\
&= \lambda_1^s\lambda_2^s + \frac{\hat{\omega}^*}{\Delta_s}\frac{A_1^*}{\Phi}y_{\omega^*}^1 A_2^* y_{k^*}^2 \hat{k}^*, \hat{\omega}^*\left[A_1^* y_{k^*}^1 - \delta\right] + \frac{\hat{\omega}}{\Delta_s}\frac{1}{\Phi}A_1 y_\omega^1 A_2 y_k^2\left[A_1 y_k^1 - \delta\right] \\
&\quad - \lambda_1^s\frac{\hat{\omega}^*}{\Delta_s}\frac{A_1^* A_2^*}{\Phi}\left[y_{k^*}^2 y_{\omega^*}^1 + \frac{1}{\Delta_s}y_{\omega^*}^2 \hat{r}'\right] - \lambda_1^s\frac{\hat{\omega}^*}{\Delta_s}\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)\frac{A_1 A_2}{\Phi}\left[y_k^2 y_\omega^1 + \frac{1}{\Delta_s}y_\omega^2 \hat{r}'^*\right] \\
&= \lambda_1^s\lambda_2^s\frac{1}{\Phi}\left[\Phi - \frac{\omega}{\Delta_s}A_2 y_\omega^2 - \frac{\omega^*}{\Delta_s}A_2 y_{\omega^*}^2\right] \\
&= \lambda_1^s\lambda_2^s\frac{\hat{C}}{\sigma\Phi}.
\end{aligned}
$$

Therefore, $\lambda_1^a\lambda_2^a$ has the same sign as that of $\lambda_1^s\lambda_2^s$, if and only if $\Phi > 0$. We also find:

$$
\begin{aligned}
\lambda_1^a + \lambda_2^a &= (\lambda_1^s + \lambda_2^s) - \frac{\hat{\omega}^*}{\Delta_s}\left[R_{\omega^*} + \left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)R_\omega\right] \\
&= (\lambda_1^s + \lambda_2^s)\frac{1}{\Phi}\left[\sigma^{-1}\hat{C} + \frac{\omega}{\Delta_s}A_2 y_\omega^2 + \frac{\omega^*}{\Delta_s}A_2 y_{\omega^*}^2\right] - \frac{\hat{\omega}^*}{\Delta_s}\frac{A_1^* A_2^*}{\Phi}\left[y_{k^*}^2 y_{\omega^*}^1 + \frac{1}{\Delta_s}y_{\omega^*}^2 \hat{r}'^*\right] \\
&\quad -\frac{\hat{\omega}^*}{\Delta_s}\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)\frac{A_1 A_2}{\Phi}\left[y_k^2 y_\omega^1 + \frac{1}{\Delta_s}y_\omega^2 \hat{r}'\right] \\
&= \frac{1}{\Phi}\left[\lambda_1^s \Phi + \lambda_2^s \sigma^{-1}\hat{C} - \frac{\hat{\omega}^*}{\Delta_s}A_1^* A_2^* y_{k^*}^2 y_{\omega^*}^1\right] - \frac{\hat{\omega}^*}{\Delta_s}\left(\frac{\hat{\omega}}{\hat{\omega}^*}\right)\frac{A_1 A_2}{\Phi}y_k^2 y_\omega^1.
\end{aligned}
$$

The above relation shows that $\lambda_1^a + \lambda_2^a < 0$ if $\Phi > 0$.

Appendix 5: *Proof of Lemma 4*

From $(15a)$, $(15b)$ and (25) we obtain:

$$
\begin{aligned}
\Phi_a &= \frac{1}{\sigma}\hat{C} + \frac{\hat{\omega}}{\Delta_s}A_2 y_\omega^2\left(\hat{k}, \hat{\omega}\right) + \frac{\hat{\omega}^*}{\Delta_s^*}A_2^* y_{\omega^*}^2\left(\hat{k}^*, \hat{\omega}^*\right) \\
&= \frac{1}{\sigma}\left[A_2 y^2\left(\hat{k}, \hat{\omega}\right) + A_2^* y^2\left(\hat{k}^*, \hat{\omega}^*\right)\right] \\
&\quad + \frac{\hat{\omega}}{\Delta_s}A_2 y_\omega^2\left(\hat{k}, \hat{\omega}\right) + \frac{\hat{\omega}^*}{\Delta_s^*}A_2^* y_{\omega^*}^2\left(\hat{k}^*, \hat{\omega}^*\right) \\
&= \frac{1}{\sigma}\frac{a_1\hat{\omega} - b_1\hat{k}}{b_1 b_2 \Delta_p}A_1 p(\omega)\tilde{r}(\omega) + \frac{1}{\sigma}\frac{a_1\hat{\omega}^* - b_1\hat{k}^*}{b_1 b_2 \Delta_p}A_1^* p(\hat{\omega}^*)\tilde{r}(\hat{\omega}^*) \\
&\quad + \frac{\hat{\omega}}{\Delta_s}A_1 p(\omega)\frac{\tilde{r}(\omega)}{\omega}\left(\frac{\alpha_2 a_1\hat{\omega} + (1-\alpha_2)b_1\hat{k}}{b_1 b_2 \Delta_p}\right) \\
&\quad + \frac{\hat{\omega}^*}{\Delta_s^*}A_1^* p(\hat{\omega}^*)\frac{\tilde{r}(\hat{\omega}^*)}{\hat{\omega}^*}\left(\frac{\alpha_2^* a_1\hat{\omega}^* + (1-\alpha_2^*)b_1\hat{k}^*}{b_1 b_2 \Delta_p}\right) \\
&= \frac{A_1 p(\hat{\omega})\tilde{r}(\hat{\omega})}{b_1 b_2 \Delta_p}\left[\frac{1}{\sigma}\left(a_1\hat{\omega} - b_1\hat{k}\right) + \frac{1}{\Delta_s}\left(\alpha_2 a_1\hat{\omega} + (1-\alpha_2)b_1\hat{k}\right)\right] \\
&\quad + \frac{A_1 p(\hat{\omega}^*)\tilde{r}(\hat{\omega}^*)}{b_1 b_2 \Delta_p}\left[\frac{1}{\sigma}\left(a_1\hat{\omega}^* - b_1\hat{k}^*\right) + \frac{1}{\Delta_s^*}\left(\alpha_2^* a_1\hat{\omega}^* + (1-\alpha_2^*)b_1\hat{k}^*\right)\right].
\end{aligned}
$$

Note that $a_1\hat{\omega} - b_1\hat{k} = b_1\Delta_p y^2\left(\hat{k}, \hat{\omega}\right)$ and $a_1\hat{\omega}^* - b_1\hat{k}^* = b_1\Delta_p y^2\left(\hat{\omega}^*, \hat{k}^*\right)$. Therefore, if $\Delta_p\Delta_s > 0$ and $\Delta_p\Delta_s^* > 0$, then $\Phi_a > 0$. When $\Delta_p < 0$, $\Delta_s > 0$ and $\Delta_s^* > 0$, we see that $\Phi_a < 0$ if $(1-\alpha_2)b_1 - b_1/\sigma > 0$ and $(1-\alpha_2^*)b_1 - b_1/\sigma > 0$.

Appendix 6: *Proof of Lemma 5*

By use of the definition of Φ_a, $\lambda_2^s = \frac{\hat{\omega}}{\Delta_s}A_1\tilde{r}'(\hat{\omega})$ and $\lambda_2^{s*} = \frac{\hat{\omega}^*}{\Delta_s^*}A_1^*\tilde{r}'(\hat{\omega}^*)$, we can conduct the following manipulation:

$$
\begin{aligned}
\lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa} &= -\psi_0 = \lambda_1^s\lambda_2^s\lambda_2^{s*} - \left[\frac{\hat{\omega}}{\Delta_s}\lambda_2^{s*}(A_1 y_\omega^1 R_k - \lambda_1^s R_\omega) + \frac{\hat{\omega}^*}{\Delta_s^*}\lambda_2^s(A_1^* y_{\omega *}^1 R_{k^*} - \lambda_1^s R_{\omega^*})\right] \\
&= \lambda_1^s\lambda_2^s\lambda_2^{s*}\frac{1}{\Phi_a}\left[\frac{1}{\sigma}\hat{C} + \frac{\hat{\omega}}{\Delta_s}A_2 y_\omega^2 + \frac{\hat{\omega}^*}{\Delta_s^*}A_2^* y_{\omega^*}^2\right] \\
&\quad +\lambda_2^{s*}A_1 A_2 y_\omega^1 y_k^2 \lambda_1^s \frac{\hat{\omega}}{\Delta_s}\frac{1}{\Phi_a} + \lambda_2^s A_1^* A_2^* y_\omega^1 y_k^2 \lambda_1^s \frac{\hat{\omega}}{\Delta_s}\frac{1}{\Phi_a} \\
&\quad -\lambda_1^s\lambda_2^s A_1 A_2 \frac{\hat{\omega}}{\Delta_s}\frac{1}{\Phi_a}\left[y_k^2 y_\omega^1 + \frac{\hat{\omega}}{\Delta_s}y_\omega^2 \tilde{r}'\right] \\
&\quad -\lambda_1^s\lambda_2^{*s} A_1^* A_2^* \frac{\hat{\omega}^*}{\Delta_s^*}\frac{1}{\Phi_a}\left[y_{k^*}^2 y_{\omega^*}^1 + \frac{\hat{\omega}^*}{\Delta_s^*}y_{\omega^*}^2 \tilde{r}'^*\right] \\
&= \lambda_1^s\lambda_2^s\lambda_2^{s^*}\frac{\hat{C}}{\sigma\Phi_a} + \frac{1}{\Phi_a}\frac{\hat{\omega}}{\Delta_s}\lambda_1^s\lambda_2^{*s}A_1 A_2 y_\omega^1 y_k^2 + \frac{1}{\Phi_a}\frac{\hat{\omega}^*}{\Delta_s^*}\lambda_1^s\lambda_2^{*s}A_1 A_2 y_\omega^1 y_k^2 \\
&\quad -\frac{1}{\Phi_a}\frac{\hat{\omega}}{\Delta_s}\lambda_1^s\lambda_2^{*s}A_1 A_2 y_\omega^1 y_k^2 - \frac{1}{\Phi_a}\frac{\hat{\omega}^*}{\Delta_s^*}\lambda_1^s\lambda_2^{*s}A_1 A_2 y_\omega^1 y_k^2 \\
&= \lambda_1^s\lambda_2^s\lambda_2^{s^*}\frac{\hat{C}}{\sigma\Phi_a}.
\end{aligned}
$$

Hence, we find that sign $(\lambda_1^{aa}\lambda_2^{aa}\lambda_3^{aa}) \times \left(\lambda_1^s\lambda_2^s\lambda_2^{s^*}\right) =$ sign Φ_a.

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