Discussion Papers In Economics And Business

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この研究は「大学院経済学研究科・経済学部記念事業」
基金より援助を受けた、記して感謝する。

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October 1, 2007

Abstract

This paper develops a dynamic model of the labor market in which the degree of substitution between employment and hours of work is determined as part of a search equilibrium. Each firm chooses the demand for working hours and the number of vacancies, and the hourly wage rate is determined by Nash bargaining. A firm increases the demand for hours as recruitment becomes more costly. Labor market tightness influences the composition of labor demand through its impact on the wage rate. Restricting working hours can expand employment, but doing so is not necessarily efficient. When there are two industries that differ in their equipment costs, workers employed by firms with higher equipment costs work longer and earn more.

JEL classification: J21, J23, J31, J64.

Keywords: employment, hours of work, search frictions.

*We thank Hiromi Nosaka and participants in seminars at Kyoto University, Tohoku University, Osaka Prefecture University, Nagoya University, the JEA Meeting, and the Australasian Meeting of the Econometric Society in Brisbane for their helpful comments. Part of this research is financially supported by the Japan Society for the Promotion of Science (Grant Number 18730164).

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1 Introduction

There are extensive and intensive margins for adjusting labor input: the number of workers and hours of work per employee.[1] Understanding how firms utilize these two margins is crucial for understanding the long-run trend in hours of work, the movements of employment and hours over the business cycle, the likely effect of regulation regarding hours of work, and cross-sectional differences in hours of work, to name a few.

Despite the importance of these issues, there seems to be no consensus among economists regarding how hours of work should be modeled. Prescott (2004), for example, argued that working hours should be understood by the conventional neoclassical framework with labor supply derived by households' labor–leisure choice (Lucas and Rapping, 1969). Pissarides (2000, 2007) developed an equilibrium matching model in which workers choose hours of work. With or without search frictions, the view of these authors is that hours of work are determined from the supply side of the labor market.

On the other hand, the labor adjustment literature such as Sargent (1978) and Hamermesh (1993) considers that hours of work are determined by the demand side. In this literature, a firm faces an exogenous cost of adjusting employment and optimally chooses employment and hours of work. An intermediate case is considered by Marimon and Zilibotti (2000), who constructed a general equilibrium with search frictions in which a firm and a worker bargain over the hourly wage rate and hours of work.

In this paper, we construct a dynamic equilibrium model of labor demand under search frictions, focusing on how firms utilize employment and hours of work. The idea is that hours of work can be chosen instantly whereas employment adjustment is frictional. As in the labor adjustment literature, we assume that firms, not workers, choose hours of work. The novel feature of this paper is that the source of labor adjustment costs is search frictions, and that the adjustment cost is influenced by labor market tightness. Thus, the degree of substitution between employment and

[1] Hamermesh (1993) asserted, "Assuming the labor is the only input into production is nonsensical. Similarly silly is the assumption that the employment–hours split can be ignored in analyzing adjustment." (p. 209.)

hours of work is determined as part of the equilibrium. This sharply contrasts with the traditional models of working hours in which the degree of substitution between working hours and employment is structurally given by the production technology (Hoel, 1986; Booth and Schiantarelli, 1987; Calmfors and Hoel, 1988; Hamermesh, 1993; Cahuc and Zylberberg, 2004).

The basic model is an extension of Smith (1999), which is particularly useful for our purpose because the number of workers at each firm is determined endogenously, although modeling the firm size in a search equilibrium is generally a formidable task. We incorporate the choice of working hours into Smith's (1999) framework. Each firm chooses the number of vacancies and hours of work per employee to maximize the value of the firm. The hourly wage rate is determined by Nash bargaining. We derive the bargaining outcome explicitly and show that the hourly wage rate is positively related to the firm's marginal product of labor. In addition, as is standard in the labor market search literature, the wage rate increases with the tightness of the labor market.

In addition to the choice of hours, we modify Smith's (1999) model such that the wage bargaining takes place *before* each firm chooses the number of vacancies and hours of work. This means that a firm takes the hourly wage rate as given when it chooses the composition of employment. The order of events has a nontrivial consequence. As a firm does not have any incentive to employ more workers to influence the bargaining outcome, Smith's (1999) bargaining externality is not present.

Our main focus is on the interaction between search frictions and the determination of hours of work. The model suggests that a firm has an incentive to increase hours of work if it faces greater search frictions. In particular, a firm increases hours of work if labor market tightness increases, exogenous recruitment cost increases, or the job separation rate increases. These are all considered as measures of frictions in adjusting the number of employees. A firm substitutes employment for hours when these frictions increase.

Free entry of firms and the absence of the bargaining externality ensure that the equilibrium level of total labor inputs at each firm is efficient. However, we show that firms overemploy. The social planner does not take into account the hourly wage rate when the optimal hours of work are chosen. However, for firms, the hourly wage rate is an additional marginal cost of increasing hours of work. As hours of work are too short and the total labor input is efficient, employment is necessarily too high. Thus, even though free entry induces the efficient level of labor inputs, the presence of search frictions distorts the composition of the labor demand.

It has become increasingly important to ask whether regulating hours of work increases employment (Booth and Schiantarelli, 1987; Marimon and Zilibotti, 2000). To address this issue, we solve the model by treating hours of work as a policy parameter to see if there is an employment effect from restricting working hours. The answer is generally yes, as long as the marginal product of labor exceeds the marginal loss associated with longer hours. However, since hours of work are less than the efficient level, reducing hours from the equilibrium level will deviate from the optimum further.

Many authors have documented declines in hours of work in major developed economies (Fitzgerald, 1996; Hunt, 1998; Pissarides, 2007). Despite a decline in average working hours, a disparity in working hours across workers has gradually widened. This disparity is particularly distinguished for workers in their 30s. The Japanese Labour Force Survey (2006) documented that the share of male employees aged between 35 and 39 years who work more than 60 hours per week rose from 18.9% in 1993 to 23.5% in 2003.[2] Similarly, the share of those who work less than 35 hours per week also rose from 6.4% to 7.1%. An important question is, what is the major cause of the dispersion in hours of work?

To address this issue, we extend the basic model to generate cross-sectional differentials in hours of work. The modeling strategy is quite straightforward. We consider a two-industry version of the model to find parameters that are responsible for generating differentials in hours of work. Interestingly, the analysis reveals that the cost of recruitment is solely responsible for hours differentials.

[2] For the US economy, Kuhn and Lozano (2005) investigated this issue and documented that "highly educated, high-wage, salaried men" had the greatest increase in long working hours. See also Hamermesh (1993).

2 The Model

2.1 Environment

This section constructs a dynamic model of the demand for hours and employment with search frictions. The model developed in this section is an extension of Smith (1999). Consider an economy consisting of workers and firms. The measure of workers is normalized to one. There is a large number of firms, with the exact number to be determined by free entry. Time is discrete and all agents discount the future at the common discount rate r.

The number of matches M is determined by the standard constant returns to scale matching technology: $M = m(U, V)$, where U is the total number of job seekers and V is the number of aggregate job vacancies. A vacancy is matched with a worker during a period with probability q, where

$$q \equiv \frac{M}{V} = m\left(\frac{U}{V}, 1\right). \tag{1}$$

It is easy to verify that an increase in labor market tightness $V/U \equiv \theta$ decreases the matching probability q. Similarly, the probability that a worker is matched with a vacancy is given by $M/U = m(1, V/U) = \theta q(\theta)$. Let λ be the exogenous rate of job destruction. Then, in any steady state, the flow into employment $\theta q(\theta)U$ must equal the flow into unemployment $\lambda(1 - U)$. Thus:

$$m\,(U, V) = \lambda(1 - U), \tag{2}$$

which defines the Beveridge curve, as shown in Figure 1.

The production technology is given by $f(hl)$, where h denotes hours of work per employee and l is the number of employees at each firm. We assume that $f'(.) > 0 > f''(.)$ and $f(0) = 0$. The instantaneous payoff to a firm with l employees is given by $f(hl) - whl - e(h)l - kv - \pi$, where w is the hourly wage rate, v is the number of vacancies, $k > 0$ is the cost of each vacancy, which may be considered as an equipment cost (Acemoglu, 2001), $\pi \geq 0$ is a fixed cost, and $e(h) \geq 0$ is the cost associated with longer working hours. Function $e(.)$ is meant to capture the time constraint for each worker and the productivity loss due to longer working hours. We assume that $e'(h) > 0$, $e''(h) > 0$ and $\lim_{h\to\infty} e(h) = \infty$. It is important to note that without function $e(.)$, employment and working hours will be perfect substitutes.[3] In effect, function $e(.)$ precludes the scenario in which a firm chooses hours of work per employee to be arbitrarily large.

2.2 Firms

Timing of events has a nontrivial consequence. In particular, as documented in Pissarides (2000, chapter 7), whether the working hours are determined before or after wage determination has nontrivial consequences. In this section, we assume that wage bargaining takes place *before* hours of work and employment are determined. In other words, each firm takes the wage rate as given when it chooses the composition of the labor demand. The firm chooses working hours and the number of vacancies to maximize the value of the firm, whereas wage bargaining determines how the total surplus is split between the firm and workers. We solve the model backwards. We first solve the firm's optimization in terms of hours and vacancies. Then, we analyze the wage determination.

Let $J(l)$ be the value of an operating firm with l employees. The Bellman equation is given by

$$J(l) = \max_{h,v} \left\{ f(hl) - whl - e(h)l - kv - \pi + \frac{1}{1+r} J((1-\lambda) l + qv) \right\}, \tag{3}$$

where λ is the exogenous rate of job destruction. The first-order conditions with respect to h and v are:

$$\begin{aligned} f'(hl) - w - e'(h) &= 0, & (4) \\ -k + \frac{q}{1+r} J'((1-\lambda) l + qv) &= 0, & (5) \end{aligned}$$

respectively. The envelope condition yields

$$J'(l) = f'(hl) h - wh - e(h) + \frac{1-\lambda}{1+r} J'((1-\lambda) l + qv). \tag{6}$$

In any steady state, the flow into unemployment equals the flow into employment:

$$\lambda l = qv. \tag{7}$$

[3] Booth and Schiantarelli (1987), Calmfors and Hoel (1988), Hoel (1986), and Holzer (1994), for example, considered the production function of the form $f(g(h)hl)$, where $g(h)$ is the average productivity per hour. According to this modeling, employment and working hours are imperfect substitutes because the efficiency of labor depends on working hours. Our modeling is meant to capture both changes in efficiency and other direct costs.

Then, (5) and (6) imply $(r+\lambda)k/q = f'(hl)h - wh - e(h)$. With (4), this expression reduces to

$$\frac{(r+\lambda)\,k}{q} = e'(h)h - e(h). \tag{8}$$

Since $e''(h) > 0$, the right-hand side of (8) is increasing.

Equation (8) plays a central role in this paper. It connects the four measures of search frictions and hours of work. One is obviously q. A decrease in q means that a firm finds it harder to fill a vacancy. Similarly, an increase in k increases frictions by raising the cost of each vacancy. An increase in the discount rate r also implies greater search frictions because a firm becomes less patient, so (8) implies that impatience raises working hours. Finally, an increase in the separation rate also implies longer working hours because a firm needs to open more vacancies to maintain a steady-state level of employment. Interestingly, $(r+\lambda)k/q$ measures the degree of frictions in labor adjustment.

2.3 Workers and Wages

Let J^U and J^E denote the values of being unemployed and employed, respectively. The probability that a worker finds a job is $M/U = \theta q(\theta)$. Then, the value of being unemployed is written recursively as $J^U = \theta q(\theta)\delta J^E + (1-\theta q(\theta))\delta J^U$, where $\delta \equiv 1/(1+r)$ is the discount factor. We assume there are no unemployment benefits. The value of being employed is $J^E = wh + \lambda\delta J^U + (1-\lambda)\delta J^E$, where λ is the exogenous separation rate, and wh is the instantaneous payoff to the worker. We ignore the utility cost of longer hours of work. From these two Bellman equations, we obtain

$$J^E - J^U = \frac{(1+r)\,wh}{r+\lambda+\theta q(\theta)}. \tag{9}$$

We assume that workers and a firm share the total surplus. In addition, we assume that workers are not unionized. Consider a bargaining process between a firm and workers of measure Δ. The threat point for the firm is $J(l-\Delta)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(l) - J(l-\Delta) + \Delta(J^E - J^U)$. If the firm's share of the surplus is given by $1-\beta \in [0,1]$, then we have $\beta[J(l) - J(l-\Delta)] = (1-\beta)\Delta(J^E - J^U)$. In the limit as $\Delta \to 0$,

$$\beta J'(l) = (1-\beta)\left(J^E - J^U\right). \tag{10}$$

This is the key equation for rent sharing.[4] Substituting the envelope condition (6) and (9) into (10), we obtain

$$f'(hl)\,h - wh - e(h) = \frac{1-\beta}{\beta}\frac{(r+\lambda)\,wh}{r+\lambda+\theta q(\theta)}, \tag{11}$$

from which it is easy to verify that the hourly wage rate w and the marginal product $f'(hl)$ are positively related. Using (4), we rewrite (11) as

$$w = \frac{\beta}{1-\beta}\frac{r+\lambda+\theta q(\theta)}{r+\lambda}\frac{e'(h)\,h - e(h)}{h}. \tag{12}$$

This equation determines the wage rate w as a function of h and θ. It is easy to verify that $\partial w/\partial\theta > 0$ and $\partial w/\partial h > 0$. Using (8), we rewrite (12) further as

$$W \equiv wh = \frac{\beta k}{1-\beta}\frac{r+\lambda+\theta q(\theta)}{q(\theta)}, \tag{13}$$

where W is the earnings. Note that $dW/d\theta > 0$. That is, as in Pissarides (2000), the earnings increase with the labor market tightness. A new finding here is that as (8) implies that $dh/d\theta > 0$, working hours and earnings are positively related. Murphy and Topel (1997) found a positive cross-sectional relationship between working hours and the hourly wage rate. Murphy and Topel (1997) interpreted this positive relationship as the standard labor supply curve based on a worker's labor–leisure choice. According to our result, the positive association between hours and the wage rate is the result of market tightness. To be precise, an increase in labor market tightness raises the wage rate as a result of Nash bargaining, and it also increases working hours because of (8). Because an increase in the tightness raises both the wage rate and working hours, these two variables are positively related. However, unlike the labor–leisure choice model, the two variables are related only *indirectly*.

The key assumption of this paper is that firms, not workers, choose the hours of work. The conventional view is that workers choose how long they work. For example, in standard neoclassical macroeconomics (Lucas and Rapping, 1969; Prescott, 2004), workers face a labor–leisure choice

[4] Smith (1999) also considered wage determination in a model similar to ours, and derived the wage rate using the intuition from the sequential bargaining theory (Osborne and Rubinstein, 1990). In contrast, our method exploits the Nash sharing rule together with the envelope condition in the dynamic programming problem.

and choose hours of work optimally, taking the market wage rate as given. Such models require that working hours are divisible and that firms can employ any quantity of labor at the market wage rate. From the firms' viewpoint, it does not matter whether workers work longer or shorter hours because the competitive market ensures that the quantity of labor demanded equals the quantity of labor supplied.

With search frictions and bilateral trading, if workers choose shorter working hours, then a firm must pay extra search costs to maintain its labor input. Thus, firms do care about working hours, and we believe this is realistic. We argue that it is plausible to assume that firms choose hours of work. Our framework is related to some extent to the indivisible labor literature (Hansen, 1985; Rogerson, 1988), where workers must choose either to work full time or to be unemployed. In the indivisible labor literature, the standard working hours are treated as exogenous. In this paper, the length of "full time" is determined by individual firms.

3 Equilibrium

3.1 Characterization

Following Smith (1999), we assume free entry of firms and that a firm entering the market opens vacancies to achieve immediately the steady state level of employment l, so that there are no transitional dynamics or (transitory) size distribution of firms. Because the job finding rate is q, in order to achieve l in the next period, the firm must create exactly l/q vacancies today. Thus, the value of entry is given by

$$J(0) = -\frac{kl}{q} - \pi + \frac{1}{1+r} J(l). \tag{14}$$

Equation (14) is interpreted as follows. The new firm creates l/q vacancies in order to employ l workers in the next period, and pays kl/q and π. Because the firm will employ l workers in the next period, the continuation value is $J(l)$. Free entry of firms therefore requires that $J(0) = 0$. Thus, from (3) and (14), we obtain

$$f(hl) - whl - e(h)l - \frac{r+\lambda}{q} kl = (1+r)\pi. \tag{15}$$

Equation (7) can be rewritten as $\lambda l = [m(U,V)/V] \times v$, which implies that, assuming symmetric equilibrium, the number of firms is

$$N \equiv \frac{V}{v} = \frac{m(U,V)}{\lambda l} = \frac{1-U}{l}. \tag{16}$$

Other steady-state conditions are given by (4)–(8) and (15), which can be reduced to a system of equations in h and l:

$$f'(L) = w + e'(h), \tag{17}$$

$$f(L) = f'(L)L + (1+r)\pi, \tag{18}$$

where $L \equiv hl$. Equation (17) states that the marginal product of labor must equal the marginal cost, and (18) asserts that the total output is distributed to labor and the fixed cost. Note that (18) determines the steady-state value of L. This suggests that with free entry of firms, the total demand for labor inputs L is determined *without any reference to the market tightness*. This is because the speed of adjusting the number of firms is infinite although the labor market is frictional. The choice of h is influenced by the market tightness. However, since total labor input L is determined by (18), the labor market tightness θ affects the choice of h and l *only* through changes in the hourly wage rate w.

Let L^* denote the solution to (18). Then, the general equilibrium of the economy is characterized by $f'(L^*) = w + e'(h)$, (8), and (12), which reduce to:

$$\frac{(r+\lambda)k}{q(\theta)} = e'(h)h - e(h), \tag{19}$$

$$f'(L^*) = \frac{\beta}{1-\beta}\frac{r+\lambda+\theta q(\theta)}{r+\lambda}\frac{e'(h)h - e(h)}{h} + e'(h), \tag{20}$$

which determine the equilibrium values of θ and h. These equations have important interpretations. Equation (19) relates search frictions and hours of work. Equation (20) comes from the wage equation.

Proposition 1 *There exists a unique steady-state equilibrium if* $-\theta q'(\theta)/q(\theta) \in (0,1)$.

Proof. Equation (19) implicitly defines $\theta = F(h)$, and F is monotone-increasing. Totally differentiating (20), we obtain

$$0 = \frac{\beta}{1-\beta} \frac{q(\theta)\left[1 + \frac{\theta q'(\theta)}{q(\theta)}\right]}{r+\lambda} \frac{e'(h)\,h - e(h)}{h} d\theta + \left[\frac{\beta}{1-\beta} \frac{r+\lambda+\theta q(\theta)}{r+\lambda} \frac{e''(h)}{h} + e''(h)\right] dh.$$

Thus, if $-\theta q'(\theta)/q(\theta) \in (0,1)$, then (20) implies a negative relationship between h and θ. Cobb–Douglas matching technology, for example, satisfies this condition. Thus, (20) implicitly defines $\theta = G(h)$, and G is monotone-decreasing. Therefore, there is a unique equilibrium. ■

Determination of equilibrium is depicted in Figure 2. The $\theta = G(h)$ locus is decreasing if and only if $1 + \theta q'(\theta)/q(\theta) > 0$. This condition is violated if the matching technology exhibits a strong complementarity, in which case the locus is upward sloping and the uniqueness of the equilibrium is not guaranteed. In what follows we focus on the case in which $1 + \theta q'(\theta)/q(\theta) > 0$.

Other equilibrium values can easily be computed. The hourly wage rate is given by (12) and employment is given by $l = L^*/h$, where L^* is the solution to (18). Since $\theta = V/U$ is determined, U and V are determined by the Beveridge curve (2), as in Figure 1, and the equilibrium number of firms is given by (16).

Proposition 2 *Specify the production technology as $f(L) = AL^\alpha$, with $\alpha \in (0,1)$. (a) An increase in A raises h, reduces l, and raises θ. This implies that earnings wh increase, V increases, U decreases, and the number of firms N increases. (b) An increase in k raises h, and reduces l and θ. This implies that earnings wh decrease, V decreases and U increases. The effect on N is ambiguous.*

Proof. Under this specification, $f'(L) = \alpha A L^{\alpha-1}$ and (18) reduces to

$$L^* = \left[\frac{(1+r)\pi}{(1-\alpha)A}\right]^{\frac{1}{\alpha}}. \tag{21}$$

(a) Equation (20) implies that an increase in A shifts the $\theta = G(h)$ locus upward. It is then evident that h and θ increase. Now, (21) implies that an increase in A decreases L^*. Thus, $l = L^*/h$ decreases. From (16), it is now evident that N increases. (b) It is evident that an increase in k shifts the $\theta = F(h)$ locus downward. Thus, h increases and θ decreases. Since L^* is not affected, $l = L^*/h$ decreases. Since U increases and l decreases, the effect on the number of firms N is ambiguous. $\blacksquare$

The results are intuitive. An increase in productivity A induces potential firms to enter the market, which tightens the labor market and raises the expected cost of a vacancy. Firms will respond to this by substituting l for h. Greater tightness leads to higher earnings wh. Similarly, an increase in the cost of vacancy k directly raises the expected cost of a vacancy. Thus, firms will respond by substituting l for h, reducing job creation. Labor market tightness decreases and, as a result, the earnings decrease.

3.2 Welfare

In this section, we consider the efficiency of the economy. Following Smith (1999), we consider the social planner's problem for $r = 0$ and focus on the steady-state welfare: $N[f(hl) - e(h)l - \pi] - kV$. Thus,

$$\max_{h,l,U,V} \frac{1-U}{l}[f(hl) - e(h)l - \pi] - kV \text{ subject to } m(U,V) = \lambda(1-U).$$

The first-order conditions are:

$$\begin{aligned} f'(L) &= e'(h), & (22) \\ f(L) &= f'(L)L + \pi, & (23) \\ \frac{f(L)-\pi}{l} - e(h) &= \frac{k(m_U + \lambda)}{m_V}, & (24) \\ m(U,V) &= \lambda(1-U), & (25) \end{aligned}$$

where $L \equiv hl$. The efficiency conditions (22)–(25) are comparable with the decentralized conditions. First, we compare (23) with (18). With $r = 0$, these two conditions are identical, suggesting that the total labor input is efficient. Consider (22) and (17). These conditions coincide if and only if $w = 0$. This implies that the equilibrium hours of work are too short because each firm faces an additional marginal cost from increasing hours, w, whereas the social planner faces only $e'(h)$. Thus, the total labor input is efficient, the equilibrium hours of work are shorter than optimal, and the equilibrium employment is less than optimal.

3.3 The Employment Effect of Restricting Working Hours

So far, we have seen that the hours of work implied by this model are less than optimal. However, we cannot take this result at face value. Note that we have assumed that a worker's instantaneous payoff is simply wh. Thus, in the analyses in the preceding sections, we have underestimated the welfare loss associated with long working hours, and it is quite possible that hours of work must be reduced. In addition, there is a policy debate about whether regulating hours of work can increase the aggregate employment (Booth and Schiantarelli, 1987, Marimon and Zilibotti, 2000). Suppose that hours of work can be perfectly controlled by regulation. Does restricting working hours stimulate employment? The purpose of this section is to address this issue.

The Bellman equation is given by

$$J(l) = \max_v \left\{ f(hl) - whl - e(h)l - kv - \pi + \frac{1}{1+r} J((1-\lambda)l + qv) \right\}.$$

Here, hours of work h is a policy parameter (Marimon and Zilibotti, 2000). The first-order conditions with respect to v is $-k + q(1+r)^{-1}J'(l) = 0$, and the envelope condition is given by (6). From these conditions, we have

$$\frac{(r+\lambda)k}{q} = f'(hl)h - wh - e(h). \tag{26}$$

From the free-entry condition, $(r+\lambda)kl/q = f(hl) - whl - e(h)l - (1+r)\pi$. We combine these two to obtain $f(hl) = f'(hl)hl + (1+r)\pi$, which is identical to (18). This expression determines the total labor input of each firm L^*. This immediately implies that a reduction in h increases l. In other words, there is a positive employment effect of restricting hours of work at the individual firm level.

Consider the general equilibrium effects of restricting hours of work. From (11),

$$wh = \left[1 + \frac{1-\beta}{\beta} \frac{(r+\lambda)}{r+\lambda+\theta q(\theta)}\right]^{-1} \left[f'(L^*)h - e(h)\right].$$

Substituting this into (26) and arranging terms, we obtain

$$\frac{(r+\lambda)k}{f'(L^*)h - e(h)} = \frac{\frac{1-\beta}{\beta}(r+\lambda)q(\theta)}{r+\lambda+\theta q(\theta) + \frac{1-\beta}{\beta}(r+\lambda)} \equiv \Omega(\theta). \tag{27}$$

It is easy to verify that $\Omega(\theta)$ is decreasing. Equation (27) determines the equilibrium labor market tightness, as depicted in Figure 3. Consider a reduction in h. This reduces the left-hand side of (27) if and only if $f'(L^*) - e'(h) > 0$. In this case, the equilibrium tightness θ increases as h decreases. From the Beveridge curve (Figure 1), it is evident that the aggregate number of vacancies V increases and the unemployment rate U decreases. To summarize:

Proposition 3 *Suppose working hours h are perfectly regulated. A reduction in hours of work increases the number of employees at each firm. It increases the aggregate vacancies and decreases unemployment if and only if $f'(L^*) - e'(h) > 0$.*

3.4 Discussion

Smith (1999) considered the case of overemployment as a result of a *bargaining externality.* The mechanism is simple. As we demonstrated in (11), the wage rate and the marginal product of labor are positively related. With concave production technology, this implies that a larger labor input reduces the marginal product and hence the wage rate. The key is that in Smith (1999), the wage rate is determined *after* the firm's decision, leaving room for the firm to influence the wage bargaining outcome by employing more workers. This is the source of over-employment in Smith (1999). In the preceding sections, we have assumed that wage bargaining takes place *before* the firm chooses hours of work and vacancies. As a result, the total labor input in equilibrium turned out to be efficient.

This section demonstrates how the model may be modified to consider the bargaining externality in Smith (1999). Since the key is that the firm's choice will influence the bargaining outcome, the order of events must be modified as follows. At the beginning of each period, each firm chooses hours of work and the number of vacancies. Then, the bargaining takes place.

Consider the bargaining outcome. From (9) and (10), we have

$$wh = \frac{r + \lambda + \theta q(\theta)}{1+r} \frac{\beta}{1-\beta} J'(l) \equiv \omega(l). \tag{28}$$

In the first stage, the firm chooses hours and vacancies, taking into account how employment l

influences the bargaining outcome (28).[5] Thus, the Bellman equation is:

$$J(l) = \max_{h,v} \left\{ f(hl) - \omega(l)l - e(h)l - kv - \pi + \frac{1}{1+r} J((1-\lambda)l + qv) \right\}.$$

The first-order conditions with respect to h and v are:

$$f'(hl) - e'(h) = 0, \tag{29}$$

$$-k + \frac{q}{1+r} J'((1-\lambda)l + qv) = 0, \tag{30}$$

Similarly, the envelope condition is given by

$$J'(l) = f'(hl)h - \omega(l) - \omega'(l)l - e(h) + \frac{1-\lambda}{1+r} J'((1-\lambda)l + qv). \tag{31}$$

Conditions (30) and (31) imply that

$$J'(l) = f'(hl)h - wh - e(h) + \frac{(1-\lambda)k}{q}. \tag{32}$$

Free entry of firms implies that any entering firm earns zero profit:

$$J(0) = -\frac{kl}{q} - \pi + \frac{1}{1+r} J(l) = 0. \tag{33}$$

Arranging these expressions, we obtain the equilibrium conditions as

$$f'(L) = e'(h), \tag{34}$$

$$f(L) = f'(L)L + (1+r)\pi - \omega'(l)l^2, \tag{35}$$

$$\frac{r+\lambda}{q} k = e'(h)h - e(h) - \omega(l) - \omega'(l)l, \tag{36}$$

where we have imposed the steady-state condition $\lambda l = qv$. From (32) it is easily verified that $J''(l) = f''(hl)h^2 < 0$, implying that $\omega'(l)l < 0$ above.

First, (34) is identical to (22). The key equation is (35), which must be comparable to (23). By $\omega'(l)l < 0$, we can conclude that the equilibrium L is larger than the efficient one, replicating Smith's (1999) result.[6] Since L is too large, (34) implies that h is less than its optimum. Therefore,

[5] Since $J(l)$ is maximized with respect to h, we can ignore the effect of h on $J'(l)$.

[6] Remember that in Smith (1999), there is no employment–hours split, so too large L implies overemployment.

l is too large. The term $\omega'(l)l < 0$ is the source of the bargaining externality in Smith (1999), in the sense that a firm has an incentive to overemploy to reduce the wage rates of all employees by exploiting the relationship between the marginal product of labor and the wage rate. The model presented in Section 2 does not possess this property because wage bargaining takes place before the choice of hours and vacancies and therefore the firm cannot influence the wage rate by hiring more employees, even though the wage rate and the marginal product of labor are positively related.

Interestingly, the analysis in Section 3.2 suggests that a firm may have another incentive to overemploy. Without the bargaining externality, the total labor input is efficient. However, since the firm faces the wage rate as an extra marginal cost of increasing hours, the demand for hours of work is less than the optimal. This causes the level of employment to be too large.

4 Interindustry Differentials in Working Hours

4.1 Characterization

Hamermesh (1993) documented that there are sizable differences in working hours across industries. For example, in 1990 in the US, the average weekly hours of work were 44 hours for the mining industry, 40.8 hours for manufacturing, 32.6 hours for services, and 28.8 for retail trade. This finding is interesting because it suggests that there are large differences in working hours even among industries, each of which consists of a variety of jobs. Hamermesh's (1993) insight is that there are interindustry differentials in the relative costs of workers and hours.

Armed with the model developed in the preceding sections, we now investigate possible determinants of dispersion in working hours. There could be a variety of reasons why people work differently, and we do not intend to give a comprehensive list of those reasons. Instead, we present a simple two-industry model and explore the possibility of interindustry differentials in working hours. The key question in this section is: which structural parameters are responsible for generating working hours differentials, and which parameters are not?

Consider an environment where there are two types of firms, type 1 and type 2. As before, the matching technology is $m(U, V)$, and we define the market tightness as $\theta \equiv V/U$. The probability that a vacancy is filled is $q(\theta) \equiv m(U/V, 1)$, and the probability that a worker finds a job is $\theta q(\theta)$.

The Bellman equation for a firm of type $i = 1, 2$ is given by

$$J_i(l_i) = \max_{h_i, v_i} \left\{ A_i (h_i l_i)^\alpha - w_i h_i l_i - e(h_i) l_i - k_i v_i - \pi_i + \frac{1}{1+r} J_i((1-\lambda) l_i + q(\theta) v_i) \right\}.$$

Notice that both types of firms face the same market tightness. This reflects the assumption that the two industries are not segmented and that workers will accept all types of jobs.[7] In addition, we assume that all firms face the same separation rate.[8] The first-order conditions with respect to h_i and v_i are:

$$\alpha A_i (h_i l_i)^{\alpha-1} = w_i + e'(h_i), \tag{37}$$

$$J_i'(l_i) = \frac{(1+r) k_i}{q(\theta)}. \tag{38}$$

From the envelope condition, and (38), we obtain

$$\frac{(r+\lambda) k_i}{q(\theta)} = e'(h_i) h_i - e(h_i). \tag{39}$$

Free entry requires that $J(0) = 0$, from which it is easy to establish that $(1-\alpha) A_i (h_i l_i)^\alpha = (1+r)\pi_i$. Thus, total labor input in sector i is determined as $h_i l_i = [(1+r)\pi_i/(1-\alpha)A_i]^{\frac{1}{\alpha}}$. Substituting this into (37), we obtain

$$\alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} [(1+r)\pi_i]^{\frac{\alpha-1}{\alpha}} = w_i + e'(h_i). \tag{40}$$

Equation (39) has a very important implication. It suggests that interindustry differentials in hours of work can arise only when $k_1 \neq k_2$. Remember that k_i captures the cost of creating a vacancy, and it works similarly to the cost of creating a job in Acemoglu (2001), in which the difference in the cost of equipment for each job creates the difference in job characteristics such as wages and productivity. To emphasize:

[7] To ensure this, we need to focus on equilibria where there is a nonnegative surplus for a relationship to share. This requires that the fixed cost π_i should not be too large.

[8] Imagine the case where the separation rate λ differs across firms. Firms facing greater separation rates would choose longer working hours. A model with on-the-job search (Burdett and Mortensen, 1998) is required to generate a search equilibrium with endogenously dispersed separation rates.

Proposition 4 *Interindustry differentials in working hours arise if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies that $h_1 > h_2$.*

We quote Hamermesh (1993, p. 208):[9]

> A survey of employers in the Rochester, New York, area in 1965–66 found an average hiring cost for all occupations of $910, but an average for professional and managerial workers of $4,600. A survey in Los Angeles in 1980 found hiring and training costs of $13,790 for salaried workers, and $5,110 for production workers. [...] In a nationwide survey of large employers in 1979 the cost of hiring a secretary was $680, but for a college graduate was $2,200.

Thus, there is a sizable difference in recruitment costs, and jobs that require higher skills are more costly to fill. The model suggests that such jobs pay more, and require longer hours of work. In what follows, we assume that $k_1 > k_2$.

Let ϕ denote the equilibrium fraction of type 1 vacancies. The probability that a job seeker finds a position is $\theta q(\theta)$. Ignoring the unemployment benefit, the value of being unemployed is

$$J^U = \theta q(\theta) \left[\phi \delta J_1^E + (1-\phi) \delta J_2^E\right] + (1 - \theta q(\theta)) \delta J^U, \tag{41}$$

where $\delta \equiv 1/(1+r)$. Similarly, the value of being employed by a firm of type i is

$$J_i^E = W_i + \lambda \delta J^U + (1-\lambda) \delta J_i^E, \tag{42}$$

where W_i is the instantaneous payoff to the worker. From (42), it is easy to verify that

$$J_1^E - J_2^E = \frac{W_1 - W_2}{1 - (1-\lambda)\delta} = \frac{1+r}{r+\lambda}(W_1 - W_2). \tag{43}$$

This implies that $W_1 > W_2$ if and only if $J_1^E > J_2^E$. We solve (41) and (42) as

$$J_1^E - J^U = \frac{1+r}{r+\lambda+\theta q(\theta)} \left[W_1 - \theta q(\theta)(1-\phi)\frac{W_2 - W_1}{r+\lambda}\right], \tag{44}$$

$$J_2^E - J^U = \frac{1+r}{r+\lambda+\theta q(\theta)} \left[W_2 - \theta q(\theta)\phi \frac{W_1 - W_2}{r+\lambda}\right]. \tag{45}$$

[9] All values are in 1990 US dollars.

As before, we assume that workers and the firm share the rent. The firm's share of rent is denoted by $1-\beta$, where $\beta \in [0,1]$. The Nash sharing rule requires that $\beta[J_i(l) - J_i(l-\Delta)] = (1-\beta)\Delta(J_i^E - J^U)$, where Δ is the measure of workers. In the limit as $\Delta \to 0$, $\beta J_i'(l) = (1-\beta)(J_i^E - J^U)$ for $i = 1,2$. With (38),

$$\frac{\beta\,(1+r)\,k_i}{q\,(\theta)} = (1-\beta)\left(J_i^E - J^U\right). \tag{46}$$

Computing $J_1^E - J_2^E$ from (46) and substituting into (43), we obtain

$$W_1 - W_2 = \frac{(r+\lambda)\,\beta}{1-\beta}\frac{k_1 - k_2}{q\,(\theta)}. \tag{47}$$

It is then easy to establish that $W_1 > W_2$ holds if and only if $k_1 > k_2$. Substituting (46) and (47) into (44) and (45), we obtain

$$\frac{(r+\lambda)\,k_i}{q\,(\theta)} = \frac{1-\beta}{\beta} W_i - \theta\left[\phi k_1 + (1-\phi)\,k_2\right] \tag{48}$$

for $i = 1,2$. Remember that the worker's instantaneous payoff as $W_i = w_i h_i$. We use (39) to rewrite (48) as

$$w_i = \frac{\beta}{1-\beta}\frac{e'(h_i)h_i - e(h_i) + \theta\left[\phi k_1 + (1-\phi)\,k_2\right]}{h_i}. \tag{49}$$

for $i = 1,2$. This is the wage equation for this economy.

The steady-state equilibrium is characterized by

$$\begin{aligned}
\frac{(r+\lambda)\,k_i}{q\,(\theta)} &= e'(h_i)h_i - e(h_i), \\
\alpha\,(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} \left[(1+r)\,\pi_i\right]^{\frac{\alpha-1}{\alpha}} &= \frac{\beta}{1-\beta}\frac{e'(h_i)h_i - e(h_i) + \theta\left[\phi k_1 + (1-\phi)\,k_2\right]}{h_i} + e'(h_i).
\end{aligned}$$

These two expressions are the counterpart of (19)–(20). To simply the exposition, we specify the functional forms as $m(U,V) = aV^bU^{1-b}$ and $e(h) = h^2$. Then, we obtain

$$\begin{aligned}
\alpha\,(1-\alpha)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} \left[(1+r)\,\pi_i\right]^{\frac{\alpha-1}{\alpha}} &= \left[\frac{\beta}{1-\beta} + 2\right]\left(\frac{(r+\lambda)\,k_i}{a}\right)^{1/2}\theta^{\frac{1-b}{2}} \\
&\quad + \frac{\beta\left[\phi k_1 + (1-\phi)\,k_2\right]}{1-\beta}\left(\frac{(r+\lambda)\,k_i}{a}\right)^{-1/2}\theta^{\frac{1+b}{2}}
\end{aligned} \tag{50}$$

for $i = 1,2$. This expression implies that $d\theta/d\phi < 0$ holds for both $i = 1,2$. The equilibrium of the model is given as the values of ϕ and θ that solve (50).

4.2 Interindustry Differentials: Alternative Formulation

In Section 4.1, interindustry differentials in working hours occur as a result of the different costs for creating a job, k_i. Equation (39) suggests another potential explanation for the dispersion of working hours, namely, the different employment probabilities faced by firms. This section explores this possibility by developing a directed search version of the model in the spirit of Acemoglu (2001, Section III.B) and Moen (1997).

Suppose that the markets for the two types of jobs are segmented. A worker can apply to either the type 1 sector or the type 2 sector. We assume that the two sectors face the same matching technology. Let U_i denote the number of unemployed workers applying to a firm of type i. Similarly, V_i is the number of vacancies of type i. Thus, the probability that a worker applying to sector i finds a job in that sector is $\theta_i q(\theta_i)$, where $\theta_i \equiv V_i/U_i$. Then the value of applying to sector $i = 1, 2$ is $J_i^U = \theta_i q(\theta_i)\delta J_i^E + (1 - \theta_i q(\theta_i))\delta J_i^U$, where $\delta \equiv 1/(1 + r)$. Similarly, the value of being employed is $J_i^E = W_i + \lambda\delta J_i^U + (1 - \lambda)\delta J_i^E$ for $i = 1, 2$. From these equations,

$$rJ_i^U = \frac{(1+r)\,\theta_i q\,(\theta_i)\,W_i}{r + \lambda + \theta_i q\,(\theta_i)} = \frac{1+r}{\frac{r+\lambda}{\theta_i q(\theta_i)} + 1} W_i. \tag{51}$$

The two types of jobs coexist if and only if $J_1^U = J_2^U \equiv J^U$. This requires that

$$\frac{1+r}{\frac{r+\lambda}{\theta_1 q(\theta_1)} + 1} W_1 = \frac{1+r}{\frac{r+\lambda}{\theta_2 q(\theta_2)} + 1} W_2.$$

Thus, it is evident that $W_1 > W_2 \Leftrightarrow \theta_2 > \theta_1$. That is, workers face a tradeoff between higher payoffs and better job finding rates. The wage equation is

$$w_i = \frac{\beta}{1-\beta}\frac{r + \lambda + \theta_i q\,(\theta_i)}{r + \lambda}\frac{e'\,(h_i)\,h_i - e(h_i)}{h_i} \equiv w\,(\theta_i, h_i)\,.$$

We substitute this into (51) to obtain

$$rJ_i^U = \theta_i q\,(\theta_i)\,\frac{\beta}{1-\beta}\frac{1+r}{r+\lambda}\left[e'\,(h_i)\,h_i - e(h_i)\right] = \frac{\beta\,(1+r)\,k_i}{1-\beta}\theta_i.$$

Thus, $J_1^U = J_2^U$ implies that

$$k_1\theta_1 = k_2\theta_2. \tag{52}$$

Therefore, $k_1 > k_2 \Leftrightarrow \theta_2 > \theta_1 \Leftrightarrow W_1 > W_2$. In other words, the industry with a greater equipment cost faces a less tight labor market because this industry pays more and attract more workers.

From firms' optimization and free entry, we obtain the equilibrium conditions: $\alpha A_i(h_i l_i)^{\alpha-1} = w(\theta_i, h_i) + e'(h_i)$, $(1-\alpha)A_i(h_i l_i)^{\alpha} = (1+r)\pi_i$, $(r+\lambda)k_i/q(\theta_i) = e'(h_i)h_i - e(h_i)$ for $i = 1, 2$. From the last expression, θ_i and h_i are positively related. Thus:

Proposition 5 *Interindustry dispersion in working hours arises if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies that $h_2 > h_1$.*

This result is striking. If the labor markets for the two industries are pooled (as in Section 4.1), then greater equipment costs imply longer working hours because they directly increase search frictions for firms. On the other hand, if the two labor markets are segmented and workers choose either of the two, then firms face different equipment costs and tightness. As pointed out by Acemoglu (2001), firms facing a greater equipment cost pay more. The same mechanism works in our framework, and workers apply to the high-paying industry. As a result, the market tightness in the high-k industry is *lower*, which means firms face smaller frictions. Interestingly, this effect dominates the other.

5 Conclusion

This paper has studied the demand for employment and hours of work in a dynamic model with search frictions. The key finding is that a firm's demand for hours of work increases as it faces greater search frictions, and the frictions are captured by a simple expression. Since there is free entry of firms, the total labor input is determined without any reference to search frictions, and is at the efficient level. However, search frictions distort the *composition* of the labor demand.

We have used this model to find the source of differences in working hours. We found that differences in recruitment costs are solely responsible for dispersion in hours of work across industries. This finding is related to some extent to Acemoglu's (2001), who argued that jobs with higher equipment costs pay more. We have treated the recruitment (or equipment) costs as parameters.

The nature of these costs, and especially their determinants, must be investigated.

In this paper, we did not consider heterogeneity of workers. Kuhn and Lozano (2005) documented that the observed increase in hours of work in the US was concentrated among skilled men. A deeper understanding of dispersion in hours of work requires a model with heterogeneous skills. It is interesting to introduce skilled and unskilled workers into the model of this paper to ingestigate whether the model can account for Kuhn and Lozano's (2005) finding.

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Figure 1:

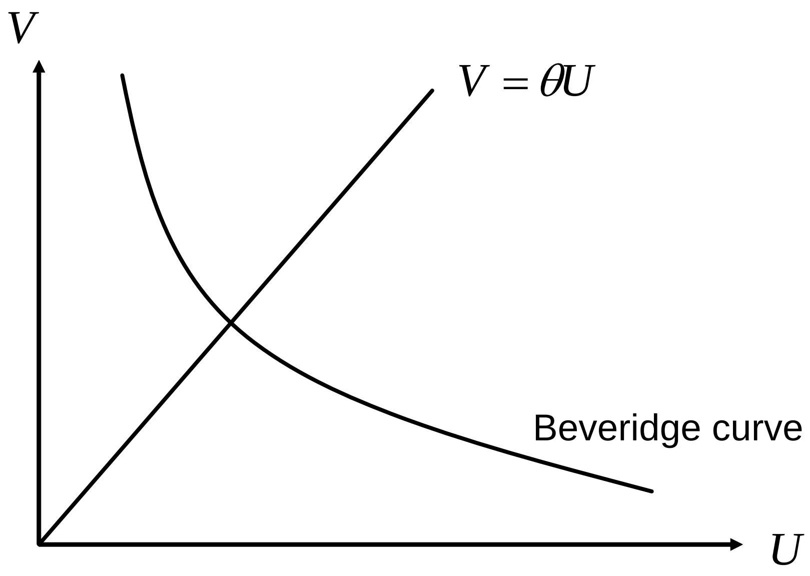

Figure 2:

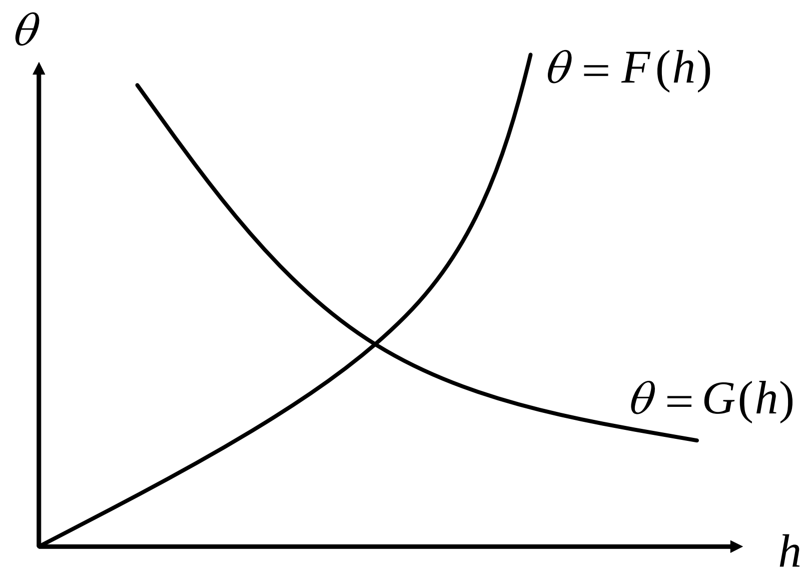

Figure 3: