# Adverse Selection, Segmented Markets, and the Role of Monetary Policy

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**Abstract**

A model is constructed in which trading partners are asymmetrically informed about future trading opportunities and where spatial and informational frictions limit arbitrage between markets. These frictions create an inefficiency relative to a full information equilibrium, and the extent of this inefficiency is affected by monetary policy. A Friedman rule is optimal under a wide range of circumstances, including ones where segmented markets limit the extent of monetary policy intervention.

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# 1 Introduction

In this paper we explore the following ideas. Spatial and informational frictions imply that arbitrage is limited across markets for goods and services, so that the same good or service may trade at different prices in different locations. In other words, there is a degree of segmentation across goods markets. Further, economic agents move among spatially separated markets in an uncoordinated fashion, so that a given agent's current potential trading partners may be different from his or her past and future trading partners. As a result, if two agents are engaged in decentralized exchange, their future trading opportunities may be quite different. If these two agents are asymmetrically informed about these future trading opportunities, then this will in general affect the terms on which they exchange goods, services, and assets. Now, monetary policy affects the relative prices of goods and services across segmented markets, for two reasons. First, given heterogeneity in the populations of buyers and sellers across different markets, market prices may respond differently in different markets to the same monetary policy intervention. Second, the central bank in general participates directly in some markets and not in others, so that a money injection by the central bank will at least initially have different effects in different markets. Given that monetary policy actions can change relative prices across markets in a persistent fashion, this will then matter for the efficiency losses due to private information frictions. We want to explore the role for monetary policy in this context, and to derive some conclusions for optimal policy.

The basic structure of the model builds on Lagos and Wright (2005), in which there is trading on centralized and decentralized markets. In our model there is segmentation in centralized markets, and the price of goods in terms of money will in general differ across these markets. In the decentralized market, there is random bilateral matching and monetary exchange, and agents who meet will be privately informed concerning their centralized market location in the next period. Thus, there is asymmetric information concerning how trading partners value money. Elements of the bargaining problem in the decentralized market conform to the features of standard adverse selection environments, such as Maskin and Riley (1984). However, a key element of the problem is that cash constraints alter the outcomes, and in this way our analysis shares something with the work of Ennis (2007).

Our model is certainly not the first to study the potential role of monetary policy in exacerbating information frictions. For example, a key contribution to the monetary policy literature was the money surprise model developed in Lucas (1972). In Lucas's competitive environment, producers can be fooled by the central bank into producing more or less than is optimal, as producers have imperfect information about relative prices. In our model, buyers of goods are imperfectly informed concerning how sellers value the money offered in exchange for goods. This implies that contracts are distorted in order to induce self-selection, and these distortions will vary with monetary intervention by the central bank.

This paper is also related to some ideas in the market segmentation literature. In particular, Williamson (2008, 2009) studies a class of models with persistent nonneutralities of money and segmentation in goods and financial markets.

The results we obtain here are the following. In general, prices will differ in equilibrium across the segmented centralized markets, and this creates a private-information inefficiency in decentralized trade. The model also contains a standard intertemporal distortion that is typically corrected by a Friedman rule, i.e. inflation causes inefficient trade resulting from under-investment in the accumulation of money balances. As it turns out (and perhaps surprisingly) a Friedman rule will correct both the private information inefficiency and the intertemporal distortion under all the alternative market arrangements we consider. In particular, first, if the central bank can intervene in all centralized markets, then a Friedman rule equalizes prices across centralized markets and corrects the standard intertemporal monetary distortion, even if the central bank is constrained to making the same lump-sum money transfer to all agents. Second, if there is financial trading (essentially a federal funds market) across centralized markets, then prices are equalized across markets and a Friedman rule is optimal, no matter who is on the receiving end of the central bank's lump-sum transfers. Third, even in the absence of financial market trading, and when the central bank can intervene in only one centralized market, a Friedman rule supports an efficient allocation.

The paper is organized as follows. In the first section the model is constructed, then features of the equilibrium related to centralized trade and decentralized trade, respectively, are determined in sections three and four. Then, in sections five through seven, an equilibrium is determined and optimal monetary policy is studied under, respectively, intervention by the central bank in all centralized markets, financial market trade across centralized markets, and intervention by the central bank in only one centralized market in the absence of cross-location financial trade. Finally, Section 8 concludes.

# 2 The Model

The basic structure of the model is derived from Lagos and Wright (2005), and we add some locational and informational frictions. Time is discrete and there is a continuum of agents with unit mass. Each agent is infinite-lived and maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - l_t],$$

where $\beta \in (0,1)$, $c_t$ is consumption of the unique perishable consumption good, and $l_t$ is labor supply. Assume that $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, with $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Let $q^*$ denote the solution to $u'(q^*) = 1$. Each agent possesses a technology which permits the production of one unit of the perishable consumption good for each unit of labor supplied, and no agent can consume his or her own output.

In periods $t = 0, 2, 4, ...$, agents are randomly allocated between two locations indexed by $i = 1, 2$. Let $\rho$ denote the probability that an agent goes to location 1, and $1 - \rho$ the probability of going to location 2, where $0 < \rho < 1$. Goods and agents cannot be moved between the two locations. Exchange occurs competitively in even periods in each location. At the beginning of periods $t = 1, 3, 5, ...$, an agent learns whether he or she will be a buyer or a seller during the current period. For an agent who is in location $i$ during period $t$, for $t$ even, the probability of being a buyer in period $t + 1$ is $\alpha_i$, and the probability of being a seller is $1 - \alpha_i$, where $0 < \alpha_i < 1$ for $i = 1, 2$. Assume that $\alpha_1 > \frac{1}{2}$, and that

$$\alpha_2 = \frac{1 - 2\alpha_1\rho}{2(1 - \rho)},$$

which guarantees that half the population consists of buyers (and the other half consists of sellers) during an odd period. We need to assume that

$$\alpha_1\rho < \frac{1}{2},$$

which assures that $\alpha_2 > 0$. Thus, agents in location 1 during an even period have a higher probability of being buyers during the next odd period than is the case for agents in location 2.

At the beginning of period $t$, for $t$ odd, each agent first learns whether he or she is a buyer or seller during the current period. At this time, sellers also learn their period $t + 1$ location, which is private information, but buyers will not learn their period $t + 1$ location until the beginning of period $t + 1$. Each buyer is randomly matched with a seller during an odd period, but each buyer/seller match occurs between a buyer and seller who will occupy the same location during the next period. Thus, in a given pairwise match in an odd period, the buyer and seller are asymmetrically informed. The seller knows his or her location next period, but the buyer does not know his or her future location, or the future location of the seller he or she is paired with. Trade is anonymous in pairwise matches, so if exchange is to take place the seller must be willing to accept money for the consumption goods that he or she can produce.

The setup of the model is illustrated in Figure 1. A key feature of the model is that there is an adverse selection problem related to decentralized trade, in that buyers and sellers are asymmetrically informed about their future trading opportunities. As we will show, monetary policy will have important effects on the nature of this adverse selection problem. There are some elements of the model that we have rigged for tractability, for example the restrictions on who meets whom and when, but we think that the ideas are quite general.

# 3 Centralized Exchange

Let $W_t^i(m)$ be the value function of an agent with $m$ units of money at location $i$, for $t = 0, 2, 4, ...$, and let $V_t^i(m)$ be the value function of an agent with $m$ units of money in the decentralized market who resided in location $i$ in period

$t-1$ (before learning period $t$ buyer/seller status), for $t = 1, 3, 5, ....$ We then have

$$W_t^i(m) = \max_{\left(c_t^i, l_t^i, \tilde{m}_{t+1}^i\right) \in \mathbb{R}_+^3} \left[ u\left(c_t^i\right) - l_t^i + \beta V_{t+1}^i \left(m_{t+1}^i\right) \right]$$

subject to

$$c_t^i + \phi_t^i \tilde{m}_{t+1}^i = l_t^i + \phi_t^i m + \phi_t^i \tau_t^i. \tag{1}$$

Here, $\phi_t^i$ is the value of money in units of consumption goods in location $i = 1, 2$, and $\tau_t^i$ is a lump-sum money transfer from the central bank which we allow at this stage to depend on the agent's location. Suppose there is an interior solution for $c_t^i$ and $l_t^i$ in every even period. Then, for each $i = 1, 2$, we have

$$W_t^i(m) = \phi_t^i m + W_t^i(0) \tag{2}$$

where

$$W_t^i(0) = u(q^*) - q^* + \phi_t^i \tau_t^i + \max_{\tilde{m}_{t+1}^i \in \mathbb{R}_+} \left[ -\phi_t^i \tilde{m}_{t+1}^i + \beta V_{t+1}^i \left(m_{t+1}^i\right) \right]. \tag{3}$$

Note from (2) that, as in Lagos and Wright (2005), the value function $W_t^i(m)$ is linear in $m$. Further, given our assumptions about the pattern of meetings in even and odd periods, the per capita stock of money must always be the same in each location in even periods. Ultimately we will show that, as in Lagos and Wright (2005), all agents in a given location choose to hold the same quantity of nominal money balances at the end of any even period.

# 4 Decentralized Exchange

There will be two kinds of meetings that can occur between buyers and sellers during an odd period $t$. In any bilateral meeting in an odd period, the buyer and seller will ultimately be in the same location in the next even period. However, the seller knows his or her location next period while the buyer does not. Let $i$ denote the seller's type, $i = 1, 2$, where the type is just period $t+1$ location. Let $q_t^i$ denote the quantity of goods provided by a type $i$ seller to the buyer, in exchange for $d_t^i$ units of money. In a meeting between a buyer and a seller, let the buyer have $m$ units of money, and assume that he or she makes a take-it-or-leave-it offer to the seller. The seller's type is private information to the buyer, and this information is critical, as revealing it would tell the buyer how the seller values the money that the buyer offers in exchange for goods. The seller's type is also the buyer's type, so the seller's type also reveals how the buyer will value the money exchanged with the seller, ex post.

The problem that the buyer faces when meeting a seller is much like the problem of a monopolist selling goods to heterogeneous buyers whose types are private information, as captured for example in the adverse selection model of Maskin and Riley (1984). A key difference in this problem, however, is that the money balances held by the buyer potentially constrain the array of contracts that can be offered to the seller (see Ennis 2007).

Now, consider the problem faced by a buyer. In general, this buyer will offer a choice of two contracts to the seller, $(q_t^1, d_t^1)$ and $(q_t^2, d_t^2)$, intended respectively for sellers of types 1 and 2. The surplus received by the buyer from an accepted contract by a type $i$ seller is $u(q_t^i) - \beta\phi_{t+1}^i d_t^i$, given (2). Buyer $i$ then chooses the two contracts to maximize his or her expected surplus

$$\rho[u(q_t^1) - \beta\phi_{t+1}^1 d_t^1] + (1-\rho)[u(q_t^2) - \beta\phi_{t+1}^2 d_t^2]. \tag{4}$$

Each contract must be *individually rational* for each type of seller, i.e. the seller receives nonnegative surplus, or

$$-q_t^i + \beta\phi_{t+1}^i d_t^i \geq 0, \text{ for } i = 1, 2, \tag{5}$$

and each contract must be *incentive compatible* for each type of seller, or

$$-q_t^i + \beta\phi_{t+1}^i d_t^i \geq -q_t^j + \beta\phi_{t+1}^i d_t^j, \text{ for } i = 1, 2 \text{ and } j \neq i. \tag{6}$$

Further, the quantities of money that can be offered in exchange to each type of seller cannot exceed $m$, that is the *cash constraints*

$$d_t^i \leq m, \text{ for } i = 1, 2, \tag{7}$$

must hold.

Now, conjecture that

$$\phi_{t+1}^1 > \phi_{t+1}^2, \tag{8}$$

which we will later show holds in equilibrium. We can then characterize the optimal contracts offered by a buyer with the following lemmas.

**Lemma 1** *The optimal contract offered by a buyer to a type* 2 *seller yields zero surplus to the seller. That is, the individual rationality constraint holds with equality for the type* 2 *seller, or*

$$-q_t^2 + \beta\phi_{t+1}^2 d_t^2 = 0. \tag{9}$$

**Proof.** Suppose $-q_t^2 + \beta\phi_{t+1}^2 d_t^2 > 0$ at the optimum. Then, from (6) and (8), we have

$$\beta\phi_{t+1}^1 d_t^1 - q_t^1 \geq \beta\phi_{t+1}^1 d_t^2 - q_t^2 > \beta\phi_{t+1}^2 d_t^2 - q_t^2 > 0$$

so that the optimal contracts offered by the buyer to each seller give both sellers strictly positive surplus. This implies that both $d_t^1$ and $d_t^2$ can be reduced, holding constant $q_t^i$, $i = 1, 2$, in such a way that constraints (5)-(7) continue to hold, while increasing the value of the objective function in (4). Thus the contracts are not optimal, a contradiction. ■

**Lemma 2** *The incentive constraint for the type* 1 *seller binds at the optimum. That is,*

$$-q_t^1 + \beta\phi_{t+1}^1 d_t^1 = -q_t^2 + \beta\phi_{t+1}^1 d_t^2. \tag{10}$$

**Proof.** Suppose $-q_t^1 + \beta\phi_{t+1}^1 d_t^1 > -q_t^2 + \beta\phi_{t+1}^1 d_t^2$ at the optimum. Then, given (8), we have

$$\beta\phi_{t+1}^1 d_t^1 - q_t^1 > 0,$$

which implies that $d_t^1$ can be reduced in such a way that the constraints (5)-(7) continue to hold, while increasing the value of the objective function in (4). Thus, the contracts are not optimal, a contradiction. ■

**Lemma 3** *The optimal contract offered to the type* 1 *seller gives the seller strictly positive surplus. That is, the individual rationality constraint for the type* 1 *seller holds as a strict inequality, or*

$$-q_t^1 + \beta\phi_{t+1}^1 d_t^1 > 0. \tag{11}$$

**Proof.** From (10), (8), and (9) we get

$$-q_t^1 + \beta\phi_{t+1}^1 d_t^1 = -q_t^2 + \beta\phi_{t+1}^1 d_t^2 > -q_t^2 + \beta\phi_{t+1}^2 d_t^2 = 0.$$

■

**Lemma 4** *At the optimum, the type* 1 *seller supplies more goods and receives more money in exchange than does the type* 2 *seller. That is,* $q_t^1 \geq q_t^2$ *and* $d_t^1 \geq d_t^2$ *at the optimum, and* $q_t^1 > q_t^2$ *if and only if* $d_t^1 > d_t^2$.

**Proof.** Adding the two incentive constraints, i.e. constraint (6) for $(i, j) = (1, 2), (2, 1)$, we obtain

$$\beta(\phi_{t+1}^1 - \phi_{t+1}^2)(d_t^1 - d_t^2) \geq \beta(\phi_{t+1}^1 - \phi_{t+1}^2)(d_t^2 - d_t^1),$$

which, given (8), implies $d_t^1 \geq d_t^2$. Then, it is immediate from equation (10) that $q_t^1 \geq q_t^2$, and that $q_t^1 > q_t^2$ if and only if $d_t^1 > d_t^2$. ■

Thus, in spite of the cash constraints (7) that make this problem different from standard adverse selection problems in the literature, from lemmas 1-4 the solution will have some standard properties. The type 2 seller, who has a low value of money in the following period, receives zero surplus from the contract offered by the buyer, while the type 1 seller, who has a high value of money, receives strictly positive surplus. The incentive constraint binds for the type 1 seller, and larger quantities are exchanged between the buyer and a type 1 seller than between the buyer and a type 2 seller. These features allow us to solve the optimal contracting problem (4) subject to (5)-(7) in a more straightforward way. In particular, substitute in the objective function in (4) and in the cash constraints (7) for $d_t^1$ and $d_t^2$ using (9) and (10), and then solve the problem as

$$\max_{q_t^1, q_t^2} \rho \left[ u(q_t^1) - q_t^1 - \left( \frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right) q_t^2 \right] + (1 - \rho) \left[ u(q_t^2) - q_t^2 \right] \tag{12}$$

subject to the cash constraints

$$q_t^1 + q_t^2 \left( \frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right) \le \beta \phi_{t+1}^1 m \tag{13}$$

$$q_t^2 \le \beta \phi_{t+1}^2 m. \tag{14}$$

From the proof of Lemma 3, since we have imposed (9) and (10), therefore both individual rationality constraints hold, and we need only check that the second incentive constraint, (6) for $(i, j) = (2, 1)$, holds. In turn, from the proof of Lemma 4, we then only need to check that the solution has the property $q_t^1 \ge q_t^2$.

## 4.1 Case 1: Cash Constraints Bind for Both Contracts

In this case the two contracts that the buyer offers the seller are both constrained by the quantity of money $m$ that the buyer possesses. That is, (13) and (14) both hold with equality. Solving for $q_t^1$ and $q_t^2$ from (13) and (14) we obtain

$$q_t^1 = q_t^2 = \beta \phi_{t+1}^2 m, \tag{15}$$

and so, since the buyer gives up all his or her money balances irrespective of the seller's type, the payoff to the buyer as a function of $m$ is

$$\psi_t^1(m) = u(\beta \phi_{t+1}^2 m). \tag{16}$$

Thus, in this case the buyer is constrained to offering the same contract to each type of seller, and the type 1 seller who values money highly extracts some surplus from the buyer.

In Figure 2, we show the equilibrium contract in Case 1. Note that both equilibrium contracts involve a distortion from full-information quantities. In this case, the buyer has sufficiently low money balances that it is inefficient for him or her to induce the seller to reveal his or her type.

## 4.2 Case 2: Cash Constraint Binds Only for the Type 1 Seller

Recall from Lemma 4 that $d_t^1 \ge d_t^2$ at the optimum, so if one cash constraint binds, it must be the one for the type 1 seller. Thus, substituting for $q_t^1$ in (12) using (13) with equality, in case 2 we can write the buyer's optimization problem as

$$\max_{q_t^2} \rho \left\{ u \left[ -q_t^2 \left( \frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right) + \beta \phi_{t+1}^1 m \right] - \beta \phi_{t+1}^1 m \right\} + (1 - \rho) \left[ u(q_t^2) - q_t^2 \right] \tag{17}$$

subject to (14). The first-order condition for an unconstrained optimum is then

$$-\rho\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right)u'\left[-q^2_t\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right)+\beta\phi^1_{t+1}m\right]+(1-\rho)\left[u'(q^2_t)-1\right]=0. \tag{18}$$

Now, let $\varphi(q^2_t, m)$ denote the function on the left-hand side of (18).

**Proposition 5** *There is a unique* $q^*_t(m)$ *that solves* $\varphi(q^*_t(m), m) = 0$, *with* $0 < q^*_t(m) < (\beta\phi^1_{t+1}m)/\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right)$.

**Proof.** Nonnegativity of consumption for the buyer implies that

$$0 \leq q^2_t \leq (\beta\phi^1_{t+1}m)/\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right).$$

Given (8), and the strict concavity of $u(\cdot)$, $\varphi(q^2_t, m)$ is strictly decreasing in $q^2_t$ on $\left(0, (\beta\phi^1_{t+1}m)/\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right)\right)$ for fixed $m > 0$. Further, $\lim_{q\to 0}\varphi(q, m) = \infty$, and $\lim_{q\to(\beta\phi^1_{t+1}m)/\left(\frac{\phi^1_{t+1}}{\phi^2_{t+1}}-1\right)}\varphi(q, m) = -\infty$. ■

**Proposition 6** *The solution* $q^*_t(m)$ *satisfies the cash constraint (14) if and only if* $\varphi(\beta\phi^2_{t+1}m, m) \leq 0$.

**Proof.** Since $\varphi(q^2_t, m)$ is strictly decreasing in $q^2_t$ and $\varphi\left[q^*_t(m), m\right] = 0$, therefore $q^*_t(m) \leq \beta\phi^2_{t+1}m$ if and only if $\varphi(\beta\phi^2_{t+1}m, m) \leq 0$. ■

Further, since at the case 2 optimum the quantity of money exchanged with the type 2 seller cannot exceed the quantity exchanged with the type 1 seller, from (10) we must have $q^1_t \geq q^2_t$, and so the incentive constraint for the type 2 seller is satisfied.

This last proposition gives us a necessary restriction on $m$ for the optimum to have case 2 characteristics. That is, from (18), $\varphi(\beta\phi^2_{t+1}m, m) \leq 0$ gives

$$\left(1-\rho\frac{\phi^1_{t+1}}{\phi^2_{t+1}}\right)u'(\beta\phi^2_{t+1}m)-(1-\rho)\leq 0. \tag{19}$$

Now, assume for now (we will later establish conditions which guarantee that this holds) that

$$1-\rho\frac{\phi^1_{t+1}}{\phi^2_{t+1}} > 0, \tag{20}$$

and let $\omega(m)$ denote the function on the left-hand side of inequality (19). Note that $\omega(m)$ is strictly decreasing and continuous in $m$ with $\omega(0) = \infty$ and $\omega(m) < 0$ for $m$ sufficiently large. Therefore, there is some $m_1 > 0$ such that $\omega(m_1) = 0$,

$\omega(m) < 0$ for $m > m_1$ and $\omega(m) > 0$ for $m < m_1$. Therefore, if the optimum is case 2, then it is necessary that $m \geq m_t^1$, where $m_t^1$ is the solution to

$$\left(1 - \rho \frac{\phi_{t+1}^1}{\phi_{t+1}^2}\right) u'(\beta \phi_{t+1}^2 m_t^1) - (1-\rho) = 0. \tag{21}$$

Finally, since when we have a case 2 optimum, the buyer gives up all of his or her cash balances to a type 1 seller and only some of his or her cash balances to a type 2 seller, the expected payoff to the buyer as a function of $m$ is

$$\begin{aligned} \psi_t^2(m) \quad = \quad & \rho u \left[ -q_t^*(m) \left( \frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right) + \beta \phi_{t+1}^1 m \right] + \\ & + (1-\rho) \left[ u\left[q_t^*(m)\right] + \beta \phi_{t+1}^2 \left( m - \frac{q_t^*(m)}{\beta \phi_{t+1}^2} \right) \right]. \end{aligned} \tag{22}$$

We illustrate the equilibrium contracts in Figure 3. Here, note that the binding cash constraint implies that the contracts for both types are distorted from what would be achieved with full information. Relative to Case 1, the buyer has enough cash that he or she optimizes by inducing self-selection by the seller, but has insufficient cash to offer a non-distorted contract to the type 1 seller.

### 4.3 Case 3: Neither Cash Constraint Binds

In this case $q_t^1$ and $q_t^2$ are chosen by the buyer to solve (12) ignoring the cash constraints. The first-order conditions characterizing an optimum are

$$u'(q_t^1) = 1, \tag{23}$$

and

$$u'(q_t^2) = 1 + \frac{\rho}{1-\rho} \left( \frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1 \right). \tag{24}$$

Now, let $\bar{q}_t^1$ and $\bar{q}_t^2$ denote the solutions to equations (23) and (24), respectively. First, notice that $\bar{q}_t^1 = q^*$. Second, note that (8) implies that $q^* > \bar{q}_t^2$, which implies that the incentive compatibility constraint for the type 2 seller is satisfied. Further, note that $q^*$ would be the quantity traded in a full information contract between the buyer and both types of sellers, unconstrained by the buyer's cash holdings. As well, given (8) $\bar{q}_t^2$ is smaller than the quantity traded with a full information contract between the buyer and a type 2 seller, again unconstrained by the buyer's cash holdings. This is a standard feature of adverse selection models with two types, whereby the type 2 contract is distorted from what it would be with full information, so as to induce the type 1 seller to self-select.

The next step is to establish conditions on $m$ that guarantee that there is a case 3 optimum. That is, we want $m$ to be sufficiently large that neither cash constraint binds. Since $q^* > \bar{q}_t^2$, a larger quantity of cash is traded in the type

1 contract, so if the cash constraint does not bind for the type 1 contract it will not bind for the other contract. Therefore, neither cash constraint binds if and only if, from (13),

$$m \geq \frac{q^*}{\beta\phi_{t+1}^1} + \bar{q}_t^2 \left( \frac{1}{\beta\phi_{t+1}^2} - \frac{1}{\beta\phi_{t+1}^1} \right), \tag{25}$$

and we let $m_t^2$ denote the quantity on the right-hand side of (25).

The payoff to the buyer if there is a case 3 optimum is

$$\begin{aligned} \psi_t^3(m) &= \rho \left\{ u\left(q^*\right) + \beta\phi_{t+1}^1 \left[ m - \frac{q^*}{\beta\phi_{t+1}^1} - \bar{q}_t^2 \left( \frac{1}{\beta\phi_{t+1}^2} - \frac{1}{\beta\phi_{t+1}^1} \right) \right] \right\} + \\ & +(1-\rho) \left[ u(\bar{q}_t^2) + \beta\phi_{t+1}^2 \left( m - \frac{\bar{q}_t^2}{\beta\phi_{t+1}^2} \right) \right]. \end{aligned} \tag{26}$$

In Figure 4, we show the equilibrium contracts in Case 3. Here, as cash constraints do not bind, the type 1 seller receives a contract that is not distorted, but the type 2 contract is distorted to induce self-selection, just as in Maskin and Riley (1984). In Figure 5, we show how contracts differ across the three cases. Note that, as the money held by the buyer declines, the surplus received by the type 1 seller falls, and the distortion in each contract rises.[1]

## 4.4 Odd-Period Value Functions

Now that we know the payoffs to the buyer as a function of the buyer's cash balances $m$, and the constraints on $m$ that are necessary to obtain the cases 1-3 above, we can proceed to construct the value functions $V_t^i(m)$, for $i = 1, 2$. Recall that $V_t^i(m)$ gives the value of money at the beginning of period $t$ (before learning buyer/seller status) of money balances $m$ to an agent who resided in location $i$ in period $t-1$, where $t$ is an odd period.

It is straightforward to show that, given (8), $m_t^1 < m_t^2$. Then, since a necessary condition for a case 2 optimum is that $m \geq m_t^1$, and a necessary condition for a case 3 optimum is $m \geq m_t^2$, we will have a case 1 optimum when $0 \leq m \leq m_t^1$, a case 2 optimum when $m_t^1 \leq m \leq m_t^2$, and a case 3 optimum when $m \geq m_t^2$. Above, we calculated the payoffs to a buyer as a function of $m$ in the three different cases. For a seller's payoff, note that the seller does not give up any money balances no matter who he or she meets in the decentralized market, and the surplus received by the seller is independent of his or her money holdings. Therefore, we can write the odd-period value function as

$$V_t^i(m) = \alpha_i v_t(m) + (1-\alpha_i) \left\{ \beta m [\rho\phi_{t+1}^1 + (1-\rho)\phi_{t+1}^2] + \sigma \right\}, \tag{27}$$

[1] Let $S_t^1(m)$ denote the surplus received by a type 1 seller. It follows that $S_t^1(m) = \beta\left(\phi_{t+1}^1 - \phi_{t+1}^2\right)m$, for $0 \leq m \leq m_t^1$, $S_t^1(m) = q_t^*(m)\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right)$, for $m_t^1 \leq m \leq m_t^2$, and $S_t^1(m) = \bar{q}_t^2\left(\frac{\phi_{t+1}^1}{\phi_{t+1}^2} - 1\right)$, for $m \geq m_t^2$.

where

$$v_t(m) = \sum_{i=1}^{3} I_t^i(m)\psi_t^i(m). \tag{28}$$

In (27), $\sigma$ is a constant, and in (28) the indicator functions $I_t^i(m)$, for $i = 1, 2, 3$, are defined by

$$I_t^1(m) = 1 \text{ if } 0 \le m \le m_t^1;\ I_t^1(m) = 0 \text{ otherwise.}$$

$$I_t^2(m) = 1 \text{ if } m_t^1 \le m \le m_t^2;\ I_t^2(m) = 0 \text{ otherwise.}$$

$$I_t^3(m) = 1 \text{ if } m \ge m_t^2;\ I_t^3(m) = 0 \text{ otherwise.}$$

**Proposition 7** *The function $v_t(m)$ is continuously differentiable for $m \ge 0$, concave for $m \ge 0$, and strictly concave for $0 \le m < m_t^2$.*

**Proof.** Note that $v_t(\cdot)$ is clearly continuously differentiable at every point $m \ge 0$, except possibly at the critical points $m_t^1, m_t^2$. It remains to show that $v_t(\cdot)$ is continuously differentiable at these points. Observe that

$$\frac{d\psi_t^1}{dm} \to \beta\phi_{t+1}^2 u'\left(\beta\phi_{t+1}^2 m_t^1\right)$$

as $m \to m_t^1$ from below. On the other hand, using (18) and (21), we find that

$$\frac{d\psi_t^2}{dm} \to \beta\phi_{t+1}^2 u'\left(\beta\phi_{t+1}^2 m_t^1\right)$$

as $m \to m_t^1$ from above. Therefore, we conclude that $v_t(\cdot)$ is continuously differentiable at $m_t^1$. Consider now the critical point $m_t^2$. As $m \to m_t^2$ from below, we have

$$\frac{d\psi_t^2}{dm} \to \beta[\rho\phi_{t+1}^1 + (1-\rho)\phi_{t+1}^2]$$

where we have used (23). For any $m > m_t^2$, it follows that

$$\frac{d\psi_t^3(m)}{dm} = \beta[\rho\phi_{t+1}^1 + (1-\rho)\phi_{t+1}^2]$$

so that we conclude that $v_t(\cdot)$ is continuously differentiable at $m_t^2$.

To show that $v_t(\cdot)$ is concave, notice that $v_t''(m) < 0$ for any $m \in \left(0, m_t^1\right) \cup \left(m_t^1, m_t^2\right)$; $v_t''(m) = 0$ for any $m > m_t^2$; and $v_t'(\cdot)$ is continuous. This implies that $v_t(\cdot)$ is concave for $m \ge 0$ and strictly concave for $0 \le m < m_t^2$. ■

We illustrate the value function in Figure 6. We will later show that $(0, m_{t+1}^2]$ is the relevant region for the agent's optimal choice of money balances in an even period $t$. The last proposition, together with this observation, then implies that, from (3), and similarly to Lagos and Wright (2005), it is optimal for each agent in a given location in an even period to hold the same quantity of money at the end of the period.

# 5 Discussion

In the model, the higher demand for money in centralized market 1 will tend to make the price of money higher in market 1 than in market 2. This difference in prices across markets matters, as economic agents are asymmetrically informed when trading in decentralized markets, concerning which centralized market they will participate in during the following period. To overcome the fundamental friction in the model, in even periods money needs to flow to the markets where it is valued more from the markets where it is valued less. In the following sections we will show how this can be achieved through central bank intervention, or through private financial trade across markets, in a manner akin to trading on the federal funds market.

# 6 Central Bank Intervention in Both Centralized Markets

Suppose that the central bank can make lump-sum transfers, but that these transfers are constrained to be the same in each location in a given period, as well as being identical across agents in a given location. This constraint could arise if, for example, the transfers are made electronically, an agent's location is private information, and the central bank has no memory of an agent's past transfers. Further, for simplicity assume that the money stock grows at a constant rate from one even period to the next. That is, let $M_t$ denote the aggregate money stock during an even period $t$, where

$$M_{t+2} = \mu^2 M_t,$$

for $t = 0, 2, 4, ...$, with $M_0$ normalized to unity and $\mu > 0$. Note that there are no money transfers in odd periods while agents are engaged in decentralized exchange. The money transfer that each agent receives in an even period $t$ is then

$$\tau_t^1 = \tau_t^2 = (\mu^2 - 1)M_{t-2}.$$

Recall that, by construction, the beginning-of-period per capita money stock is the same in each location. Since the central bank is constrained to make the same lump-sum transfer in each location, it follows that the end-of-period per capita money stock in each location is also the same. Given that there is a continuum of agents with measure one, it follows that the aggregate money stock in an even period $t$ equals the per capita money stock in each location.

Now, confine attention to stationary equilibria having the property that $\phi_t^i = \frac{\phi^i}{\mu^t}$, for $i = 1, 2$, where $\phi^i$ is a constant for $i = 1, 2$. From (3) and (27), the following first-order conditions must be satisfied for each $t = 0, 2, 4, ...$,

$$\frac{\phi^i}{\mu^t} = \beta \left\{ \alpha_i v'_{t+1}(\tilde{m}_{t+1}^i) + \frac{(1-\alpha_i)\beta[\rho\phi^1 + (1-\rho)\phi^2]}{\mu^{t+2}} \right\}, \text{ for } i = 1, 2, \tag{29}$$

where $\tilde{m}^i_{t+1}$ is the quantity of money acquired by the agent in period $t$ and available to spend in decentralized trade in period $t+1$. Then, imposing the equilibrium condition that $\tilde{m}^i_{t+1} = M_t = \mu^t$ for $i = 1, 2$, and rearranging, we get

$$1 = \frac{\alpha_i \beta \mu^t v'_{t+1}(\mu^t)}{\phi^i} + (1-\alpha_i)\frac{\beta^2}{\mu^2}\frac{\left[\rho\phi^1 + (1-\rho)\phi^2\right]}{\phi^i}, \text{ for } i = 1, 2. \tag{30}$$

**Proposition 8** *If $\mu > \beta$, then $\phi^1 > \phi^2$ in a stationary equilibrium.*

**Proof.** If $\mu > \beta$, then we must have $\tilde{m}^i_{t+1} \leq m^2_{t+1}$ for each $i = 1, 2$, with at least one strict inequality. To see this, note that, as $m \to m^2_{t+1}$ from below,

$$-\phi^i_t + \beta \frac{dV^i_{t+1}}{dm} \to \frac{1}{\mu^t}\left\{-\phi^i + \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right]\right\}$$

where we have used (23). When $\mu > \beta$, any stationary monetary equilibrium must satisfy

$$\phi^i \geq \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right]$$

for each $i = 1, 2$. If $\phi^1 \neq \phi^2$, then there must be at least one strict inequality. Therefore, the optimal choice of money balances in location $i$ in an even period $t$ is such that $\tilde{m}^i_{t+1} \leq m^2_{t+1}$ for each $i = 1, 2$, with at least one strict inequality.

Notice that $v'_{t+1}(\cdot)$ is a decreasing function and that

$$v'_{t+1}(m) \geq \frac{\beta}{\mu^{t+2}}\left[\rho\phi^1 + (1-\rho)\phi^2\right]$$

for all $m \geq 0$. In fact, it holds as a strict inequality when $m < m^2_{t+1}$. In a stationary equilibrium, we have $\tilde{m}^1_{t+1} = \tilde{m}^2_{t+1} = \mu^t$, and

$$\beta\mu^t v'_{t+1}(\mu^t) > \frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right].$$

Since $\alpha_1 > \alpha_2$, it follows that

$$\begin{aligned} & \alpha_1\beta\mu^t v'_{t+1}(\mu^t) + (1-\alpha_1)\frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right] \\ > \quad & \alpha_2\beta\mu^t v'_{t+1}(\mu^t) + (1-\alpha_2)\frac{\beta^2}{\mu^2}\left[\rho\phi^1 + (1-\rho)\phi^2\right], \end{aligned}$$

so $\phi^1 > \phi^2$ in a stationary equilibrium as claimed. ■

If $\mu > \beta$, this implies that some cash constraint must bind in equilibrium, and that a buyer faces a higher marginal payoff to holding money than does a seller in the decentralized market. Since an agent in location 1 in an even period has a higher probability of being a buyer in the next decentralized market, this agent then must have a higher expected marginal payoff to holding money in an even period. Since the quantities of money per capita are identical in the two locations in an even period, money must have a higher value in location 1 than in location 2 in equilibrium.

Thus, when the rate of money growth is larger than the discount rate, prices are different in the two locations, and we know that this induces a private information friction in monetary exchange in this model. That is, there is a friction here, in addition to what would occur with full information, due to the fact that a seller with a high value of money can extract some surplus from the buyer because the buyer needs to induce self-selection.

**Proposition 9** *$\mu = \beta$ yields an optimal equilibrium allocation.*

**Proof.** As $\mu \to \beta$ from above, it follows that

$$\beta \mu^t v'_{t+1}(\mu^t) \to \rho\phi^1 + (1-\rho)\phi^2.$$

$>$From (30), it follows that

$$\phi^i = \rho\phi^1 + (1-\rho)\phi^2$$

for each $i = 1, 2$, which holds if and only if $\phi^1 = \phi^2 = \phi$. Then, as $\mu \to \beta$ from above, we have

$$\frac{\beta \mu^t v'_{t+1}(\mu^t)}{\phi} \to 1 = u'(q^*)$$

so that agents in both locations acquire enough money to get $q^*$ in the next decentralized market if they are buyers. ■

Under a Friedman rule, all cash constraints are relaxed, and there is a stationary equilibrium where $\phi^1 = \phi^2$ so that prices are equalized in the two locations in even periods. The private information friction is eliminated and the economy collapses to essentially the same allocation studied by Lagos and Wright (2005), for the special case where buyers have all the bargaining power. The efficient quantity of output is produced and consumed in every bilateral match in the decentralized market. Therefore, with the ability to intervene in all centralized markets, the central bank is able to effectively saturate centralized markets with real money balances, relax cash constraints, and accommodate differences in money demand across markets. Thus, prices are equated across markets at the optimum. However, when monetary policy departs from the Friedman rule, not only does the standard intertemporal distortion come into play, whereby agents economize too much relative to the optimum on money balances and consume too little, but there is a difference in prices across markets which induces a private information friction.

# 7 Financial Market Trade Between Locations

We have assumed that, in even periods, there is no trade between agents in location 1 and those in location 2. Here, we will continue to assume that neither goods nor people can move across the two locations. However, we will permit a bond market in even periods where agents in the two locations can exchange outside money (say, in electronic form) for claims to money in the next even period. This of course requires that a bond issuer in period $t$ can be found in period $t+2$ and that the financial claim can be enforced.

Assume a market in an even period $t$ for two-period bonds, each of which sells for one unit of money and is a claim to $R_{t+2}$ units of money in period $t+2$. We can then rewrite the budget constraint (1) of an agent in location $i$ in an even period as

$$c_t + \phi_t^i \tilde{m}_{t+1}^i + \phi_t^i b_{t+2}^i = l_t^i + \phi_t^i m + \phi_t^i R_t b + \phi_t^i \tau_t^i, \tag{31}$$

where $b$ denotes the quantity of bonds acquired by the agent in period $t-2$ that mature in period $t$. Given quasilinear utility, equilibrium requires that each agent in each location be indifferent about the bond holdings in any even period $t$, or

$$\phi_t^i = \beta^2 R_{t+2} \left[\rho \phi_{t+2}^1 + (1-\rho)\phi_{t+2}^2\right] \text{ for } i = 1, 2. \tag{32}$$

But these two conditions clearly imply that $\phi_t^1 = \phi_t^2$ in equilibrium, so that prices are equalized across the two locations. This economy then collapses to a basic Lagos-Wright structure with take-it-or-leave-it offers by buyers, and with no private information friction.

Now, if the aggregate money stock grows at a constant rate in a stationary equilibrium, as in the previous section, then (29) must hold, but now $\phi^1 = \phi^2 = \phi$ in equilibrium, and the stocks of money in each location are endogenous. That is, in a stationary equilibrium, the per capita quantity of money in location $i$ is $M^i \mu^t$ in an even period $t$, where from (30) and the equilibrium condition $\rho M^1 + (1-\rho)M^2 = 1$, we obtain

$$\alpha_1 \beta \mu^t v_{t+1}'(\mu^t M^1) + (1-\alpha_1)\frac{\beta^2}{\mu^2}\phi = \alpha_2 \beta \mu^t v_{t+1}'\left(\mu^t \frac{1-\rho M^1}{1-\rho}\right) + (1-\alpha_2)\frac{\beta^2}{\mu^2}\phi, \tag{33}$$

which solves for $M^1$, giving us the equilibrium distribution of money balances between locations 1 and 2.

**Proposition 10** *If $\mu > \beta$, then $M^1 > M^2$ in equilibrium, and the equilibrium allocation is inefficient.*

**Proof.** If $\mu > \beta$, we have that $\tilde{m}_{t+1}^i < \mu^{t+2} q^* / (\beta\phi)$, for $t$ even, and

$$\frac{\beta \mu^t v_{t+1}'(\tilde{m}_{t+1}^i)}{\phi} > \frac{\beta^2}{\mu^2}$$

for each $i = 1, 2$. Since $\alpha_1 > \alpha_2$ and $\tilde{m}_{t+1}^i = M^i \mu^t$ for each $i = 1, 2$ in equilibrium, it follows from (33) that

$$v_{t+1}'(M^1 \mu^t) < v_{t+1}'(M^2 \mu^t)$$

Since $v'_{t+1}(\cdot)$ is strictly decreasing for $0 \leq m < \mu^{t+2}q^*/(\beta\phi)$, we have that $M^1 > M^2$. The fact that $\mu > \beta$ implies that it is not optimal for agents in each location to take enough money to the decentralized market in order to get $q^*$ if they are buyers. ■

**Proposition 11** *If $\mu = \beta$, there is an optimal equilibrium allocation.*

**Proof.** When $\mu \to \beta$ from above, we have

$$\frac{\beta\mu^t v'_{t+1}(\mu^t M^i)}{\phi} \to 1 = u'(q^*)$$

for each $i = 1, 2$. This implies that the efficient quantity is traded in each bilateral match in the decentralized market. ■

Just as in the previous section, a Friedman rule is optimal, but trading in this cross-location bond market serves to equalize prices in the two locations by moving money balances to where they would otherwise have a higher value. Thus, there is no private information friction, even when money growth is higher than the Friedman rule rate. The bond market plays a role much like the federal funds market in the United States, except that in our model we have assumed that all economic agents have access to this market. Note that, given trading on the bond market, it is irrelevant what market the central bank intervenes in. Agents could receive money transfers from the central bank in location 1, location 2, or both locations, but the actions of the central bank can have no effect on the end-of-period distribution of money balances between locations 1 and 2 in an even period.

# 8 No Inter-Location Trade, and Central Bank Intervention in Only One Location

In practice, there is financial market segmentation that may be important for the effects and conduct of monetary policy. In particular, not all economic agents are on the receiving end of central bank actions, and we can capture this in a simple way in our environment. As well, in practice not all economic agents can trade on the federal funds market or something comparable. In this section, as an example to show the effects of limited intervention by the central bank, and limited financial market participation, we will assume that there is no trade between locations during an even period, and that the central bank can intervene at only one location, through lump-sum money transfers.

Suppose, without loss of generality, that the central bank intervention is confined to location 1. Let $M_t^i$ denote the even-period $t$ per capita money stock at location $i$. Given that the central bank intervenes only at location 1, $M_t^1$ can be treated as exogenous, and we will have

$$M_{t+2}^2 = \rho M_t^1 + (1-\rho)M_t^2, \ t = 0, 2, 4, ... \tag{34}$$

Now, consider monetary policies such that $M_{t+2}^i = \mu^2 M_t^i$ for $t = 0, 2, 4, ...$, with $\frac{M_t^1}{M_t^2} = \delta$, where from (34), we have

$$\delta = \frac{\mu^2 - 1 + \rho}{\rho}. \tag{35}$$

As should be clear, equation (35) reflects the fact that the central bank cannot independently determine the money growth rate and the distribution of money balances across the two locations. Normalize $M_0^1$ to unity.

**Proposition 12** *Suppose that $\beta$ is sufficiently close to one. There exists a stationary equilibrium at the Friedman rule $\mu = \beta$ where $\phi^1 = \phi^2 = q^*$ provided that $\rho \in (1 - \beta^2, \hat{\rho}]$, where $\hat{\rho}$ solves*

$$\hat{\rho} = \left(1 - \beta^2\right) \frac{1 + \beta - \beta^2}{1 + \beta - 2\beta^2}. \tag{36}$$

**Proof.** In any stationary equilibrium, the first-order conditions for the optimal choice of money balances in each location imply

$$1 = \frac{\alpha_1 \beta \mu^t v_{t+1}'(\mu^t)}{\phi^1} + (1 - \alpha_1) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^1} \tag{37}$$

and

$$1 = \frac{\alpha_2 \beta \mu^t v_{t+1}'\left(\mu^t \frac{\rho}{\mu^2 - 1 + \rho}\right)}{\phi^2} + (1 - \alpha_2) \frac{\beta^2}{\mu^2} \frac{[\rho\phi^1 + (1-\rho)\phi^2]}{\phi^2} \tag{38}$$

for all $t = 0, 2, 4, ....$ Conjecture that $\phi^1 = \phi^2 = q^*$ is a stationary equilibrium at $\mu = \beta$. Then, (37) and (38) become

$$1 = \alpha_1 \frac{\beta^{t+1} v_{t+1}'(\beta^t)}{q^*} + 1 - \alpha_1 \tag{39}$$

and

$$1 = \alpha_2 \frac{\beta^{t+1} v_{t+1}'(\beta^t \frac{\rho}{\beta^2 - 1 + \rho})}{q^*} + 1 - \alpha_2. \tag{40}$$

Conjecture also that neither cash constraint binds. Notice that the right-hand side of (25) becomes

$$m_{t+1}^2 = \frac{q^*}{\beta \phi_{t+2}^1} = \beta^{t+1}$$

at the conjectured solution. From (26) and (27), we have that

$$v_{t+1}(m) = u(q^*) - q^* + q^* \beta^{-t-1} m$$

for all $m \geq m_{t+1}^2$, so that

$$v_{t+1}'(m) = q^* \beta^{-t-1}$$

for all $m \geq m^2_{t+1}$. Therefore, both (39) and (40) are satisfied if both $M^1_t = \beta^t$ and $M^2_t = \rho\beta^t / \left(\beta^2 - 1 + \rho\right)$ are greater than or equal to $m^2_{t+1}$, which involves a case 3 optimum. Then, we have that

$$\frac{\rho}{\beta^2 - 1 + \rho}\beta^t > \beta^t > m^2_{t+1},$$

so that the optimality conditions (39) and (40) are indeed satisfied at the conjectured solution.

It remains to check whether the budget constraints are satisfied. In any bilateral meeting in the odd-period $t+1$, it follows that $d^1_{t+1} = d^2_{t+1} = \beta^{t+1}$. This means that each seller receives $\beta^{t+1}$ units of money in exchange for $q^*$ units of the good. Money holdings and nominal transfers are

$$\tilde{m}^1_{t+1} = M^1_t = \beta^t,$$

$$\tilde{m}^2_{t+1} = M^2_t = \beta^t \frac{\rho}{\beta^2 - 1 + \rho},$$

$$\tau^1_t = M^1_t - \rho M^1_{t-2} - (1-\rho) M^2_{t-2} = \beta^t \frac{\left(\beta^2 - 1\right)}{\beta^2 - 1 + \rho},$$

and

$$\tau^2_t = 0.$$

for $t = 0, 2, 4, ....$[2] Consider an agent who leaves location 1 in the even-period $t$ with $\beta^t$ units of money and becomes a buyer in period $t+1$. He spends $\beta^{t+1}$ in the decentralized market in exchange for $q^*$. In period $t+2$, he ends up in location $i$ and works $l^i_{t+2}$ such that

$$l^i_{t+2} = q^* + \phi^i_{t+2} M^i_{t+2} - \phi^i_{t+2}\tau^i_{t+2} - \phi^i_{t+2}\beta^t (1-\beta).$$

Then, we have

$$l^1_{t+2} = l^2_{t+2} = q^* \left(1 + \frac{\rho}{\beta^2 - 1 + \rho} - \frac{1-\beta}{\beta^2}\right) > 0.$$

If the same agent were a seller in period $t+1$, his budget constraint in period $t+2$ would be

$$l^i_{t+2} = q^* + \phi^i_{t+2} M^i_{t+2} - \phi^i_{t+2}\tau^i_{t+2} - \phi^i_{t+2}\beta^t (1+\beta).$$

Then, we would have

$$l^1_{t+2} = l^2_{t+2} = q^* \left(1 + \frac{\rho}{\beta^2 - 1 + \rho} - \frac{1+\beta}{\beta^2}\right) > 0.$$

Consider now an agent who leaves location 2 in the even-period $t$ with $\rho\beta^t / \left(\beta^2 - 1 + \rho\right)$ units of money and becomes a buyer in period $t+1$. He

[2] Notice that $\rho M^1_t + (1-\rho) M^2_t$ is the per capita stock of money in each location at the *beginning* of period $t+2$.

spends $\beta^{t+1}$ in the decentralized market in exchange for $q^*$. In period $t+2$, he ends up in location $i$ and works $l^i_{t+2}$ such that

$$l^i_{t+2} = q^* + \phi^i_{t+2} M^i_{t+2} - \phi^i_{t+2} \tau^i_{t+2} - \phi^i_{t+2} \beta^t \left( \frac{\rho}{\beta^2 - 1 + \rho} - \beta \right).$$

Then, we have

$$l^1_{t+2} = l^2_{t+2} = q^* \left( \frac{\beta^2 - 1 + 2\rho}{\beta^2 - 1 + \rho} \right) - q^* \frac{1}{\beta^2} \left( \frac{\rho}{\beta^2 - 1 + \rho} - \beta \right) > 0.$$

If the same agent were a seller in period $t+1$, his budget constraint in period $t+2$ would be

$$l^i_{t+2} = q^* + \phi^i_{t+2} M^i_{t+2} - \phi^i_{t+2} \tau^i_{t+2} - \phi^i_{t+2} \beta^t \left( \frac{\rho}{\beta^2 - 1 + \rho} + \beta \right).$$

Then, we would have

$$l^1_{t+2} = l^2_{t+2} = q^* \left( \frac{\beta^2 - 1 + 2\rho}{\beta^2 - 1 + \rho} \right) - q^* \frac{1}{\beta^2} \left( \frac{\rho}{\beta^2 - 1 + \rho} + \beta \right) \geq 0.$$

■

**Proposition 13** *At the Friedman rule $\mu = \beta$, there is no stationary equilibrium with $\phi^1 \neq \phi^2$.*

**Proof.** In any stationary equilibrium with $\mu = \beta$, the optimality conditions (37) and (38) must hold for all $t = 0, 2, 4, ....$ Recall that $v_{t+1}(\cdot)$ is a concave function such that

$$v'_{t+1}(m) \geq \beta^{-t-1} \left[ \rho\phi^1 + (1-\rho)\phi^2 \right], \tag{41}$$

for all $m \geq 0$. Notice that (37) and (38), together with (41), imply that both

$$\phi^1 \geq \rho\phi^1 + (1-\rho)\phi^2$$

and

$$\phi^2 \geq \rho\phi^1 + (1-\rho)\phi^2$$

must hold in a stationary equilibrium at the Friedman rule. But these conditions are simultaneously satisfied if and only if $\phi^1 = \phi^2$. ■

These results are perhaps surprising. The fundamental friction which gives rise to the private information friction in the model arises because of limitations on the flows of money across markets. In this example, in spite of the fact that there is no financial market that permits flows of outside money across markets, and the central bank is limited to intervention in only one market, a Friedman rule achieves the equalization of prices across markets, and also eliminates the standard intertemporal distortion. Our intuition might tell us that two policy instruments are needed to correct the two distortions (the private information distortion and the intertemporal distortion), but in fact one instrument is all that is needed. At the Friedman rule, the central bank withdraws outside money from market 1 in each even period, so that the quantity of money per person is smaller in market 1 than in market 2 in each even period. Thus, since no contracts are cash-constrained at the Friedman rule, all agents from market 2, including the ones who are buyers, take more cash with them to trade in the decentralized market than what they will trade in exchange for goods. They are happy to hold this excess cash in equilibrium, as the implicit nominal interest rate is zero.

# 9 Conclusion

In the model constructed here, the key frictions are market segmentation and private information. Money demand differs across spatially separated markets implying that, in the absence of central bank intervention and financial trade between markets, there will be price dispersion across markets. Then, given asymmetric information concerning future trading opportunities, there exists a private information inefficiency that in general will be affected by monetary policy.

When there is financial market trade across markets, then this eliminates price dispersion across markets in all circumstances, and the optimal central bank policy conforms to a Friedman rule, which acts to eliminate an intertemporal distortion, as is typical in many monetary models. However, given incompleteness in private financial market participation, the central bank can still achieve an efficient allocation by implementing a Friedman rule, but in this case the Friedman rule eliminates not only the standard intertemporal distortion but also the private information friction. Perhaps surprisingly, this result holds even if the central bank's participation in markets is limited.

In this paper we have explored a mechanism by which informational frictions matter for the effects of monetary policy and for optimal policy. In contrast to Lucas (1972), this theory does not rely on imperfect information concerning aggregate shocks, but on asymmetric information at the level of individual exchange.

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Figure 1: Locational Itineraries

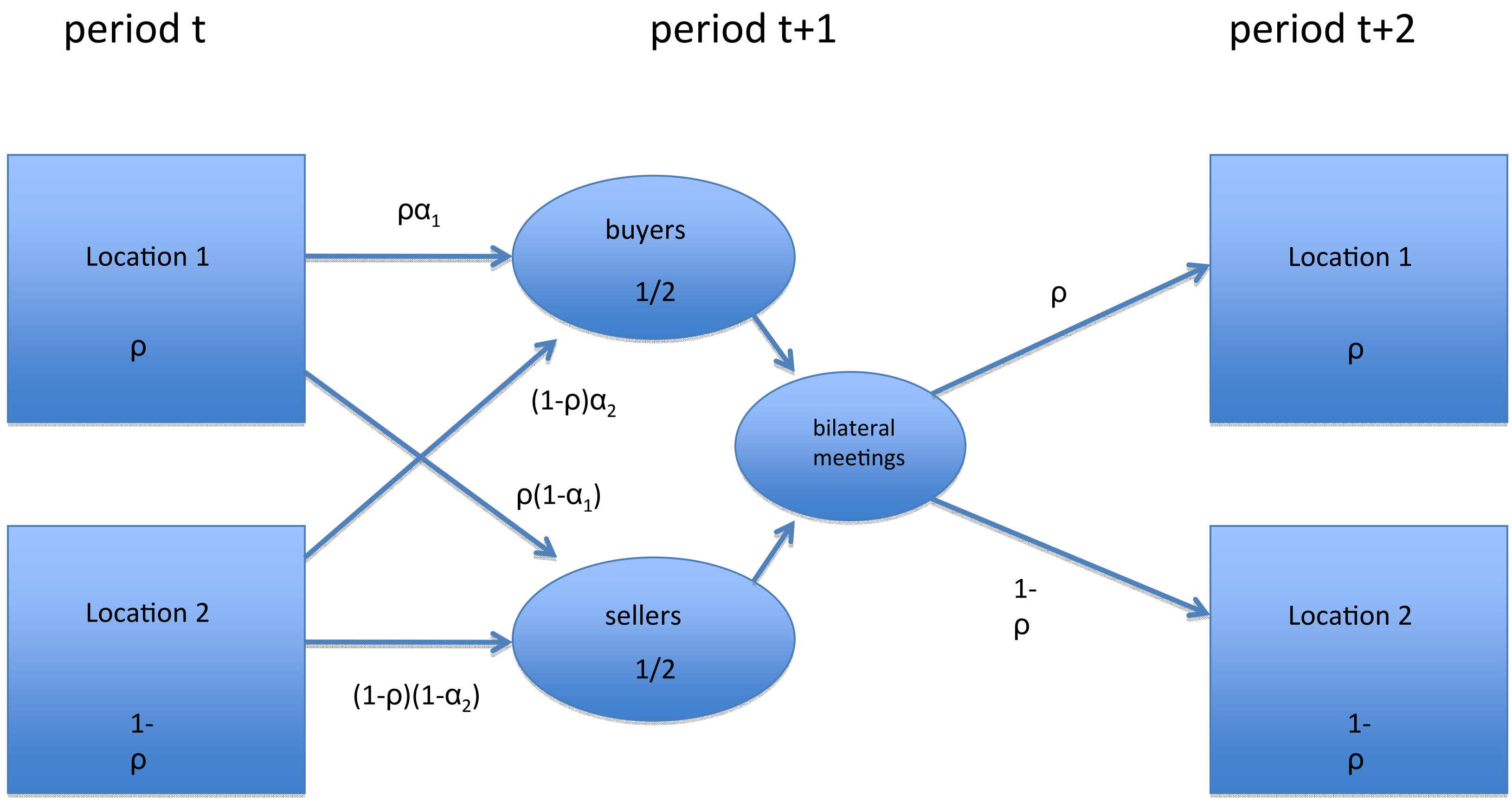

**Figure 2: Case 1**

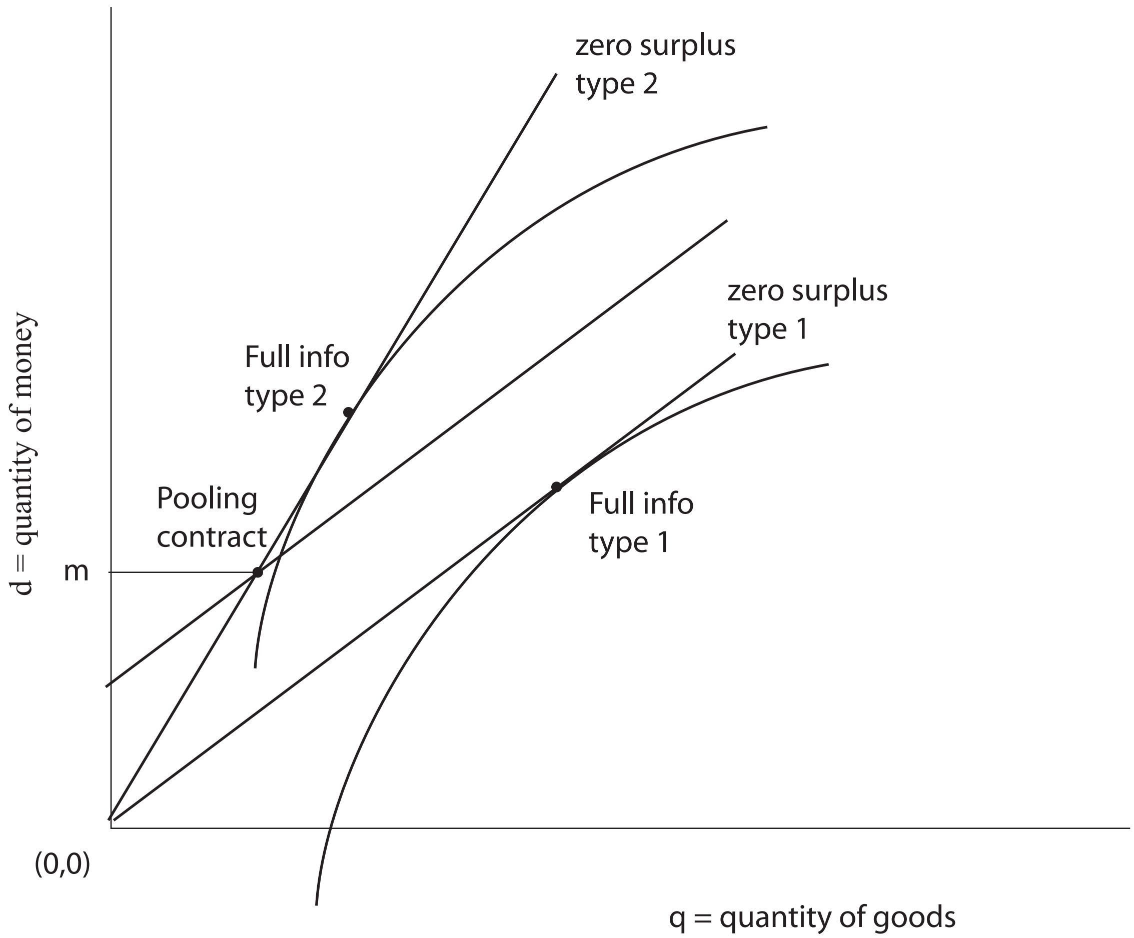

**Figure 3: Case 2**

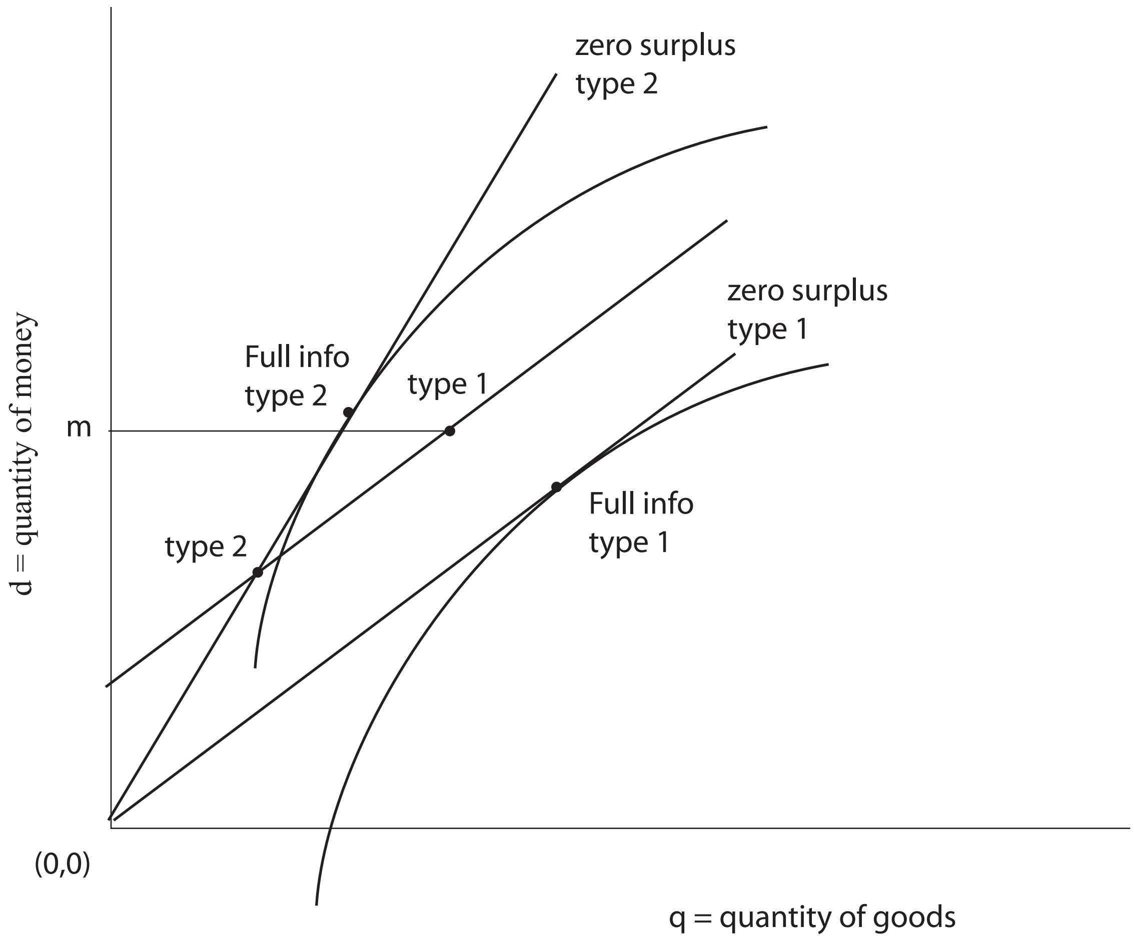

**Figure 4: Case 3**

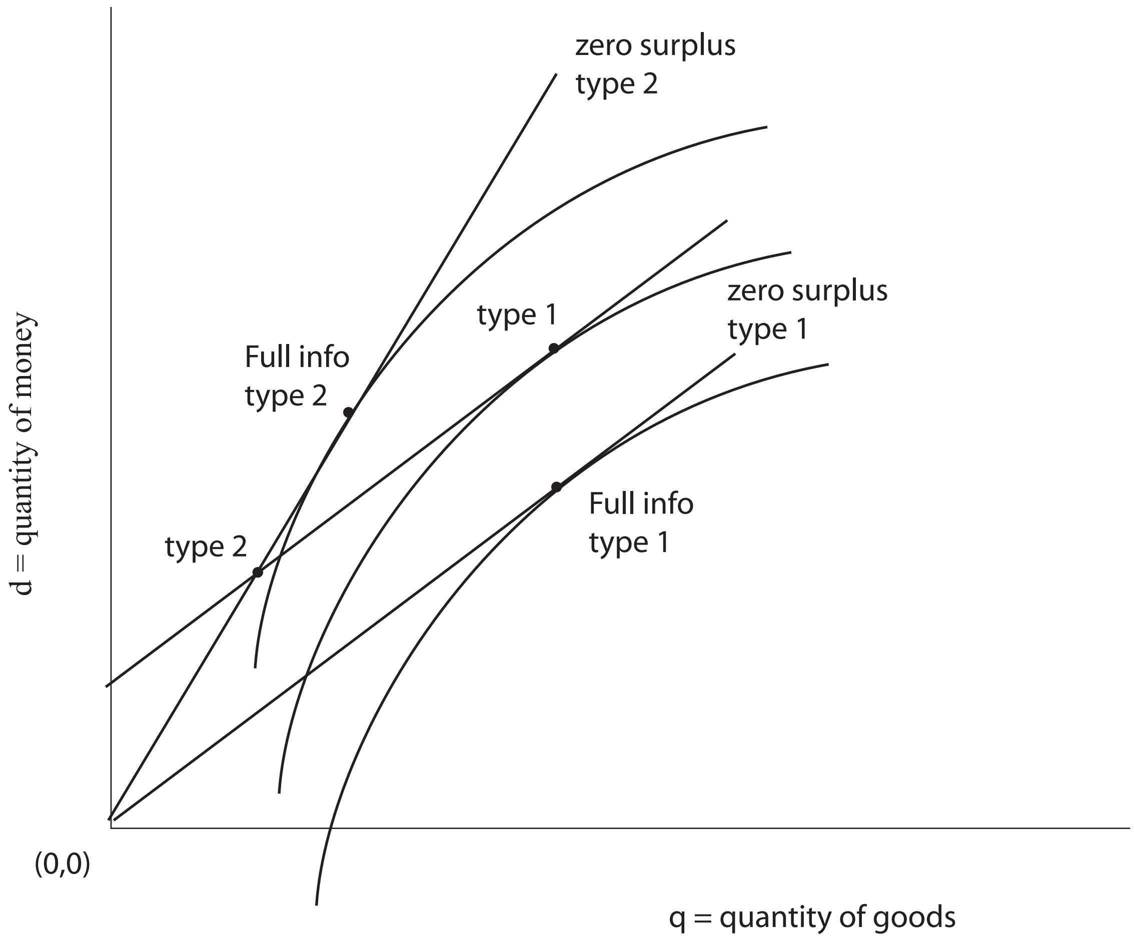

**Figure 5**

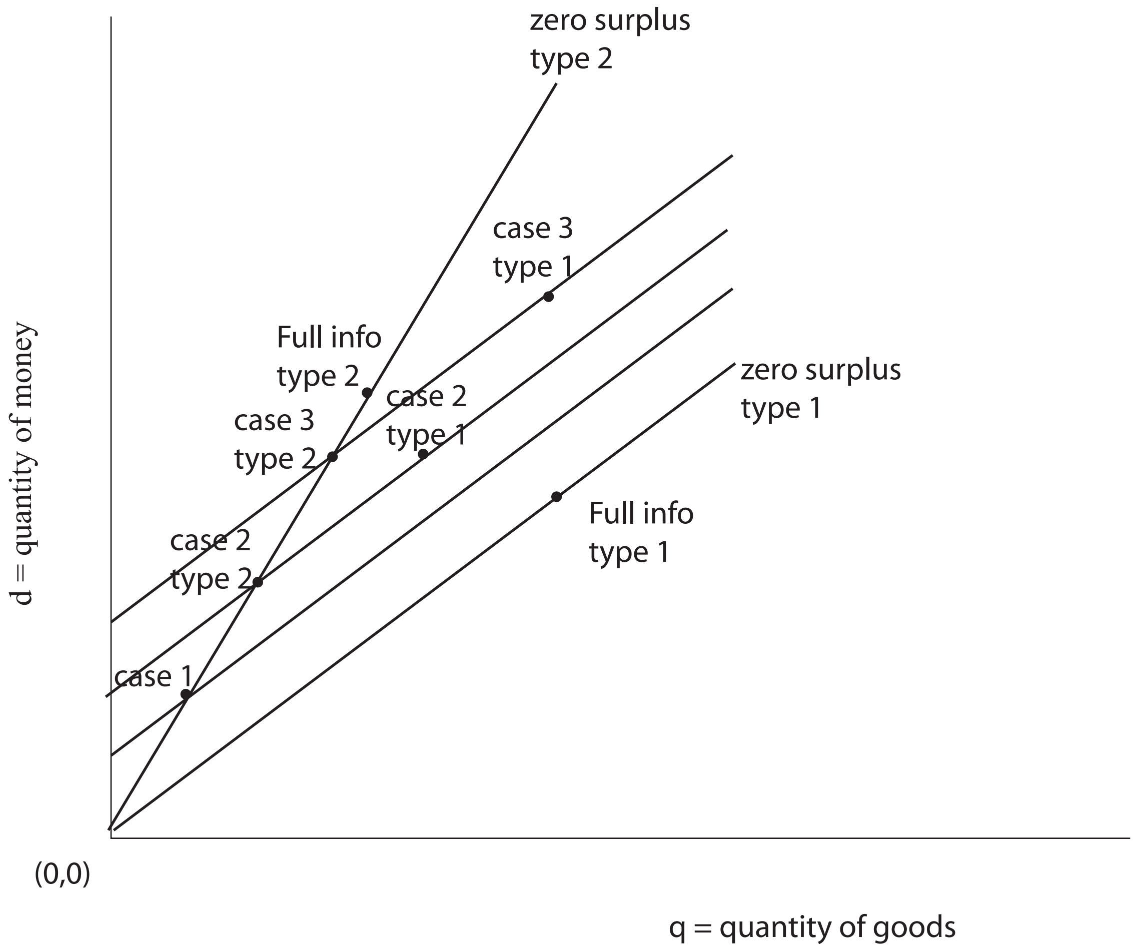
zero surplus
type 2
case 3
type 1
Full info
type 2
case 2
type 1
case 3
type 2
zero surplus
type 1
Full info
type 1
case 2
type 2
case 1
d = quantity of money
(0,0)
q = quantity of goods

**Figure 6: Value Function**