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SAMUELSON'S FULL DUALITY AND THE USE OF DIRECTED ACYCLICAL GRAPHS

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To date, mixed demand systems have been all but ignored in empirical work. A possible reason for the scarcity of such applications is that one needs to know a priori which prices and quantities are endogenous in the mixed demand system. By using a directed acyclical graph (DAG), causal relationships among price and quantity variables are identified giving rise to a causally identified mixed demand system. A statistical comparison is made of the traditional Rotterdam model, a synthetic demand system, which subsumes the traditional Rotterdam model, and a Rotterdam mixed demand system identified through the use of a DAG. In this analysis, the respective demand systems consist of five products: steak, ground beef, beef roasts, pork, and chicken.

JEL classification codes: D11, D12
Key words: directed acyclic graphs, mixed demand systems

I. Introduction

Generally, most demand systems are mono-dependent, that is, they constitute a set of either quantity-dependent demand relations or price-dependent demand relations. Quantity-dependent demand functions are the usual representation of preferences for individual consumers whose task is to make optimal consumption decisions at given levels of prices and income. The behavioral implications of

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demand theory typically are assumed to hold at the aggregate or market-level, that is, over a collective set of individuals. At the market level, the use of quantitydependent demand functions also is tantamount to assuming that supplies are perfectly elastic. Price-dependent or inverse demand functions are particularly useful in markets for agricultural and natural resource commodities where, in the short-run, it is reasonable to argue that supplies are perfectly inelastic (Wong and McLaren 2005; Park et al. 2004; Barten and Bettendorf 1989).

However, at the market level, it is often the case that both prices and quantities of commodities are endogenous variables. The sources of endogeneity in prices and quantities largely are attributed to simultaneity of demand and supply relationships as well as aggregation across economic agents. Theil (1976) provides a general discussion of price endogeneity in demand systems. Thurman (1986, 1987), Wahl and Hayes (1990), and Eales and Unnevehr (1993) have examined the issue of price versus quantity endogeneity in demand analysis using the Wu-Hausman test (Hausman 1970 and 1977). La France (1991), Attfield (1985, 1991), Brown et al. (1994) and Capps et al. (1994) also question the endogeneity of total expenditure in conditional demand models. The commonality of the works in dealing with these endogeneity issues is that consistent estimates of demand parameters are obtained by using instrumental variables. The difficulty then arises in empirical application with the justification of selected instruments.

In accord with Heien (1977), depending on the characteristics of the particular market, one may want to specify some demand relationships as price dependent and others as quantity dependent. The use of full duality in the words of Samuelson (1965) allows for the specification of a mixed demand system. In the case of mixed demand functions, prices of some goods are predetermined such that the respective quantities demanded adjust to clear the market, whereas, for the remaining set of goods, quantities supplied are predetermined and prices must adjust to clear the market. Bottom line, in addition to the polar cases of direct (quantity-dependent) and inverse (price-dependent) demand functions, seemingly one must at least consider Samuelson's mixed demand functions, particularly when studying aggregate consumption behavior (Chavas 1984).

Moschini and Vissa (1993) used a Rotterdam mixed demand system to study retail price and quantity relationships of beef, pork and chicken in Canada. They justified the mixed demand system approach because "for chicken, equilibrium is characterized by exogenously determined supply with price adjusting to clear the market. Their assumption that beef and pork prices are exogenous to the Canadian market seems tenable" (p.5).

Matsuda (2004) extended the model of Moschini and Vissa (1993) by incorporating in it a generalized form of marginal budget shares. That is, the constant marginal budget shares in the Rotterdam mixed demand system of Moschini and Vissa (1993) are replaced with those derived from originally defined and specified mixed Engel curves that take a generalized functional form using the Box-Cox transformation. Matsuda (2004) empirically illustrated the use of this mixed demand system in analyzing the Japanese demand for fresh and processed fruits and vegetables.

If mixed demand systems are theoretically consistent, as shown by Samuelson (1965), Chavas (1984), and Cunha-e-Sá and Ducla-Soares (1999), and, technically feasible, as demonstrated by Barten (1992), Moschini and Vissa (1993), Gao et al. (1996), and Matsuda (2004), why have they been largely ignored in empirical work? Moschini and Vissa (1993) suggested that one reason for the scarcity of applications of mixed demand systems is that knowledge of both direct and indirect utility functions is required to characterize the demand properties. Commonly used flexible functional forms such as the translog models (Christensen et al. 1975) and the

Almost Ideal Demand System (AIDS) model (Deaton and Muellbauer, 1980) do not have a closed form dual representation, making it impossible to express these flexible functional forms as mixed demand systems.

Another possible reason for the scarcity of such applications is that one needs to know a priori which prices and quantities are endogenous in the mixed demand system. Through the works of Pearl (2000) and Spirtes et al. (2000), techniques have been developed to determine causality among variables. One specific technique is the use of a directed acyclical graph (DAG). The causal relationships, represented by the DAG, then are used as a guide in specifying the left-hand side (endogenous) and right-hand side (predetermined) variables of the equations in a demand system, giving rise to a causally identified demand system (CIDS).

Using scanner data consisting of 113 weekly observations on quantities and prices of various meats -steak, beef roasts, ground beef, pork and chicken- (Capps 1989; Nayga and Capps 1994), a statistical comparison is made of the traditional Rotterdam model, a synthetic demand system which subsumes the traditional Rotterdam model (Barten, 1993) and a Rotterdam mixed demand system. Through the use of the DAG, in the mixed demand system, we show that steak, ground beef, pork and chicken are quantity-dependent variables but beef roast is a pricedependent variable. Hence, the use of the DAG suggests the specification of a mixed demand system.

The intent of this analysis is to propose a method whereby the full theory of demand may be exploited when estimating a system of equations. The use of mixed demand systems, according to Chavas (1984), "is very attractive for the investigation of consumption decisions since it is derived without sacrificing the elegance of theory." A more precise understanding of the relationships of prices and quantities leads to more precise measurement of elasticities and/or flexibilities.

The organization of this paper is as follows. In the next section, we describe the notion of a directed acyclical graph (DAG). Subsequently, in Section III, guided by the work of Moschini and Vissa (1993), we lay out the details of the Rotterdam mixed demand system. We then present the data used in this analysis, along with the results associated with the DAG (Section IV). In Section V, empirical results associated with the various demand systems are presented, with emphasis placed on uncompensated and compensated elasticities. Finally, we offer concluding remarks and recommendations for further work.

II. Directed acyclic graphs and PC algorithm

A directed graph represents pictorially causal flows among a set of variables. Edges (arrows) are used to represent flows; for example A→B indicates that variable A causes variable B. The graph is acyclic, as we do not consider graphs that return to their origin; for example, the graph A→ B → A is not permitted. The idea that allows detection of the direction of causal flows among variables from observational or non-experimental data is that of screening-off phenomena and their formal representations as d-separation (Pearl 2000). Two variables are said to be d-separated if the information flow between them is blocked.

For three variables A, B and C, if we have variable A as a common cause of B and C, that is (B←A→C), then the unconditional association between B and C is non-zero, as both have a common cause in A (this graph is labeled a 'causal fork', Pearl 2000). If we measure association (linear association) by correlation, then B and C have a non-zero correlation. If we condition on A, the partial correlation between B and C (given we know the value of A) is zero. Simply put, knowledge of the common cause (A) "screens-off" association between its effects (B and C). Variables B and C are d-separated.

Alternatively, suppose that we have variables D, E and F such that D→E←F. In this case, E is a common effect of D and F (this graph is also labeled a 'causal inverted fork'). D and F have no association (zero correlation if we consider linear association). If we condition on E, the association between D and F is non-zero (the partial correlation between D and F, given knowledge of E is non-zero).

Knowledge of the common effect does not "screen-off" association between its causes.

Finally, if we have variables G, H and I forming a 'causal chain', G→H→I, the unconditional association (correlation) between G and I is non-zero, but the conditional (partial) correlation between G and I, given we know the value of H, is zero. Again, knowledge of H "screens off" association between G and I.

The "screening off" conditions summarized above provide potentially useful information about the causal structures generated directly from the data. Recently computer scientists have constructed programs to take advantages of such information. Pearl (2000), as well as Spirtes et al. (2000) through their work, provide two such algorithms. The latter is labeled PC algorithm, embedded in the software TETRAD II, III, and IV (see the offering at http://www.phil.cmu.edu/projects/tetrad/ and described in Spirtes et al. 2000); the former is IC algorithm presented in Pearl (2000, pp.50-51). We offer a brief description of PC algorithm. Essentially, this algorithm sequentially computes zero-order correlations (by zero order partial correlation we mean the unconditional correlation between, say, X and Y) or higherorder partial correlations (by higher order partial correlations we mean conditional correlations between, say, X and Y given knowledge of variables W and Z) among a set of variables to determine causal relationships.

One begins with a set of causally sufficient variables. Causal sufficiency relates to the issue of completeness. That is, there is no omitted variable from the specified set that causes two or more of the variables in the set. The primary source of information on our causally sufficient set is economic theory. One forms a complete undirected graph on this set of variables. In demand analysis, these variables correspond to quantities and prices of various commodities as well as their total expenditure. Say we have variables X, Y and Z. Form the complete undirected graph as:

```
   X
 /   \
Y  —  Z .
```

This graph has an edge connecting each variable with every other variable in the predetermined (causally sufficient) set. Edges between variables are removed sequentially based upon vanishing unconditional (zero-order) correlation or higherorder partial correlation at some pre-specified significance level. The Fisher Z statistic, distributed asymptotically as standard normal, is used in ascertaining the significance of the respective correlations among the variables.

Edges that survive these attempts at removal are directed by using the notion of sepset (separating set). The conditioning variable(s) on removed edges between two variables is called the sepset of the variables whose edge has been removed (for vanishing zero-order conditioning information the sepset is an empty set). PC algorithm directs the edges between X and Y into variable Z if Z is not in the sepset of X and Y. For our X, Y, Z example, suppose we have removed the edge between X and Y not conditional on Z (that is, the unconditional correlation between X and Y is zero). We then direct $X — Z — Y$ as $X \rightarrow Z \leftarrow Y$. Had Z been used to remove the edge between X and Y (if PC algorithm removed the edge because the correlation between X and Y conditional on Z is zero) then PC algorithm would not be able to direct the edges between X, Y and Z as the underlying model may have been a causal fork, that is $X \leftarrow Z \rightarrow Y$ or a causal chain $X \rightarrow Z \rightarrow Y$. In such cases of ambiguity, PC algorithm leaves the remaining edges undirected: $X — Z — Y$.

If we have at least one other variable in the pre-specified set in addition to X, Y and Z (say W), the aforementioned ambiguity may be resolved. Suppose that after removing edges on the four-variable set, we are left with the undirected graph on X, Y, Z and W:

$$\begin{array}{ccccc} X & — & Z & — & Y \\ & & | & & \\ & & W. & & \end{array}$$

If the sepset of X and W does not contain Z, but Z is in the sepset of X and Y, then X – Z – W may be characterized as an inverted fork:

$$\begin{array}{ccccc} X & \rightarrow & Z & — & Y \\ & & \uparrow & & \\ & & W. & & \end{array}$$

The inverted fork relation among X, Z and W, resolves the ambiguity on X, Z, Y directions. The causal fork possibility ($X \leftarrow Z \rightarrow Y$) does not hold. Consequently, PC algorithm returns the directed acyclic graph:

$$\begin{array}{ccccc} X & \rightarrow & Z & \rightarrow & Y \\ & & \uparrow & & \\ & & W. & & \end{array}$$

PC algorithm has been studied in Monte Carlo simulations (Spirtes et al. 2000; Demiralp and Hoover 2003). The algorithm may make mistakes of two types: edge inclusion or exclusion and edge direction (orientation); the latter appears to be more likely than the former. Spirtes et al. (p.116, 2000) state that "in order for the methods to converge to correct decisions with probability 1, the significance level used in making decisions should decrease as the sample size increases; the use of higher significance levels (e.g., .2 at sample sizes less than 100 and .1 at sample sizes between 100 and 300) may improve performance at small sample sizes." Hence in PC algorithm, the caveat is that the orientation (edge direction) decision is less reliable than the edge inclusion decision.[1]

III. Rotterdam mixed demand system

A flexible mixed demand system can be specified by approximating the demand equations directly through a differential approach. This process leads to a Rotterdam mixed demand system. The literature on mixed demand functions is limited. Not many empirical applications deal with mixed demand systems. The system introduced by Mochini and Vissa (1993) is both theoretically consistent and empirically manageable in the sense that it is relatively easy to impose theoretical restrictions and to compute elasticities.[2]

Assume m quantity-dependent demand equations and n-m price-dependent demand equations:

$$w_i d \ln q_i = w_i \eta_i d \ln y + \sum_{j=1}^{m} w_i \varepsilon_{ij} d \ln p_j + \sum_{k=m+1}^{n} w_i \psi_{ik} d \ln q_k \,, \qquad (1)$$

$$w_k d \ln p_k = w_k \theta_k d \ln y + \sum_{j=1}^{m} w_k \rho_{kj} d \ln p_j + \sum_{s=m+1}^{n} w_k \theta_{ks} d \ln q_s \,, \qquad (2)$$

where y corresponds to total expenditure $y = \sum_{r=1}^{m} p_r q_r + \sum_{r=m+1}^{n} p_r q_r$; w_i and w_k are expenditure shares $w_i = \frac{p_i q_i}{y}$ and $w_k = \frac{p_k q_k}{y}$ and $i, j = 1, 2, ..., m$ and $k, s = m + 1,$

---

[1] For an interesting extension of DAGs in the use of instrumental variables for identification of structural models we recommend the reader to the recent paper by Chalak and White (2006).

[2] Matsuda (2004) indeed extends the model of Moschini and Vissa (1993). We employ the latter model because of its relative simplicity. Of course, it is possible to extend our work using the Matsuda (2004) version of a mixed demand systems model.

…, n. Note that ε_{ij} is the uncompensated own-price elasticity (if $i = j$) and uncompensated cross-price elasticity ($i \neq j$) associated with goods i and j. Mathematically, $\varepsilon_{ij} = \left(\frac{\partial q_i}{\partial p_j}\right)\left(\frac{p_j}{q_i}\right)$; $\psi_{ik} = \left(\frac{\partial q_i}{\partial q_k}\right)\left(\frac{q_k}{q_i}\right)$; $\rho_{ki} = \left(\frac{\partial p_k}{\partial p_i}\right)\left(\frac{p_i}{p_k}\right)$; $\theta_{ks} = \left(\frac{\partial p_k}{\partial q_s}\right)\left(\frac{q_s}{p_k}\right)$; $\eta_i = \left(\frac{\partial q_i}{\partial y}\right)\left(\frac{y}{q_i}\right)$; and $\theta_k = \left(\frac{\partial p_k}{\partial y}\right)\left(\frac{y}{p_k}\right)$.

Furthermore,

$$\varepsilon_{ij} = \varepsilon_{ij}^c - \eta_i\left(w_j + \sum_{k=m+1}^{n} w_k \rho_{kj}^c\right), \tag{3}$$

$$\psi_{ik} = \psi_{ik}^c - \eta_i\left(\sum_{s=m+1}^{n} w_s \theta_{sk}^c\right), \tag{4}$$

$$\rho_{ki} = \rho_{ki}^c - \theta_k\left(w_i + \sum_{s=m+1}^{n} w_s \rho_{si}^c\right), \text{ and} \tag{5}$$

$$\theta_{ks} = \theta_{ks}^c - \theta_k\left(w_i + \sum_{r=m+1}^{n} w_r \theta_{rs}^c\right). \tag{6}$$

Bt choosing the parameterization that

$$\alpha = w_i \eta_i, \tag{7}$$

$$\beta_\kappa = \omega_\kappa \theta_\kappa\,, \tag{8}$$

$$\alpha_{ij} = w_i \varepsilon_{ij}^c, \tag{9}$$

$$\beta_{ks} = w_k \theta_{ks}^c\,, \tag{10}$$

$$\gamma_{ki} = -w_k \rho_{ki}^c \text{ and} \tag{11}$$

$$\delta_{ik} = w_i \psi_{ik}^c\,, \tag{12}$$

it is possible to obtain the Rotterdam mixed demand system,

$$w_i d \ln q_i = \alpha_i d \ln \bar{y} + \sum_{j=1}^{m} \left[\alpha_{ij} + \alpha_i \left(\sum_{k=m+1}^{n} \gamma_{kj} \right) \right] d \ln p_j + \tag{13}$$

$$+ \sum_{k=m+1}^{n} \left[\delta_{ik} - \alpha_i \left(\sum_{s=m+1}^{n} \beta_{sk} \right) \right] d \ln q_k \,,$$

$$w_k d \ln p_k = \beta_k d \ln \bar{y} + \sum_{j=1}^{m} \left[-\gamma_{kj} + \beta_k \left(\sum_{s=m+1}^{n} \gamma_{sj} \right) \right] d \ln p_j + \tag{14}$$

$$+ \sum_{s=m+1}^{n} \left[\beta_{ks} - \beta_k \left(\sum_{r=m+1}^{n} \beta_{rs} \right) \right] d \ln q_s \,,$$

where $d \ln \bar{y} = \left[d \ln y - \sum_{i=1}^{m} w_i d \ln p_i \right]$ represents the change in nominal total expenditure adjusted by the changes in exogenous prices.

Because of this parameterization, the homogeneity, symmetry, and adding-up properties can be set in terms of parametric restrictions.

Homogeneity, adding-up and symmetry conditions are satisfied when

$$\sum_{j=1}^{m} \alpha_{ij} = 0 \text{ and } \sum_{i=1}^{m} \gamma_{ki} = -w_k . \tag{15}$$

$$\sum_{i=1}^{m} \alpha_i + \sum_{k=m+1}^{n} \beta_k = 1, \sum_{i=1}^{m} \alpha_{ij} = 0, \text{ and } \sum_{i=1}^{m} \delta_{ik} = -w_k . \tag{16}$$

$$\alpha_{ij} = \alpha_{ji}, \, \beta_{ks} = \beta_{sk}, \text{ and } \gamma_{ik} = \delta_{ki} . \tag{17}$$

The adding-up and homogeneity condition involve shares and thus, in general, can be satisfied only locally (at a point).

IV. Data description and data analysis

A. Data description

Scanner data from a retail food firm in Houston are aggregated to form weekly time-series observations from September 1986 to November 1988 (Capps 1989; Nayga and Capps 1994). These data correspond to point-of-sale purchases and involve only fresh meat items, namely ground beef, beef roasts, steak, chicken, and pork.

To obtain the quantities of the various fresh meat products, we sum the respective quantities in the commodity group. The corresponding prices are weighted averages of the prices in the particular commodity group. The weights correspond to the quantity shares of each product in the relevant group. Quality effects may result from commodity aggregation (Houthakker 1952; Cox and Wohlgenant 1986). Although the use of these implicit prices potentially limits the analysis, quality effects attributable to commodity aggregation could be assumed negligible given that the meat products in question are relatively homogeneous.

Descriptive statistics associated with the quantities, prices, and expenditures shares of the respective meat products are exhibited in Table 1. Chicken and ground beef are the most important items in terms of volume of purchases. Beef roasts are the least important in terms of volume of purchases. In terms of prices, steak is the most expensive commodity ($3.92 per pound on average), and chicken is the least expensive commodity ($1.42 per pound on average).

Total meat expenditures associated with these products are slightly more than $1 million per week on average for this retail food firm. In terms of expenditure shares, pork and steak comprise nearly 50% of the total dollar sales of the various meat products. Ground beef and chicken comprise slightly more than 40% of the total dollar sales, and beef roasts constitute roughly the remaining 7% of total dollar sales.

B. Data analysis

We use the TETRAD II program to determine the directed acyclical graph (DAG) for all prices, quantities, and total expenditure indigenous to this analysis. The resulting DAG is exhibited in Figure 1. Arrows associated with prices for ground beef, steak, chicken, and pork point to the corresponding quantities of these meat products. Consequently, for ground beef, steak, chicken, and pork, price is casual to quantity. However, for beef roasts, the reverse is true as the quantity arrow for this commodity points to its corresponding price.[3] Bottom line, the DAG provides empirical evidence that this demand system of meat products is not mono-dependent but mixed.[4]

Table 1. Descriptive statistics associated with weekly quantities, prices, and expenditure shares of meat products

	Mean	Median	St. dev	Minimum	Maximum
A. Quantities (pounds)					
Ground beef	119,273	112,629	29,066	68,341	261,449
Beef roasts	31,310	20,827	22,931	13,190	142,663
Steak	63,640	58,463	17,311	27,415	121,822
Chicken	172,460	143,094	124,480	72,162	1,296,539
Pork	95,021	72,561	140,510	46,784	1,543,418
B. Prices (cents/pound)					
Ground beef	190.42	197.26	23.12	134.37	223.86
Beef roasts	267.34	286.26	52.93	124.34	341.39
Steak	391.95	395.74	49.47	251.38	486.87
Chicken	142.03	144.15	27.41	68.27	193.74
Pork	299.71	297.58	38.44	198.13	385.65
C. Expenditure shares					
Ground beef	0.2224	0.2281	0.0368	0.0296	0.3038
Beef roasts	0.0724	0.0644	0.0247	0.0077	0.1558
Steak	0.2413	0.2388	0.0390	0.0395	0.3326
Chicken	0.2181	0.2172	0.0365	0.1090	0.3105
Pork	0.2459	0.2177	0.0810	0.1682	0.6181

V. Empirical results

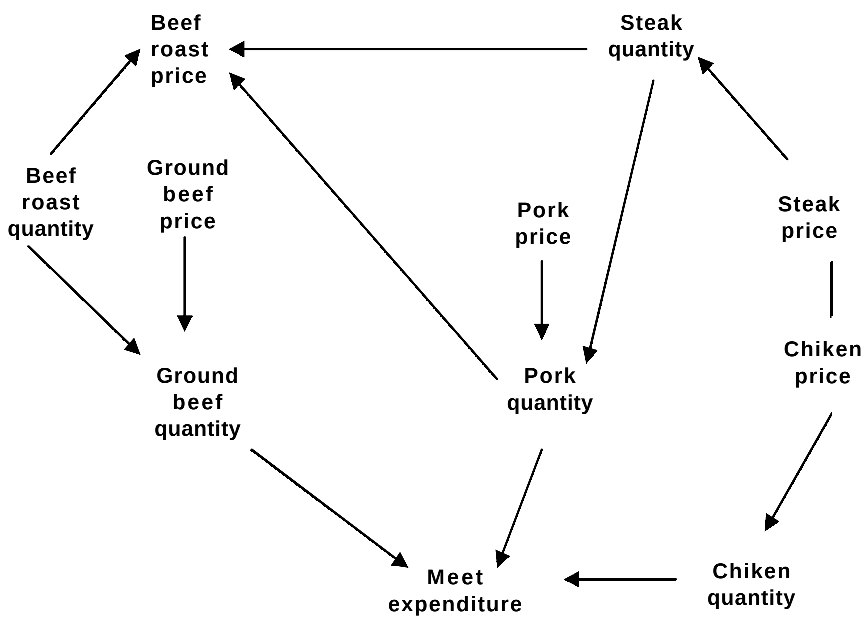
A. Rotterdam mixed demand model

We assume that the meats group is weakly separable from other

[3] The data used in our analysis are from a single retailer located in Houston, Texas. The customers associated with this retailer are upscale, with incomes well above the norm. A hypothesis consistent with our empirical findings is that beef roasts are used as a loss leader for this retailer.

[4] One may develop a system of equations based solely on the DAG exhibited in Figure 1. We do not attempt to estimate this system, however, because of its inconsistency with demand theory.

Figure 1. Directed acyclic graph of prices, quantities, and total expenditure of the various meat products



commodities.[5] We initially describe the estimation of the mixed demand system based on the analysis of the DAG, wherein the quantity-dependent demand relations correspond to ground beef, steak, chicken, and pork and the price-dependent demand relation corresponds to beef roasts. Implementing the Rotterdam mixed demand system requires converting the differential terms in equations (13) and (14) to finite logarithmic changes. That is, $d\ln q_i = \ln q_{it} - \ln q_{it-1}$, $d\ln p_j = \ln p_{jt} - \ln p_{jt-1} = \ln(p_{jt} / p_{jt-1})$ and $d\ln y = \ln y_t - \ln y_{t-1} = \ln(y_t / y_{t-1})$.

The stochastic version of the model is obtained by adding error terms that are assumed to be multi-normally distributed and contemporaneously correlated. Adding-up holds only at a particular point, and so the demand system is *not* singular. Thus, *all* equations are used in the estimation.

[5] This type of econometric utilization of weak separability is standard practice in demand analysis. However, owing to the endogenity of total meat expenditure, simultaneous-equation bias may arise (LaFrance 1991; Capps et al 1994).

The mixed differential model is estimated using SHAZAM 9.0 with the inclusion of intercepts for each equation as well as a correction for first-order autocorrelation. To satisfy adding-up, intercepts of the equations are constrained to sum to zero. Also, the autocorrelation coefficient is constrained to be the same in all equations.

Parameter estimates, standard errors, and t-statistics associated with the Rotterdam mixed demand system are exhibited in Table 2. The number of independent parameters estimated is 19; we recover the remaining estimates through the use of the restrictions from homogeneity, symmetry, and adding-up in equations (15), (16), and (17). The goodness-of-fit associated with the respective demand relationships range from 0.5596 (ground beef) to 0.9029 (chicken).

Table 2. Parameter estimates with the Rotterdam mixed demand system

Coefficient	Parameter estimate	Standard error	Coefficient	Parameter estimate	Standard error
Ground beef, equation (1)	0.0085	0.0035 **	α_{14} [2]	0.0938	0.0201 ***
Steak, equation (2)	0.0039	0.0033	α_{22}	-0.3388	0.0143 ***
Chicken, equation (3)	0.0008	0.0032	α_{23}	0.0998	0.0097 ***
Pork, equation (4)	-0.0088	0.0044 **	α_{24} [3]	0.1730	0.0167 ***
Beef roasts, equation (5)[1]	-0.0044	0.0011 ***	α_{33}	-0.2636	0.0118 ***
α_1	0.1211	0.0159 ***	α_{34} [4]	0.0910	0.0137 ***
α_2	0.1206	0.0146 ***	α_{44} [5]	-0.3578	0.0295 ***
α_3	0.2862	0.0145 ***	β_{55}	-0.0386	0.0020 ***
α_4	0.4600	0.0196 ***	$\gamma_{51}(\delta_{15})$ [6]	-0.0045	0.0048
α_5	0.0121	0.0051 **	$\gamma_{52}(\delta_{25})$	-0.0278	0.0040 ***
α_{11}	-0.2326	0.0198 ***	$\gamma_{53}(\delta_{35})$	-0.0138	0.0035 ***
α_{12}	0.0661	0.0125 ***	$\gamma_{54}(\delta_{45})$	-0.0262	0.0051 ***
α_{13}	0.0727	0.0122 ***	Rho	-0.2989	0.0469 ***

Notes: [1]Derived from the restriction that the sum of the intercepts equals zero. [2]Derived from the restriction that $\alpha_{14} = -\alpha_{11} - \alpha_{12} - \alpha_{13}$. [3]Derived from the restriction that $\alpha_{24} = -\alpha_{12} - \alpha_{22} - \alpha_{23}$. [4]Derived from the restriction that $\alpha_{34} = -\alpha_{13} - \alpha_{23} - \alpha_{33}$. [5]Derived from the restriction that $\alpha_{44} = -\alpha_{11} + \alpha_{22} + \alpha_{33} + 2\alpha_{12} + 2\alpha_{13} + 2\alpha_{23}$. [6]Derived from the restriction that $\gamma_{51} = -\gamma_{52} - \gamma_{53} - \gamma_{54}$ – average expenditure share of beef roasts. ** denotes significance at 5% and *** at 1%.

With the exception of the intercept estimates for the steak and chicken equations and of the estimate of γ_{51}, the remaining parameter estimates are significantly different from zero. If we divide the estimates of the intercepts in the respective equations by their corresponding expenditure share, we determine the rate of change of quantity demanded for each of the meat products. Hence, over the sample period and for reasons not attributable to meat prices and total meat expenditures, the quantity demanded of ground beef, steak, and chicken rose by 3.8 percent, 1.6 percent, and 0.4 percent respectively. On the other hand, the quantity demanded of pork and beef roasts fell by 3.6 percent and 6.1 percent respectively.

Marshallian mixed elasticities retrieved from the use of equations (3)-(8) are given in Table 3. The mixed expenditure elasticities are all positive and statistically significant in accord with prior expectations. These elasticities for ground beef, steak, chicken, and pork indicate the percentage change in consumption (0.54 percent, 0.50 percent, 1.31 percent and 1.87 percent) attributed to a one percent change in total meat expenditure. For beef roasts, the expenditure elasticity indicates

Table 3. Elasticities estimated from the Rotterdam mixed demand system

Commodity	Price ground beef	Price steak roasts	Price chicken	Price pork	Quantity beef	Meat expenditure	Meat share
A. Marshallian elasticities♣							
Quantity ground beef	-1.1695	0.1507	0.2008	0.2735	0.0006	0.5445	0.2224
p-value	0.0000	0.0132	0.0002	0.0029	0.9795	0.0000	
Quantity steak	0.1605	-1.5390	0.2977	0.5809	-0.0959	0.4999	0.2413
p-value	0.0035	0.0000	0.0000	0.0000	0.0000	0.0000	
Quantity chicken	0.0357	0.1045	-1.5129	0.0603	-0.0129	1.3124	0.2181
p-value	0.5464	0.0330	0.0000	0.3493	0.4108	0.0000	
Quantity pork	-0.0432	0.2000	-0.0637	-1.9639	-0.0343	1.8707	0.2459
p-value	0.6036	0.0048	0.2609	0.0000	0.1213	0.0000	
Price beef roasts	0.0250	0.3393	0.1527	0.3165	-0.5264	0.1666	0.0724
p-value	0.7184	0.0000	0.0018	0.0001	0.0000	0.0185	
B. Compensated elasticities ♣							
Quantity ground beef	-1.0459	0.2972	0.3271	0.4217	-0.0204		
p-value	0.0000	0.0000	0.0000	0.0000	0.3458		
Quantity steak	0.2739	-1.4044	0.4136	0.7169	-0.1152		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity chicken	0.3336	0.4576	-1.2086	0.4174	-0.0635		
p-value	0.0000	0.0000	0.0000	0.0000	0.0001		
Quantity pork	0.3814	0.7034	0.3701	-1.4548	-0.1065		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Price beef roasts	0.0628	0.3841	0.1913	0.3618	-0.5329		
p-value	0.3458	0.0000	0.0001	0.0000	0.0000		

Notes: Asymptotic standard errors (not reported) are computed via the use of the Delta method. The p-values associated with the significance of the respective elasticities are reported. These p-values are based on the relation that the ratios of the elasticities to their respective standard errors are asymptotically normally distributed. ♣ Calculations at the sample means.

the percentage change in price (0.17 percent) when total meat expenditure changes by one percent. The own-price elasticities for the commodities range from –1.17 (ground beef) to –1.96 (pork). The own-price flexibility for beef roasts is –0.52.

Estimated mixed compensated elasticities obtained by the use of equations (9)-(12) and their associated p-values are given in Table 3. The ratios of the elasticities to their standard errors are asymptotically normally distributed. Ground beef, steak, chicken, and pork are net substitutes for each other as indicated by the positive mixed compensated elasticities. The compensated cross-price flexibilities of beef roasts with respect to steak, chicken, and pork are negative and statistically different from zero.

To facilitate comparisons to quantity-dependent demand systems, it is desirable to retrieve direct Marshallian elasticities and direct compensated elasticities from the mixed elasticities. Let $\varepsilon = \lfloor \varepsilon_{ij} \rfloor_{mxm}$, $\psi = [\psi_{ik}]_{mx(n-m)}$, $\rho = \lfloor \rho_{kj} \rfloor_{(n-m)xm}$ and $\theta = [\theta_{kk}]_{(n-m)x(n-m)}$ correspond to the matrix of elasticities derived from equations (3) - (6), $i, j=1, 2, \ldots m$ and $k = m+1, \ldots, n$. Then the Marshallian and mixed elasticities are related by:

$$\varepsilon_{AA} = \lfloor \varepsilon - \psi\theta^{-1}\rho \rfloor, \tag{18}$$

$$\varepsilon_{BB} = \lfloor \psi\theta^{-1} \rfloor, \tag{19}$$

$$\varepsilon_{BA} = -\theta^{-1}\rho, \text{ and} \tag{20}$$

$$\varepsilon_{BB} = \theta^{-1}, \tag{21}$$

where $\varepsilon, \psi, \theta$, and ρ denote the aforementioned matrices of mixed elasticities and ε_{AA}, ε_{AB}, ε_{BA}, and ε_{BB} denote submatrices of the direct Marshallian price elasticities. In our analysis, ground beef, steak, chicken and pork are in group A and beef roasts are in group B.

Direct elasticities retrieved from the Rotterdam mixed demand system are reported in Table 4. We initially compute the direct Marshallian elasticities. Then, through the homogeneity restrictions, we retrieve the expenditure elasticities. Finally, via the use of Slutsky's equation, we obtain the direct compensated elasticities. For the quantity-dependent demand functions, the set of mixed elasticities exhibited in Table 3 are very similar to the set of direct elasticities exhibited in Table 4. The direct elasticities associated with the price-dependent demand relation for beef roasts retrieved from the mixed Rotterdam model suggest that: (1) beef roasts are substitutes (in the Hicksian sense) for the meat commodities; (2) the expenditure elasticity for beef roasts is about 0.32, smaller than that of steak (0.47), of ground beef (0.54), chicken (1.31), and pork (1.86); and (3) the own-price elasticity of demand for beef roasts is –1.90, on par with the elastic demands of the other meat items in the system.

Table 4. Direct elasticities retrieved from the Rotterdam mixed system

Commodity	Price ground beef	Price steak roasts	Price chicken	Price pork	Quantity beef	Meat expenditure	Meat share
A. Marshallian elasticities ♣							
Quantity ground beef	-1.1695	0.1510	0.2010	0.2739	-0.0011	0.5447	0.2224
p-value	0.0000	0.0163	0.0002	0.0022	0.9795	0.0000	
Quantity steak	0.1559	-1.6008	0.2698	0.5809	0.1823	0.4696	0.2413
p-value	0.0064	0.0000	0.0000	0.0000	0.0000	0.0000	
Quantity chicken	0.0351	0.0962	-1.5167	0.0244	-0.0129	1.3084	0.2181
p-value	0.5533	0.0464	0.0000	0.4113	0.4108	0.0000	
Quantity pork	-0.0448	0.1779	-0.0737	-1.9845	-0.0343	1.8599	0.2459
p-value	0.5889	0.1357	0.1964	0.0000	0.1213	0.0000	
Quantity beef roasts	0.0250	0.6445	0.2900	0.6012	-0.5264	0.3164	0.0724
p-value	0.7204	0.0000	0.0016	0.0000	0.0000	0.0179	
B. Compensated elasticities♣							
Quantity ground beef	-1.0484	0.2824	0.3198	0.4078	0.0384		
p-value	0.0000	0.0000	0.0000	0.0000	0.3556		
Quantity steak	0.2604	-1.4875	0.3722	0.6387	0.2162		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity chicken	0.3261	0.4118	-1.2314	0.3743	0.1191		
p-value	0.0000	0.0000	0.0000	0.0000	0.0001		
Quantity pork	0.3688	0.6266	0.3319	-1.5271	0.1999		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity beef roasts	0.1179	0.7208	0.3590	0.6790	-1.8767		
p-value	0.3556	0.0000	0.0001	0.0000	0.0000		

Notes: Asymptotic standard errors (not reported) are computed via the use of the Delta method. The p-values associated with the significance of the respective elasticities are reported. These p-values are based on the relation that the ratios of the elasticities to their respective standard errors are asymptotically normally distributed. ♣ Calculations at the sample means.

B. Synthetic demand system

The choice of demand systems can potentially have a material effect on the estimation of elasticities. For comparison purposes, we estimate a synthetic demand system (quantity-dependent) developed by Barten (1993).[6] Lee, Brown and Sale (1994) provide details of this quantity-dependents synthetic demand system. The synthetic demand nests four differential demand systems: The traditional Rotterdam model (Theil 1965; Theil 1980; Mountain 1988); (2) the Linear Approximate Almost Ideal Demand System (LA/AIDS) (Deaton and Muellbauer 1980); (3) the CBS model (named after the Dutch Central Bureau of Statistics); and (4) the NBR model (named after the National Bureau of Research). The use of the Almost Ideal Demand System (AIDS) model and the use of the Rotterdam model are common in demand system estimation using scanner data (Nayga and Capps 1994; Seo and Capps 1997; Capps, Seo, and Nichols 1997). That is, given the nature of scanner data, often the assumption is made that prices are exogenous. Under this assumption, retailers set their prices, and consumers may purchase all they want at the given prices.

Initially, one estimates the following model

$$w_i d \ln q_i = (b_1 + \delta w_i) d \ln Q + \sum_j \left[c_{ij} - \gamma w_i (\delta_{ij} - w_i) \right] \cdot d \ln p_j , \tag{22}$$

where δ_{ij} denotes the Kronecker δ such that δ_{ij} =1 if i=j and δ_{ij} =0 if $i \neq j$, $d \ln Q$ denotes the Divisa volume index; w_i denotes the expenditure share of the ith good; q_i denotes the volume purchased of the ith good; and p_j denotes the price of the jth good. The parameters to be estimated in this synthetic system are b_i, c_{ij}, δ and γ. The indices i and j run from 1 to n, where n corresponds to the number of commodities in the demand system.

Theoretical demand restrictions in the synthetic model are adding-up, homogeneity, and symmetry:

$$\sum_i b_i = 1 - \delta \text{ and } \sum_i c_{ij} = 0 \text{ for all j.} \tag{23}$$

$$\sum c_{ij} = 0, \text{ and} \tag{24}$$

$$c_{ij} = c_{ji} \text{ for all } i\text{,j.} \tag{25}$$

[6] Indeed, one may compare a mixed demand system with inverse demand systems (Brown et al. 1995), as well as with direct demand systems. Eales and Unnevehr (1993) and others estimated U.S. meat demand using inverse demand functions. Because the majority of the demand relationships were found to be quantity dependent, we compare our mixed demand model to the synthetic Barten direct demand system.

Restricting the values of δ and γ yield the following demand systems:

Rotterdam: $\delta = \gamma = 0$; (26)

LA/AIDS: $\delta = \gamma = 1$; (27)

CBS: $\delta = 1,\ \gamma = 0$; (28)

NBR: $\delta = 0,\ \gamma = 1$; (29)

Maynard and Veeramani (2003) reject the conventional demand models (i.e., Rotterdam model, LA/AIDS model, CBS model, and NBR model) in their investigation of the demand for U.S. frozen dairy products. Their test results illustrate the strength of the synthetic model in helping to avoid inadequate functional form choices that could lead to specification bias. Likelihood ratio tests evaluated with two degrees of freedom allow one to choose which set of restrictions (if any) adequately describes the appropriate functional form of the quantity-dependent demand functions. The expressions for the calculation of expenditure elasticities, compensated price elasticities, and uncompensated price elasticities are:

$$\eta_i = (b_i + \delta w_i) / w_i, \tag{30}$$

$$\varepsilon_{ij}^c = \left[c_{ij} - \gamma w_i (\delta_{ij} - w_j) \right] / w_i, \tag{31}$$

$$\varepsilon_{ij} = \varepsilon_{ij}^c - w_j \eta_i. \tag{32}$$

Standard errors of the elasticity estimates are calculated using the Delta method.

To operationalize the synthetic demand system, we add random error terms to the system. The model then is estimated using an iterated seemingly unrelated regression (ITSUR) technique, with an allowance for AR(1) serial correlation in the disturbance terms of all equations (Berndt and Savin 1975). Because this demand system is singular, the pork equation is dropped in estimation. The autocorrelation coefficient, however, is constrained to be the same in all equations.

The parameter estimates, standard errors, and t-statistics associated with the synthetic Barten demand system are exhibited in Table 5. The number of independent parameters to be estimated in this system is 21. The goodness-of-fit statistics range from 0.8492 (ground beef) to 0.9490 (chicken). In light of the parameter estimates associated with delta (0.7396) and gamma (2.0735), we reject the conventional or traditional demand models in this analysis. Specifically, the synthetic Barten model offers a statistically superior representation over the traditional Rotterdam model.

Table 5. Parameter estimates with the synthetic Barten demand system

Coefficient	Parameter estimate	Standard error	Coefficient	Parameter estimate	Standard error
Ground beef, equation (1)	0.0028	0.0019	c_{15} [3]	-0.0226	0.0138
Beef roasts, equation (2)	0.0032	0.0013 **	c_{22}	-0.0038	0.0113
Steak, equation (3)	0.0028	0.0018	c_{23}	-0.0020	0.0065
Chicken, equation (4)	-0.0049	0.0021 **	c_{24}	0.0041	0.0054
Pork, equation (5)[1]	-0.0039	0.0028	c_{25} [4]	0.0084	0.0078
b_1	0.0119	0.0137	c_{33}	0.0684	0.0254 ***
b_2	0.0051	0.0075	c_{34}	-0.0373	0.0097 ***
b_3	0.0100	0.0013	c_{35} [5]	0.0054	0.0137
b_4	0.1153	0.0212 ***	c_{44}	0.1128	-0.0234 ***
b_5 [2]	0.1182	0.0343 ***	c_{45} [6]	-0.0331	0.0123 ***
c_{11}	0.1104	0.0251 ***	c_{55} [7]	0.0045	0.0313
c_{12}	-0.0068	0.0064	Delta	0.7396	0.0680 ***
c_{13}	-0.0346	0.0108 ***	Gamma	2.0735	0.1213 ***
c_{14}	-0.0465	0.0097 ***	Rho	-0.4691	0.0501 ***

Notes: [1]Derived from the restriction that the sum of the intercepts equals zero. [2]Derived from the restriction that $b_5 = 1 - b_1 - b_2 - b_3 - b_3 - delta$. [3] Derived from the restriction that $c_{15} = -c_{11} - c_{12} - c_{13} - c_{14}$. [4] Derived from the restriction that $c_{25} = -c_{12} - c_{22} - c_{23} - c_{24}$. [5] Derived from the restriction that $c_{35} = -c_{13} - c_{23} - c_{33} - c_{34}$. [6]Derived from the restriction that $c_{45} = -c_{14} - c_{22} - c_{34} - c_{44}$. [7]Derived from the restriction that $c_{55} = c_{11} + c_{22} + c_{33} + 2c_{12} + 2c_{13} + 2c_{14} + 2c_{23} + 2c_{24} + 2c_{34}$. **denotes significance at 5% and ***at 1%.

Uncompensated and compensated own-price and cross-price elasticities as well as expenditure elasticities obtained from the synthetic Barten model are exhibited in Table 6. The uncompensated own-price elasticities for ground beef and beef roasts are higher in the synthetic Barten model than in the Rotterdam mixed demand system. The reverse is true for steak, chicken, and pork. In addition, the expenditure elasticities for ground beef, steak, and beef roasts are much higher in the synthetic Barten model than in the Rotterdam mixed demand system. The expenditure elasticities for chicken and pork are lower in the Barten model than in the Rotterdam mixed demand system. Noticeable differences also exist for the magnitude of the uncompensated cross-price elasticities across the two demand systems. While the same is true in regard to the magnitude of the compensated cross-price elasticities associated with the respective demand systems, empirical evidence exists from both models to indicate that the meat products in question are substitutes for each other.

Table 6. Direct elasticities estimated from the synthetic Barten demand system

Commodity	Price ground beef	Price steak	Price chicken	Price pork	Price roasts	Meat expenditure	Meat share
A. Marshallian elasticities ♣							
Quantity ground beef	-1.2923	0.1535	0.0703	0.2133	0.0623	0.7929	0.2224
p-value	0.0000	0.0002	0.0414	0.0002	0.0199	0.0000	
Quantity steak	0.1441	-1.4780	0.1274	0.3402	0.0852	0.7811	0.2413
p-value	0.0001	0.0000	0.0000	0.0000	0.0008	0.0000	
Quantity dhicken	-0.0340	0.0235	-1.3807	0.0460	0.0771	1.2681	0.2181
p-value	0.3520	0.5112	0.0000	0.3550	0.0007	0.0000	
Quantity pork	0.0979	0.2278	0.0513	-1.6933	0.0961	1.2203	0.2459
p-value	0.0654	0.0000	0.2377	0.0000	0.0016	0.0000	
Quantity beef roasts	0.1878	0.2772	0.3323	0.4274	-2.0341	0.8095	0.0724
p-value	0.0268	0.0017	0.0000	0.0000	0.0000	0.0000	
B. Compensated elasticities♣							
Quantity ground beef	-1.1160	0.3448	0.2432	0.4083	0.1197		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity steak	0.3178	-1.2895	0.2977	0.5323	0.1417		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity chicken	0.2480	0.3294	-1.1042	0.3579	0.1689		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity pork	0.3693	0.5222	0.3174	-1.3932	0.1844		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		
Quantity beef roasts	0.3678	0.4725	0.5088	0.6265	-1.9755		
p-value	0.0000	0.0000	0.0000	0.0000	0.0000		

Notes: Asymptotic standard errors (not reported) are computed via the use of the Delta method. The p-values associated with the significance of the respective elasticities are reported. These p-values are based on the relation that the ratios of the elasticities to their respective standard errors are asymptotically normally distributed. ♣ Calculations at the sample means.

VI. Model selection

The synthetic Barten model subsumes the traditional Rotterdam model, among others. We provide statistical evidence that the synthetic Barten model dominates statistically the traditional Rotterdam model. However, the synthetic Barten model and the Rotterdam mixed demand system are non-nested specifications. To determine the superiority of non-nested demand specifications, we rely on three criteria: (1) the Akaike criterion (Akaike 1973); (2) the Schwarz criterion (Schwarz 1978); and (3) the Likelihood Dominance Criterion (Pollak and Wales 1991).

Let L_1 denote the log-likelihood value of the Rotterdam mixed demand system, and let L_2 denote the log-likelihood value of the synthetic Barten demand system. As well, let n_1 and n_2 denote the number of independent parameters associated with the Rotterdam mixed demand system and the synthetic Barten demand system respectively. Finally, let $C(\iota)$denote the critical values of the χ^2 distribution with ô degrees of freedom at some fixed significance level.

Then, the Rotterdam mixed demand system is preferred to the synthetic Barten demand system if according to the

(i) Akaike criterion, $L_1 - L_2 > n_1 - n_2$;

(ii) Schwarz criterion, $L_1 - L_2 > (n_1 - n_2)(½\ln T)$, where T is the number of observations used in the estimation of the demand system; and

(iii) Likelihood Dominance criterion, $L_2 – L_1 < [C(n_2+1) - C(n_1+1)] /2$.

In our analysis, 112 observations are used in the estimation of each demand system. The log-likelihood value of the Rotterdam mixed demand system (L_1) is 1079.397, and the log-likelihood value of the synthetic Barten demand system (L_2) is 1069.566. The number of independent parameters in the mixed demand model is 19, and the number of independent parameters in the synthetic Barten model is 21. Thus, by the Akaike and Schwarz criteria, clearly the Rotterdam mixed demand model is preferred to the synthetic Barten demand model. Now, assume a level of significance of 0.05; then C(22) = 33.9 and C(20) = 31.4. Subsequently, by the Likelihood Dominance Criterion, the Rotterdam mixed demand system is preferred to the synthetic Barten model. Bottom line, we provide statistical evidence to demonstrate that the Rotterdam mixed demand system dominates statistically the synthetic Barten demand system.

VII. Conclusion

To date, mixed demand systems have received scant attention in empirical work. A limited literature exists pertaining to mixed demand functions. Mixed demand systems offer an alternative approach to consumer demand analysis other than the more heavily used direct and inverse demand systems. The difficulty in the empirical application of mixed demand systems is that analysts need to know a priori which prices and quantities are endogenous. Until recently, no scientific and theoretically tenable methodology had been accessible to combine with economic theory that reveals the causal relationships from observational or non-experimental data. A specific application of the theory of causality is made using PC-algorithm, which creates a directed acyclical graph (DAG) from various correlations among variables. The causal relationships, represented by the DAG procedure, then are used as a guide in specifying the left-hand and right-hand side variables of the equations in the demand system. Consequently, the use of a DAG gives rise to causally-identified demand systems.

A statistical comparison is made of the traditional Rotterdam model, the synthetic Barten model that subsumes the tradition Rotterdam model, and a Rotterdam mixed demand system identified through the use of a DAG. In this analysis, the respective demand systems consist of five products: ground beef, steak, beef roasts, chicken, and pork. We demonstrate that the magnitudes of the direct elasticities retrieved from the Rotterdam mixed demand system and those from the synthetic Barten model indeed are different, in some cases substantially. We also offer statistical evidence to support the contention that in our analysis, the Rotterdam casually-identified mixed demand system is preferred to the synthetic Barten model. One may extend our analysis to the more general Matsuda mixed demand system. Also, one may compare mixed demand systems to inverse demand systems.

Given that, at the market level, it is often the case that both prices and quantities of commodities are endogenous variables, we recommend the consideration of casually-identified demand systems. Through the use of a DAG indigenous to the demand system, we may determine a priori which price and quantity variables in the system are endogenous. A more precise understanding of the relationships of prices and quantities results in a more precise measurement of elasticities and/or flexibilities.

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