Wage Bargaining in a Multiple Application Search Model with Recall*

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February 15, 2010

Abstract

In this paper I extend the multiple application urn-ball game structure, introduced by Gautier and Moraga-Gonzalez (2004) and Albrecht, Gautier, and Vroman (2006), to an scenario where firms can, after a rejection, make additional wage offers. This expands the game structure from a one-shot set up to a sequential game. A firm, after being rejected by an applicant, can choose another applicant to make him a new wage offer. This possibility gives firms an outside option after a rejection. This increases the bargaining power of firms, implying a change in their wage offer behavior. The resulting wage distribution is hump-shaped with the density of wage offers concentrated on central values, rather than in extreme values.

Keywords: Matching, Labor Market, Multiple application, Wage distribution.

JEL classification: J31, J65, D83

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†I am indebted to my supervisor Iñigo Iturbe-Ormaetxe for his advice, comments and corrections. I thank the Spanish Ministry of Education for financial support, FPU grant AP2003-0563.

1 Introduction

The matching process is one of the most relevant features of the labor market. The matching process describes the mechanism that is required to join firms with workers. This mechanism is complex and not necessarily efficient. In fact, there is strong evidence of involuntary unemployment together with the existence of empty job vacancies. The literature mostly uses exogenous matching functions to describe this mechanism. See Petrongolo and Pissarides (2001) for an extensive survey of the matching literature.

A microfounded and intuitive way to describe the labor market, the Urn-Ball game structure, was introduced by Butters (1977), Hall (1979) and Pissarides (1979). This structure describes the matching process as a game where workers submit applications, symbolically described as balls, to job vacancies, symbolically described as urns. Each worker has a single ball that she must introduce in one of the urns, simultaneously with the rest of workers. An urn with at least one ball chooses randomly one of them, forming a match with that particular worker. A coordination problem arises when workers choose simultaneously to which vacancies they apply, since they are uniformed about the decisions of the rest of the workers. There is a positive probability that a vacancy does not receive any application remaining unfilled, while at the same time some workers are left unemployed, since they were not randomly chosen to fulfill the vacancy where they applied.

This kind of structure has been used to describe unemployment, unemployment duration, and also wage differentials and wage dispersion (Montgomery (1991), Lang (1991), Blanchard and Diamond (1994)).

Perhaps the most significant drawback in the standard urn-ball game is the fact that workers submit a single application, being then constrained to receive a unique offer that must be accepted. Lang (1991) constructs a model where workers apply to at least two firms, but the complete multiple application structure is described in Albrecht, Gautier and Vroman (2006), AVG hereafter. In a multiple application structure, workers send applications to several firms, being then possible to receive multiple offers. This makes the wage bargaining mechanism highly relevant, since firms may compete for the same worker. The bargaining mechanism proposed by AVG is a wage posting set up. Firms post the reservation wage and, in the case that more than one firm is interested in the same worker, they engage in a Bertrand competition. Bertrand competition yields a wage offer equal to the value of production. The result is a wage distribution concentrated in the two extreme values, the reservation wage and the full production value. This structure implicitly implies that firms have information about the existence of a competitor. Gautier and Moraga-Gonzalez (2005), GM-G hereafter, develop a more natural informational structure. Firms are not aware of other possible competitors, but they know that there is a positive probability that their chosen worker receives additional offers. Firms do not post wages, but vacancies, and once they choose randomly one of the received applications, they make a single take-it-or-leave-it wage offer. Workers collect offers and accept the highest one. This structure is identical to a sealed bid first-price auction with an unknown number of bidders, where all bidders value identically the worker. The result is a mixed strategy of firms. They extract their wage offer from a continuous distribution that goes from the reservation wage to an upper bound that is strictly lower than the production value. The resulting wage distribution seems more intuitive than the one obtained by AVG, with just two possible outcomes. However, the resulting wage distribution has a strictly increasing density function, with most of the density concentrated around the highest value. Halko, Kultti and Virrankoski (2008) obtain a hump-shaped wage distribution using a similar mechanism. They assume that agents make infinite applications and they can choose the direction in which the market develops. Workers can choose to send applications to firms, receiving then wage offers from them, or to be contacted by firms, making then workers a wage offer to firms. In this second case, firms are the ones who collect wage offers, choosing the worker that offers the lowest wage. The result of such game is the mentioned hump-shaped wage offer structure with a flat area in the middle.

In this paper I follow the work of GM-G. Firms will not post wages, just vacancies. Firms make take-it-or-leave-it wage offers simultaneously, without information about the offers that the agent might receive. The intention is to extend that model to a more natural scenario. In GM-G the resulting game is a one-shot game. This leads to a situation where workers have a large bargaining power with respect to firms. Workers control a given number of applications that might end up in multiple wage offers, while firms compete for a single worker. Firms do not have any outside option, even in the case that they have a large set of applications. Firms that are rejected do not find a worker and their job vacancy remains unfilled. It seems natural to think that a firm with several applications can, after a rejection, take another application from the pool and make a new offer, having then an outside option. Allowing recall, firms can make further offers, depending on the number of received applications. Recall is treated in Kircher (2008) in a directed search, simultaneous, labor network, where firms post wages. The efficiency result obtained by Kircher will not hold in the sequential set up that I propose. Moreover, there is also a significant difference in the fact that in my model firms wage offers are not posted. The wage structure will depend on the applications made by workers, while in Kircher the applications depend on the posted wages.

In this paper I present a model where workers and firms are ex-ante identical. However, ex-ante identical firms behave differently once their type is revealed. The type is given by the number of rounds in which the firm can make wage offers, if needed. Since workers cannot coordinate in the number of applications they send, neither on where they apply, not all firms receive the same number of applications. The type of the firm is then stochastically determined, after workers have send their applications.

Workers observe the number of job vacancies and the total number of workers looking for a job. They choose the number of applications they wish to make and send them to different job vacancies.[1] Firms collect applications and list them randomly, since agents are ex-ante homogenous. The list gives the order in which firms will make wage offers. Firms offer then, simultaneously and privately, a take-it-or-leave-it wage offer to the first applicant in the list. Workers collect wage offers and accept the highest one, forming a match. Both the matched firm and the worker exit the market, but this information is not public. This first round is as in the model presented in GM-G. The extension is to allow unmatched firms to proceed through

[1] In the presented paper the number of applications is treated as an exogenous choice.

their application list, making further rounds of wage offers. This implies that any firm that was rejected in a wage offer round and that still has applications in her list, can make another wage offer to the next applicant in the list. This sequential wage offer process goes on until the firm wage offer is accepted, forming a match, or the list is exhausted. If the list ends, the vacancy remains unfilled. There is then a positive probability that a vacancy is unfilled and a worker remains unemployed, unless all workers apply to all vacancies. The number of wage offer rounds can be very large, making very difficult to deal with the full model. However, the number of wage offer rounds can be exogenously determined. The result of the model is clearly affected by this choice.

Firms are, therefore, ex-post heterogenous, depending on the number of applications they receive. This number determines how many bargaining rounds they can be active in the market. This implies a different behavior according to the type of the firm, since the value of the outside option depends on the type. This is equivalent to say that firms do not value identically the worker. In fact they give a different value in each round, depending on the value of the outside option. Consequently, wage offers are different across types. Firms solve their problem mainly through mixed strategies, but the support of the wage distributions differs across firms. In fact, wage offer supports do not overlap, since firms compete only with those of their own type, losing against all firms of a lower type and winning against higher type firms. Also the number of firms and the number of workers that are active in the market gets lower as rounds go on. Workers leave the market if they form a match or if their applications are all in firms that have already found a worker. Firms leave the market if they make a match or if they do not have more applications in their list.

This exclusion of types is due to the differences in the expected profit related to further wage offer rounds. In any bargaining round, a particular type of firm will be playing her last round, making an offer to the last application in her list. This implies that she has no outside option and if she does not make a match in that round, her vacancy remains unfilled. The rest of active firms have at least one additional round, so they still expect a positive profit if rejected. Firms in their last round will behave aggressively, making a high wage offer in order to increase the probability to get a worker. Those with additional rounds behave less aggressively, since they still have a positive outside option. Their wage offer supports do not overlap, implying that firms are not interested in a competition between types. Lower types will make wage offers that overrun any offer made by higher type firms.

This implies that in each round the wage offers can be separated, according to types, in different supports. Since the distribution of firms types follows a binomial distribution, the wage offers will follow a similar distribution. This leads to a hump-shaped wage offer distribution where most of wage offers are concentrated on central values rather than on extreme values, i.e. in the first round, firms with just one applicant will offer high wages, those with two applications will offer wages in a support just below the offers made by firms with a single application, firms with three applications in a support below those offers made by firms with two applications and so on. Since the amount of firms of each type follows a binomial distribution, related to market tightness and the number of applications, the amount of offers made in each subset of wages will follow a similar distribution.

The number of applications made by workers affects the wage offer distribution, affecting the wage that workers expect to receive. This relation is not monotonic provided that the number of workers is higher than the number of vacancies. If workers make a single application it is optimal for firms to offer the reservation wage. If all workers apply to all firms it is also optimal for firms to offer the reservation wage, since at some round the reservation wage will be accepted. This implies that, if there are more workers than firms, even if sending applications has no cost, it is never optimal to apply to all firms or to make an infinite number of applications.

In the description of the model, the behavior of workers accepting wages may be seen as too naive. They take the highest offer received in a particular round ignoring the possibility of receiving a better offer in the future. This seems counterintuitive because workers might wish to reject an offer if it is below the wage they expect to get in a later round. There is an informational asymmetry that explains this fact. Workers are not aware of the bargaining round as long as firms do not provide information about the fulfillment of their vacancies. A worker is active in a particular round because she has not received any offer in previous rounds, so she has no information about her position in the list nor about the round that is currently going on. This informational asymmetry is enough to make her accept the highest offer received in a particular bargaining round.

2 Model

The model is intuitively simple but mathematically complex to solve. The matching mechanism of the economy is described as an urn-ball game with multiple applications and recall. Recall is considered as the option of firms to make additional wage offer rounds if they were previously rejected. The process is sequential, but within each step, all acting agents make simultaneous choices.

The economy is composed by N workers looking for a job and V firms posting a single job vacancy. All firms and workers are ex-ante homogenous. Moreover, N and V are observable and large.[2] Once job vacancies are posted, workers send a number S of applications to different vacancies. Workers choose randomly where to send their applications. Each application has a cost c for the agent. Firms collect applications and list them randomly. Then, a set of sequential rounds of contacts starts. Each firm contacts workers sequentially, according to her list, making a take-it-or-leave-it wage offer W to the corresponding applicant until the offer is accepted or the list ends.[3] In each of these rounds, workers collect wage offers and accept the highest one they have at that moment, forming a match.[4] All workers and firms that have formed a match exit the market. Firms that have not formed a match start a new wage offer round if they still have applications in their list. Firms without a match and without further applications exit the market as unfilled vacancies.

Step by step, the market works as follows:[5]

[2] Large enough to apply asymptotic properties for the probability distributions.

[3] In each round the offer can be different, a round subscript is omitted for simplicity.

[4] Ties are solved randomly, an offer below the reservation wage $\underline{w}$ is rejected.

[5] T subscript denotes the current round of offers, K subscript denotes firm or application type.

1. The number of workers looking for a job, N, and the number of firms that open a single job vacancy, V, is observed by all participants. The market tightness θ, expressed as the ratio $\frac{V}{N}$, is therefore known by all agents.

2. Workers choose the number S of applications they make. Each application has a cost c for the worker.

3. Firms collect applications and order them randomly in a list. The list can have a maximum length of N, but the number of rounds where the firm can make offers might be lower. The maximum number of rounds where a firm can be active, that is, the highest possible type, is given by $\overline{R} = \min\{V, N\}$. However, the number of rounds can be exogenously fixed to any R lower than $\overline{R}$. Firms learn their type K, where K is the number of rounds where the firm can make offers. The type is equal to the length of the application list as long as it is shorter than R. The type is R if the list is larger or equal than R. Firms type is private information.

4. Firms make a take-it-or-leave-it wage offer $W_{T,K}$ to the corresponding applicant in the list.[6]

5. Workers that are still in the market collect offers. If they receive at least one offer above their reservation wage $\underline{w}$, they accept the highest offer received and exit the market. This happens with probability D_T.

6. Firms with an accepted offer form a match and exit the market. This happens with probability $M_{T,K}$.

7. A formed match produces a value Q. Firms gain $Q - W_{T,K}$ and workers $W_{T,K} - Sc$.

8. Firms with no remaining applications in their list exit the market. This happens to all firms of type $K = T$ that are still in the market.

9. Rejected firms of type $K > T$ go to the next round, starting again from step 4.

[6] The particular wage offer will be random, the distribution depends on round and type.

Workers are assumed to take the highest offer received in the current round. This behavior seems naive, since workers can reject all offers below the highest expected offer of the next round. That is, they could update their reservation wage every round. If they do not do so, it is because they do not know in which round they have received the offer. The unique information that workers have are the received offers, but they do not now how many rounds have gone without receiving offers. This lack of information forces workers to accept the highest current offer.[7]

Workers and firms commit to the formed matches and to the accepted wages. This implies that a formed match cannot be broken. Matched workers reject all offers received in further rounds, even if they imply a higher wage. Matched firms do not make further wage offers trying to find a cheaper worker.

If the maximum number of active rounds is set to one, $R = 1$, this model boils down to the model by GM-G. Increasing the number of rounds above one gives firms an outside option after a rejection. Firms bargaining power is, therefore, increased with respect to GM-G and their wage offer behavior changes. Firms will offer wages according to their ex-post revealed type. Firms of lower types act more aggressively than those of higher types, as there are fewer rounds in which they can be active in the market. They are more eager to get a worker, making then higher wage offers in order to overrun the offers made by firms with a higher type.

In fact, the equilibrium result is such that the optimal wage offer distributions of different types do not overlap in a particular round. Firms compete in each round with the firms of their own type. They lose against firms of lower type and they win against firms of higher type.

The model becomes highly complicated due to the construction of the probabilities involved. The probability structure must be solved recursively starting from the first round until round R.

[7]The reservation wage is exogenously given. However, given the probabilistic structure of the game, it might be possible to endogenously determine a constant reservation wage $\underline{w}$ to be used in all rounds. This does not change the fact that agents accept the highest offer received at the current round.

2.1 Probability construction

Agents send S applications to V firms. The probability that a particular agent sends an application to a particular firm is $\frac{S}{V}$. The number of applications that a firm receives is stochastically determined. The probability distribution of firms ex-post types follows a censored binomial from 0 to R. Type R accumulates the probabilities that correspond to firms with at least R applications. The probability distribution of firms types is given by:

$$F_K = \binom{N}{K} \left(\frac{S}{V}\right)^K \left(1 - \frac{S}{V}\right)^{N-K} \quad \text{if } K < R \tag{1}$$

$$F_R = 1 - \sum_{i=0}^{R-1} \binom{N}{i} \left(\frac{S}{V}\right)^i \left(1 - \frac{S}{V}\right)^{N-i} \quad \text{if } K = R, \tag{2}$$

where F_K denotes the probability that a particular firm is a type K firm and the type denotes the number of rounds where the firm can be active, if she did not exit the market previously.

Since firms are ex-post of different types, applications are also different because they have been received by firms of different types. That is, a particular application may end up in a firm with no other applicants, a type 1 firm, or it may end up in a firm with 9 other applicants, a type 10 firm.[8] The probability distribution of applications ex-post types also follows a censored binomial distribution, now from 1 to R, since an application cannot end up in a firm with no applications at all. The probability distribution of applications type is given by:

$$A_K = \binom{N-1}{K-1} \left(\frac{S}{V}\right)^{K-1} \left(1 - \frac{S}{V}\right)^{N-K} \quad \text{if } 0 < K < R \tag{3}$$

$$A_R = 1 - \sum_{i=0}^{R-1} \binom{N-1}{i-1} \left(\frac{S}{V}\right)^{i-1} \left(1 - \frac{S}{V}\right)^{N-i} \quad \text{if } K = R, \tag{4}$$

[8] This implies that R must be higher or equal than 10. If not, the type of the firm would be R.

where A_K denotes the probability that a particular application is a type K application and the type denotes the type of the firm that received the application.

Both firms and applications types are ex-post types. They are revealed in step three and only to firms. Workers are not aware of types in any moment, so they are not aware of it when making their choice. In fact it is the choice of workers what determines the type of firms and applications. Therefore, workers do not know neither the types of the firms where they have sent their applications, neither the types of applications they hold. However, workers can anticipate the probability distributions when they make their choices. Firms learn their own type and the type of the applications they hold, but they do not know the type of the firms with which they compete for a particular worker neither the type of applications that the worker holds.

An application of type K, given that it was not successful in previous rounds, has a probability of being successful in round T that is given by:

$$O_{T,K} = \begin{cases} 0 & \text{if } K < T \text{ and } K < R \\ \frac{1}{1+K-T} & \text{if } K \geq T \text{ and } K < R \\ \frac{1}{A_R} \sum_{i=R}^{N} \frac{1}{1+i-T} \binom{N-1}{i-1} \left(\frac{S}{V}\right)^{i-1} \left(1 - \frac{S}{V}\right)^{N-i} & \text{if } K = R. \end{cases} \quad (5)$$

In a given round T, a firm of type K has a probability $G_{T,K}$ of being active. This probability can be computed as:

$$G_{T,K} = \begin{cases} 1 & \text{if } T = 1 \\ \prod_{i=1}^{T-1} (1 - M_{i,K}) & \text{if } 1 < T \leq K \\ 0 & \text{if } T > K. \end{cases} \quad (6)$$

Here $M_{i,K}$ is the probability that a type K firm has left the market in round i. Basically $G_{T,K}$ computes the probability that a firm of type K has not left the market in any of the previous $T-1$ rounds.

A worker is active in round T with probability B_T, given by:

$$B_T = \begin{cases} 1 & \text{if } T = 1 \\ \prod_{i=1}^{T-1}(1 - D_i) & \text{if } T > 1, \end{cases} \quad (7)$$

where D_i is the probability that an agent has left the market in period i. Again, B_T computes the probability that a worker has not left the market in the previous $T - 1$ rounds.

In a particular round T, an agent can receive wage offers from different types of firms that are still in the market. A particular application, given that it was not successful in previous rounds, has a probability $L_{T,K}$ of receiving an offer from a type K firm. This probability takes into account the probability that the application is a type K application, the probability that a type K application is successful in round T and the probability that the particular K type firm is still in the market. $L_{T,K}$ is expressed as:

$$L_{T,K} = O_{T,K} A_K G_{T,K}. \quad (8)$$

In a given round T, a worker that is still in the market, receives an offer from a firm of type K or lower with probability:

$$R_{T,K} = 1 - \left(1 - \sum_{i=T}^{K} L_{T,i}\right)^S. \quad (9)$$

A worker that is still in the market has not received an offer in any previous round. This implies that all her S applications may receive offers in the current round. However, the probability that a particular application is successful changes in each round.

The highest offer that the agent receives, that might form a match, comes from a firm of type K with probability:

$$Z_{T,K} = R_{T,K} - R_{T,K-1}. \quad (10)$$

To form a match the agent must be still in the market at round T. This implies that a match with a type K firm is formed with probability $M^{-}_{T,K} = Z_{T,K}B_T$. In a given round the total number of matches with type K firms is given by $M^{*}_{T,K} = M^{-}_{T,K}N$. Since there are $G_{T,K}F_KV$ type K firms that are active in the market at period T, the probability that a firm of type K forms a match in round T is then given by:

$$M_{T,K} = \frac{M^{-}_{T,K}N}{G_{T,K}F_KV}. \tag{11}$$

In a given round T a worker, that was still in the market, receives at least one offer, forming a match and leaving the market with probability:

$$D_T = R_{T,R}. \tag{12}$$

Since $R_{T,R}$ represents the probability of receiving at least one offer from type R or lower, it is also the probability of receiving an offer from any firm type.

An active firm in round T must take into account that she holds one of the applications of the workers in her list. This implies that the probability that this particular worker receives offers is different than the one exposed above. That particular agent might have received, in previous rounds, an offer from a firm type K or lower with probability:

$$R^{F}_{T,K} = 1 - \left(1 - \sum_{i=1}^{K} L_{T,i}\right)^{S-1}. \tag{13}$$

The difference with $R_{T,K}$ is due to the fact that the number of active applications is lower, in particular there is one less application. For an outside observer, a worker can receive wage offers from all her S applications. However, the firm holds one application, so she knows that the worker can receive offers only from $S - 1$ applications. This fact reduces her probability of receiving offers in previous rounds. From the firms point of view, a worker that is in her list, exits the market with probability:

$$D_T^F = R_{T,R}^F. \tag{14}$$

Given these facts, from firms point of view, the worker to which they make an offer is in the market with probability B_T^F, that must be computed as:

$$B_T^F = \begin{cases} 1 & \text{if } T = 1 \\ \prod_{i=1}^{T-1}(1 - D_i^F) & \text{if } T > 1. \end{cases} \tag{15}$$

Following a similar construction, it turns out that this result holds for all rounds if $R < V - S$. If $R > V - S$ there are distortions in the probabilities for high types at high rounds. When $T > V - S$, those firms that are still active know that there cannot be more than $V - T$ competing firms in the market. The extreme case is when $T = R = V$. A firm of type $K = V$ knows that in the last round she will hold at least one application and that all those applications belong to workers that are still in the market. This is because her list had more applications than competing firms in the market. The applicants that rejected her offer in each of the $V - 1$ previous rounds have formed matches with the $V - 1$ competing firms. The remaining applicants must then be unemployed. Since there are no more competing firms in the market, any of the applicants will accept the reservation wage offer. The optimal strategy for a firm of type V is to offer $\underline{w}$ in all subsequent rounds. In a round where $T > V - S$, the probability that an agent receives an offer from a type K or lower is computed, from the firms point of view, as:

$$R_{T,K}^F = 1 - \left(1 - \sum_{i=1}^{K} O_{i,K} A_i G_{i,K}\right)^{V-T-1}. \tag{16}$$

Then, in a situation where $\theta < 1$ and from the workers point of view, it is not optimal to make V applications, even if applications were for free. If all workers apply to all vacancies and $R = V$, the optimal behavior of firms is to offer the reservation wage in all rounds. Workers are better off if the do not apply at all. This construction implies a non-monotonicity in the probabilities computed from the point of view of firms. The probability that a particular worker is in the market decreases from the first round to the round $V - S$ and increases from round $V - S$ till round R, reaching 1 if $R = V$.

3 Firms choice

In each round T, firms choose wage offers from the set:

$$\Omega_{T,K}(W) = \arg\max \{(Q - W) P_T(W) - (1 - P_T(W))\Pi_{T,K}\}. \tag{17}$$

This can be rewritten as:

$$\Omega_{T,K}(W) = \arg\max \{(Q - \Pi_{T,K} - W) P_T(W) + \Pi_{T,K}\}. \tag{18}$$

In both cases $P_T(W)$ is the probability that a wage offer W is accepted in the current round and $\Pi_{T,K}$ is the profit that, at round T, a type K firm expects to obtain from further rounds. This expected profit is strictly positive as long as there are applications in her list and zero otherwise.

$$\Pi_{T,K} \begin{cases} > 0 & \text{if } K < T \\ = 0 & \text{otherwise.} \end{cases} \tag{19}$$

The expected profit is increasing with K and decreasing with T, as long as $K < V$. It is constant if $K = V$.

This implies that the higher is the type of the firm, the higher is the expected profit from further rounds. Firms with a high type behave then less aggressively that those with fewer applications. Firms choose their wage offers from non-overlapping wage offer sets, competing among firms of the same type. They lose against offers from lower types and they win against offers from higher types. For example, those firms that have just one application left in their list do not receive any expected profit from the next round. If they do not form a match in that round, their vacancy is not fulfilled. They have to do very aggressive wage offers, offers that are higher or equal than the highest offer that the rest of firms types make, as they are more eager to form a match.

In each round, wage offers are extracted from different wage offer sets. In a particular round T there are $1+R-T$ different wage offer sets, one for each type of firm that might be still in the market. These wage offer sets have supports that do not overlap, except in their boundaries. From the point of view of workers or of an external observer, wage offers are extracted from a continuous distribution function. The distribution function is composed by the wage offer distributions related to each type of firm.

The wage offer behavior in each round is defined according to the type of firm, the probabilities related to that particular period and the expected profit relative to further rounds. In a particular round T, active firms behave according to their type, making offers according to a distribution function:

$$J_{T,K}(W) = \left(\left(\frac{Q - \Pi_{T,K} - \overline{w}_{T,K+1}}{Q - \Pi_{T,K} - W} \right)^{\frac{1}{S-1}} - 1 \right) \frac{(1 - L_{T,K})}{L_{T,K}}, \tag{20}$$

where $\overline{w}_{T,K+1}$ is the highest wage offer made by $K+1$ type firms, $\Pi_{T,K}$ is the expected profit from further rounds and $L_{T,K}$ the probability that a particular application receives an offer from a type K firm in round T. Firms of type K make offers from $\overline{w}_{T,K+1}$ to $\overline{w}_{T,K}$. The upper bound is set as:

$$\overline{w}_{T,K} = (Q - \Pi_{T,K}) - (Q - \Pi_{T,K} - \overline{w}_{T,K+1})(1 - L_{T,K})^{S-1}. \tag{21}$$

In each round, firms of type R make the lowest offers. For a firm of type R, the lowest offer cannot be $\overline{w}_{T,K+1}$, since there are no higher types. For a firm of type R the lowest offer is $\underline{w}$. Firms of type R make offers from $\underline{w}$ to $\overline{w}_{T,R}$, where $\overline{w}_{T,R}$ is computed as :

$$\overline{w}_{T,R} = (Q - \Pi_{T,R}) - (Q - \Pi_{T,R} - \underline{w})(1 - L_{T,R})^{S-1}. \tag{22}$$

The expected profit from this round, to be used in previous rounds, can be computed as:

$$\Pi_{T-1,K} = (Q - \Pi_{T,K} - \overline{w}_{T,K+1}) B_T^F \prod_{i=T}^{K} \left(1 - R_{T,i}^F\right). \tag{23}$$

To define firms wage offer behavior for a given round, both $\Pi_{T,K}$ and $L_{T,K}$ are required. The second one depends on the probabilities related to previous rounds and the first one depends on the probabilities and the behavior in further rounds. This implies that probabilities for all possible rounds must be solved and then we have to solve for the behavior in each round, starting from round R and proceeding backwards until round 1. The number of bargaining rounds can be very large. Then, to illustrate how the process works I present the solution for a two-period model.

4 Two period model

The length of the list is at most two, $R = 2$. This divides firms into just three types: i) Firms with no applications, called type 0 firms; ii) firms with one application, type 1 firms and iii) firms with two or more applications, type 2 firms.

The corresponding probabilities are:

$$F_0 = \left(1 - \frac{S}{V}\right)^N, \tag{24}$$

$$F_1 = \frac{SN}{V}\left(1 - \frac{S}{V}\right)^{N-1}, \tag{25}$$

$$F_2 = 1 - \left(1 - \frac{S}{V}\right)^N - \frac{SN}{V}\left(1 - \frac{S}{V}\right)^{N-1}. \tag{26}$$

Recall that F_0 is the probability that a particular firm is a type 0 firm, that is, the probability that a firm does not receive any application, being not active in any round; F_1 is the probability that a firm is a type 1 firm, that is, the probability that a firm has received only one application, being then active only in the first round;

F_2 is the probability that a firm is a type 2 firm. Type 2 includes all firms with two or more applications and not just those firms with two applications.

Applications can be of just two types, according to the type of firm where they end up. A type 1 application is received by a firm that has no other applicants, while a type 2 application is received by a type 2 firm. The probabilities corresponding to application types are:

$$A_1 = \left(1 - \frac{S}{V}\right)^{N-1}, \tag{27}$$

$$A_2 = 1 - \left(1 - \frac{S}{V}\right)^{N-1}, \tag{28}$$

where A_1 is the probability that a particular application is the unique application received by the firm and A_2 includes all other possible cases.

In the first round $G_{1,1} = G_{2,1} = 1$. That is, all firms with applications are active in the first round. Moreover, $B_1 = 1$, meaning that all workers are active in the first round.

An application is successful in round 1, given the type, according to probabilities:

$$O_{1,1} = 1, \tag{29}$$

$$O_{1,2} = \frac{1}{A_2} \sum_{i=2}^{N} \frac{1}{i} \binom{N-1}{i-1} \left(\frac{S}{V}\right)^{i-1} \left(1 - \frac{S}{V}\right)^{N-i}. \tag{30}$$

An application of type 1 is successful for sure in the first round, since there are no other applicants. An application of type 2 is successful in the first round, depending on the number of other applicants that a type 2 firm may have.

The probability that a type 2 application is successful in round 2, given that it was not successful in round one, is:

$$O_{2,2}=\frac{1}{A_2}\sum_{i=2}^{N}\frac{1}{i-1}\binom{N-1}{i-1}\left(\frac{S}{V}\right)^{i-1}\left(1-\frac{S}{V}\right)^{N-i}. \tag{31}$$

Given that construction of probabilities, in the first round an application receives an offer from a type K firm with probability:

$$L_{1,K}=O_{1,K}A_K G_{1,K} \text{ for } K\in\{1,2\}. \tag{32}$$

That is:

$$L_{1,1}=O_{1,1}A_1G_{1,1}=A_1, \tag{33}$$

and

$$L_{1,2}=O_{1,2}A_2G_{1,2}=O_{1,2}A_2. \tag{34}$$

This implies that a particular worker receives at least one offer from a type 1 firm with probability:

$$\begin{aligned} R_{1,1} &= 1-\left(1-\sum_{i=1}^{1}L_{1,i}\right)^{S} \qquad (35)\\ &= 1-(1-L_{1,1})^{S}\\ &= 1-(1-A_1)^{S}. \end{aligned}$$

In the same way, it receives at least one wage offer from a firm of type 2 or lower with probability:

$$\begin{aligned} R_{1,2} &= 1-\left(1-\sum_{i=1}^{2}L_{1,i}\right)^{S} \qquad (36)\\ &= 1-(1-L_{1,1}-L_{1,2})\\ &= 1-(1-A_1-O_{1,2}A_2)^{S}. \end{aligned}$$

The highest offer received comes from a type 1 firm with probability:

$$Z_{1,1} = R_{1,1}, \tag{37}$$

and from a type 2 firm with probability:

$$\begin{aligned} Z_{1,2} &= R_{1,2} - R_{1,1} \\ &= (1 - A_1)^S - (1 - A_1 - O_{1,2}A_2)^S . \end{aligned} \tag{38}$$

These probabilities are also the probabilities of a match, because all workers are in the market in the first round. This implies that $M^{-}_{T,K} = Z_{T,K}$. The total number of matches with firms of types 1 and 2 in the first round are:

$$M^{*}_{1,1} = Z_{1,1}N = \left(1 - (1 - A_1)^S\right) N, \tag{39}$$

and

$$M^{*}_{1,2} = Z_{1,2}N = \left((1 - A_1)^S - (1 - A_1 - O_{2,2}A_2)^S\right) N, \tag{40}$$

respectively.

At the end of round 1, all type 1 firms exit the market, since they cannot be active in round 2. At the end of round 1, a type 2 firm exists the market if she had formed a match. She leaves the market in round 1 with probability:

$$M_{1,2} = \frac{Z_{1,2}N}{F_2V}. \tag{41}$$

Then, a type 2 firm is still active in the second round with probability $G_{2,2} = 1 - M_{1,2}$.

Workers exit the market in the first round with probability $D_1 = R_{1,2}$. Then, a worker is still in the market in the second round with probability:

$$B_2 = 1 - R_{1,2}.$$

In the second round, an application receives an offer from a type 2 firm with probability:

$$L_{2,2} = O_{2,2} A_2 G_{2,2}.$$

In the second round workers receive a type 2 offer with probability:

$$\begin{aligned} R_{2,2} &= 1 - \left(1 - \sum_{i=2}^{2} L_{2,i}\right)^S \\ &= 1 - (1 - L_{2,2})^S, \end{aligned} \tag{42}$$

forming a match with probability:

$$M_{2,2} = R_{2,2} B_2. \tag{43}$$

This is the general probability construction of the model.

Firms behave according to their perception of the probabilities, that are slightly different since they hold one of the applications send by the worker. When choosing the wage offer in the second round, a firm knows that this particular worker may have formed a match in the previous period with probability:

$$\begin{aligned} R_{1,2}^F &= 1 - \left(1 - \sum_{i=1}^{2} L_{1,i}\right)^{S-1} \\ &= 1 - (1 - L_{1,1} - L_{1,2}) \\ &= 1 - (1 - A_1 - O_{1,2} A_2)^{S-1}, \end{aligned} \tag{44}$$

being still in the market with probability $B_2^F = 1 - R_{1,2}^F$.

I can compute also the probability that a worker receives an offer from other type 2 firms in round 2 as:

$$R^F_{2,2} = 1-\left(1-\sum_{i=2}^{2} L_{2,i}\right)^{S-1} = 1-(1-L_{2,2}) = 1-(1-O_{2,2}A_2G_{2,2})^{S-1}. \quad (45)$$

In the second round, a type 2 firm chooses her wage from:

$$\Omega_{2,2}(W) = \arg\max\{(Q-W)\,P_2(W)\}. \quad (46)$$

The profit associated to the reservation wage is given by:

$$\Pi(\underline{w}) = (Q-\underline{w})\left(1-R^F_{2,2}\right)B^F_T. \quad (47)$$

The profit associated to any wage offer is given by:

$$\Pi(W) = (Q-W)P_2(W). \quad (48)$$

Since all wage offers must yield the same profit then:

$$P_2(W) = \frac{(Q-\underline{w})}{(Q-W)}\left(1-R^F_{2,2}\right)B^F_T. \quad (49)$$

The probability that a particular wage offer is accepted can also be constructed as:

$$P_2(W) = \left((1-L_{2,2})^{S-1} + \sum_{i=1}^{S-1}\binom{S-1}{i}(L_{2,2})^i(1-L_{2,2})^{S-1-i}J_{2,2}(W)^i\right)B^F_T, \quad (50)$$

that can be expressed as:

$$P_2(W) = \left((1 - L_{2,2}) + L_{2,2} J(W)\right)^{S-1} B_T^F. \tag{51}$$

Combining Equations (49) and (50), the optimal wage offer strategy for type 2 firms in round 2 is solved as the following cumulative distribution function:

$$J_{2,2}(W) = \left(\left(\frac{(Q - \underline{w})}{(Q - W)} \right)^{\frac{1}{S-1}} - 1 \right) \left(\frac{1 - L_{2,2}}{L_{2,2}} \right). \tag{52}$$

Type 2 firms expected profit form the second round, when they are in the first round, is given by:

$$\Pi_{1,2} = (Q - \underline{w}) \left(1 - R_{2,2}^F\right) B_T^F. \tag{53}$$

In the first round type 2 firms choose their wage offer from:

$$\Omega_{1,2}(W) = \arg\max \left\{ (Q - W) P_1(W) - (1 - P_1(W)) \Pi_{1,2} \right\}, \tag{54}$$

and a type 1 firm from:

$$\Omega_{1,1}(W) = \arg\max \left\{ (Q - W) P_1(W) \right\}. \tag{55}$$

Type 2 firms will offer wages from $\underline{w}$ to $\overline{w}_{1,2}$ where:

$$\overline{w}_{1,2} = (Q - \Pi_{1,2}) - (Q - \Pi_{1,2} - \underline{w}) (1 - L_{1,2})^{S-1}, \tag{56}$$

and type 1 firms will offer wages from $\overline{w}_{1,2}$ to $\overline{w}_{1,1}$ where:

$$\overline{w}_{1,1} = Q - (Q - \overline{w}_L) (1 - L_{1,1})^{S-1}. \tag{57}$$

Type 2 firms offer wages in round 1 according to:

$$J_{1,2}(W) = \left(\left(\frac{Q - \Pi_{1,2} - \underline{w}}{Q - \Pi_{1,2} - W} \right)^{\frac{1}{S-1}} - 1 \right) \frac{(1 - L_{1,2})}{L_{1,2}}, \tag{58}$$

and type 1 firms according to:

$$J_{1,1}(W) = \left(\left(\frac{Q-\overline{w}_{1,2}}{Q-W}\right)^{\frac{1}{S-1}} - 1\right)\frac{(1-L_{1,1})}{L_{1,1}}, \tag{59}$$

This implies that from the point of view of workers, the expected wage is distributed according to:

$$H_{1,1}(W) = \left(\frac{Q-\overline{w}_{1,2}}{Q-W}\right)^{\frac{S}{S-1}} (1-L_{1,1})^S, \tag{60}$$

from $\overline{w}_{1,2}$ to $\overline{w}_{1,1}$, and

$$H_{1,2}(W) = \left(\frac{Q-\Pi_{1,2}-\underline{w}}{Q-\Pi_{1,2}-W}\right)^{\frac{S}{S-1}} (1-L_{1,1})^S (1-L_{1,2})^S, \tag{61}$$

from $\underline{w}$ to $\overline{w}_{1,2}$, if the offer is received in the first period, and

$$H_{2,2}(W) = \left(\frac{(Q-\underline{w})}{(Q-W)}\right)^{\frac{S}{S-1}} (1-L_{2,2})^S, \tag{62}$$

from $\underline{w}$ to $\overline{w}_{2,2}$, in the case that the offer is received in the second period.

The expected return associated to a particular number of applications S is then computed as:

$$\begin{aligned} Rt(S) &= \int_{\underline{w}}^{\overline{w}_{1,2}} W\frac{dH_{1,1}(W)}{dW}dW + \int_{\overline{w}_{1,2}}^{\overline{w}_{1,1}} W\frac{dH_{1,2}(W)}{dW}dW \\ &\quad + (1-R_{1,2})\int_{\underline{w}}^{\overline{w}_{2,2}} W\frac{dH_{2,2}(W)}{dW}dW - Sc. \end{aligned} \tag{63}$$

To illustrate this results I present some numerical examples. The presented results are simulations of models with two and five wage offer rounds, respectively. In particular I set exogenously $S = 5$, $N = 100$, $V = 100$ and the mentioned two or five wage offer rounds, $R = 2$ or $R = 5$. Production is normalized to one, $Q = 1$, and the reservation wage is set to zero, $\underline{w} = 0$. In figures 1 to 5, I present the results obtained in a simulation of a model with two wage offer rounds. In Figure 6 and tables 1 to 3, I present the results obtained in a simulation of a model with five wage offer rounds.

In a model with only two wage offer rounds there are only two kind of firms that can interact, since there is no interaction at all with type 0 firms. The probability of being a type 1 firm is low in both absolute and relative terms. Part of the mentioned effect on the wage behavior corresponds to the behavior of type 1 firms during the first round. The parameters of the simulation are low in order to obtain a relatively high number of type 1 firms, implying a clearer effect on the wage offer behavior and on the expected return of workers. This is done just with illustrative purposes, since the effect exists in any case. In the case of five rounds, the effects on both the wage offers and the expected return of workers is very much amplified, as we see below.

In Figure 1 I present the cumulative distribution function of the wage offer of type 2 firms in round 2 , that is $J_{2,2}(W)$. In period two, type 2 firms offer wages from the reservation wage to a value slightly above 0.3 (0.3028).

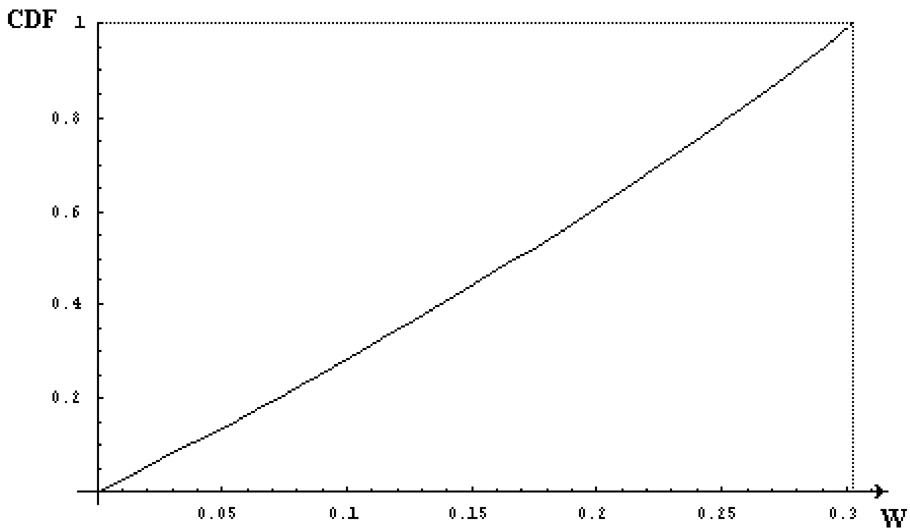

Figure 1: Wage offer in second period.

The shape of the obtained CDF is close to a straight line. Wage offers are, therefore, close to a uniform distribution.

In Figure 2 I present the cumulative distribution function of the wage offer of type 2 firms in round 1, that is $J_{1,2}(W)$. In period one, type 2 firms offer wages from the reservation wage to a value close to 0.4 (0.4098).

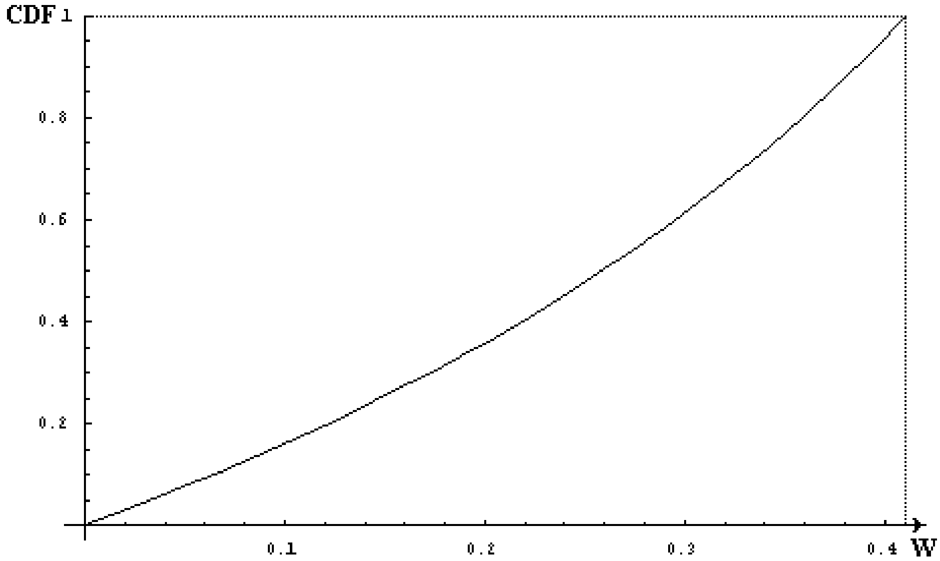

Figure 2: Type 2 firms wage offer in the first round.

Type 2 firms make higher wage offers in the first round than in the second one, and this is due to the reduction in competition in the second round. There are less competing firms so the agent has a lower probability of receiving offers. However, there are also less workers in the market. This has an effect in the expected profit, but it has no clear effect on the wage offer behavior. The shape of the CDF is close to be linear and shows a high density concentration on higher wage offers.

In Figure 3 I present the cumulative distribution function of the wage offer for a type 1 firm in round 1, that is $J_{1,1}(W)$. In period 1, type 1 firms offer wages from the upper bound of type 2 firms (0.4098), up to a value close to 0.4 (0.4244). Since there are few type 1 firms in the market, they do not expect much competition. The probability that a particular worker receives two offers from type 1 firms is very low.

The wage offers of type one firms are then concentrated on a narrower set of values.

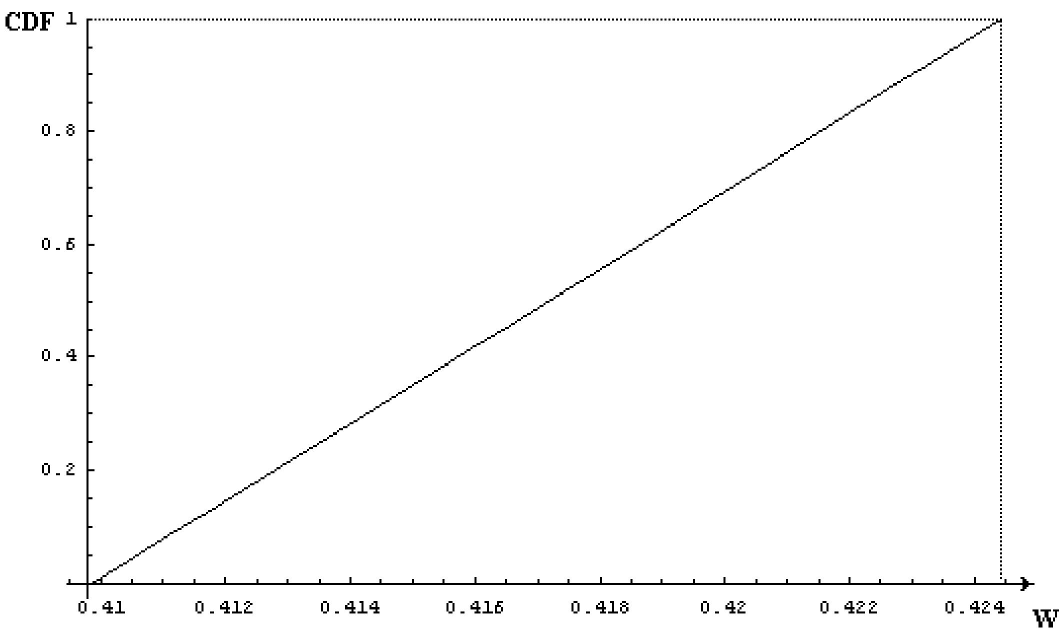

Figure 3: Type 1 Firms wage offer in the first round.

Again the shape of the function is close to a line, which implies that it is well approximated by a uniform distribution.

In Figure 4 I present the cumulative distribution that the worker expects to get from a particular offer, taking into account that the offer can be made in the first or the second period, being then a combination of $J_{1,1}(W)$, $J_{1,2}(W)$ and $J_{2,2}(W)$.

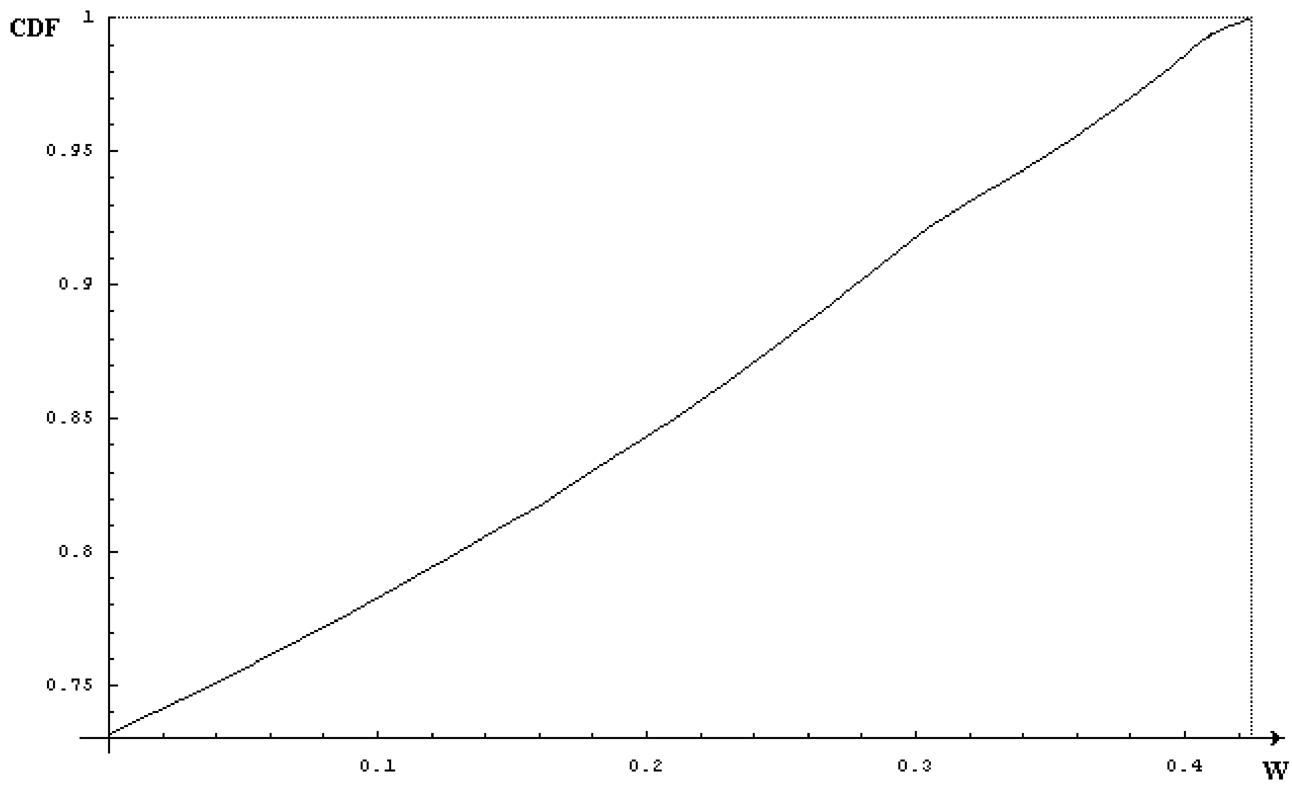

Figure 4: Expected wage offer distribution related to a particular application.

In Figure 5 I present the distribution of the expected wage for the worker, that is, the distribution of the highest offer. This is also a combination of $J_{1,1}(W)$, $J_{1,2}(W)$ and $J_{2,2}(W)$.

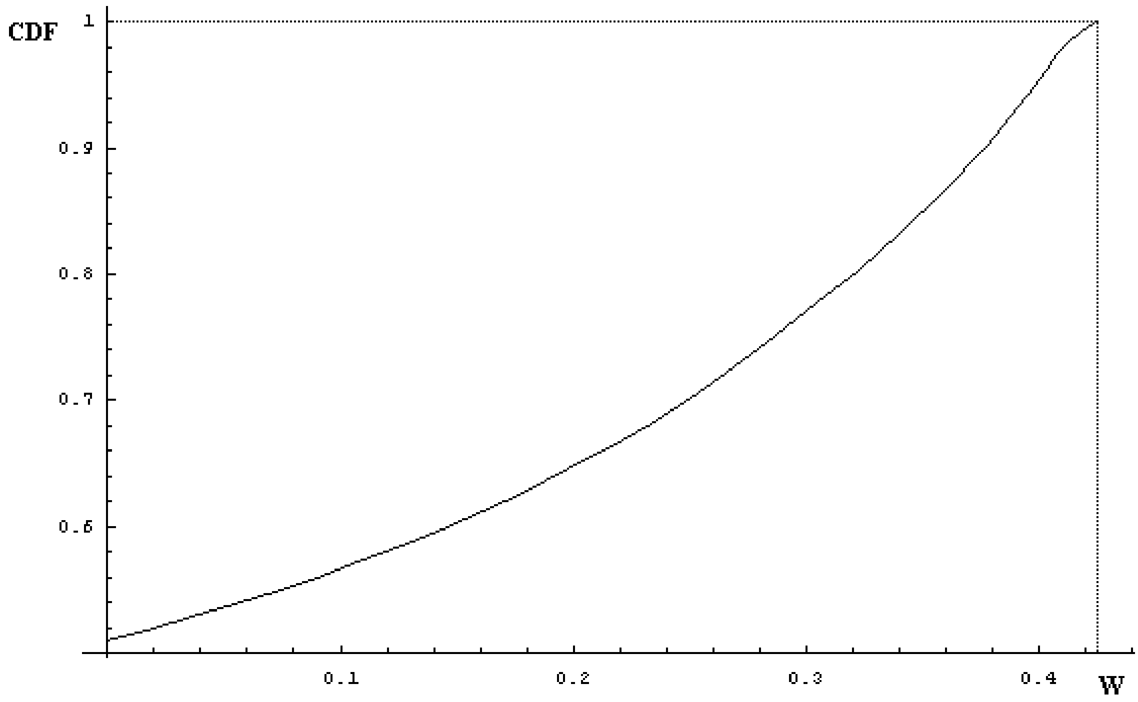

Figure 5: Expected wage distribution.

We see that, for values close to the upper bound a less steep curvature is obtained, denoting a lower density concentration on extreme values. Still the wages are heavily concentrated close to the upper bound of the wage offers. If more wage offer rounds are allowed, the wage offer behavior of firms is such that there is less concentration on the extreme wage values. This can be observed in a model with five wage offer rounds.

In Figure 6 I present the expected offer from a particular application if there are five rounds of wage offers. I use the same parameter values as above.

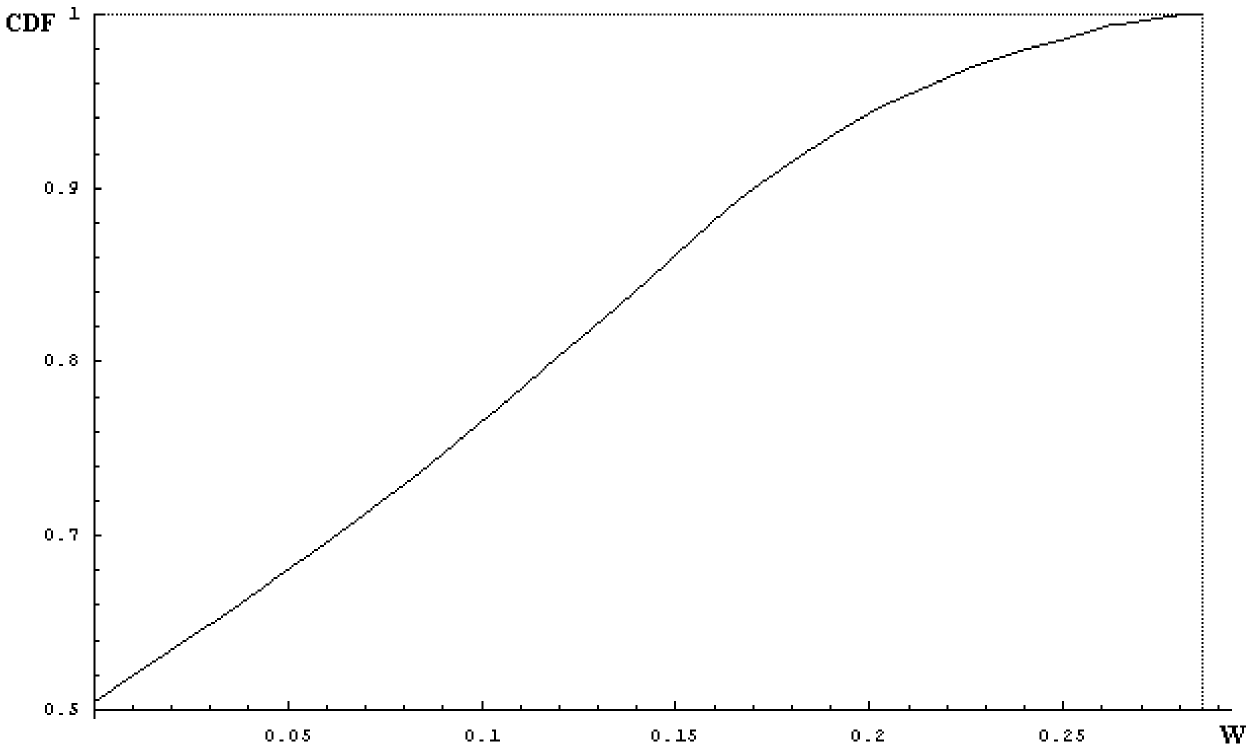

Figure 6: The expected offer from a particular application in a five offer-round market.

When there are five rounds, the highest wage offered is slightly above 0.28 (0.2808), which is much lower than the upper bound obtained in a two round game. The existence of further rounds gives more bargaining power to firms. Their wage offers are lower and the shape of the expected return suggests a higher concentration on middle values of the wage domain.

In Table 1 I present the probabilities of firms types (F) and applications types (A) in a five round model:

Types	0	1	2	3	4	5
Firms (F)	0.0059	0.0312	0.0812	0.1396	0.1781	0.5640
Applications (A)	·	0.0062	0.0325	0.0837	0.1425	0.7350

(64)

Table 1: Distribution of firms and applications types

Firms of type 5 include more than half of all firms. However, they receive nearly three fourths of all applications. A particular application is a type 1 application, receiving then a type 1 firm offer, with a probability slightly over 0.6%. There is nearly a 0.6% of firms that do not receive any application, remaining then unfilled.

In Table 2 I present the probabilities that firms form a match in a given round, provided that they are active:

Round	1	2	3	4	5
Firm type 1	0.9876	·	·	·	·
Firm type 2	0.9439	0.6578	·	·	·
Firm type 3	0.8624	0.4859	0.6335	·	·
Firm type 4	0.7544	0.4170	0.4153	0.5965	·
Firm type 5	0.5456	0.3118	0.2726	0.2635	0.3187

(65)

Table 2: Matching probabilities by round

For example, a type 2 firm forms a match in the first round with probability 0.9439. All those type 2 firms that do not form a match in the first round, go to the second round forming a match with probability 0.6578.

There is a strong last-round effect in probabilities, related to the fact of being the lowest type in a given round. For firms of type 3 or higher, the probability to form a match in the last active round is higher than the probability of forming a match in the previous period. This effect is due to the aggressive bidding that overruns the bid of all other types, or in the case of type 5 firms, to the lack of competition from other types of firms.

In Table 3 I present the probability that a worker leaves the market, if it was active, and the probability that a particular worker is still active in a given round :

Round	1	2	3	4	5
Is active in the market	1	0.3301	0.2171	0.1527	0.1105
Leaves the market	0.6699	0.3425	0.2963	0.2764	0.2685

(66)

Table 3: Workers probabilities

Even in the fifth round, workers have more than one fourth of chances to form a match and more than 10% of workers are still actively searching for a job. After the fifth round, there is still roughly an 8% of workers unemployed. This implies that after five rounds, an 8% of the vacancies remain unfilled. Of this 8%, a 0.6% corresponds to firms that did not receive any application and a 7.4% to firms that received applications, but failed to form a match during the five offer rounds.

5 Conclusion

A hump shaped wage distribution can be obtained in a labor market where both workers and firms are ex-ante homogeneous.

This requires to develop the labor market in a quite intuitive way, a multiple application model with recall. Computing all the probability distributions and the optimal wage offers for firms I obtain a hump shaped wage distribution, even when firms and workers are perfectly homogenous, ex-ante.

Since recall is allowed, firms are ex-post heterogeneous, depending on the number of bargaining rounds where they can be active. This implies that each type of firm gives a different value to a match. This is equivalent to say that in each period the bargaining is equivalent to a sealed bid first price auction with an unknown number of bidders, where bidders can have different valuations of the good.

The result is a set of mixed strategies, corresponding to different type of firms in different rounds. Each firm makes wage offers as extractions of an endogenously determined distribution function, that is different in each round. The distribution functions are such that in a given round the different domains do not overlap, firms with higher type make lover wage offers and loose to all active firms with lower type, and wins against an active firm with a higher type.

Each type of firm, in each round, uses a mixed strategy that yields a offer distribution similar to the one obtained in GM-G. The expected wage of an agent is the higher offer expected, that is a composition of the offer distributions, weighted according the probabilities of receiving offers from such firms, yielding the obtained hump shaped wage distribution.

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