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Discussion Paper 07-04

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Abstract

In this paper, we use a two-period overlapping generations model to examine the behavior of an economy that incorporates intergenerational transfers of time. In the first part, we describe the dynamics and steady state of the economy in which there is no government. We show that the rate of life expectancy has negative impact on the steady-state level of the capital stock. In the second part, we study the role and the effect of public long-term care policy. We also show that public long-term care lowers the steady-state level of the capital stock but enhances the welfare when the rate of tax is small.

Classification Numbers: E60, I12, J14, J22

Key words: time transfers, household production, overlapping generations

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1 Introduction

Since the seminal papers of Becker (1965) and Barro (1974), there have been many studies of intergenerational transfers (see, for example, Weil (1987) and Abel (1987)) Almost all argue that altruism and the way in which transfers are made are important determinants of dynamic allocation. Any theory of intergenerational transfers depends on the way in which transfers are made. One involves transferring goods and the other involves transferring time. In the context of these studies, intergenerational transfers of time are likely to become more prominent given that the economy is 'graying' at a rapid rate.[1] Because physical and mental health tend to deteriorate with age, the number of people needing more time-intensive transfers such as health care and other health services increases. For example, in the developed countries, the average density of practicing nurses per thousand people was about 2.5 in 1960 and around 8.1 in 2003 (OECD (2005)). In Japan, about 94.8% of people needing care are over 65 years of age. In particular, family care accounts for 74.4% of the total. Care undertaken by children accounts for 39.1% of all care (Japan Ministry of Health, Labor and Welfares (2004)).

When labor supply decisions are endogenous, given that young agents can choose to transfer their time between to work in the household contributing to household-produced health or can work in the market producing market goods, intergenerational transfers of time become an important issue when the population is aging. This is because the demand for care increases with population aging. Therefore, in this paper, we focus on the intergenerational transfer of time allocated to health care and examine the interaction between intergenerational transfers of time and the macroeconomy when there is population aging.

Population aging, along with increasing associated health care costs, has received much attention Almost all studies have been empirical (see, for example, Lakdawalla and Philipson (2002)). Despite the importance of empirical work, intergenerational transfers of time allocated to health care have received little theoretical attention. In recent theoretical work, Bednarek and Pecchenino (2002) and Tabata (2005) have theoretically analyzed the health care issue using goods-related transfers. Unlike these studies, this paper examines time-related transfers. Specifically, our study examines the impact of intergenerational transfers of time and public long-term care policy on capital stock and welfare. For this purpose, we employ a two-period overlapping generations model that incorporates uncertainties about lifespan and

[1] According to the United Nations (2006), the total number of persons aged 60 years or older has tripled over the last 50 years, and is projected to more than triple again over the next 50 years.

illness in old age.

In the model developed, we assume that old agents have a production technology for health and produce own health in the household by using time. When agents derive utility from their aged parents' health status, intergenerational transfers of time are modeled by allowing young agents to contribute to the work undertaken in the household on household-produced health.[2] In what follows, we refer to intergenerational transfers of time synonymously with providing care to aged parents and synonymously with care provision.

In our model, the population of every generation is assumed to be the same size. Thus, a rise in life expectancy increases the ratio of the old-age population to the young-age population, and thereby constitutes population aging. Using our model, in the first part of the paper, we describe the dynamics and steady state of the economy in which there is no government. By analyzing this model, we show that the rate of life expectancy has two opposing effects on the steady-state level of the capital stock. One is the 'demographic effect', which operates through aggregate care provision to aged parents. Because aggregate care provision increases with population aging, through this effect, an increase in life expectancy reduces the steady-state level of the capital stock. The other is the 'time preference effect'. In our model, the interest rate is an increasing function of life expectancy and, thus, an increase in life expectancy in our model is synonymous with the rate of time preference as incorporated in models such as those of Yaari (1965) and Blanchard (1985). Therefore, through this effect, an increase in life expectancy increases the steady-state level of the capital stock. At the steady state, the negative demographic effect is exclusive the positive time preference effect, thus the net effect becomes negative.

In the second part of the paper, we examine the role and the effect of public long-term care policy, that is, LTC. When we assume that LTC is financed by taxing the income of young agents, and when young agents transfer their time to provide public care, LTC lowers labor supply to firms. A decrease in labor supply also reduces savings, and thereby lowers the steady-state level of the capital stock.

In addition, we show the effect of LTC on welfare. The effect of LTC is considered to comprise a 'health status effect', a 'capital stock effect', and a 'subsidy effect'. The health status effect shows how health status affects welfare. Because LTC improves health status,

[2] Intergenerational transfers of time using household production have been studied by Cardia and Michel (2004). Since the seminal paper of Grossman (1972), many papers have been devoted to analyzing household health production (see, for example, Wagstaff (1986), Jacobson (2000)).

this effect on welfare is positive. LTC also has a capital stock effect on welfare. Because LTC lowers the steady-state level of the capital stock, the steady-state level of welfare is lowered. In addition to these two effects, we have the subsidy effect on welfare. Because old agents can receive public care provision without being taxed, this effect improves the steady-state level of welfare of old agents. By comparing these effects, we show that LTC is welfare enhancing when the rate of tax is small.

The remainder of this paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the equilibrium in which there is no government. Section 4 examines the role and effect of LTC on the dynamic equilibrium. Section 5 shows the effect of LTC on welfare. Section 6 concludes this paper.

2 The Model

We consider a two-period overlapping generations model that incorporates uncertainty about lifespan and illness status in old age. Time is discrete and the time horizon is infinite. The economy begins operating in period 1, and the cohort born in period t is known as generation t. Generation t is composed of a continuum of $N_t > 0$ agents who live for a maximum of two periods, that is, young and old. At each date, new generations, each consisting of a continuum of agents with a unit measure, are born. They are endowed with one unit of time when young and old. Those who are old in period 0 are the initial old.

Agents

Agents have altruism and derive utility from the state of their aged parents' health. The probability that an agent lives through the period of old age is $p \in (0, 1)$. The probability that an individual dies at the beginning of the period of old age, after having had a child is $1 - p$. If an individual is alive in his or her old age, he or she also has some probability of being in poor health. The probability of an individual being in good health throughout his or her old age is $\psi \in (0, 1)$; the corresponding probability of poor health is $1 - \psi$. Therefore, there are three different states in two periods of life: good health, poor health, and death.

A fraction $p\psi$ of young agents is of type g, whose parents have good health. Type b agents, who constitute $p(1 - \psi)$ of young agents, have parents who are in poor health. The fraction $1 - p$ of young agents are of type d, whose parents die. In addition, the fraction $p\psi$ of old agents are of type g, who have good health. Type b agents, who constitute $p(1 - \psi)$ of old agents, have poor health. The fraction $1-p$ of old agents are of type d. We express the death–illness status of each agent's parents by using the index i; each agent's own status is indexed by j. To simplify the analysis, we assume that the probability of death and illness status is not serially correlated across generation. Thus, the probabilities of the death–illness states θ_s $(s = i, j)$ are:

$$\theta_s = \begin{cases} p\psi & \text{if} \quad s = g, \\ p(1-\psi) & \text{if} \quad s = b, \\ (1-p) & \text{if} \quad s = d. \end{cases} \tag{1}$$

We assume that each old agent has the following household-produced health technology and produces his or her own health by using his or her own time and that transferred by his or her children:

$$h_{j,t} \quad = \quad dq_{i,t}^{\,t} + \gamma, \quad j = g, b \tag{2}$$

where $d \geq 1$ is a productivity parameter such that $d > 1$ if $j = g$ and $d = 1$ if $j = b$. In addition, $q_{i,t}^{\,t}$ represents care provision from type i children to parents, and γ represents the productivity of old agents. Each old agent supplies one unit of time to produce household health, although productivity is $\gamma = \gamma \in (0,1)$ if $j = g$; and $\gamma = 0$ if $j = b$, which is given exogenously.

We assume that each agent of generation $t \geq 1$, whose parents' death–illness status is i, has the following expected utility function:

$$\begin{aligned} \max_{h_{i,t}, c_{i,t+1}^{\,t}} U_i \quad &\equiv \quad Eu(h_{i,t}, c_{i,t+1}^{\,t}, h_{j,t+1}; p, \psi) \\ &= \quad \beta \ln h_{i,t} + p[c_{i,t+1}^{\,t} + \psi \ln h_{g,t+1} + (1-\psi) \ln h_{b,t+1}], \quad i, j = g, b, d, \end{aligned} \tag{3}$$

where $c_{i,t+1}^{\,t}$ is the consumption of market goods during old age, and $\beta \in (0,1)$ measures the degree of altruism towards parents. p and ψ are realized at the beginning of each period. Because an agent is endowed with one unit of time, he or she allocates time to working in the household on household-produced health, $q_{i,t}^{\,t}$, or to the market place producing market goods, $l_{i,t}^{\,t}$. A young agent earns wage income of $w_t l_{i,t}^{\,t}$ by working for a firm, and saves all that wage income for his or her old age. When old, an agent receives the proceeds of these savings and allocates his or her endowed time to generating household-produced health. Following Yaari (1965) and Blanchard (1985), we assume the existence of actuarially fair insurance companies. These companies collect funds and invest them for firms. Returns on investments are repaid to the insured household members who are still living. In other words, the contract offered by the insurance company redistributes income from the dead to the living. Suppose that the insurance company collects e_t from each young agent (and thereby $E_t \equiv e_t N_t$ in the aggregate) in period t. (Note that old agents have no incentive to buy the annuity because they are not alive in the subsequent period.) The company invests the funds for the firms and acquires total proceeds of $R_{t+1}E_t$ in period $t+1$. Given that only pN_t old people survive in period $t+1$, each receives $R_{t+1}e_t/p$ from the insurance company (because of perfect competition between companies). Thus, the rate of return on the annuities is R_{t+1}/p for the living and 0 for those who die at the end of period t. On the other hand, if young agents in period t invest e_t directly for their firms, they receive $R_{t+1}e_t$ whether they are alive or dead. (For agents who do not live to the next period, their children inherit the funds.) Thus, the rate of return on self-investment is R_{t+1}. Because we assume that households have no bequest motive, they accept the insurance contract, which yields a higher interest rate than does self-investment. Thus, the budget constraints of generation t, and their parents' death–illness status $i = g, b, d$, are $c_{i,t+1}^{t} = R_{t+1}w_t l_{i,t}^{t}/p$. By combining the time constraints $l_{i,t}^{t} + q_{i,t}^{t} = 1$, we have the following lifetime budget constraints:

$$c_{i,t+1}^{t} = \frac{R_{t+1}}{p} w_t (1 - q_{i,t}^{t}), \quad i = g, b, d \tag{4}$$

In each period, the time spent working in the household on household-produced health or that spent in the market place producing market goods must be nonnegative and must not exceed unity, as follows.[3]

$$0 \leq q_{i,t}^{t} \leq 1, \quad \text{and} \quad 0 \leq l_{i,t}^{t} \leq 1, \quad i = g, b, d \tag{5}$$

Taking R_{t+1}, w_t, p, and ψ as given, each young agent maximizes the expected utility of (3) subject to (2), (4), and (5). The optimal allocation is derived as follows.

$$q_{g,t}^{t} = \begin{cases} 0 & \text{if } \frac{d\beta}{\gamma} \leq R_{t+1}w_t, \\ \dfrac{\beta}{R_{t+1}w_t} - \dfrac{\gamma}{d} & \text{if } \frac{d\beta}{d+\gamma} \leq R_{t+1}w_t \leq \frac{d\beta}{\gamma}, \\ 1 & \text{if } R_{t+1}w_t \leq \frac{d\beta}{d+\gamma}, \end{cases} \tag{6}$$

$$q_{b,t}^{t} = \begin{cases} \dfrac{\beta}{R_{t+1}w_t} & \text{if } \beta \leq R_{t+1}w_t, \\ 1 & \text{if } R_{t+1}w_t \leq \beta \end{cases} \tag{7}$$

When $R_{t+1}w_t$ is sufficiently large, the opportunity cost of care provision becomes high; thus, young agents reduce their care provision. If the parents of young agents die, agents derive no

[3]See Appendix A for derivation.

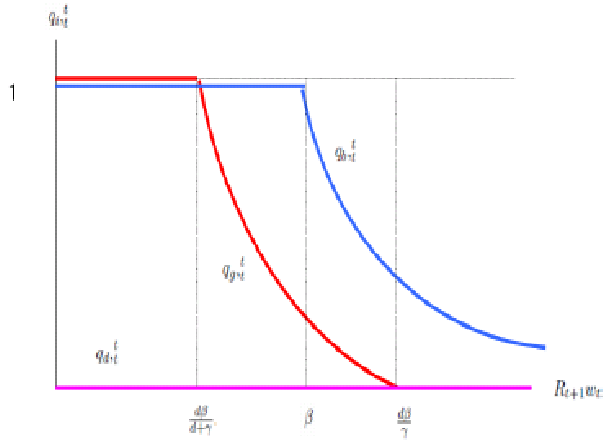

Figure 1: The care provision

utility from the state of their aged parents' health and, thus, we obtain:

$$q_{d,t}^{\ t} = 0. \tag{8}$$

Figure 1 illustrates the relation of care provision.

Noting that $d\beta/(d+\gamma) < \beta < d\beta/\gamma$, we derive aggregate care provision by summing up equations (6) to (8) and by using (1), as follows.

$$\begin{aligned} Q_t \equiv q_t N_t &= \sum_{i=g,b,d} \theta_i q_{i,t}^{\ t} N_t \\ &= \begin{cases} p\Big((1-\psi)\dfrac{\beta}{R_{t+1}w_t}\Big)N_t & \text{if } \frac{d\beta}{\gamma} \le R_{t+1}w_t, \\ p\Big(\dfrac{\beta}{R_{t+1}w_t} - \dfrac{\psi\gamma}{d}\Big)N_t & \text{if } \beta \le R_{t+1}w_t \le \frac{d\beta}{\gamma}, \\ p\Big(1-\psi+\dfrac{\psi\beta}{R_{t+1}w_t} - \dfrac{\psi\gamma}{d}\Big)N_t & \text{if } \frac{d\beta}{d+\gamma} \le R_{t+1}w_t \le \beta, \\ pN_t & \text{if } R_{t+1}w_t \le \frac{d\beta}{d+\gamma} \end{cases} \end{aligned} \tag{9}$$

From (9), we find that, for a given level of $R_{t+1}w_t$, aggregate care provision increases with life expectancy p. From the resource constraints, we obtain the following aggregate labor supply.[4]

$$L_t \equiv l_t N_t = \begin{cases} \Big(1 - \dfrac{p\beta(1-\psi)}{R_{t+1}w_t}\Big)N_t & \text{if } \frac{d\beta}{\gamma} \le R_{t+1}w_t, \\ \Big(1 + \dfrac{p\psi\gamma}{d} - \dfrac{p\beta}{R_{t+1}w_t}\Big)N_t & \text{if } \beta \le R_{t+1}w_t \le \frac{d\beta}{\gamma}, \\ \Big(1 + \dfrac{p\psi\gamma}{d} - p(1-\psi) - \dfrac{p\psi\beta}{R_{t+1}w_t}\Big)N_t & \text{if } \frac{d\beta}{d+\gamma} \le R_{t+1}w_t \le \beta, \\ (1-p)N_t & \text{if } R_{t+1}w_t \le \frac{d\beta}{d+\gamma} \end{cases} \tag{10}$$

[4] Aggregate labor supply is derived as $L_t \equiv l_t N_t = [p\psi(1-q_{g,t}^{\ t}) + p(1-\psi)(1-q_{b,t}^{\ t}) + (1-p)(1-q_{d,t}^{\ t})]N_t$.

Because young agents save all their wage income, aggregate savings are $S_t \equiv s_t N_t = w_t l_t N_t$. Dividing both sides by N_t yields the following savings functions.

$$
\begin{aligned}
s_t &= w_t l_t \\
&= \begin{cases}
w_t\Big(1 - \dfrac{p\beta(1-\psi)}{R_{t+1}w_t}\Big) & \text{if } \frac{d\beta}{\gamma} \le R_{t+1}w_t, \\
w_t\Big(1 + \dfrac{p\psi\gamma}{d} - \dfrac{p\beta}{R_{t+1}w_t}\Big) & \text{if } \beta \le R_{t+1}w_t \le \frac{d\beta}{\gamma}, \\
w_t\Big(1 + \dfrac{p\psi\gamma}{d} - p(1-\psi) - \dfrac{p\psi\beta}{R_{t+1}w_t}\Big) & \text{if } \frac{d\beta}{d+\gamma} \le R_{t+1}w_t \le \beta, \\
w_t(1-p) & \text{if } R_{t+1}w_t \le \frac{d\beta}{d+\gamma}
\end{cases}
\end{aligned}
$$

Firms

Firms are perfectly competitive profit maximizers that produce output according to a Cobb–Douglas production function of the form, $Y_t = AK_t^{\alpha}(l_t N_t)^{1-\alpha}$, where Y_t is aggregate output, $A > 0$ is a productivity parameter, and K_t is the aggregate capital stock. The production function can be rewritten in intensive form as $y_t = A(k_t)^{\alpha}(l_t)^{1-\alpha}$, where $k_t \equiv K_t/N_t$ is the per capita level of capital. We assume that capital depreciates fully in the process of production. Given that firms are price takers, they take the wage w_t and the real rental price of capital R_t as given and then hire labor and capital so that their marginal products equal their factor prices:

$$w_t = (1-\alpha)A\tilde{k}_t^{\alpha}, \quad R_t = \alpha A\tilde{k}_t^{\alpha-1}, \tag{11}$$

where $\tilde{k}_t \equiv k_t/l_t$ is the capital–labor ratio.

3 Equilibrium and Welfare

As a benchmark case, we first describe an economy in which there is no government. In Section 4, we analyze an economy in which there is public policy. We first derive equilibrium in the goods market. The equilibrium condition for the capital market is given by $K_{t+1} = s_t N_t = w_t l_t N_t$, which implies that the savings of young agents in generation t forms the aggregate capital stock in period $t+1$. Dividing both sides by N_t and using (11) yields the following equilibrium condition for the capital market.

$$\tilde{k}_{t+1} = \frac{(1-\alpha)A\tilde{k}_t^{\alpha} l_t}{l_{t+1}} \tag{12}$$

In period 1, there are the young agents of generation 1 and the initial old agents of generation 0. The initial old agents of generation 0 are endowed with k_1 units of capital. Each old agent rents his or her capital to the insurance firms and earns an income of $(R_1/p)k_1$, which is then consumed. The measure of initial old individuals is $pN_0 > 0$. The utility obtained by each individual in generation 0 is $c_1^0 + \psi \ln h_{g,1}^{0} + (1-\psi)\ln h_{b,1}^{0}$.

Definition 1 *The economic equilibrium is a sequence of allocations and prices,* $\{\{c_{i,t}^{t}, q_{i,t}^{t}, l_{i,t}^{t}, s_{i,t}^{t}\}_{i=g,b,d}, k_t, y_t, w_t, r_t\}_{t=1}^{\infty}$*, given the initial condition* $k_1 = K_1/N_1 > 0$*, such that all individual's utility levels are maximized, firms' profits are maximized, and all markets are cleared.*

By substituting (11) and (12) into (10), we obtain the following dynamics of labor supply $\{l_t\}_{t=1}^{\infty}$.

$$l_{t+1}^{1-\alpha} = \begin{cases} \dfrac{p\beta(1-\psi)l_t^{1-\alpha}}{\alpha A(1-l_t)[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}}, & \text{if } \frac{d\beta}{\gamma} \le R_{t+1}w_t \quad \text{< Regime I >} \\ \dfrac{p\beta l_t^{1-\alpha}}{\alpha A(1+\frac{p\psi\gamma}{d}-l_t)[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}}, & \text{if } \beta \le R_{t+1}w_t \le \frac{d\beta}{\gamma} \quad \text{< Regime II >} \\ \dfrac{p\psi\beta l_t^{1-\alpha}}{\alpha A[1+\frac{p\psi\gamma}{d}-p(1-\psi)-l_t][(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}}, & \text{if } \frac{d\beta}{d+\gamma} \le R_{t+1}w_t \le \beta \quad \text{< Regime III >} \\ (1-p)^{1-\alpha}, & \text{if } R_{t+1}w_t \le \frac{d\beta}{d+\gamma} \quad \text{< Regime IV >} \end{cases} \tag{13}$$

The following dynamics of the capital–labor ratio $\{\tilde{k}_t\}_{t=1}^{\infty}$ are obtained from (12) and (13).

$$\tilde{k}_{t+1}^{1-\alpha} = \begin{cases} \dfrac{\alpha A(1-l_t)(1-\alpha)A\tilde{k}_t^{\alpha}}{p\beta(1-\psi)}, & \text{if } \frac{d\beta}{\gamma} \le R_{t+1}w_t \quad \text{< Regime I >} \\ \dfrac{\alpha A(1+\frac{p\psi\gamma}{d}-l_t)(1-\alpha)A\tilde{k}_t^{\alpha}}{p\beta}, & \text{if } \beta \le R_{t+1}w_t \le \frac{d\beta}{\gamma} \quad \text{< Regime II >} \\ \dfrac{\alpha A[1+\frac{p\psi\gamma}{d}-p(1-\psi)-l_t](1-\alpha)A\tilde{k}_t^{\alpha}}{p\psi\beta}, & \text{if } \frac{d\beta}{d+\gamma} \le R_{t+1}w_t \le \beta \quad \text{< Regime III >} \\ \left((1-\alpha)A\tilde{k}_t^{\alpha}\right)^{1-\alpha}, & \text{if } R_{t+1}w_t \le \frac{d\beta}{d+\gamma} \quad \text{< Regime IV >} \end{cases} \tag{14}$$

Before describing the equilibrium, we consider the border lines between the regimes. By using (11), (12), and (14), we can express these border lines as follows.

$(a) : \frac{d\beta}{\gamma} = R_{t+1}w_t$

$$(a) : l_t = 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d}$$

$(b) : \beta = R_{t+1}w_t$

$$(b) : l_t = 1 + \frac{p\psi\gamma}{d} - p$$

$(c) : \frac{d\beta}{d+\gamma} = R_{t+1}w_t$

$$(c) : l_t = 1 - p$$

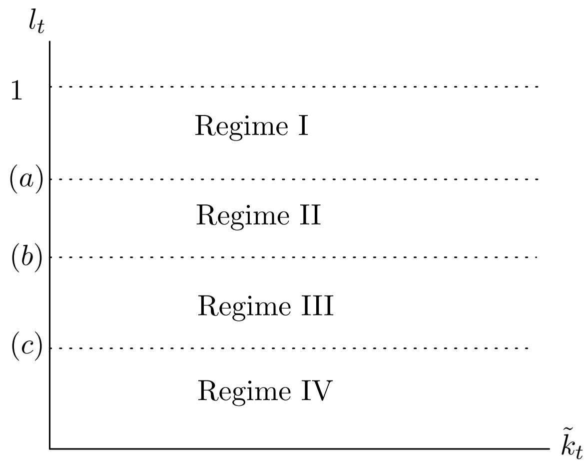

Figure 2: The regime without LTC

These regimes are depicted in Figure 2.

Equations (13) and (14) characterize the economic equilibria that are represented by sequences of $\{\tilde{k}_t, l_t\}_{t=1}^{\infty}$ and by the initial condition $(\tilde{k_1}, l_1) > 0$. We draw the phase diagram on the $(\tilde{k}_t, l_t)$ plane. The locus on the $(\tilde{k}_t, l_t)$ plane representing $\tilde{k}_{t+1} = \tilde{k}_t$ is referred to as the KK locus and the one representing $l_{t+1} = l_t$ is referred to as the LL locus. Given that the minimum level of labor supply is $1-p$, the KK and LL loci obtained from (13) and (14) are as follows.

Regime I: $1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d} \leq l_t$

$$LL_1 \;:\; l_t = 1 - \frac{p\beta(1-\psi)}{\alpha A\{(1-\alpha)A\tilde{k}_t^{\alpha}\}^{\alpha}},$$
$$KK_1 \;:\; \tilde{k}_t = \left(\frac{(1-\alpha)A\alpha A(1-l_t)}{p\beta(1-\psi)}\right)^{\frac{1}{1-2\alpha}}$$

The LL_1 and KK_1 loci intersect border line (a) at the A^{ll} and A^{kk}, respectively, where: $A^{ll} \equiv (\tilde{k}_t, l_t) = ([(\frac{\beta d}{\gamma\alpha A})^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d})$ and $A^{kk} \equiv (\tilde{k}_t, l_t) = ([\frac{(1-\alpha)A\alpha A\gamma}{\beta d}]^{\frac{1}{1-2\alpha}}, 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d})$. These loci in Regime I are depicted in Figure 3.

Regime II: $1 + \frac{p\psi\gamma}{d} - p \leq l_t \leq 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d}$

$$LL_2 \;:\; l_t = 1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}},$$
$$KK_2 \;:\; \tilde{k}_t = \left(\frac{(1-\alpha)A\alpha A(1+\frac{p\psi\gamma}{d} - l_t)}{p\beta}\right)^{\frac{1}{1-2\alpha}}$$

The LL_2 and KK_2 loci intersect border line (a) at points A^{ll} and A^{kk}, respectively, and inter-

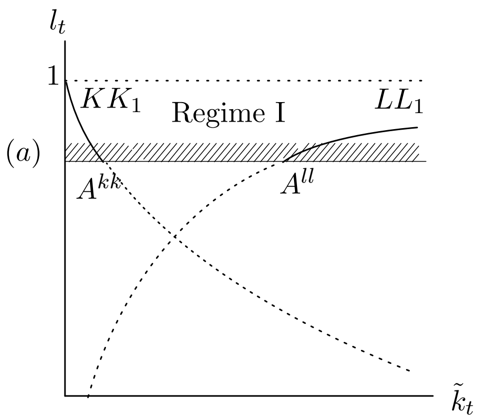

Figure 3: The relationship in Regime I

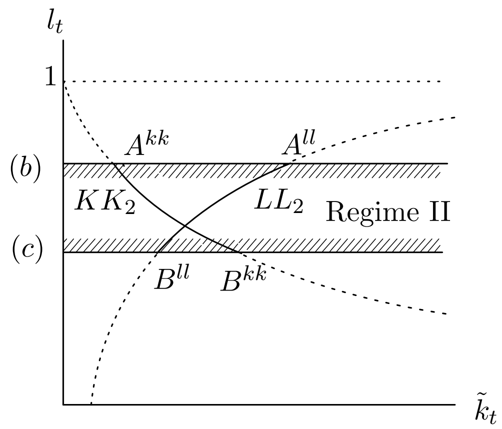

Figure 4: The relationship in Regime II

sect border line (b) at points B^{ll} and B^{kk}, respectively, where $B^{ll} \equiv (\tilde{k}_t, l_t) = ([(\frac{\beta}{\alpha A})^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, 1+\frac{p\psi\gamma}{d} - p)$ and $B^{kk} \equiv (\tilde{k}_t, l_t) = ([\frac{(1-\alpha)A\alpha A}{\beta}]^{\frac{1}{1-2\alpha}}, 1+\frac{p\psi\gamma}{d} - p)$. These loci in Regime II are depicted in Figure 4.

Regime III: $1 - p \leq l_t \leq 1 + \frac{p\psi\gamma}{d} - p$

$$LL_3 \quad : \quad l_t = 1 + \frac{p\psi\gamma}{d} - p(1-\psi) - \frac{p\psi\beta}{\alpha A[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}},$$

$$KK_3 \quad : \quad \tilde{k}_t = \left(\frac{(1-\alpha)A\alpha A[1 + \frac{p\psi\gamma}{d} - p(1-\psi) - l_t]}{p\psi\beta} \right)^{\frac{1}{1-2\alpha}}$$

The LL_3 and KK_3 loci intersect border line (b) at points B^{ll} and B^{kk}, respectively, and intersect border line (c) at points C^{ll} and C^{kk}, respectively, where $C^{ll} \equiv (\tilde{k}_t, l_t) = ([(\frac{\beta\gamma}{\alpha A(d+\gamma)})^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, 1-p)$, and $C^{kk} \equiv (\tilde{k}_t, l_t) = ([\frac{(1-\alpha)A\alpha A(1+\gamma)}{\beta}]^{\frac{1}{1-2\alpha}}, 1-p)$. These loci in Regime III are depicted in Figure 5.

Regime IV: $l_t \leq 1 - p$

$$LL_4 \quad : \quad l_t = 1 - p$$

$$KK_4 \quad : \quad \tilde{k}_t = ((1-\alpha)A)^{\frac{1}{1-\alpha}}$$

The LL_4 and KK_4 loci intersect border line (c) at points C^{ll} and C^{kk}, respectively. These loci in Regime IV are depicted in Figure 6.

For analytical simplicity, we assume the following.

Assumption 1 $\alpha < \frac{1}{2}$

Assumption 2 $\underline{A} \equiv \left(\frac{\beta}{\alpha}\right)^{1-\alpha} \left(\frac{1}{1-\alpha}\right)^{\alpha} < A < \left(\frac{\beta d}{\gamma\alpha}\right)^{1-\alpha} \left(\frac{1}{1-\alpha}\right)^{\alpha} \equiv \bar{A}$

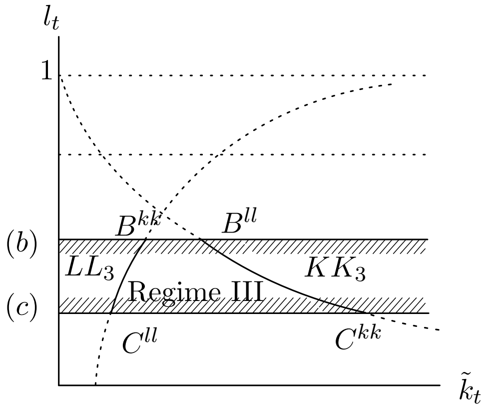

Figure 5: The relationship in Regime III

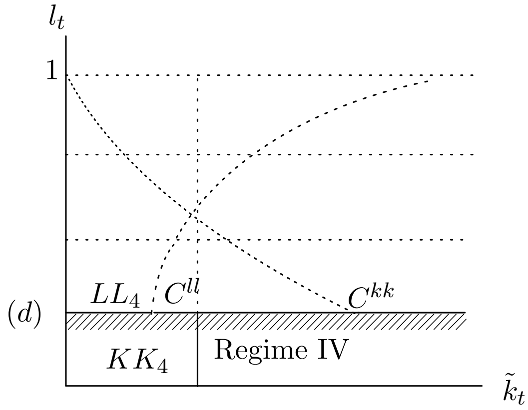

Figure 6: The relationship in Regime IV

Assumption 1 implies that the KK locus is decreasing in l. Assumption 2 implies that the KK and LL loci intersect in Regime II, where each agent adopts an interior solution to the problem of care provision to aged parents.[5] Figure 7 summarizes Figures 3 to 6 and represents the phase diagram for this economy.

The initial point from which the economy starts can be derived from the following.

$$l_1 = \frac{k_1}{\tilde{k}_1} \tag{15}$$

The surface representing (15) can be drawn in (k_t, l_t) space because the initial per capita capital stock, $k_1 \equiv K_1/N_1$, is given exogenously. Therefore, the economy must initially be on the curve (15). As (15) confirms, the contours of (15) when drawn on the (k_t, l_t) plane are downward sloping.

For later reference, we first consider the case in which the initial level of capital is sufficiently large. In this case, the initial contour is drawn in the upper-right corner of Figure 7, and the trajectory is drawn as J. Because any trajectory that is above J does not satisfy the time constraints (see (5)), we can exclude these trajectories as potential equilibria. Thus, contour J represents the boundary trajectory of this economy.

Next, we consider, for example, the case in which $l_1 = K_1/k_1N_1$ in Figure 7. If the economy initially happens to be on the SS line, it converges to E_2. Given Assumption 2, this equilibrium is located in Regime II.

When the economy initially happens to be above the SS line, at point G for example, in Figure 7, the initial level of capital is sufficiently high and, thus, so is the opportunity cost of care provision. In the economy, both labor supply and the capital–labor ratio initially increase.

[5]See Appendix B for the derivation of Assumption 1 and Assumption 2.

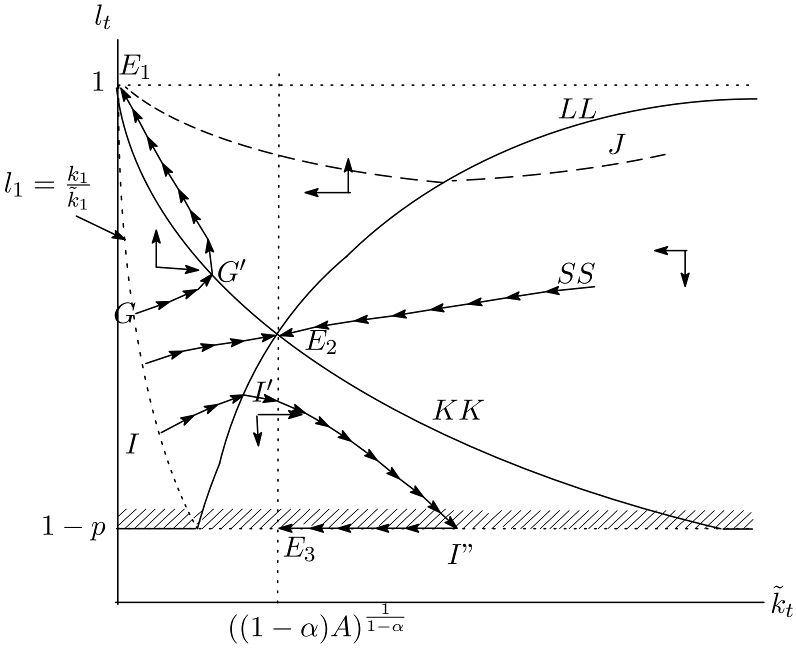

Figure 7: Phase diagram analysis

However, when the KK locus is crossed at point G' in Figure 7, the increases in labor supply leads to a decrease in the capital–labor ratio, $\tilde{k}_t$. Because the boundary trajectory of this economy is J, it follows that the trajectory for this regime converges to E_1. Assumption 2 is not satisfied by the equilibrium E_1, thus, we exclude it as a potential equilibrium.

When the economy initially happens to be below the SS line, at point I, for example, in Figure 7, the initial level of capital is sufficiently low and, thus, so is the opportunity cost of care provision. In the economy, both labor supply and the capital–labor ratio initially increase. However, when the LL locus is crossed at point I' in Figure 7, the increase in the capital stock leads to a decrease in labor supply, l_t. When the trajectory reaches point I'' in Figure 7, where labor supply is at its lowest, the capital–labor ratio falls and approaches E_3. Assumption 2 does not satisfied by the equilibrium E_3, thus, we exclude it as a potential equilibrium.

The following proposition shows the equilibrium of the economy in the absence of a government. Stability is discussed in the Appendix.

Proposition 1 *The steady state of the economy in the absence of a government is derived as* *follows:*

$$(\tilde{k}_2^*, l_2^*) = \left(((1-\alpha)A)^{\frac{1}{1-\alpha}},\ 1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}} \right)$$

Proof. See Appendix C. ■

From Proposition 1, we have the steady-state level of the capital stock $k^* = l^*\tilde{k}^*$. From (4), (6), (7), and (11), the steady-state levels of consumption $c_{j},_{t+1}^{t}$, care provision to aged parents $q_{i},_{t}^{t}$, and factor prices w_t and R_t depend on the steady-state level of the capital stock. The steady-state level of the capital stock depends on the parameters A, p, ψ, and β. We examine how these parameters affect the steady-state level of the capital stock. The following proposition shows comparative static results for the steady-state level of the capital stock.

Proposition 2

$$(a) : \frac{\partial k_2^*}{\partial p} < 0 \quad (b) : \frac{\partial k_2^*}{\partial \beta} < 0, \quad (c) : \frac{\partial k_2^*}{\partial \psi} > 0, \quad (d) : \frac{\partial k_2^*}{\partial A} > 0.$$

Proof. (i) Because $k_2^* \equiv \tilde{k}^* l^* = (1 + \frac{p\psi\gamma}{d})^{\frac{1}{1-\alpha}}((1-\alpha)A)^{\frac{1}{1-\alpha}} - \frac{p\beta(1-\alpha)}{\alpha}$, thus $\frac{\partial k_2^*}{\partial p} = \frac{\psi\gamma}{d}((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}\alpha A - \frac{\beta(1-\alpha)}{\alpha} > (<)0$, if and only if $A > (<)(\frac{\beta d}{\alpha\gamma})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha}(\frac{1}{\psi})^{1-\alpha} \equiv \check{A}$. Under the Assumption 2, because $\bar{A} < \check{A}$, only the $A < \check{A}$ is effective. Thus we have $\frac{\partial k_2^*}{\partial p} < 0$. ■

What is interesting about Proposition 2 is that the comparative static effects of life expectancy on the steady-state level of the capital stock. There are two effects of life expectancy on the steady-state level of the capital stock. One is the 'demographic effect', which operates through aggregate care provision to aged parents. Because aggregate care provision for household-produced health is formalized as $Q_t \equiv q_t N_t = \sum_{i=g,b,d} \theta_i q_{i},_{t}^{t} N_t$, care provision increases with life expectancy p, therefore, a rise in life expectancy lowers aggregate labor supply and savings. Therefore, the demographic effect on the steady-state level of the capital stock is negative. The other effect is the 'time preference effect'. The interest rate is an increasing function of life expectancy p; thus, an increase in life expectancy can be interpreted as the rate of time preference, as incorporated in models such as those of Yaari (1965) and Blanchard (1985).[6] Therefore, an increase in life expectancy lowers each agent's rate of time preference

[6]By using equations (11) and (12), the rate of interest in Regime II can be rewritten as:

$$R_{t+1} = \frac{p\beta}{(1 + \frac{p\psi\gamma}{d} - l_t)(1-\alpha)Ak_t^{\alpha}}.$$

Differentiating this with respect to life expectancy p, we have:

$$\frac{\partial R_{t+1}}{\partial p} = \frac{\beta(1-\alpha)Ak_t^{\alpha}}{\{(1 + \frac{p\psi\gamma}{d} - l_t)(1-\alpha)Ak_t^{\alpha}\}^2}(1 - l_t) > 0.$$

and increases savings. Under Assumption 2, the negative demographic effect outweighs the positive time preference effect and, thus, an increase in life expectancy lowers the steady-state level of the capital stock.

In addition to the preceding analysis of life expectancy, it is worth determining the effects of illness status. Because the care provision of type b agents is higher than that of type g agents, an increase in the fraction of people with good health status, ψ, reduces aggregate care provision. Thus, the steady-state level of the capital stock increases when the proportion of those with good health status increases.

4 Public Long-term Care Policy

In the preceding sections, we described the economy that prevail when there is no government. In this section, we introduce a government that provides public long-term care policy, that is, LTC, to agents who have poor health in their old age.

We assume that LTC is implemented as follows. First, the government levies a payroll tax τ on young agents (generation t). Second, the government employs each young agent's (generation t) time $z_{i,t}^{t}$ and transfers time $\hat{z}_t$ to old agents (generation $t-1$) who have poor health. When old agents supply their one unit of time to the production of household health status, type g agents have a productivity level of $\gamma > 1$, whereas type b agents do not. Thus, LTC complements the productivity of type b old agents by providing $\hat{z}_t \leq \gamma/d$ units of public care provision to those agents.

Because each young agent supplies $z_{i,t}^{t}$ time to the public care market in an economy with LTC, aggregate public care provision Z_t is determined as $Z_t \equiv z_t N_t = \sum_{i=g,b,d} \theta_i z_{i,t}^{t} N_t$. Aggregate public care provision is divided among old agents who have poor health. Thus, an old agent who has poor health can receive an amount of public care provision of $\hat{z}_t = z_t/p(1-\psi)$ through LTC.

Government collects payroll taxes from young workers who work in the market place and in the public care market, and then repays these revenues to young agents who work in the public care market. For analytical simplicity, we assume that perfect substitution prevails between the labor market and the public care market, that is, the wage rates are equalized in both markets. Thus we have the following budget constraint.

$$\tau(l_t + z_t)w_t = z_t w_t \tag{16}$$

The left-hand side represents aggregate tax income collected by the government. The right-hand side represents aggregate expenditure on the public care provision.

Given that the government transfers time to old agents (generation $t-1$) so that a type b old agent (of generation $t-1$) can be provided with public care and so that a young agent (of generation t) can transfer his or her time to the public care market, the household produced-health technology function of type b old agents and the budget constraints of young agents can be rewritten as follows.

$$h_{b,t} = q_{b,t}^{\,t} + \hat{z}_t, \tag{17}$$

$$l_{i,t}^{\,t} + q_{i,t}^{\,t} + z_{i,t}^{\,t} = 1, \qquad i = g, b, d, \tag{18}$$

$$c_{i,t+1}^{\,t} = \frac{R_{t+1}}{p}(1-\tau)w_t(l_{i,t}^{\,t} + z_{i,t}^{\,t}), \qquad i = g, b, d, \tag{19}$$

$$0 \le q_{i,t}^{\,t} \le 1, \quad 0 \le l_{i,t}^{\,t} \le 1, \quad 0 \le z_{i,t}^{\,t} \le 1, \qquad i = g, b, d \tag{20}$$

We obtain the lifetime budget constraint from (20) and (19) as follows.

$$c_{i,t+1}^{\,t} = \frac{R_{t+1}}{p}(1-\tau)w_t(1 - q_{i,t}^{\,t}), \qquad i = g, b, d \tag{21}$$

Taking $R_{t+1}, w_t, \tau, \hat{z}_t, p$ and ψ as given, each young agent whose parents' death–illness status is $i = g$ (or $i = b$) maximize (3) subject to (2) (or (17)), (20), and (21). Solving this problem by using a similar method to that used in Section 2, we can describe the optimal allocation as follows.

$$q_{g,t}^{\,t} = \begin{cases} 0 & \text{if } \frac{d\beta}{\gamma(1-\tau)} \le R_{t+1}w_t, \\ \dfrac{\beta}{R_{t+1}w_t(1-\tau)} - \dfrac{\gamma}{d} & \text{if } \frac{d\beta}{(d+\gamma)(1-\tau)} \le R_{t+1}w_t \le \frac{d\beta}{\gamma(1-\tau)}, \\ 1 & \text{if } R_{t+1}w_t \le \frac{d\beta}{(d+\gamma)(1-\tau)}, \end{cases} \tag{22}$$

$$q_{b,t}^{\,t} = \begin{cases} 0 & \text{if } \frac{\beta}{\hat{z}_t(1-\tau)} \le R_{t+1}w_t, \\ \dfrac{\beta}{R_{t+1}w_t(1-\tau)} - \hat{z}_t & \text{if } \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \le R_{t+1}w_t \le \frac{\beta}{\hat{z}_t(1-\tau)}, \\ 1 & \text{if } R_{t+1}w_t \le \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \end{cases} \tag{23}$$

If the parents of young agents die, the agents derive no utility from the state of their aged parents' health, thus, we obtain:

$$q_{d,t}^{\,t} = 0. \tag{24}$$

When

$$\tau < \frac{z_t}{p(1-\psi) + z_t},$$

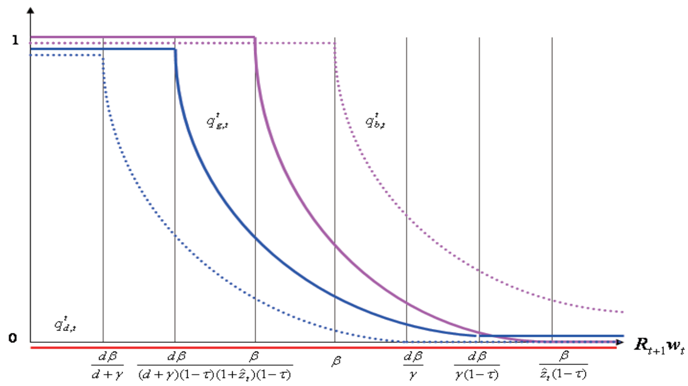

Figure 8: The care provision with LTC

the care provision of type g young agents increases when there is LTC, though that of type b young agents decreases. The borderline in Figure 8 shows the level of care provision for an economy in which there is LTC; the solid line corresponds to an economy that has no LTC. Noting that $\hat{z}_t \leq \gamma/d$, aggregate private care provision is derived by summing up (22) to (24) and by using (1) as follows.

$$
\begin{aligned}
Q_t \equiv q_t N_t &= \sum_{i=g,b,d} \theta_i q_{i,t}^{\ t} N_t \\
&= \begin{cases}
0, & \text{if } \frac{\beta}{\hat{z}_t(1-\tau)} \leq R_{t+1}w_t, \\
p\Big(\frac{\beta(1-\psi)}{R_{t+1}w_t(1-\tau)} - (1-\psi)\hat{z}_t\Big)N_t, & \text{if } \frac{d\beta}{\gamma(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{\hat{z}_t(1-\tau)}, \\
p\Big(\frac{\beta}{R_{t+1}w_t(1-\tau)} - \frac{\psi\gamma}{d} - (1-\psi)\hat{z}_t\Big)N_t, & \text{if } \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \leq R_{t+1}w_t \leq \frac{d\beta}{\gamma(1-\tau)}, \\
p\Big(\frac{\psi\beta}{R_{t+1}w_t(1-\tau)} + 1 - \psi - \frac{\psi\gamma}{d}\Big)N_t, & \text{if } \frac{d\beta}{(d+\gamma)(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{(1-\tau)(1+\hat{z}_t)}, \\
pN_t, & \text{if } R_{t+1}w_t \leq \frac{d\beta}{(d+\gamma)(1-\tau)}
\end{cases}
\end{aligned}
\tag{25}
$$

From (20) and (25), we obtain the following aggregate labor supply.[7]

$$
L_t = l_t N_t = \begin{cases}
(1-z_t)N_t, & \text{if } \frac{\beta}{\hat{z}_t(1-\tau)} \leq R_{t+1}w_t, \\
\Big(1 - \frac{p\beta(1-\psi)}{R_{t+1}w_t(1-\tau)}\Big)N_t, & \text{if } \frac{d\beta}{\gamma(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{\hat{z}_t(1-\tau)}, \\
\Big(1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{R_{t+1}w_t(1-\tau)}\Big)N_t, & \text{if } \frac{\beta}{(1-\tau)(1+\hat{z}_t)} \leq R_{t+1}w_t \leq \frac{d\beta}{\gamma(1-\tau)}, \\
\Big(1 + \frac{p\psi\gamma}{d} - p(1-\psi) - z_t - \frac{p\psi\beta}{R_{t+1}w_t(1-\tau)}\Big)N_t, & \text{if } \frac{d\beta}{(d+\gamma)(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{(1-\tau)(1+\hat{z}_t)}, \\
(1-p)(1-\frac{z_t}{1-p})N_t, & \text{if } R_{t+1}w_t \leq \frac{d\beta}{(d+\gamma)(1-\tau_t)}
\end{cases}
\tag{26}
$$

[7]Aggregate labor supply is derived as $L_t \equiv l_t N_t = [p\psi(1-q_{g,t}^{\ t} - z_{g,t}^{\ t}) + p(1-\psi)(1-q_{b,t}^{\ t} - z_{b,t}^{\ t}) + (1-p)(1-z_{d,t}^{\ t})]N_t$. When $R_{t+1}w_t \leq \frac{d\beta}{(d+\gamma)(1-\tau)}$, public care supply of type g and type b agents $z_{g,t}^{\ t} = z_{b,t}^{\ t} = 0$. Thus, aggregate labor supply is derived as $L_t \equiv l_t N_t = (1-p)(1-z_{d,t}^{\ t}) = (1-p)(1-\frac{z_t}{1-p})$.

4.1 Equilibrium with LTC

In this subsection, we derive the equilibrium that prevails when there is LTC. To find the equilibrium, we examine the equilibrium condition for the capital market. The savings of young agents form the aggregate capital stock in period $t+1$, and are thus derived as $K_{t+1} = s_t = (1-\tau)w_t(l_t + z_t)N_t$. Dividing both sides by N_t and substituting $z_t = \tau l_t/(1-\tau)$ into this equilibrium condition (see (16)) yields the following equilibrium condition for the capital market.

$$\tilde{k}_{t+1} = (1-\alpha)A\tilde{k}_t^{\alpha}\frac{l_t}{l_{t+1}} \tag{27}$$

Noting that $z_t = \tau l_t/(1-\tau)$, we obtain the following dynamics for labor supply $\{l_t\}_{t+1}^{\infty}$ by using (11), (26), and (27).

$$l_{t+1}^{1-\alpha} = \begin{cases} (1-\tau)^{1-\alpha}, & \text{if } \frac{\beta}{\hat{z}_t(1-\tau)} \leq R_{t+1}w_t \quad \text{< Regime I >,} \\ \dfrac{p\beta(1-\psi)l_t^{1-\alpha}}{\alpha A\{(1-\alpha)A\tilde{k}_t^{\alpha}\}^{\alpha}(1-l_t)(1-\tau)} & \text{if } \frac{d\beta}{\gamma(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{\hat{z}_t(1-\tau)} \quad \text{< Regime II >} \\ \dfrac{p\beta l_t^{1-\alpha}}{\alpha A\{(1-\alpha)A\tilde{k}_t^{\alpha}\}^{\alpha}(1+\frac{p\psi\gamma}{d}-l_t)(1-\tau)}, & \\ & \text{if } \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \leq R_{t+1}w_t \leq \frac{d\beta}{\gamma(1-\tau)} \quad \text{< Regime III >} \\ \dfrac{p\psi\beta l_t^{1-\alpha}}{\alpha A\{(1-\alpha)A\tilde{k}_t^{\alpha}\}^{\alpha}[\{1+\frac{p\psi\gamma}{d}-p(1-\psi)\}(1-\tau)-l_t]} & \\ & \text{if } \frac{d\beta}{(d+\gamma)(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \quad \text{< Regime IV >} \\ ((1-\tau)(1-p))^{1-\alpha}, & \text{if } R_{t+1}w_t \leq \frac{d\beta}{(d+\gamma)(1-\tau)} \quad \text{< Regime V >} \end{cases} \tag{28}$$

We obtain a sequence for the capital–labor ratio $\{\tilde{k}_t\}_{t=1}^{\infty}$ by using (27) and (28), as follows.

$$\tilde{k}_{t+1}^{1-\alpha} = \begin{cases} \left((1-\alpha)A\tilde{k}_t^{\alpha}\right)^{1-\alpha}, & \text{if } \frac{\beta}{\hat{z}_t(1-\tau)} \leq R_{t+1}w_t, \quad \text{< Regime I >} \\ \dfrac{(1-\alpha)A\tilde{k}_t^{\alpha}\alpha A(1-\tau)(1-l_t)}{p\beta(1-\psi)}, & \\ & \text{if } \frac{d\beta}{\gamma(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{\hat{z}_t(1-\tau)} \quad \text{< Regime II >} \\ \dfrac{(1-\alpha)A\tilde{k}_t^{\alpha}\alpha A(1-\tau)(1+\frac{p\psi\gamma}{d}-l_t)}{p\beta}, & \\ & \text{if } \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \leq R_{t+1}w_t \leq \frac{d\beta}{\gamma(1-\tau)} \quad \text{< Regime III >} \\ \dfrac{(1-\alpha)A\tilde{k}_t^{\alpha}\alpha A\{[1+\frac{p\psi\gamma}{d}-p(1-\psi)](1-\tau)-l_t\}}{p\psi\beta}, & \\ & \text{if } \frac{d\beta}{(d+\gamma)(1-\tau)} \leq R_{t+1}w_t \leq \frac{\beta}{(1+\hat{z}_t)(1-\tau)} \quad \text{< Regime IV >} \\ \left((1-\alpha)A\tilde{k}_t^{\alpha}\right)^{1-\alpha}, & \text{if } R_{t+1}w_t \leq \frac{d\beta}{(d+\gamma)(1-\tau)} \quad \text{< Regime V >} \end{cases} \tag{29}$$

Before describing the equilibrium, we examine the above border lines in a similar way to that used in Section 3. The border lines can be rewritten as follows.

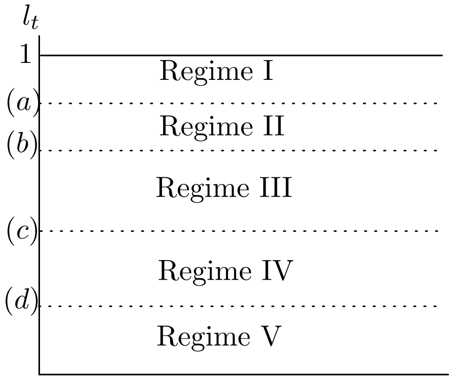

Figure 9: The regime with LTC

$(a) : \frac{\beta}{\hat{z}_t(1-\tau)} = R_{t+1}w_t$

$$l_t = 1 - \tau,$$

$(b) : \frac{d\beta}{\gamma(1-\tau)} = R_{t+1}w_t$

$$l_t = 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d},$$

$(c) : \frac{\beta}{(1+\hat{z}_t)(1-\tau)} = R_{t+1}w_t$

$$l_t = \Big(1 + \frac{p\psi\gamma}{d} - p\Big)\Big(\frac{(1-\psi)(1-\tau)}{1-\psi(1-\tau)}\Big),$$

$(d) : \frac{d\beta}{(d+\gamma)(1-\tau)} = R_{t+1}w_t$

$$l_t = (1-p)(1-\tau)$$

The above five regimes are depicted in Figure 9.

Equations (28) and (29) define economic equilibria that are represented by sequences of $\{\tilde{k}_t, l_t\}_{t=1}^{\infty}$ and by the initial condition that $(\tilde{k}_1, l_1) > 0$. We refer to the locus on the $(\tilde{k}_t, l_t)$ plane that represents $\tilde{k}_{t+1} = \tilde{k}_t$ as the KK locus and refer to that representing $l_{t+1} = l_t$ as the LL locus. Noting that the minimum level of labor supply is $(1-p)(1-\tau)$, and noting that the maximum level of labor supply is $1-\tau$, equations (28) and (29) imply the following KK and LL loci.

Regime I: $1 - \tau \leq l_t$

$$LL_1 \quad : \quad l_t = 1 - \tau,$$

$$KK_1 \quad : \quad \tilde{k}_t = ((1-\alpha)A)^{\frac{1}{1-\alpha}}$$

The LL_1 and KK_1 loci intersect border line (a) at points A^{ll} and A^{kk}, respectively, where $A^{ll} \equiv (\tilde{k}_t, l_t) = ([(\frac{p\beta(1-\psi)}{\tau\alpha A(1-\tau)})^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, 1-\tau)$, and $A^{kk} \equiv (\tilde{k}_t, l_t) = ((\frac{(1-\alpha)A\alpha A(1-\tau)\tau}{p\beta(1-\psi)})^{\frac{1}{1-2\alpha}}, 1-\tau)$.

Regime II: $1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d} \leq l_t \leq 1-\tau$

$$
\begin{aligned}
LL_2 \quad &: \quad l_t = 1 - \frac{p\beta(1-\psi)}{\alpha A[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}(1-\tau)}, \\
KK_2 \quad &: \quad \tilde{k}_t = \Big(\Big(\frac{(1-\alpha)A\alpha A(1-\tau)}{p\beta(1-\psi)}\Big)(1-l_t)\Big)^{\frac{1}{1-2\alpha}}
\end{aligned}
$$

The LL_2 and KK_2 loci intersect border line (a) at points A^{ll} and A^{kk}, respectively. The LL_2 and KK_2 loci intersect border line (b) at points B^{ll} and B^{kk}, respectively, where $B^{ll} \equiv (\tilde{k}_t, l_t) = ([(\frac{\beta d}{\gamma\alpha A(1-\tau)})^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, 1+\frac{p\psi\gamma}{d}-\frac{p\gamma}{d})$, and $B^{kk} \equiv (\tilde{k}_t, l_t) = ((\frac{(1-\alpha)A\alpha A(1-\tau)\gamma}{\beta d})^{\frac{1}{1-2\alpha}}, 1+\frac{p\psi\gamma}{d}-\frac{p\gamma}{d})$.

Regime III: $(1 + \frac{p\psi\gamma}{d} - p)(\frac{(1-\psi)(1-\tau)}{1-\psi(1-\tau)}) \leq l_t \leq 1 + \frac{p\psi\gamma}{d} - \frac{p\gamma}{d}$

$$
\begin{aligned}
LL_3 \quad &: \quad l_t = 1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}(1-\tau)}, \\
KK_3 \quad &: \quad \tilde{k} = \Big(\Big(\frac{(1-\alpha)A\alpha A(1-\tau)}{p\beta}\Big)\Big(1 + \frac{p\psi\gamma}{d} - l_t\Big)\Big)^{\frac{1}{1-2\alpha}}
\end{aligned}
$$

The LL_3 and KK_3 loci intersect border line (b) at points B^{ll} and B^{kk}, respectively. The LL_3 and KK_3 loci intersect border line (c) at points C^{ll} and C^{kk}, respectively, where $C^{ll} \equiv (\tilde{k}_t, l_t) = ([\{\frac{p\beta(1-\psi+\psi\tau)}{\alpha A(1-\tau)[\tau(1+\frac{p\psi\gamma}{d})+p(1-\psi)(1-\tau)]}\}^{\frac{1}{\alpha}} \frac{1}{(1-\alpha)A}]^{\frac{1}{\alpha}}, (1 + \frac{p\psi\gamma}{d} - p)(\frac{(1-\psi)(1-\tau)}{1-\psi(1-\tau)}))$, and $C^{kk} \equiv (\tilde{k}_t, l_t) = (\{\frac{(1-\alpha)A\alpha A(1-\tau)[\tau(1+\frac{p\psi\gamma}{d})+p(1-\psi)(1-\tau)]}{p\beta(1-\psi+\psi\tau)}\}^{\frac{1}{1-2\alpha}}, (1 + \frac{p\psi\gamma}{d} - p)(\frac{(1-\psi)(1-\tau)}{1-\psi(1-\tau)}))$.

Regime IV: $(1-p)(1-\tau) \leq l_t \leq (1 + \frac{p\psi\gamma}{d} - p)(\frac{(1-\psi)(1-\tau)}{1-\psi(1-\tau)})$

$$
\begin{aligned}
LL_4 \quad &: \quad l_t = \Big(1 + \frac{p\psi\gamma}{d} - p(1-\psi)\Big)(1-\tau) - \frac{p\psi\beta}{\alpha A[(1-\alpha)A\tilde{k}_t^{\alpha}]^{\alpha}}, \\
KK_4 \quad &: \quad \tilde{k} = \Big(\Big(\frac{(1-\alpha)A\alpha A}{p\psi\beta}\Big)\Big(1 + \frac{p\psi\gamma}{d} - p(1-\psi)\Big)(1-\tau) - l_t\Big)^{\frac{1}{1-2\alpha}}
\end{aligned}
$$

The LL_4 and KK_4 loci intersect border line (c) at points C^{ll} and C^{kk}, respectively. The LL_4 and KK_4 loci intersect border line (d) at points D^{ll} and D^{kk}, respectively, where $D^{ll} \equiv (\tilde{k}_t, l_t) = ([\frac{\beta}{\alpha A(1+\frac{\gamma}{d})(1-\tau)})^{\frac{1}{\alpha}}(\frac{1}{(1-\alpha)A})]^{\frac{1}{\alpha}}, (1-p)(1-\tau))$, and $D^{kk} \equiv (\tilde{k}_t, l_t) = ([\frac{(1-\alpha)A\alpha A(1-\tau)(1+\frac{\gamma}{d})}{\beta}]^{\frac{1}{1-2\alpha}}, (1-p)(1-\tau))$.

Regime V: $l_t \leq (1-p)(1-\tau)$

$$
\begin{aligned}
LL5 \ &: \ l_{t+1} = (1-\tau)(1-p) \\
KK5: \ & \tilde{k}_t = ((1-\alpha)A)^{\frac{1}{1-\alpha}}
\end{aligned}
$$

The $LL5$ and $KK5$ loci intersect border line (d) at the points D^{ll} and D^{kk}, respectively.

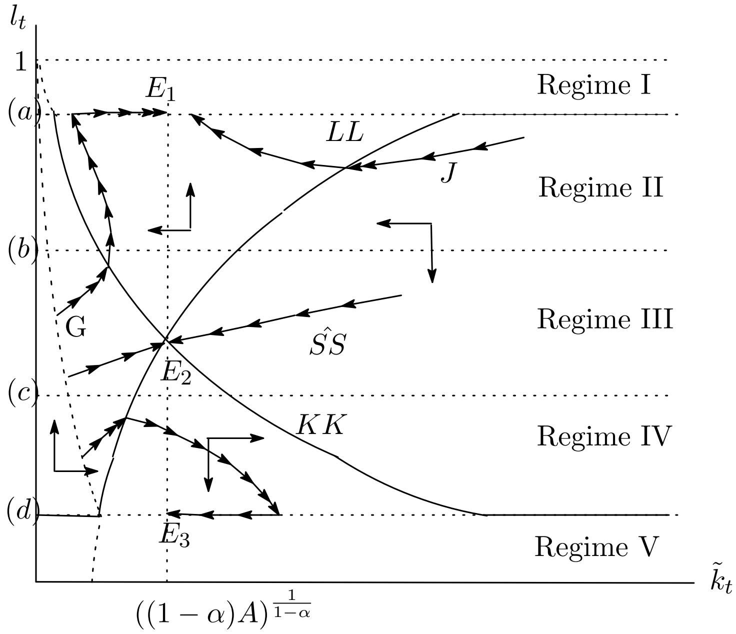

Figure 10: The dynamics of an economy with LTC

The noteworthy feature of the economy with LTC is its equilibrium dynamics. As shown in Section 3.1, an economy that initially happens to be above the SS line (for example, at point G in Figure 7), the economy converges to the point in which the capital labor ratio $\tilde{k}_t$ is zero. However, in an economy with LTC, the level of labor supply to firms is limited to $1-\tau$. Therefore, when the economic regime is one in which labor supply is at its highest, the capital stock increases and approaches E_1 in the long run.

Therefore, for a given initial condition, the economy converges to E_1, E_2, or E_3 (see Figure 10) in the long run. From Assumption 2, E_2 is in Regime III.[8] The equilibria E_1 and E_3 does not satisfied under Assumption 2, thus the following Proposition shows the steady state equilibrium of the economy with LTC.

Proposition 3 *The steady state of the economy with LTC is as follows:*

$$(\tilde{k}_2^*, l_2^*) = \left(((1-\alpha)A)^{\frac{1}{1-\alpha}},\ 1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A((1-\alpha)A(1-\tau))^{\frac{\alpha}{1-\alpha}}} \right)$$

We are also interested in whether LTC has a positive impact on the steady-state level of the capital stock. At the economy with LTC, young agents must provide time to public care provision, thus, the aggregate labor supply decreases.

[8]This proof is given by Appendix D.

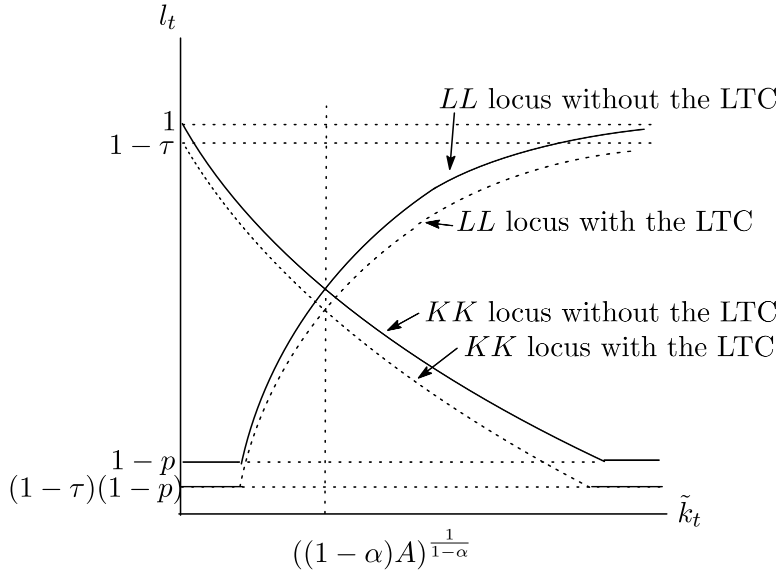

Figure 11: A comparison of the dynamics of economies with and without LTC

An decrease in labor supply shifts both the KK and LL lines downwards, as shown in Figure 11. Therefore, the steady-state level of the capital stock decreases in an economy with LTC.

5 Welfare Analysis

In this section, we examine the welfare impact of LTC. To do so, we compare the steady-state levels of welfare in economies without LTC (in which $\tau = 0$) and with LTC (in which $\tau > 0$). Because it is difficult to analyze the allocation, we limit our attention to the steady state in the rest of this section. We define the steady-state level of welfare as the sum of agents' lifetime utilities, which are given by (3). This sum is formulated as follows:

$$\begin{aligned} W &= \sum_{i=g,b,d} \theta_i u_i \\ &= p\psi(1+\beta)\ln h_g + p(1-\psi)(1+\beta)\ln h_b + pc, \end{aligned} \tag{30}$$

where h_g, h_b, and $c = \Sigma_{j=g,b,d}\theta_j c_j$, respectively, represent the steady-state levels of the health status of agents $s = g, b$ $(s = i, j)$, and the steady-state levels of aggregate consumption.

To facilitate comparison, we denote the steady-state level of welfare at the economy without LTC as W^c and the economy with LTC as W^s. W^c is measured by substituting the steady-state values of $\ln h_g = \ln\{\frac{d\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}\}, \ln h_b = \ln\{\frac{\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}\}$ and $c = \frac{1}{p}[\alpha A((1-$

$\alpha)A)^{\frac{\alpha}{1-\alpha}}(\frac{1+p\psi\gamma}{d}) - p\beta]$ into (30).[9]

$$W^c = p(1+\beta)\ln\left\{\frac{d^\psi\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}\right\} + \alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}\left[\left(1+\frac{p\psi\gamma}{d}\right) - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}\right]$$

Given that $\ln h_g = \ln\{\frac{d\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)}\}$, $\ln h_b = \ln\{\frac{\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)}\}$, and $c = \frac{1}{p}[\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1+\frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)})]^{10}$, W^s is measured as follows.

$$\begin{aligned} W^s &= p(1+\beta)\ln\left\{\frac{d^\psi\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)}\right\} \\ &+ \alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}\left(1+\frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)}\right). \end{aligned}$$

To determine the benefit (or harm) of LTC, we subtract W^s from W^c, as follows.

$$W^c - W^s = p\Big[\underbrace{(1+\beta)\ln\{1-\tau\}}_{-} + \underbrace{\beta\frac{\tau}{1-\tau}}_{+}\Big] \tag{31}$$

Noting that $W^c - W^s$ is a linear function of p, and noting that $W^c - W^s = 0$ when $p = 0$, the term inside the braces in (31) determines the value of $W^c - W^s$. Differentiating $W^c - W^s$ with respect to τ yields

$$\frac{\partial(W^c - W^s)}{\partial\tau} = \frac{1}{1-\tau}(-(1-\tau-\beta\tau))\begin{cases} < 0 & \text{if } \tau \in (0, \frac{1}{1+\beta}), \\ > 0 & \text{if } \tau \in (\frac{1}{1+\beta}, 0). \end{cases} \tag{32}$$

Defining the term inside the braces in (31) as $\Delta(\tau) \equiv (1+\beta)\ln(1-\tau) + \frac{\beta\tau}{1-\tau}$, implies that $\Delta(0) = 0$ and $\Delta(1) \to \infty$. From these results and from (32), we find that there exists a unique $\tau^* \in (\frac{1}{1+\beta}, 1)$, such that $\delta(\tau) = 0$. Figure 12 illustrates this relationship.

Therefore, we can state the following.

$$W^c - W^s \begin{cases} < 0 & \textit{for} \quad \tau \in (0, \tau^*), \\ > 0 & \textit{for} \quad \tau \in (\tau^*, 1) \end{cases}$$

The welfare gain (or loss) that arises in an economy with LTC (in which $\tau > 0$) can be interpreted as comprising a 'health status effect', a 'capital stock effect', and a 'subsidy effect'. The health status effect is represented by the first term on the right-hand side of (31). The steady-state level of health status improves when LTC is introduced and, thus, a negative value in (31) would be obtained. LTC also has a capital stock effect on welfare. Because LTC implies a lower steady-state level of the capital stock, it leads to lower steady-state levels of

[9] See Appendix E for the derivation of the steady-state values.

[10] See also Appendix E for the derivation of the steady-state values.

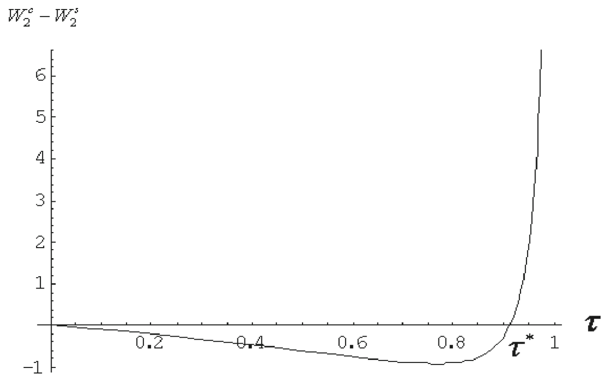

Figure 12: The relationship between W^c and W^s

income and welfare. This effect is represented by the second term on the right-hand side of (31). In addition to these two effects, we have the subsidy effect on welfare. Because old agents can receive public care provision without paying tax, this effect improves the steady-state level of welfare of old agents.

When the tax rate is low, the negative health status and subsidy effects dominate the positive capital stock effect. However, when the tax rate is sufficiently high, labor supply falls in the economy with LTC. Consequently, LTC has a larger positive effect on the capital stock. Thus, the positive capital stock effect outweighs the negative health status and subsidy effects.

6 Conclusion

In this paper, our aim was to analyze the impact of intergenerational transfers of time from children to their aged parents on the capital stock and welfare in an aging population. To examine these issues, we used a two-period overlapping generations model that incorporated uncertainties about lifespan and health status in old age. In addition to this, we assume that (i) young agents derive utility from their aged parents' health status; (ii) household health status is produced by using time; and (iii) intergenerational transfers of time, that is, providing care to aged parents, are modeled by allowing young agents to participate in the production of household health status.

Using this model, we first described an economy in which there is no government. We showed that life expectancy has a negative impact on the steady-state level of the capital stock. On the other hand, an increase in the proportion of healthier old agents increases the steady-state level of the capital stock.

In the second part of the paper, we studied the role and the effect of public long term care policy, that is, LTC. We showed that LTC lowers the steady-state level of the capital stock. However, when the tax rate is small, LTC enhances welfare.

Appendix

Appendix A: Utility maximization problem

Let λ_1 be the nonnegative multipliers associated with the constraints (4). The optimal solution is obtained by setting up the following Lagrangian function.

$$
\begin{aligned}
\pounds \;=\;& p\psi\Big[\beta\ln(dq_{g,t}^{\,t}+\gamma)+p\{c_{g,t+1}^{\,t}+\psi\ln(dq_{g,t+1}^{\,t+1}+\gamma)+(1-\psi)\ln q_{b,t+1}^{\,t+1}\}\Big]\\
+\;& p(1-\psi)\Big[\beta\ln(q_{b,t}^{\,t})+p\{c_{b,t+1}^{\,t}+\psi\ln(dq_{g,t+1}^{\,t+1}+\gamma)+(1-\psi)\ln q_{b,t+1}^{\,t+1}\}\Big]\\
+\;& (1-p)\Big[p\{c_{d,t+1}^{\,t}+\psi\ln(dq_{g,t+1}^{\,t+1}+\gamma)+(1-\psi)\ln q_{b,t+1}^{\,t+1}\}\Big]\\
+\;& \lambda_1\Big[p\psi\Big\{\frac{R_{t+1}}{p}w_t(1-q_{g,t}^{\,t})-c_{g,t+1}^{\,t}\Big\}+p(1-\psi)\Big\{\frac{R_{t+1}}{p}w_t(1-q_{b,t}^{\,t})-c_{b,t+1}^{\,t}\Big\}\\
+\;& (1-p)\Big(\frac{R_{t+1}}{p}w_t-c_{d,t+1}^{\,t}\Big)\Big]+\lambda_2(1-q_{g,t}^{\,t})+\lambda_3 q_{g,t}^{\,t}+\lambda_4(1-q_{b,t}^{\,t})
\end{aligned}
$$

Then, the Kuhn–Tucker conditions below, together with (2), (4), and (5), are necessary and sufficient for the maximization problem. Because young agents are categorized based on their parents' death–illness status ($i=g,b,d$), we derive each type of agent's utility maximization problem.

<The young agent whose parents' death–illness status is $i=g$>

$$p-\lambda_1\le 0 \qquad \text{with equality if } c_{g,t+1}^{\,t}>0 \tag{33}$$

$$\frac{\beta d}{dq_{g,t}^{\,t}+\gamma}-\lambda_1\frac{R_{t+1}}{p}w_t-\lambda_2+\lambda_3\le 0 \quad \text{with equality if } q_{g,t}^{\,t}>0 \tag{34}$$

$$\lambda_1 p\psi\Big(\frac{R_{t+1}}{p}w_t(1-q_{g,t}^{\,t})-c_{g,t+1}^{\,t}\Big)=0 \tag{35}$$

$$\lambda_2(1-q_{g,t}^{\,t})=0 \tag{36}$$

$$\lambda_3 q_{g,t}^{\,t}=0 \tag{37}$$

Because (33) can be rewritten as $0<p\le\lambda_1$, λ_1 takes a positive value. Noting that $p\psi>0$, (35) can be rewritten as follows.

$$\frac{R_{t+1}}{p}w_t(1-q_{g,t}^{\,t})=c_{g,t+1}^{\,t} \tag{38}$$

The solution is found by using a 'guess-and-verify' method.

The first guess is that $q_{g,t}^{t} = 1$. From (38), $c_{g,t+1}^{t} = 0$, and from (36) and (37), $\lambda_2 > 0$ and $\lambda_3 = 0$. Because $q_{g,t}^{t} > 0$, we have $d\beta/(d+\gamma) = \lambda_1 R_{t+1}w_t/p + \lambda_2$ from (34). Given that $\lambda_1 \geq p$ and $\lambda_2 \geq 0$, this last equation can be rewritten as $R_{t+1}w_t \leq d\beta/(d+\gamma)$. Thus, if and only if $R_{t+1}w_t \leq d\beta/(d+\gamma)$, the guess $q_{g,t}^{t} = 1$ is correct and $c_{g,t+1}^{t} = 0$.

The next guess is that $q_{g,t}^{t} \in (0,1)$. From (38), $c_{g,t+1}^{t} > 0$, then $\lambda_1 = p > 0$ from (33). (36) and (37) imply that $\lambda_2 = \lambda_3 = 0$, then we have $q_{g,t}^{t} = \beta/R_{t+1}w_t - \gamma/d$ from (34). Given that $q_{b,t}^{t} > 0$, this last condition can be rewritten as $R_{t+1}w_t < d\beta/\gamma$. Thus, if and only if $R_{t+1}w_t < d\beta/\gamma$, the guess $q_{g,t}^{t} \in (0,1)$ is correct.

The final guess is that $q_{g,t}^{t} = 0$. From (36), (37), and (38), $\lambda_2 = 0$, $\lambda_3 > 0$, and $c_{g,t+1}^{t} = R_{t+1}w_t/p$. Substituting these values into (34), we have $\beta d/\gamma \leq R_{t+1}w_t$. Thus, if and only if $\beta d/\gamma \leq R_{t+1}w_t$, the guess $q_{b,t}^{t} = 0$ correct.

<The young agent whose parents' death–illness status is $i = b$ >

$$p - \lambda_1 \leq 0 \qquad \text{with equality if } c_{b,t+1}^{t} > 0 \tag{39}$$

$$\frac{\beta}{q_{b,t}^{t}} - \lambda_1 \frac{R_{t+1}}{p} w_t - \lambda_4 = 0 \tag{40}$$

$$\lambda_1 p(1-\psi)\Big(\frac{R_{t+1}}{p} w_t(1 - q_{b,t}^{t}) - c_{b,t+1}^{t}\Big) = 0 \tag{41}$$

$$\lambda_4(1 - q_{b,t}^{t}) = 0 \tag{42}$$

Because (39) can be rewritten as $0 < p \leq \lambda_1$, λ_1 takes a positive value. Noting that $p\psi > 0$, (41) can be rewritten as follows.

$$\frac{R_{t+1}}{p} w_t(1 - q_{b,t}^{t}) = c_{b,t+1}^{t} \tag{43}$$

The solution is found by using the guess-and-verify method.

The first guess is that $q_{b,t}^{t} = 1$. From (41),(39) and (42), we have $c_{b,t+1}^{t} = 0$, $\lambda_1 \geq p$ and$\lambda_4 \geq 0$, respectively. Because $q_{b,t}^{t} > 0$, we have $\beta = \lambda_1 R_{t+1}w_t/p + \lambda_4$ from (40). Given that $\lambda_1 \geq p$ and $\lambda_2 \geq 0$, this last equation can be rewritten as $R_{t+1}w_t \leq \beta$. Thus, if and only if $R_{t+1}w_t < \beta$, the guess $q_{b,t}^{t} = 1$ is correct.

The next guess is that $q_{b,t}^{t} \in (0,1)$. (43) imply that $c_{b,t+1}^{t} > 0$, then we have $p = \lambda_1$ and $\lambda_4 = 0$ from (39) and (42). Substituting these values into (41), we have $q_{b,t}^{t} = \beta/R_{t+1}w_t$. Thus, if and only if $0 < R_{t+1}w_t < \beta d/\gamma$, the guess is correct and $q_{b,t}^{t} = \beta/R_{t+1}w_t$.

Appendix B: The derivation of Assumption 1 and Assumption 2

In this appendix, we assume that certain conditions are satisfied in order to derive the equilibrium.

<The derivation of Assumption 1>

We first assume that KK locus is decreasing in l_t. To derive this condition, we differentiate the KK locus with respect to l_t.

$$
\begin{aligned}
KK1 \quad &: \quad \frac{\partial \tilde{k}_t}{\partial l_t} = -\frac{1}{1-2\alpha}\Big(\frac{(1-\alpha)A\alpha A(1-l_t)}{p\beta(1-\psi)}\Big)^{\frac{2\alpha}{1-2\alpha}}, \\
KK2 \quad &: \quad \frac{\partial \tilde{k}_t}{\partial l_t} = -\frac{1}{1-2\alpha}\Big(\frac{(1-\alpha)A\alpha A(1+\frac{p\psi\gamma}{d}-l_t)}{p\beta(1-\psi)}\Big)^{\frac{2\alpha}{1-2\alpha}}, \\
KK3 \quad &: \quad \frac{\partial \tilde{k}_t}{\partial l_t} = -\frac{1}{1-2\alpha}\Big(\frac{(1-\alpha)A\alpha A\{1+\frac{p\psi\gamma}{d}-p(1-\psi)-l_t\}}{p\beta(1-\psi)}\Big)^{\frac{2\alpha}{1-2\alpha}}
\end{aligned}
$$

These results show that if $\alpha < 1/2$, each KK locus is decreasing in l_t.

<The derivaion of Assumption 2>

Next, we assume that the equilibrium of the economy is determined under a regime in which each agent adopts an interior solution to the problem of care provision. That is, we assume that the economy is under Regime II. When $A^{kk} < A^{ll}$ and $B^{ll} < B^{kk}$ hold simultaneously, the equilibrium falls under Regime II (see Figure 4). Thus, we derive the following condition. Supposing that $A^{kk} < A^{ll}$, we have $A < (\frac{\beta d}{\gamma\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha}$. Supposing that $B^{ll} < B^{kk}$, we have $(\frac{\beta}{\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} < A$. Therefore, when $\underline{A} \equiv (\frac{\beta}{\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} < A < (\frac{\beta d}{\gamma\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} \equiv \bar{A}$, the equilibrium falls under Regime II.

Appendix C

In this appendix, we demonstrate the stability of the steady state.

The dynamics of E_3 are derived as $\tilde{k}_{t+1} = (1-\alpha)A\tilde{k}_t^{\alpha}$. Given that $\alpha < 1$, the stability of E_3 is straightforward. We examine the local dynamics of E_2. We take a first-order Taylor expansion of the system around the steady state $(\tilde{k}^*, l^*)$. By letting $\check{k}_t \equiv k_t - k^*$ and $\check{l}_t \equiv \tilde{l}_t - \tilde{l}^*$, this linearization can be expressed as:

$$
\begin{pmatrix} \check{k}_{t+1} \\ \check{l}_{t+1} \end{pmatrix} = \begin{pmatrix} \dfrac{\alpha}{(1-\alpha)} & -\dfrac{\alpha A}{p\beta}\cdot\dfrac{((1-\alpha)A)^{\frac{1+\alpha}{1-\alpha}}}{1-\alpha} \\ -\dfrac{l^*\alpha^2}{(1-\alpha)((1-\alpha)A)^{\frac{1}{1-\alpha}}} & 1+\dfrac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}l^*\alpha A}{(1-\alpha)p\beta} \end{pmatrix} \begin{pmatrix} \check{k}_t \\ \check{l}_t \end{pmatrix},
$$

where $l^* \equiv 1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}$. Letting κ be the eigenvalue, the characteristic polynomial is as follows.

$$
P(\kappa) = \kappa^2 - T\kappa + D,
$$

$$
\begin{aligned}
T &= \frac{\alpha}{1-\alpha} + 1 + \frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{(1-\alpha)p\beta}, \\
D &= \frac{\alpha}{1-\alpha} + \frac{\alpha}{(1-\alpha)} \cdot \frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{p\beta}
\end{aligned}
$$

Azariadis (1993) shows that the steady state is a saddle, when $1 - T + D < 0$ holds. It is clear that

$$
1 - T + D = -\frac{((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} l^* \alpha A}{p\beta} < 0.
$$

Therefore, E_2 is a saddle point.

Appendix D

When $B^{kk} < B^{ll}$ and $C^{kk} < C^{ll}$ hold simultaneously, the equilibrium with LTC is under Regime III. To derive this condition, we first assume that $B^{kk} < B^{ll}$. Solving this, we have $A < (\frac{d\beta}{\gamma\alpha(1-\tau)})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} \equiv \hat{A}$. Next, assuming that $C^{kk} < C^{ll}$, we have $\check{A} \equiv (\frac{p\beta(1-\psi)}{[p(1-\psi)+\tau](1-\tau)\alpha})^{1-\alpha}(\frac{1}{1-\alpha})^{\alpha} < A$. Because $\hat{A} < \bar{A}$ and $\check{A} < \underline{A}$, it follows that, given Assumption 1-(ii), the equilibrium with LTC is under Regime III.

Appendix E

To derive the steady-state level of welfare, in this appendix, we derive the steady-state values of care provision and consumption.

<The derivation of steady state value in the absence of a government>

Because the equilibrium E_2 is located in Regime II, each value is derived by substituting in the steady-state levels of the following values.

$$
\begin{aligned}
h_g &= dq_g + \gamma = d\Big(\frac{\beta}{Rw} - \frac{\gamma}{d}\Big) + \gamma = \frac{d\beta}{Rw}, \\
h_b &= q_b = \frac{\beta}{Rw}, \\
c &= p\psi c_g + p(1-\psi)c_b + (1-p)c_d, \\
&= p\psi\Big(\frac{Rw}{p}\Big(1 - \frac{\beta}{Rw} + \frac{\gamma}{d}\Big)\Big) + p(1-\psi)\frac{Rw}{p}\Big(1 - \frac{\beta}{Rw}\Big) + (1-p)\frac{Rw}{p}(1), \\
&= \frac{Rw}{p}\Big(1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{Rw}\Big)
\end{aligned}
$$

Substituting the steady-state values into $Rw = \alpha A\tilde{k}^{\alpha-1}(1-\alpha)A\tilde{k}^{\alpha}$, yields $Rw = \alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}$. Therefore, each steady-state level is determined as follows.

$$
h_g = \frac{d\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}, h_b = \frac{\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}, \text{ and } c = \frac{1}{p}\Big[\alpha A((1-\alpha)A^{\frac{\alpha}{1-\alpha}}\Big(1 + \frac{p\psi\gamma}{d}\Big) - p\beta\Big]
$$

<The derivation of the steady state value when there is LTC>

To derive the steady-state level of welfare when there is LTC, we derive the steady-state values of care provision and consumption. Because the equilibrium E_2 is located in Regime III, each value is derived by substituting the steady-state values into each constraint, as follows.

$$\begin{aligned}
h_g &= dq_g + \gamma = d\Big(\frac{\beta}{Rw(1-\tau)} - \frac{\gamma}{d}\Big) + \gamma = \frac{d\beta}{Rw(1-\tau)}, \\
h_b &= q_b + \hat{z} = \frac{\beta}{Rw(1-\tau)}, \\
c &= p\psi c_g + p(1-\psi)c_b + (1-p)c_d, \\
&= p\psi\Big(\frac{Rw(1-\tau)}{p}\Big(1 - \frac{\beta}{Rw(1-\tau)} + \frac{\gamma}{d}\Big)\Big) \\
&\quad + \ p(1-\psi)\frac{Rw(1-\tau)}{p}\Big(1 - \frac{\beta}{Rw(1-\tau)} + \hat{z}\Big) + (1-p)\frac{Rw(1-\tau)}{p}(1), \\
&= \frac{Rw(1-\tau)}{p}\Big(\underbrace{1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{Rw(1-\tau)}}_{l} + \underbrace{p(1-\psi)\hat{z}}_{z}\Big)
\end{aligned}$$

Given that $z = \frac{\tau l}{1-\tau}$, $l + z$ becomes $\frac{l}{1-\tau}$ and $c = \frac{Rw}{p}(1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{Rw(1-\tau)})$. Substituting steady-state values into $Rw = \alpha A\tilde{k}^{\alpha-1}(1-\alpha)A\tilde{k}^{\alpha}$, yields $Rw = \alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}$. Therefore, each steady-state level is derived as follows.

$$h_g = \frac{d\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)}, h_b = \frac{\beta}{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}(1-\tau)},$$

and

$$c = \frac{\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}{p}\Big(1 + \frac{p\psi\gamma}{d} - \frac{p\beta}{(1-\tau)\alpha A((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}}\Big)$$

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