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Should the Public Sector Conduct Genomics R&D?

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Abstract: The nature of the observed market structure and R&D competition in genomics research is used as the basis for a comparative analysis of research under a mixed oligopoly, pure oligopoly and monopoly when the timing of the innovation outcome is uncertain (as in an R&D race), the winner-take-all assumption is relaxed and the profits in later stages are a function of the R&D expenditures of prior stages. The sufficient conditions under which a mixed oligopoly performs more R&D than the pure oligopoly and monopoly markets are derived and are shown to be a function of a) that public firm's objective is strictly greater than in the winning state then in the losing state, b) profits for the winning and losing private firms in the private duopoly are equal, post innovation, and c) the objective function of the firms in the mixed duopoly are increasing in research faster than they are for firms in the other two cases. It is suggested that when these conditions are met, the public firm can play a role in increasing the level of research in genomics.

Keywords: Genomics, Mixed Oligopoly, Public Research, Innovation Race
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1. Introduction

Over the past two and a half decades biotechnology has revolutionized major portions of the human health and agriculture industries, transforming them into the life sciences industry. Yet the most revolutionary biotechnologies are still in the early stages. Perhaps the most potent of these technologies is genomics—the science of sequencing all the genes of a given species to study the structure, function and evolution of diverse organism. Genome research, however, is more than biology; it is also about developing better drugs, foods, industrial products, and, in the case of agriculture, improving plant and animal productivity and quality.

A striking feature of genomic research is the significant levels of investment by a few dominant private firms in competition with an equally well-funded public sector that seek to discover and subsequently patent important gene sequences. This observation is made most pellucid by the private sector's Celera Genomics Group challenge to the longer-lived and more expensive, publicly funded Human Genome Project (HGP). The Department of Energy and National Institutes of Health started HGP in 1990, at a cost of approximately $2.2 billion over the course of the project. In 1992 Craig Venter, a scientist with the HGP, left to form his own private company, Celera Genomics, and claimed that the firm could sequence, using a different technique from the HGP, the whole genome in less than three years and at a fraction of the cost (approx. $200 million). Celera's challenge to the publicly funded HGP signaled the start of the race for the sequencing of the human genome, which was joined by numerous other start-up companies looking to capitalize on the potential of genomic research. The competition between Celera and HGP so accelerated sequencing efforts that by late 2000 both projects were essentially complete ahead of schedule.

Although less confrontational, the private sector is also at the forefront of characterizing plant and animal genomes, sometimes subcontracting from the public sector. In 1998, a consortium led by the International Rice Genome Research Program (IRGSP) in Tsukuba, Japan, began efforts to sequence the rice genome. The participants, which included primarily government and research foundation sponsored labs, took a traditional approach to genome mapping known as the 'stepwise sequence analysis' (Bennetzen, 2002). This approach, while expensive and slow, provides the most precise and complete sequence with a goal of 99.99% accuracy.[1] Soon after the IRGSP was initiated, the Monsanto Co. began funding research to sequence the same variety of rice as that by IRGSP. The Monsanto sequencing strategy was slightly different from that of IRGSP, allowing it to sequence more of the genome with less time and cost. However the strategy does not provide enough information for highly accurate assembly

[1] By early 2002, the project had sequenced 15% of the genome.

(Bennetzen, 2002).[2] Much more recently, the Beijing Genomics Institute (BGI) and Syngenta, a Switzerland based agricultural biotechnology company, independently produced draft sequences of the rice genome by the quickest and least costly method, the 'shotgun sequence analysis.' Syngenta obtained 99.8% sequence accuracy, identifying more than 99% of the genes at 10% of the cost of the IRGSP strategy (Bennetzen, 2002)[3]. On January 26, 2001 Syngenta and Myriad Genetics announced that they had sequenced the rice genome and planned to provide their database to commercial customers, such as seed companies or agricultural biotechnology companies. The competition from the private sector has in turn spurred the IRGSP into advancing its calendar by almost four years (to 2004) and increasing its budget. Japan pledged to increase its annual rice genome research $60 million in 2000; a threefold increase over the previous year.

The Human and Rice Genome Projects are classic examples of a research race between firms where the objective is to be the first to discover, and subsequently obtain patents, on important gene sequences. However, current racing models do not conform well to the type of behavior observed in genomics research for two reasons. First, most models have assumed that the research race is between profit maximizing private agents (Sabido, 1994; Reinganum, 1985). This is appropriate for a variety of industrial research in which the public sector is not involved. However in the agricultural sector, or in genomic research, there is a dominant public research sector whose objective, it can be argued, is to maximize not profits but welfare.[4] Thus, such research is best characterized as a race between a social welfare maximizing public organization(s) and a profit maximizing private firm(s). In the context of genomics research, a private firm's objective is to patent the genetic code for important proteins and obtain royalties on the patent. Whereas the objective of a publicly funded entity is to promote further innovations (and hence increase welfare) which it does by making the genetic information more widely available, without regard to maximizing royalty revenues. The asymmetric objectives of these two types of firms' gives rise to a different behaviors from the case wherein all the firms in the analysis are private.

A second reason why earlier patent racing models fail to capture the intricacies of research such as genomics is in their assumption that the value of the prize is exogenously determined (Sabido, 1994). Genomics is not simply the identification of a sequence of genes, but also involves understanding the

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[2] In 2000, Monsanto abandoned its rice sequencing effort and donated its data to IRGSP.
[3] A useful analogy in comparing the different strategies is to imagine the whole genome as being a large puzzle. Without the knowledge of what the whole genome (puzzle) resembles, the genome is first broken down into pieces (DNA strands) and the individual pieces subsequently sequenced. After the identification (sequencing) of the smaller pieces has occurred the task of determining how the individual pieces relate to each other and where the fall on the map follows (this is akin to putting puzzle together). While it is easier to sequence smaller DNA strands, it is much more difficult to 'rebuild' the map with so many pieces.
[4] Following the literature on mixed oligopolies, we abstract here from moral hazard and internal organization issues and define a public firm to be an entity whose objective is to maximize social welfare, whereas a private firm would aim to maximize profit.

properties and relationships of the genetic code embodied in those genes. How accurately and in what manner the genetic sequence has been identified has bearing on the ease of interpreting the functions of the genes in the later stages. The amount of research done and the method employed in the sequencing stage can also influence later stages of genomics research if, for example, one assumes that more expenditure in the sequencing stage will, on average, lower the cost of doing research in the later stages. For example, the scientific knowledge and tools used in the sequencing the plant DNA have the potential of lowering the cost of breeding varieties with agronomicaly desirably traits. These cost reductions primarily result in more precision in transferring desirable genes to crops and reducing the time to breed specific varieties. The implication for the winning firm is that it gains knowledge in the process of the race that is useful and can be productively employed in further research (by it or others). That is, the more research it expends on the race today, the greater is the likelihood of winning *and* the greater will be the cost savings on future research or production. Moreover if we do not assume a winner-take all situations the endogeneity of the prize value also has repercussions for the losing firm[5]. That is, the losing firm, which moves on to the next stage but uses the now inferior technology, will face lower profits due to a decline in market share. The decline in market share for the losing firm, and by extension profits, is a function then of the cost reduction implied by winning the prize (for the winning firm) and how much research was expended to achieve that prize (by the winning firm)[6].

To capture more accurately the microfoundations of an R&D race observed in genomics research, this paper characterizes research as a two-stage process. The research effort in the first stage reduces the cost of applied research or production in the second stage. We use this framework to gain insights into a traditional theme in industrial organization research that of the relationship between market structure and innovation. Economists have been interested in this issue ever since Joseph Schumpeter's seminal work

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[5] The notion of 'winning' and 'losing' in the context of genome research may seem inappropriate as firms cannot obtain patents for simply sequencing a certain DNA strand nor are there any immediate commercial benefits from the knowledge of such sequences. Nevertheless, it has been observed that private firms are more reluctant to disclose their sequences (e.g. both Syngenta and Celera did not make their discoveries public through a commonly use public database). By effectively using trade secrecy to protect their sequences (especially large assembled sequences), the private firms hope to appropriate any return that may arise at later stages. This is in contrast to public efforts in genomic research, which have made the sequences available to all without any strings attached.

[6] For example, assume a strategic game between two firms in the output stage where the cost of production for firm *i* before the innovation is $c_i[q_i, \gamma]$ where q_i is the amount produced and γ a technology parameter. After the innovation race, the winning firm will maximize its profits $\pi_w = P[q_w, q_l]q_w - c[q_w, \gamma_w, x_w]$ choosing q_w (where x_w is the research expended in the racing stage by the winner) and the losing firm maximizes $\pi_l = P[q_w, q_l]q_l - c[q_l, \gamma_l = \gamma]$ choosing q_l (assume $\frac{\partial c_i}{\partial \gamma_i}, \frac{\partial c_w}{\partial x_w} < 0$ and $\gamma_w > \gamma_l$). Solving for the equilibrium properties it can be shown that profits of the winning firm will be greater than that of the losing firm $\pi_w^* = \pi_w^*[\gamma_l^*, \gamma_l^*, x_w^*] > \pi_l^* = \pi_l^*[\gamma_l^*, \gamma_l^*, x_w^*]$

hypothesizing a positive correlation between market power and innovation. Schumpeter (1934) argued that a few firms were more likely efficiently to develop and employ more advanced technology than a competitive industry. Formal models of firms' innovation-seeking behavior have evolved, that have either confirmed or refuted the so-called 'Schumpeterian tradeoff.'[7] Similar to the mixed theoretical results, the findings of the vast empirical literature on the Schumpeterian tradeoff are mixed and ambiguous as no obvious relationship between industrial concentration and R&D performance emerges from the data.

Specifically we ask what kind of market structure, and under what circumstances, promotes R&D when the nature of R&D is as described for genomics. Does the public sector serve a useful purpose by performing genomics R&D when all evidence suggests that there are willing private firms that can do the same type of research more quickly and at lower cost? Does public sector efforts have a role to play in genomics R&D? We show that this question can be answered in the affirmative, if certain sufficient conditions regarding the prize value and the nature of the innovation process are met. In the process of deriving these conditions we also obtain conditions under which a monopoly research market performs more R&D than a pure duopoly market and vice versa. These market structures are chosen as they represent the observed (mixed duopoly) and alternative (duopoly and monopoly) market structures in genomics research[8]. Note that it is not the intention of our analysis to comment on the desirability of a mixed market over the other two (which requires comparison to the first-best outcome), but to provide a set of sufficient conditions whereby one would *expect* the public sector to conduct more R&D relative to firms in the other regimes. It is in this sense that we suggest that the public sector has a role to play in conducting research of the kind observed in genomics R&D.

The paper is organized as follows. The next section briefly reviews earlier the R&D models on which our analysis is based. Section three introduces the modeling framework, the assumptions and our approach in the comparing the different market structures. Section four discusses the comparative analysis. Section five concludes the paper.

2. Literature Review

Before proceeding to describe our modeling framework, we briefly review the relevant R&D models of Loury (1979), Lee and Wilde (1980) and Delbono and Denicolo (1991). Loury (1979) modeled a one-shot non-cooperative game in which *n* identical firms invest in R&D to innovate first. The first innovator is awarded an exogenously determined prize. The probability of success by firm *i* is an

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[7] For a review of this literature see Kamien and Schwartz (1982) or van Cayseele (1998). Among the many writers who subscribe to Schumpeterian Kamien and Schwartz (1982). The claim has been challenged by Arrow (1962) and Dasgupta and Stiglitz (1980).
[8] The high fixed costs associated with genomics R&D serves as an entry barrier, and hence a competitive environment is not analyzed here.

exponential function of its hazard rate ($h[x_i]$) given no success to date. The loser of the race gets nothing and thus suffers a loss equal to the cost of its R&D expenditure. The R&D cost for each firm is a lump-sum x, expended at the beginning of the race. Lee and Wilde (1980) reformulate Loury's model assuming that the R&D expenditure is a flow cost that the firm i pays until any one firm is successful. The different specification of the cost of R&D results in a different impact of an increase in the number of firms on the equilibrium individual R&D effort. Whereas such an effect is negative in Loury's model, it is positive in Lee and Wilde's model.

The contribution of the aforementioned models has been to illuminate the relationship between the intensity of rivalry and R&D performance. However, since neither Loury (1979) or Lee and Wilde (1980) explicitly model the product market, the relationship between the structure of the product market and incentives to invest in R&D was not examined. One rationalization for not allowing for an explicit specification of a product market has been that firms compete in prices in a homogenous product market so that a Bertrand equilibrium results. In such a market, pre-innovation profits are zero (as all firms share the same technology) and post-innovation, the winner, which has reduced its own cost, will be the only active firm (Delbono and Denicolo, 1991).

Delbono and Denicolo (1991) show that when firms make positive profits in the pre- and post-innovation Cournot-equilibrium markets (where the losers also make positive profits, post innovation) the Loury result—that there is a positive relationship between profits and equilibrium R&D effort—holds. This result is significant in that it highlights the relationship between the number of firms and the equilibrium R&D effort to be much more complex than in models where the prize value, or expected returns from R&D, are exogenously given.

Now while the Delbono and Denicolo model allows for pre- and post-innovation profits for all firms, the nature of the innovation and how much of an improvement it is on a prior technology is still exogenously given. That is at the end of the race, the winner obtains the rights to technology that bestows on it an exogenously determined cost advantage over the prior technology. This cost advantage is exogenously given and has no relationship to the actual amount of research that the firm conducted in the R&D race. As we have argued, there exists the possibility where the R&D effort of the racing stage of the game results in further cost advantages for later stages, such that a higher amount of research leads to a higher amount of product value for the winning firm. It is this aspect of the R&D process, and the point of departure from other models, that we shall highlight in our modeling framework and comparative analysis.

3. The Model

Consider a two-stage game. In the first stage, firms choose their R&D investment and in the second stage they compete further by conducting more applied R&D or competing in the product market[9]. The first stage is modeled as an innovation race where firms compete for the rights to an infinitely lived patent. The innovation embodied in the patent allows firms to lower the cost of research in the second stage (or the cost of production, if the second stage is modeled as a product market). Through backward induction, the profits from the second stage determine the value of the first stage patent. The firm that innovates first is awarded the patent and gets the exclusive right to use the more productive technology forever. The losing firm, on the other hand, has to continue using the pre-innovation race technology in the second stage and hence accrues a lower profit than the winning firm and possibly even lower than its own profits prior to innovation.

The research effort employed in the R&D race not only determines the outcome of the race, but also results in the generation of knowledge that is valuable to the winning firm. This knowledge can be used in later stages to complement with the winning technology and lower costs in those stages even further. In this respect the value of the prize for the winner is endogenous and an increasing function of research expenditure in the R&D race. R&D effort thus has a two pronged direct effect on the winning firm; allowing it to win the race *and* lowering the its cost in later stages. The losing firm is also affected by the amount of research effort employed by the winning firm in the first stage (see footnote 6). Since we assume the strategic game in the second stage as well, an increased market share for the winning firm from the lowering of its cost will imply lower profits for the losing firm, *cetris paribus*.

To fix these ideas, assume that two firms play the following two-stage game. In the first stage firm i independently takes action, denoted x_i, regarding the current research market. In a patent race set-up, x_i represents the flow cost of research where its probability of being successful at or prior to date t is $1-e^{-th[x_i]}$. The instantaneous conditional probability that firm i will be first to innovate at time t, given no success to date, is therefore $h[x_i]$. Firm i's expected benefits after a discovery are determined by both firms actions (b_i,b_j) and are denoted by $W_{Wi}(b_i[x_i],b_j[x_i])$ if the firm emerges as the winner and $W_{Li}(b_i[x_j],b_j[x_j])$ when it losses the race. In a Cournot set-up, b_i would represent output whereas in a Bertrand game it would be price. A strategy of firm i in this entire game can then be written as $s_i \equiv (x_i, b_i(\bullet))$ where $b_i(\bullet)$ is a function specifying firm i's post innovation action conditional on first

[9] The specific characteristics of the second stage is not of concern here. We note only that the payoff (or prize) from undertaking R&D in the first stage is function of the market structure in the second stage and the amount of research performed in the first stage.

stage actions, in particular on the amount of research done by the winning firm. Given (s_i, s_j), the payoff to the private firm i is:

$$V_i[W_{Wi}, W_{Li}, \{x_i, x_j\}] = \int_0^\infty e^{-(h[x_i]+h[x_j]+r)t}(h[x_i]W_{Wi}[x_i]/r + h[x_j]W_{Li}[x_j]/r + W_i - x_i)dt \quad (1a)$$

$$= \frac{(1/r)\left(h[x_i]W_{Wi}[x_i] + h[x_j]W_{Li}[x_j]\right) + W_i - x_i}{h[x_i] + h[x_j] + r} \quad (1b)$$

where

r is the discount rate

x_i is firm i's R&D expenditure

$h[x_i]$ is firm i's instantaneous probability of innovating or the hazard rate. The hazard rate, $h[x_i]$, is twice differentiable, strictly increasing and satisfies 1) $h[0] = 0 = \lim_{x\to\infty} h'[x]$, 2) $h'[x_i] > 0$, 3) $h''[x_i] < 0$.

$W_{Wi}[x_i]$ is the value of innovation accruing to firm i if it wins the race

$W_{Li}[x_j]$ is the value of innovation accruing to firm i if it loses the race where j is the winning firm.

W_i is the pre-innovation benefits accruing to firm i

As we shall see, the value of the innovation will be different for the private and public firms. For the public firm the value of the an innovation is the total welfare generated by it. For the private firm it is the value of the private benefits or profits. Next we proceed to characterize the equilibrium condition in R&D for the market structures of interest, namely monopoly, pure duopoly, and mixed duopoly. We make progress by first characterizing the best response function for each firm in the three markets.

3.1. Monopoly equilibrium condition

The monopolist maximizes its payoff (eqn (1)) where i=1=M), by choosing x_M.[10] This defines the first order condition for a maximum for the monopolist, $\frac{\partial V_M}{\partial x_M}$=0, which is true if and only if $R_M[x_M] = 0$

where

$$R_M[x_M] \equiv \begin{bmatrix} h'[x_M](W_M[x_M] - W_M) - r - h[x_M] + x_M h'[x_M] + \\ h[x_M]W_{WM}'[x_M](1 + h[x_M]/r) \end{bmatrix} = 0 \quad (2)$$

Equation (2) determines the equilibrium value of the R&D expenditure by the monopolist x_M^*.

[10] The monopolist faces no rival, therefore x_j=0.

Following Delbono and Denicolo (1993), the difference between the profit from winning and the firm's current profit $(W_{WM}[x_M]-W_M)$ measures the incentive to innovate in the absence of rivalry and is referred to as the 'profit incentive.' If we assume that $W_{WM}[x_M]>W_M>0$, then presence of current profits in this model induces firms to delay the expected date of innovation[11].

3.2. Pure duopoly equilibrium condition

In the pure duopoly case (two profit maximizing firms), firm 1 chooses x_1 and firm 2 chooses x_2 to maximize the payoff function. Due to symmetry (n=2, therefore, x_1=x_2=x_D), the best response function for each firm is defined by the condition $\frac{\partial V_D}{\partial x_D}=0$, which is true, if and only if, $R_D[x_D]=0$, where

$$R_D[x_D]\equiv\left[\frac{h'[x_D](W_{WD}[x_D]-W_D)+(1/r)h'[x_D]h[x_D](W_{WD}[x_D]-W_{LD}[x_D])-}{r-2h[x_D]+x_Dh'[x_D]+h[x_D]W_{WD}'[x_D](1+(2/r)h[x_D])}\right]=0 \tag{3}$$

As in the monopolist case, the duopolist also faces a 'profit incentive' $(W_{WD}[x_D]-W_D)$ which measures the incentive to invest in R&D in the absence of rivalry. However in a duopoly there is rivalry as each firm anticipates research by the other. In the presence of rivalry the incentive to invest in R&D is also reflected in the difference between the flow of profits should it win the race and should it not, $W_{WD}[x_D]-W_{LD}[x_D]$. Call this the 'rivalry incentive.' The presence of both pre-innovation profits and post-innovation profits for the loser induce firms to delay the expected date of innovation, that is higher pre-innovation and loser profits decrease the profit and rivalry incentives. The smaller the profit and rivalry incentive, the more time it will take for innovations to arrive.

3.3. Mixed duopoly equilibrium condition

In the mixed duopoly the private and public firms choose x_i but maximize different payoffs. The private firm maximization problem remains unchanged from that of a firm in the pure duopoly case. That is, it maximizes equation (III.1) (for i=2), where we denote firm 1 as the private firm (P) and firm 2 as the public (S). Since symmetry no longer holds (as the public firm's payoff is different), the private firm's best response function is defined by the condition $\frac{\partial V_P}{\partial x_P}=0$, which is true if and only if $R_P[x_P,x_S]=0$ where

[11] Stated differently, if there were no current profits such that $W_M=0$, then $W_{WM}[x_M]-W_M<W_{WM}[x_M]$, increasing the LHS of equation (2).

$$R_P[x_P, x_S] \equiv \begin{bmatrix} h'[x_P](W_{WP}[x_P]-W_P)+(1/r)h'[x_P]h[x_S](W_{WP}[x_P]-W_{LP}[x_S])-r- \\ h[x_P]-h[x_S]+x_P h'[x_P]+h[x_P]W_{WP}^{'}[x_P]\left(1+(1/r)(h[x_P]+h[x_S]\right) \end{bmatrix}=0 \qquad (4)$$

The best response function of the private firm in the mixed duopoly is similar to the one in pure duopoly. In both cases, there exists a profit incentive as well as a competitive threat, faced by the private firm. The only difference between the two is that the source of the competitive threat in the pure duopoly case is another, identically specified private firm, whereas in the mixed duopoly case it is the welfare-maximizing public firm.

The public firm in our model is a welfare maximizer to whom the value of the prize is not just the private profits it embodies but the social welfare as well. This means, first, that the "W's" in the payoff function are greater for the public firm than they will be for the private (more discussion on the assumption and relative values of the prize follows). Second, the public firm, as a social welfare maximizer, takes into account the flow cost of research incurred by all firms in the economy. In the duopoly case where one firm is public and the other private, the public firm's payoff function is written as:

$$V_S = \int_0^\infty e^{-(h[x_P]+h[x_S]+r)t}\,(h[x_S]W_{WS}[x_S]/r + h[x_P]W_{LS}[x_P]/r + W_S - x_S - x_P)dt \qquad (5a)$$

$$= \frac{(1/r)\left(h[x_S]W_{WS}[x_S]+h[x_P]W_{LS}[x_P]\right)+W_P - x_P - x_S}{h[x_S]+h[x_P]+r} \qquad (5b)$$

The maximization of the equation III.5 yields the public firm's reaction curve which is defined by the condition $\frac{\partial V_S}{\partial x_S}=0$, which is true, if and only if, $R_S[x_S, x_P]=0$

$$R_S[x_S, x_P] \equiv \begin{bmatrix} h'[x_S](W_{WS}[x_S]-W_S)+(1/r)h'[x_P]h[x_S](W_{WS}[x_S]-W_{LS}[x_P])- \\ r-h[x_P]-h[x_S]+h'[x_P](x_P+x_S)+ \\ h[x_S]W_{WS}^{'}[x_S]\left(1+(1/r)\left(h[x_P]+h[x_S]\right)\right) \end{bmatrix}=0 \qquad (6)$$

The reaction curve of the public firm in the mixed oligopoly also reveals that the public firm faces the 'profit incentive' and the 'rivalry threat' from the opposing firm. For the public firm the 'profit incentive' is a misnomer (since the value of the prize to it is total welfare, and not private profits as the name implies), although it is still the incentive to innovate in the absence of a rival. As with the earlier case, the smaller the rivalry and profit incentives the later is the date of innovation

In summary, equations (2), (3), (4) and (6) are, respectively, the best response functions for a monopolist, duopolist (in a pure duopoly), private firm in the mixed duopoly and the public firm (in a mixed duopoly). In all these cases we see that each firm faces a profit incentive and a competitive threat (except in the case of the monopolist, where it does not face a rival). The profit incentive is a function of how large the difference is between current profits and profits if the firm wins, and similarly the competitive threat is a function of how big the difference in profits is between winning and losing. Clearly, if the differences are small, then the firms will be conducting less research, which will delay the expected date of innovation. For the public firm in the mixed duopoly, the profit incentive and the competitive threat also matter (only that it is not profits that the public-sector firm is after but welfare). But since it takes into account the total R&D cost, the effect of R&D cost on the public firm $(x_P + x_S)$, relative to private firm (x_P), is to bring closer the expected date of innovation.

Lastly we note that in each of the four best response functions the presence of the term $\frac{\partial W_{Wi}}{\partial x_i}$. This term, which reflects the marginal change in the prize value due to a change in own research, is a direct consequence of our assumptions regarding the endogeneity of the value of the prize. If we assume that profits are concave with respect to own research then the endogenous nature of the prize value brings closer the expected date of innovation. This implies all firms carry out a greater amount of research relative to the case where the prize value is exogenous. To put it differently, when R&D complements the innovation at a later stage, firms have an incentive to increase their research effort. High amounts of research or the so-called over-investment problem (Dasgupta and Stiglitz, 1980) could therefore be explained by the presence of such a complementary effect.

4. Comparative Analysis

Having established the nature of the game and market structure we now turn our attention to the relative ranking of the equilibrium research effort by individual firms (x_M , x_D, x_P, and x_S) as well the industry (x_M , $2x_D$, and x_P+x_S). First, however, we need to make certain assumptions about the relative value of the prize in the different markets and for the different firms. Since the relative values of W will determine the equilibrium values of research effort for all the firms, we need to make our assumptions regarding them explicit.

Assumption 1: The winning firm profits (for the private firms) or welfare (for the public firm) is greater than current profits/welfare. That is $W_{Wi}[x_i] > W_i$. This assumption ensures that the 'profit incentive' to innovate is always positive.

Assumption 2: In the pure duopoly the profits in the winning state are greater than or equal to in the losing state. Moreover profits are increasing in R&D for firm winning and decreasing for firm the losing firm. Assume that firm 1 emerges as the winner and it gets the rights to a superior cost reducing technology. In the second stage, the two firms play a Cournot game. If we assume increasing costs of production in the second stage, then it can be shown that in equilibrium the winning firm will produce more than the losing firm. Consider, for example, a second stage product market where $P = a - bQ$ is the inverse demand function (for $Q = q_{WD} + q_{LD}$). Assume that prior to the race, the cost of production for both firms was $C[\gamma, q] = \gamma q^2 / 2$ where the parameter γ represents the technological opportunity due to the successful research, such that $\partial C / \partial \gamma < 0$. At the completion of the race, the losing firm will continue to produce at the pre-innovation cost, but the winner obtains a better technology such that it lowers its costs to $C[\gamma, q] = \overline{\gamma} q^2 / 2(1 + x_D)$ where $\overline{\gamma} < \gamma$. The $\left(1 + x_D\right)$ term reflects the fact that the winning firm also gains from its research effort of the first stage. The post-innovation maximization problems for the winning and losing firm are therefore, respectively

$$\max_{q_{WD}} W_{WD} = P q_{WD} - C[\overline{\gamma}, q_{WD}, x_D^*] \tag{7}$$

$$\max_{q_{LD}} W_{LD} = P q_{LD} - C[\gamma, q_{LD}] \tag{8}$$

The two firms acting simultaneously and non-cooperatively solve their maximization problem. From the first order condition for a maximum it can be easily shown (see appendix) that the standard Cournot equilibrium when costs are asymmetric will prevail, that is

$$q_{WD}^*[x_D^*] \geq q_{LD}^*[x_D^*],\ W_{WD}^*[x_D^*] \geq W_{LD}^*[x_D^*],\ \partial W_{WD}^*[x_D^*] / \partial x_D^* > 0,\ \partial W_{LD}^*[x_D^*] / \partial x_D^* < 0$$

Assumption 3: In the mixed duopoly, no public production or further research takes place in the second stage. Should the public firm win the race, it licenses its technology to the private firm. Alternatively, if the private firm wins the race in the mixed case, it does not face a rival in the second stage and assumes the role of a monopolist. As we are assuming the same demand and technology characteristics across the different market structures, it follows that the profits of the monopolist, should it succeed in innovating (in the monopoly case), equal the profits of the winning firm in the mixed duopoly, if and only if, the research effort by the two firms was equal in the first stage[12]. On the other hand if the public firm wins the first stage race, and in the absence of further research or production by the public firm in the second stage, it licenses the technology to the private firm. While the private firm is still a monopolist even with the license (by virtue of the fact that no rival exists), the terms of the license are such that it is not allowed to produce at the profit maximizing level (where marginal cost (*MC)* equal marginal revenue (*MR)*) but rather at a level where the welfare losses associated with monopoly production are minimized, though not necessarily eliminated. This is because the terms of the license also

[12] The claim here is that if $x_M = x_P = x$ then $W_{WM}[x] = W_{WP}[x]$

have to be incentive compatible in the sense that the profits for the private firm from production using the new licensed technology are greater than or equal to the profits associated with the older technology.

Therefore it is reasonable to think that the welfare attained with a public firm innovating is higher than the welfare attained when the innovator is a private firm, simply because a public firm would license and contract its innovation to the private firm which would have to price its innovation in order to maximize social welfare taking into account consumer surplus. With $W_{WS}^*[x_S^*]$, as the total welfare generated by the innovation when the innovator is the public firm, and $W_{LS}^*[x_P^*]$ as the total welfare generated when the innovator is the private firm. Finally, $W_{WP}^*[x_P^*]$ is the value of the private benefits when the private firm innovates, and $W_{LP}^*[x_S^*]$ when it does not. Given these definitions, one has $W_{WS}^*[x_S^*] > W_{LS}^*[x_P^*] \geq W_{WP}^*[x_P^*] \geq W_{LP}^*[x_S^*]$ and $q_{LP}^*[x_S^*] \geq q_{WP}^*[x_P^*]$ [13].

The generalized functional form of the hazard rate and the prize value does not permit us to solve explicitly for equilibrium research effort in each case that could be compared across the three different market structures. However if we assume that the equilibrium research condition x_M^*, x_D^*, x_P^*, and x_S^* solves their respective best response function, we can derive a set of sufficient conditions to evaluate the relative magnitude of research among the firms. To illustrate this approach, assume that, in a two firm strategic game, x_i^* and x_j^* solve the best response function $R_i[x_i, x_j]$ and $R_j[x_i, x_j]$ for firms *i* and *j*, respectively (that is $R_i[x_i^*, x_j^*] = R_j[x_i^*, x_j^*] = 0$). Assume also that there is an asymmetric relationship between the two firms such that, *a priori*, we are unable to determine the relative levels of research i.e. x_i^* š x_j^*. In such cases it is possible to derive a set of sufficient conditions that will satisfy $x_i^* > x_j^*$, $x_i^* = x_j^*$ and $x_i^* < x_j^*$. We do so by taking the difference of the best response function of one firm evaluated at the other firm's optimal research levels (i.e. $R_j[x_i^*, x_i^*]$) and the best response of the other firm evaluated at its optimal level (i.e $R_i[x_i^*, x_j^*] = 0$). If we assume that the underlying value function for both firms is concave with a relative maximum at x_i^* and x_j^*, and that the second order condition is satisfied ($\partial^2 V_i[x_i] / \partial x_i^2 < 0$), then we can claim that the conditions under which $R_j[x_i^*, x_i^*] - R_i[x_i^*, x_j^*]$ š0 imply x_i^* š x_j^*.

[13] For a more formal treatment of this, refer to the appendix.

We apply this strategy separately to compare and derive the sufficient conditions to evaluate the equilibrium research effort between a) a monopolist and firms in a pure duopoly, b) a monopolist and firms in the mixed duopoly, and c) firms in a duopoly and firms in the mixed duopoly.

4.1. Monopoly vs. Pure Duopoly

The first order condition of a monopolist evaluated at the equilibrium value for the duopolist is expressed as:

$$R_M[x_D^*] = \begin{bmatrix} h'[x_D^*](W_M[x_D^*] - W_M) - r - h[x_D^*] + x_D^* h'[x_D^*] + \\ h[x_D^*] W_{WM}'[x_D^*](1 + h[x_D^*]/r) \end{bmatrix} \tag{9}$$

From this we subtract the best response function of a duopolist (equation(3)) evaluated at the equilibrium value of the R&D expenditure, x_D^*:

$$R_D^*[x_D^*] = \begin{bmatrix} h'[x_D^*](W_{WD}[x_D^*] - W_D) + (1/r)h'[x_D^*]h[x_D^*](W_{WD}[x_D^*] - W_{LD}[x_D^*]) \\ -r - 2h[x_D^*] + x_D^* h'[x_D^*] + h[x_D^*] W_{WD}'[x_D^*](2 + (1/r)h[x_D^*]) \end{bmatrix} = 0 \tag{10}$$

Evaluating and collecting terms in the expression $R_M[x_D^*] - R_D^*[x_D^*]$, we get

$$\begin{aligned} R_M[x_D^*] - R_D^*[x_D^*] = {} & h[x_D^*]h'[x_D^*](1/r)(W_{LD}[x_D^*] - W_{WD}[x_D^*]) + \\ & h[x_D^*](W_M'[x_D^*] - W_{WD}'[x_D^*]) + \\ & h'[x_D^*]\big((W_D - W_{WD}[x_D^*]) + (W_M[x_D^*] - W_M)\big) + \\ & h[x_D^*](1 + (1/r)h[x_D^*](W_M'[x_D^*] - 2W_{WD}'[x_D^*]) \end{aligned} \tag{11}$$

We consider first the conditions under which the optimal research effort of the duopolist in a (pure) duopoly is greater than that of the monopolist (i.e. $x_D^* > x_M^*$), which is implied by $R_M^*[x_D^*] - R_D^*[x_D^*] < 0$. For this inequality to be satisfied it suffices that each of the four terms (lines) in equation (11) be less than zero (the sufficient conditions). Assumption 2 unambiguously implies that the first term is negative; the profits of the winner are greater than that of the loser in a duopoly. The second term will be negative ($h[x_D^*](W_M'[x_D^*] - W_{DW}'[x_D^*])$<0) iff $W_M'[x_D^*] < W_{DW}'[x_D^*]$; the marginal changes in second period profits (at the duopolist's optimal level) are greater for the winning duopoly firm than it is for the monopolist. For the third term to be negative requires that the condition

$|W_D - W_{DW}[x_D^*]| > |W_M - W_M[x_D^*]|$ be met[14]. For the final term in equation (11) to be negative requires that $\frac{r}{h[x_D^*]} < 2W_{DW}^{'}[x_D^*] - W_M^{'}[x_D^*]$.[15]

The implication and interpretation of these conditions is as follows. Prior to the innovation the firms in a duopoly make equal but lower profits than the sole firm in a monopoly. If the innovation in question results in significantly higher profits for the winning firm (relative to the losing) in a duopoly than these conditions are sufficient to ensure that the optimal research effort by each firm in a duopoly will be greater than that of a monopolist. It follows than that if $x_D^* > x_M^*$, than the industry wide research effort in a duopoly will be greater than in a monopoly as well (i.e. $2x_D^* > x_M^*$).

Consider now the reverse case where a monopolist undertakes more research than both firms in a duopoly (i.e. $2x_D^* < x_M^*$)[16]. The conditions that would imply this result would require that $R_M[2x_D^*] - R_D^*[x_D^*]$ >0, which is expressed as:

$$\begin{aligned}R_M[2x_D^*] - R_D^*[x_D^*] = {} & (2h[x_D^*] - h[2x_D^*] + 2x_D^* h'[2x_D^*] - x_D^* h'[x_D^*]) + \\ & (h'[x_D^*](W_{D1} - W_{WD}[x_D^*]) + h'[2x_D^*](W_M[2x_D^*] - W_{M1}) + \\ & (1/r)h[x_D^*]h'[x_D^*](W_{LD}[x_D^*] - W_{WD}[x_D^*]) + \\ & (h[2x_D^*]W_M^{'}[2x_D^*] - h[x_D^*]W_{WD}^{'}[x_D^*] + \\ & (1/r)(h[2x_D^*]^2 W_M^{'}[2x_D^*] - 2h[x_D^*]^2 W_{WD}^{'}[x_D^*])\end{aligned} \tag{12}$$

A sufficient condition for (12)>0 is that each term in (12) be non-negative. The first term is always positive due to the assumptions on the hazard rate.[17] For the second term to be greater than zero, we first note that it can be expressed as the following inequality:

$$\begin{gathered}(h'[x_D^*](W_{D1} - W_{DW}[x_D^*]) + h'[2x_D^*](W_M[2x_D^*] - W_{M1}) \\ > \\ h'[x_D^*]\Big((W_{D1} - W_{DW}[x_D^*]) + (1/2)(W_M[2x_D^*] - W_{M1})\Big)\end{gathered} \tag{13}$$

If the right hand side of the inequality in (13) is non-negative that it follows that left hand side is also non-negative. For $h'[x_D^*]((W_{D1} - W_{DW}[x_D^*]) + (1/2)(W_M[2x_D^*] - W_{M1}))$ >0 implies that

[14] Consider the extreme case where the innovation is drastic allowing the winning firm in the duopoly to serve the whole market then $W_{DW}[x_D^*] > W_{DL}[x_D^*] = 0$. Having driven off the other firm, the duopoly winner will act as a monopolist, such that $W_{DW}[x_D^*] = W_M[x_D^*]$.

[15] This being a consistent, but stronger condition to $W_{WD}^{'}[x_D^*] > W_M^{'}[x_D^*]$

[16] Which implies that $x_D^* < x_M^*$

[17] Note that $2h[x_D^*] > h[2x_D^*]$ and $2x_D^* h'[2x_D^*] > x_D^* h'[x_D^*]$

$\left|W_{D1} - W_{DW}[x_D^*]\right| < \left|(1/2)(W_M[2x_D^*] - W_{M1})\right|$. That is the change in the profits for the winning duopoly firm relative to pre-innovation profits is less than half the change for the monopolist.

The third term will be less than or equal to zero since from assumption 2, we know that $W_{LD}[x_D^*] \le W_{WD}[x_D^*]$. Therefore a sufficient condition for (12)>0 would necessarily require that $W_{LD}[x_D^*] - W_{WD}[x_D^*] = 0$, implying that the gains from winning and losing are the same in the duopoly. Such a situation would arise if there are perfect spillovers allowing the losing duopolist to appropriate all the returns of the winning firm's prize. A weaker condition would require that the winning duopolist prize in the second stage allow it only slightly higher profits than the losing firm's profits $W_{WL}[x_D^*] - W_{WD}[x_D^*] \approx 0$. One implication of this condition would be that the innovation is non-drastic and "small".

As with the second term, the fourth term in eqn (12) can be expressed as an inequality such that $h[2x_D^*]W_M^{'}[2x_D^*] - h[x_D^*]W_{DW}^{'}[x_D^*] > h[x_D^*]\left((1/2)W_M^{'}[2x_D^*] - W_{DW}^{'}[x_D^*]\right) \geq 0$. Thus, for the fourth term to be non-negative requires that following condition be met: $\frac{\partial W_M[2x_D^*]}{\partial x_D^*} \ge 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}$

This condition implies the slope of the profit function for the monopolist is relatively constant (relative to the duopoly case). This sufficient condition is consistent with the others in that innovation for the monopolist is significant but insignificant for the duopoly winner.

The last sufficient condition for (12)>0 requires that the following inequality be satisfied: $(1/r)(h[2x_D^*]^2 W_M^{'}[2x_D^*] - 2h[x_D^*]^2 W_{WD}^{'}[2x_D^*]) > 0$. Assume that the change in profits for the monopolist due to first stage research is constant, such that $\partial W_i\ [\alpha x^*]/\partial x^* = \alpha c$ for all $\alpha > 0$. We have established that for the fourth term in (12) to be non-negative requires $\frac{\partial W_M[2x_D^*]}{\partial x_D^*} \ge 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}$. If we assume that $\frac{\partial W_M[2x_D^*]}{\partial x_D^*} = 2\frac{\partial W_M[x_D^*]}{\partial x_D^*} = 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}$ than the last term in eqn (12) can be reduced to $(1/r)(h[2x_D^*]^2 - h[x_D^*]^2)W_M^{'}[x_D^*]$. This term will be non-negative if and only if $h[2x_D^*]^2 > h[x_D^*]^2$, which is always true. Table III.1 provides a summary of the sufficient conditions for ranking the research effort of a monopolist and firms in a duopoly.

4.2. Monopoly vs. Mixed Duopoly

We next derive the sufficient conditions under which $x_P^* \sim x_M^*$ and $x_S^* \sim x_M^*$ (which taken together would imply that $x_P^* + x_S^* \sim x_M^*$). We first derive the sufficient conditions for the equilibrium research of the private firm (in the mixed duopoly) to be greater than that of the monopolist. This condition, $x_P^* > x_M^*$, is implied by $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$>0 where $R_P[x_M^*, x_S^*]$ is the best response of the private firm in the mixed duopoly (4) evaluated at the equilibrium research level of the monopolist and the public firm, and $R_M^*[x_M^*]$ is the monopolist best response (eqn (9)). Evaluating and collecting the terms in the expression $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$, we get

$$\begin{aligned} R_P[x_M^*, x_S^*] - R_M^*[x_M^*] = {} & h'[x_M^*](W_{M1} - W_{P1}) + \\ & h'[x_M^*](W_{WP}[x_M^*] - W_M[x_M^*]) + \\ & h[x_M^*] + (1/r)h[x_M^*]^2)(W_{WP}^{'}[x_M^*] - W_M^{'}[x_M^*] + \\ & (1/r)h'[x_M^*]h[x_S^*](W_{WP}[x_M^*] - W_{LP}[x_M^*]) + \\ & h[x_S^*]\left(h[x_M^*]W_{WP}^{'}[x_M^*] - (1/r)\right) \end{aligned} \tag{14}$$

A sufficient condition for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$>0 is that each term in eqn (14) be non-negative. From our assumptions on the prize value, we note that $W_{M1} = W_{P1}$ (pre-innovation profits for the both the monopolist and the private firm in the mixed duopoly are equal) and that $W_{WP}[x_M^*] = W_M[x_M^*]$ (post-innovation profits, when both monopolist and the private firm are successful in research, are equal as well). These assumptions imply that the first three terms in (14) all converge to zero. The fourth term is non-negative and follows from assumption 1 $(W_{WP}[x] \geq W_{LP}[x])$. Thus the sufficient condition for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$>0 is reduced to $rh[x_M^*]W_{WP}^{'}[x_M^*] > 1$.

For equilibrium research of the monopolist to be greater than the private firm in the mixed duopoly simply requires that we reverse the previous condition. That is for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$<0 (which implies $x_P^* < x_M^*$) suffices that the sufficient condition $rh[x_M^*]W_{WP}^{'}[x_M^*] \leq 1$ be met[18]. Further, since the profits from winning for the private firm in the mixed duopoly will never be less than the profits from losing (i.e. $W_{WP}[x] \geq W_{LP}[x]$), another sufficient condition that would guarantee $x_P^* < x_M^*$ is $W_{WP}[x] = W_{LP}[x]$. Recall that the when the private firm losses the race to the public firm, it can either continue using the pre-innovation technology in the second stage or can license the technology from the public firm (which does not produce in the second stage), and produce at level determined from the

[18] Note that the first three terms in (14) equal to zero

maximization of the public firm's objective (eqn **(27)**). The implication of the condition $W_{PW}[x] = W_{PL}[x]$ is that the public firm essentially allows the private firm to earn rents that the private firm would earn under the scenario where private firm actually had won the race.

Next we derive the conditions for which the equilibrium research by the public sector is greater than the monopolist (i.e. $x_S^* > x_M^*$) which is now implied by $R_S[x_P^*, x_M^*] - R_M^*[x_M^*] > 0$. Evaluating and collecting the terms in this expression, we get

$$\begin{aligned} R_S[x_P^*, x_M^*] - R_M^*[x_M^*] = {} & h'[x_M^*]\left((W_{M1} - W_M[x_M^*]) + (W_{WS}[x_M^*] - W_{S1})\right) + \\ & h[x_M^*](W_{WS}'[x_M^*] - W_M'[x_M^*]) + \\ & (1/r)\left(h[x_M^*]^2 (W_{WS}'[x_M^*] - W_M'[x_M^*]) + h[x_M^*]h[x_P^*]W_{WS}'[x_M^*]\right) + \\ & x_P^* h'[x_M^*] - h[x_P^*] + \\ & (1/r)h[x_P^*]h'[x_M^*](W_{WS}[x_M^*] - W_{LS}[x_P^*]) \end{aligned} \tag{15}$$

As before each line in equation (15) is a term, and it suffices that it be non-negative to satisfy the inequality $x_S^* > x_M^*$. The first term will be non-negative if and only if $\left|W_{M1} - W_M[x_M^*]\right| \le \left|W_{WS}[x_M^*] - W_{S1}\right|$, that is the absolute gains in profits for the monopolist are less than the welfare gains to the public firm due to the innovation. This sufficient condition is consistent with $W_{WS}'[x_M^*] \ge W_M'[x_M^*]$ which ensures that the second and third terms are non-negative.

The fourth term will be non-negative if and only if $x_P^* h'[x_M^*] \ge h[x_P^*]$. To guarantee that the fourth term is non-negative it suffices that curvature of the hazard rate be constant (i.e. a linear hazard rate $h[x] = x$). The last term in eqn (15) is non-negative for $W_{WS}[x_M^*] \ge W_{LS}[x_P^*]$. While welfare will be greater when the public sector wins as opposed to when it losses (assumption 3), this assumption requires that research effort, *x*, across the two states be equal. Clearly when $x_M^* \ge x_P^*$, then $W_{WS}[x_M^*] \ge W_{LS}[x_P^*]$ will hold. However if $x_M^* < x_P^*$, then the comparison between the welfare in the two states remains ambiguous therefore one has to assume that $W_{WS}[x_M^*] \ge W_{LS}[x_P^*]$ or that the public firm is always larger when the public firm wins as opposed to when it losses.

Lastly we state the sufficient conditions for the case where the equilibrium research by the monopolist is greater than the public firm (i.e. $x_S^* < x_M^*$)[19]. Thus, relative to public's welfare, the gains to the monopolist are now larger such that $W_{WS}'[x_M^*] < W_M'[x_M^*]$ and $\left|W_{M1} - W_M[x_M^*]\right| > \left|W_{WS}[x_M^*] - W_{S1}\right|$

[19] As one would expect this implies reversing the sufficient conditions for $x_S^* > x_M^*$.

(which would now ensure that the first three terms in eqn (15)). For the last two terms in (15) to be negative simply suffices that $x_P^* < x_M^*$, and the sufficient conditions that are implied by it. Table III.2 summarizes these results for the comparison between the monopolist and the firms in the duopolist. Note that while $x_P^* > x_M^*$ and $x_S^* > x_M^*$ implies $x_P^* + x_S^* > x_M^*$, we are unable to establish whether $x_P^* < x_M^*$ and $x_S^* < x_M^*$ implies $x_P^* + x_S^* < x_M^*$. The best we can do here is to say that sufficient conditions for $x_P^* < x_M^*$ and $x_S^* < x_M^*$ are the necessary conditions for $x_P^* + x_S^* < x_M^*$.

4.3 Mixed Duopoly vs. Pure Duopoly

Our final comparative analysis is between the firms in the mixed duopoly and firms in a pure duopoly. Here we seek to derive the sufficient conditions under which x_D^* ~ x_P^* and x_D^* ~ x_S^*. If the sufficient conditions for $x_D^* > x_P^*$ are not inconsistent for those that would satisfy $x_D^* > x_S^*$, then we can claim to have found the sufficient condition for $2x_D^* > x_S^* + x_P^*$. Similarly if the sufficient conditions for $x_D^* < x_P^*$ and $x_D^* < x_S^*$ are not inconsistent, then those conditions would imply that the aggregate research effort in the mixed market is greater than that in the duopoly market (i.e. $2x_D^* < x_S^* + x_P^*$).

We first examine the sufficient conditions that would satisfy $x_P^* > x_D^*$ implied by $R_P[x_D^*, x_S^*] - R_D^*[x_D^*]$>0, where

$$\begin{aligned} R_P[x_D^*, x_S^*] - R_D^*[x_D^*] = h[x_D^*] - h[x_S^*] + \\ & (1/r)h'[x_D^*]h[x_D^*]\left(W_{LD}[x_D^*] - W_{WD}[x_D^*]\right) + \\ & (1/r)h'[x_D^*]h[x_S^*]\left(W_{WP}[x_D^*] - W_{LP}[x_S^*]\right) + \\ & h'[x_D^*]\left((W_D - W_{WD}[x_D^*]) + (W_{WP}[x_D^*] - W_P)\right) + \\ & h[x_D^*]\left(W'_{WP}[x_D^*] - W'_{WD}[x_D^*]\right) + \\ & (1/r)h[x_D^*]\left(W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) - W'_{WD}[x_D^*](2h[x_D^*])\right) \end{aligned} \tag{16}$$

As before the sufficient conditions for $R_P[x_D^*, x_S^*] - R_D^*[x_D^*]$>0 require that each term in (16) be non-negative. For the first term in (16) to be non-negative would require that the equilibrium research effort by a firm in the duopoly be greater or equal than that of public firm ($x_D^* \geq x_S^*$). The second term will be non-negative if we assume that profits for firms in the duopoly are equal, win or lose (i.e. $W_{LD}[x_D^*] = W_{WD}[x_D^*]$). The third term is always non-negative and follows from assumption 2 and the earlier sufficient condition $x_D^* \geq x_S^*$ (hence, $W_{WP}[x_D^*] \geq W_{LP}[x_S^*]$). For the fourth and fifth terms to be non-negative implies that the profits from winning in the mixed duopoly case increases faster than in the pure duopoly case, such that $W_{WP}^{'}[x_D^*] \geq W_{WD}^{'}[x_D^*]$ and $\left|W_D - W_{WD}[x_D^*]\right| \leq \left|W_{WP}[x_D^*] - W_P\right|$. The last term will be non-negative if and only if $W_{WP}^{'}[x_D^*](h[x_D^*] + h[x_S^*]) \geq W_{WP}^{'}[x_D^*](2h[x_D^*])$.

A key sufficient condition in establishing that $x_D^* < x_P^*$ has been the assumption that $x_D^* \geq x_S^*$. Are the sufficient conditions that would satisfy $x_D^* \geq x_S^*$ consistent with those for $x_D^* < x_P^*$? We examine this next. For $x_D^* \geq x_S^*$ to hold implies that $R_S[x_P^*, x_D^*] - R_D^*[x_D^*] \leq 0$ also needs to hold, where

$$\begin{aligned} R_S[x_P^*, x_D^*] - R_D^*[x_D^*] = {} & (1/r)h'[x_D^*]h[x_D^*]\left(W_{LD}[x_D^*] - W_{WD}[x_D^*]\right) + \\ & (1/r)h'[x_D^*]h[x_P^*]\left(W_{WS}[x_D^*] - W_{LS}[x_P^*]\right) + \\ & h'[x_D^*]\left((W_D - W_{WD}[x_D^*]) + (W_{WS}[x_D^*] - W_S)\right) + \\ & h[x_D^*](W_{WS}^{'}[x_D^*] - W_{WD}^{'}[x_D^*]) + \\ & (1/r)h[x_D^*]\left(1 + W_{WS}^{'}[x_D^*](h[x_D^*] + h[x_P^*]) - W_{WD}^{'}[x_D^*](2h[x_D^*])\right) \\ & x_P^* h'[x_D^*] - h[x_P^*] \end{aligned} \tag{17}$$

For $R_S[x_P^*, x_D^*] - R_D^*[x_D^*] \leq 0$, a sufficient condition is that each term in the equation be less than or equal to zero. The first term will be less then or equal to zero by assumption 2, $W_{LD}[x_D^*] \leq W_{WD}[x_D^*]$. However to be consistent with the sufficient conditions that satisfy $x_D^* < x_P^*$ requires the stronger condition $W_{LD}[x_D^*] = W_{WD}[x_D^*]$, which is maintained here as well. The sign on the second term is ambiguous, as assumption 2 no longer holds due to asymmetric research effort. That is since $x_D^* < x_P^*$, the term [$W_{WS}[x_D^*] - W_{LS}[x_P^*]$] cannot be signed without further assumptions on the value of the welfare in the losing and winning states. Therefore we require the stronger condition that the public's welfare in the winning state (evaluated at the research effort of the duopolist) be equal to or less than the losing state (evaluated at the research effort of the private firm in the mixed duopoly). The third and fourth terms, taken together imply that relative to the pre-innovation profits, the gains from innovation are greater to the winning duopolist than they are for the public firm should it win. That is $W_{WS}^{'}[x_D^*] \leq W_{WD}^{'}[x_D^*]$ and $\left|W_D - W_{WD}[x_D^*]\right| \geq \left|W_{WS}[x_D^*] - W_S\right|$. The fifth term will be less than or equal to zero if and only if $W_{WS}^{'}[x_D^*](h[x_D^*] + h[x_P^*]) < W_{WD}^{'}[x_D^*](2h[x_D^*]) + 1$. Note that this sufficient condition is consistent with our assumptions that $x_P^* > x_D^*$ and $W_{WS}^{'}[x_D^*] > W_{WD}^{'}[x_D^*]$ which implies

$W_{WS}^{'}[x_D^*](h[x_D^*]+h[x_P^*]) > W_{WD}^{'}[x_D^*](2h[x_D^*])$. The last term will be less than or equal to zero if and only if $x_P^* h'[x_D^*] < h[x_P^*]$.

To derive the sufficient conditions that would guarantee the reverse relationship, i.e. $x_P^* < x_D^* < x_S^*$, simply implies that we reverse the above condition. For the sake of brevity we present those results and a summary of the sufficient conditions in Table III.3a. The conditions summarized in table three cannot, however, be used to make statements on the aggregate research relationship between the mixed and pure duopoly. To do so requires that we derive conditions for $x_D \,\tilde{}\, x_S, x_P$, which implies $2x_D \,\tilde{}\, x_S + x_P$. Table III.3b summarize these sufficient conditions and follows from equation (17). That is to derive the sufficient conditions that would simultaneously satisfy $x_D^* > x_S^*$ and $x_D^* > x_P^*$ requires that both (16) and (17) be non-negative. These sufficient are summarized in the first column of table 3b. The second column lists those sufficient conditions that would reverse this relationship, such that they satisfy $x_D^* < x_S^*$ and $x_D^* < x_P^*$

5. Discussion

The sufficient conditions that have been derived, which allow us to rank the research effort in the three markets, are primarily a function of three properties. First is how the profits/welfare are distributed, post-innovation, in the pure and mixed duopoly. Second, how the profits/welfare are increasing, for the winning firm, in *x*. Lastly how hazard rate changes in *x* (or the curvature properties of the hazard rate). In all the comparison we note that our assumptions about these three properties allow us to derive the sufficient conditions and hence a particular ranking. What do the sufficient conditions mean and what are the implications?

Consider first our comparison of mixed and pure duopoly where our interest was to explore the conditions under which a mixed duopoly would perform more research than a pure duopoly (or vice versa; refer to Table III.3b). For the research intensity of the mixed market to be greater than that of the pure duopoly it is observed that four conditions need to be satisfied. The first condition, that profits be distributed equally across both the winning and losing states in the pure duopoly, will be satisfied if one assumes that the losing firm easily imitates the innovation. This would arise if property rights protection is weak that results in an inability of the winning firm to appropriate the returns to the innovation[20].

[20] A simple example illustrates this point. Consider that post innovation, the winning duopolist is able to lower its cost to $C[\underline{\gamma}, q_{WD}, x_D^*]$. If the losing firm is able to imitate the technology than it too will have the same technology and cost function, that would result in equilibrium conditions where both firms produce the same amount and make equal profits (i.e. with all firms having the same cost function, the symmetric result would hold).

The second and third conditions relate to the profits/welfare function in the winning state are strictly greater than the losing state. The second condition is satisfied if we assume that the first-stage R&D enters into the firm's second stage profit function as a autonomous cost reduction. For example, if we assume that the cost of production, post innovation, for the winning firm is $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma})q_{WD} - x_D^*$, and for the losing firm $C[\gamma, q_{LD}] = (\gamma)q_{LD}$ then it can be shown that $W_{Li}[x_j^*] < W_{Wi}[x_g^*]$ for $i = S, P, D$ and $\forall x_g, x_j > 0$ (see appendix).

The third condition relates to the curvature properties of the profit (for the private firm) and welfare (for the public) functions. That is if the profit function of the winning private firm (in the mixed duopoly) and the welfare function of the public firm are "more" concave in *x* than the profit function of the winning duopolist, then this condition is satisfied.

The last sufficient condition for the mixed duopoly to perform more R&D than that in the pure duopoly can be restated as such:

$$1 + W_{WP}^{'}[x_D^*](h[x_D^*] + h[x_S^*]) - W_{WD}^{'}[x_D^*](2h[x_D^*]) > \frac{h[x_S^*]}{h[x_D^*]} > 1 \text{ for } x_S^* > x_D^* > 0$$

A case where this condition is satisfied is when we assume that the hazard rate is bounded such that $h[x] \to 1$ as $x \to \infty$ (e.g. a logarithmic function) and that equilibrium R&D is "high" for both the firm in the private duopoly and the public firm (i.e. $\frac{h[x_S^*]}{h[x_D^*]}$ approaches 1 from above). Since $W_{WP}^{'}[x_D^*](h[x_D^*] + h[x_S^*]) > W_{WD}^{'}[x_D^*](2h[x_D^*])$, it follows that the left hand side will be greater than $\frac{h[x_S^*]}{h[x_D^*]}$, iff, $W_{WP}^{'}[x_D^*] \gg W_{WD}^{'}[x_D^*]$. An interpretation of this sufficient condition is that the innovation is drastic for the winning private firm in the mixed duopoly and non-drastic for the winning duopolist (in the pure duopoly).

We next turn to the interpretation of the conditions under which a mixed duopoly would perform more R&D than a monopolist (refer to Table III.2). These conditions are also related to the properties of the profit/welfare function and hazard rate. If for example, the public firm's welfare function is more concave in R&D than the monopolist than the first condition is satisfied. The second sufficient condition is satisfied if we assume that the hazard rate is relatively constant, for example a linear hazard rate. The third condition requires that public welfare be strictly greater in the winning state than in the losing state, an example of which was given earlier.

The last sufficient condition ($rh[x_M^*]W_{WP}^{'}[x_M^*] > 1$) follows from the earlier sufficient conditions and is met when those conditions suffice. Recall that when the winning firm in the mixed duopoly wins it becomes a monopolist in the second stage. Hence $W_{WP}[x_M^*] = W_M[x_M^*]$. Since for the first sufficient condition to be met, it suffices that the public firm's welfare function be more concave than the monopolist, it follows that it should be also more concave than the winning private firm's profit function in the mixed duopoly.

In our comparison of the mixed duopoly with that of pure duopoly we also derived a set of sufficient conditions, which, if met, ranked individual research effort at the firm level (Table III.3a). The rankings, however, do not allow for an unambiguous statement on whether at the industry level one market performs more R&D than the other. Focusing on the sufficient conditions which ranks the amount of research performed by the public firm above that of the private firms (either in the mixed duopoly or the pure duopoly) we note that all the sufficient conditions relate to the losing and winning profits of the private firms (which need to be equal) and the concavity properties of the public firm's welfare function in the winning state (the public firm's welfare function has to be more concave in R&D than the profit function of the private firms).

Our analysis also presents a comparison of the pure duopoly market with that of a monopoly market (Table III.1). Here, too, the sufficient conditions for ranking the industry R&D in the two markets is function of curvature properties of the profit function and the hazard rate as well as the distribution of profits in the duopoly. For example, when the profits after the race are distributed evenly among winners and losers and the profits for the monopolist is larger than it is for the winning duopolist, then it suffices that the sole firm in the monopoly market performs more research than the firms in the duopoly.

6. Concluding Remarks

The analysis of this paper was motivated by the observation that in genomics research, the amount of R&D expended to win an R&D race affects not only the probability of success but also downstream profits. Further we observe that the public research sector is engaged in fierce competition with private firms in a variety of genomics projects. Since it was not clear, a priori, the reasons for the public firm to undertake genomics research (especially in light of the fact that activities are similar to those of the private firms), we set out in this paper to derive a set of sufficient conditions under which R&D across different plausible and observed markets in genomics research could be ranked. It was found that the sufficient conditions relate to

- the concavity properties of the profit/welfare function with respect to first stage research;
- the distribution of profits across firms in the duopoly markets;
- the magnitude of the gains in the second (profit/welfare) from the innovation relative to other firms as well as pre-innovation profits/welfare; and

- the curvature properties of the hazard rate.

One interpretation of the concavity of profit/welfare is how useful first stage R&D (or the knowledge gained in the racing stage) complements the innovation that is eventually employed in applied research or production. For the firm for which the complementarity effect is the greatest, the incentive to conduct more research will also be larger.

When comparing with the duopoly market, how the profits are distributed across the two firms affects has implication for the sufficient conditions. Distribution of profits across firms can be affected by the nature of the innovation (whether drastic or non-drastic) and/or the ease to which the winning innovation can be imitated by the losing firm. For example, if the innovation is minor such that the distribution of profits of the two firms remains relatively unchanged then the incentives to innovate are less. Similarly if the innovation is easily copied, due to weak property rights perhaps, then the incentives remain weak. In such cases, we find that the mixed market undertakes more R&D then the pure duopoly.

For firms in one market to perform more research relative to firms in another market (e.g. firms in mixed relative to firms in pure duopoly), it also suffices that the absolute profit/welfare gains (relative to the pre-innovation case) be greater than the other market. For example, in the comparison between the mixed duopoly with that of the monopoly market, a sufficient condition under which the public sector performs more research than the monopolist is if the gains to public firm's welfare are larger than they are to the monopolist's profits. This occurs if pre-innovation public firm welfare is low or that the post-innovation welfare is large. It is not difficult to imagine cases where this would be true. For example, if a monopoly exists pre-innovation, then the public welfare is calculated as the summation of the monopolist profits and consumer surplus (e.g. areas A and C in Figure A.1). If the public firm wins and licenses the technology to the monopolist and we assume marginal cost pricing, then the gains to the public welfare will be larger than the profits to the monopolist.

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Appendix

More on Assumption 2:

The second stage post innovation profits of the winning and losing doupolist is given by equations III.7 and III.8:

$$\max_{q_{WD}} W_{WD} = P[Q]q_{WD} - C[\overline{\gamma}, q_{WD}, x_D^*] \tag{18}$$

$$\max_{q_{LD}} W_{LD} = P[Q]q_{LD} - C[\gamma, q_{LD}] \tag{19}$$

where $P[Q] = a - (q_{WD} + q_{LD})$ is the inverse demand, $C[\underline{\gamma}, q_{WD}, x_D^*]$ is the cost of production for the winning firm and $C[\gamma, q_{LD}]$ is the cost of production for the losing firm. Consider three cost regimes, one where marginal cost is increasing (decreasing returns to scale), and two others where marginal cost is constant but differ in how first stage R&D affects production cost (a lump sump reduction in one case and a unit reduction in another). Mathematically the three cost regimes are specified as:

1. Increasing costs: $C[\underline{\gamma}, q_{WD}, x_D^*] = \dfrac{\underline{\gamma}}{(1+x_D)} \dfrac{(q_{WD})^2}{2}$, $C[\gamma, q_{LD}] = \gamma \dfrac{(q_{LD})^2}{2}$, where $\underline{\gamma} < \gamma$,

   $x_D^* \geq 0$
2. Constant costs with per unit cost decreasing with R&D: $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma} - x_D^*)q_{WD}$,

   $C[\gamma, q_{LD}] = (\gamma)q_{LD}$. Assume, for the constant cost cases, that $a > \gamma > \underline{\gamma} > 0$ and $\underline{\gamma} > x_D$
3. Constant costs with lump sump reduction in total cost: $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma})q_{WD} - x_D^*$,

   $C[\gamma, q_{LD}] = (\gamma)q_{LD}$

Substituting the different cost specifications into equations (III.7) and (III.8), and from the first order condition for a maximum of the profit expression, we can solve for the equilibrium quantities produced by the two firms in the second stage given x_D^* and that a winner to the R&D race has been determined.

1. If increasing costs then

$$q_{WD}^*[x_D^*] = \frac{a(1+x_D^*)(1+\gamma)}{(1+x_D^*)(3+2\gamma)+\underline{\gamma}(2+\gamma)} \tag{20}$$

$$q_{LD}^*[x_D^*] = \frac{a(1+x_D^*+\underline{\gamma})}{(1+x_D^*)(3+2\gamma)+\underline{\gamma}(2+\gamma)} \tag{21}$$

2. If constant costs than

$$q_{WD}^*[x_D^*] = \frac{a - 2\underline{\gamma} + \gamma + 2x_D^*}{3} \tag{22}$$

$$q_{LD}^{*}[x_D^{*}] = \frac{a + \underline{\gamma} - 2\gamma - x_D^{*}}{3} \tag{23}$$

3. If constant costs with lump sump reduction due to R&D

$$q_{WD}^{*}[x_D^{*}] = \frac{a - 2\underline{\gamma} + \gamma}{3} \tag{24}$$

$$q_{LD}^{*}[x_D^{*}] = \frac{a + \underline{\gamma} - 2\gamma}{3} \tag{25}$$

It is easily verified that under the three regimes, $q_{WD}^{*}[x_D^{*}] \geq q_{LD}^{*}[x_D^{*}]$, $W_{WD}^{*}[x_D^{*}] \geq W_{LD}^{*}[x_D^{*}]$, $\partial W_{WD}^{*}[x_D^{*}] / \partial x_D^{*} > 0$ and $\partial W_{LD}^{*}[x_D^{*}] / \partial x_D^{*} < 0$.

More on Assumption 3:

When the mixed duopoly private firm wins, it faces no rival and its maximization problem is simply the maximization of a monopolist. That is it maximizes the following profit expression

$$\max_{q_{WP}} W_{WP} = Pq_{WP} - C[\overline{\gamma}, q_{WP}, x_P^{*}] \tag{26}$$

From the first order condition, the private firm chooses that output level for which $MR = MC$. At that output level, profits and the public welfare can be calculated[21]. On the other hand if the public firm emerges as the winner, since it does not participate in the second stage it necessarily must introduce the technology through a licensing arrangement with the private firm. The licensing arrangement is such that it maximizes the public welfare subject to the incentive compatibility condition of the private firm. The output level that comes from solving this maximization is then used to calculate the equilibrium profits for the private firm and welfare for the public firm. The maximization problem for the public firm then is written as

$$\max_{\mathrm{q_{LP}}} \quad W_{WS} = \int_0^{Q^*} P(Q)dQ - P^*Q^* + W_{LP}[x_S] + F \quad \text{s.t. } W_{LP}[x_S] \geq W_P \tag{27}$$

where

$\int_0^{Q^*} P[Q]dQ - P^*Q^*$ is the consumer surplus

F is the licensing fee charged by the public firm

$W_{LP}[x_D] = Pq_{LP} - C[\overline{\gamma}, q_{LP}, x_M^{*}] - F$ is the private firm's second stage profits when it licenses the technology from the public firm (the winner)

[21] Public firm's welfare in the second stage is defined as the summation of consumer and producer surplus

$W_P = Pq_P - C[\gamma, q_P] = W_M$ is the private firms pre-innovation profits, where we assume that pre-innovation profits of a private firm is equivalent to that of the monopolist, pre-innovation.

From the first order conditions for a maximum of the public firm's objective function eqn. (27), the equilibrium levels of quantity produced and profits can be calculated. It can be shown (see appendix) that in equilibrium $W^*_{WS}[x^*_S] > W^*_{LS}[x^*_P] \geq W^*_{WP}[x^*_P] \geq W^*_{LP}[x^*_S]$ and $q^*_{LP}[x^*_S] \geq q^*_{WP}[x^*_P]$.

To show that $W^*_{WS}[x^*_S] > W^*_{LS}[x^*_P] \geq W^*_{WP}[x^*_P] \geq W^*_{LP}[x^*_S]$, consider Figures A.1 and A.2 below which assumes a linear inverse demand curve and increasing marginal cost. If the winner of the R&D race is the private firm than it takes on a monopoly position (in the absence of another firm). Monopoly profits are maximized where $MR = MC$ (figure A.1), in which case monopoly profits ($W^*_{WP}[x^*_P]$) are given by area *C* and consumer surplus is given by area *A*. The public firm's welfare is simply the aggregation of monopolist's profits and consumer surplus, i.e. $W^*_{LS}[x^*_P] = A + C$.

Next consider the case where the winner of the R&D race is the public firm that must license the technology to the private firm. However to prevent monopoly pricing, and the associated welfare loss, the public firm dictates the terms of the licenses such that welfare is maximized (the first best welfare outcome) which occurs where $P = MC$. In this case, the licensees' profits ($W^*_{LP}[x^*_P]$) are given by area *E* and the consumer surplus is given by area *D* in Figure A.2. Comparing the two figures, it is clear that $W^*_{WS}[x^*_P] = D + E > W^*_{LS}[x^*_P] = A + C$. Moreover $W^*_{LP}[x^*_P] = E < W^*_{WP}[x^*_P] = E$.

Figure A.1: Second stage mixed market if private firm is the winner of the R&D race

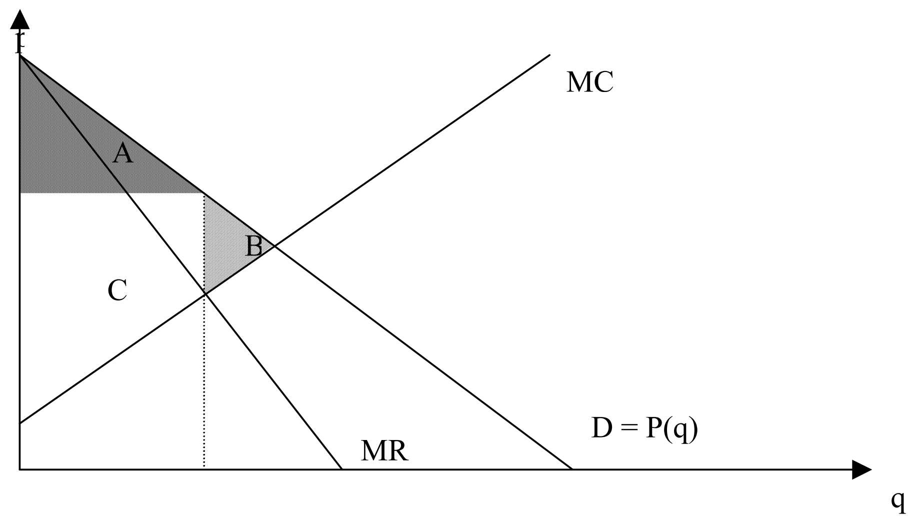

Figure A.2: Second stage mixed market if public firm is the winner of the R&D race

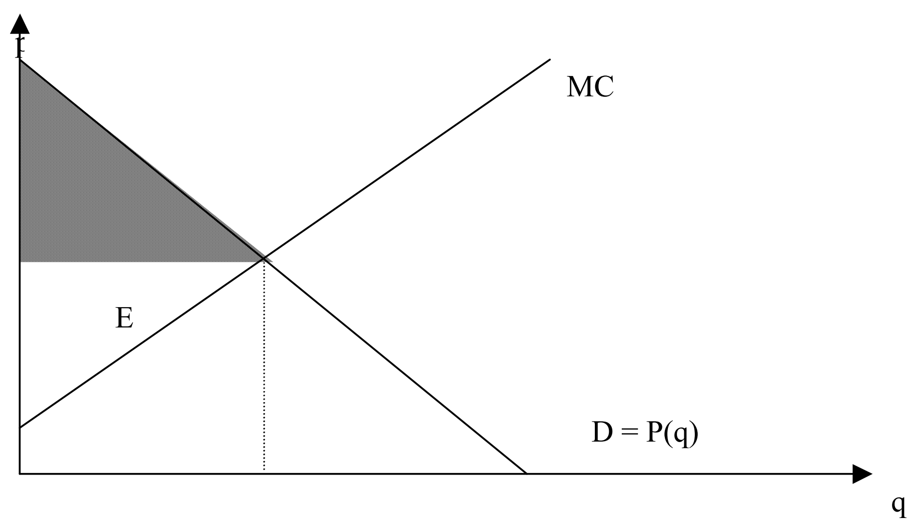

Table III.1. Sufficient conditions for ranking monopoly and pure duopoly research effort

$x_M^* < x_D^*$ (implies $x_M^* < 2x_D^*$)	$x_M^* > 2x_D^*$
1. The profit function of the winning doupolist is increasing at a faster rate in x than the monopolist's profit function (i.e $\frac{\partial W_M[x_D^*]}{\partial x} < \frac{\partial W_{WD}[x_D^*]}{\partial x}$) 2. Relative to their respective pre-innovation profits, the duopoly winner makes more profits than a monopolist. $\lvert W_D - W_{WD}[x_D^*] \rvert > \lvert W_M - W_M[x_D^*] \rvert$ 3. $\frac{r}{h[x_D^*]} < 2W_{WD}'[x_D^*] - W_M'[x_D^*]$	1. The difference between the duopoly winner and loser is not too big (i.e. $W_{LD}[x_D^*] - W_{WD}[x_D^*] \approx 0$) 2. Relative to their respective pre-innovation profits, the duopoly winner makes lower profits than a monopolist. $\lvert W_D - W_{WD}[x_D^*] \rvert < \lvert W_M - W_M[x_D^*] \rvert$ 3. The slope of the profit function is relatively constant (with respect to x such that $\frac{\partial W_M[2x_D^*]}{\partial x} \geq 2\frac{\partial W_{WD}[x_D^*]}{\partial x}$ holds

Table III.2. Sufficient conditions for ranking monopoly and mixed duopoly research effort

$x_P^* > x_M^*$ and $x_S^* > x_M^*$ (implies $x_P^* + x_S^* > x_M^*$)	$x_P^* < x_M^*$ and $x_S^* < x_M^*$ (implies $x_P^* + x_S^* < x_M^*$)
1. Post-innovation, public welfare increases faster than monopoly profits (i.e. $W_{WS}^{'}[x_M^*] \geq W_M^{'}[x_M^*]$ and $\left\|W_{M1} - W_M[x_M^*]\right\| < \left\|W_{WS}[x_M^*] - W_{S1}\right\|$) 2. The hazard rate is relatively constant such that $x_P^* h'[x_M^*] \geq h[x_P^*]$ 3. Public welfare is (strictly) larger when the public firm wins relative to when it loses. $W_{WS}[x_M^*] > W_{LS}[x_P^*]$ 4. $rh[x_M^*]W_{WP}^{'}[x_M^*] > 1$	1. Post-innovation, monopoly profits increases faster than public welfare (i.e. $W_{WS}^{'}[x_M^*] \leq W_M^{'}[x_M^*]$ and $\left\|W_{M1} - W_M[x_M^*]\right\| > \left\|W_{WS}[x_M^*] - W_{S1}\right\|$) 2. Post innovation profits for the private firm is the same regardless of whether it wins or losses (i.e. $W_{WP}[x] = W_{LP}[x]$) 3. $rh[x_M^*]W_{WP}^{'}[x_M^*] < 1$

Table III.3a. Sufficient conditions for ranking pure duopoly and mixed duopoly research effort

$x_P^* > x_D^* > x_S^*$	$x_P^* < x_D^* < x_S^*$
1. Post innovation profits in the duopoly are equal across states ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$).	1. Post innovation profits in the duopoly are equal across states ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$).
2. Post innovation welfare for the public firm in the mixed duopoly is equal across states ($W_{LS}[x_D^*] = W_{WS}[x_P^*]$).	2. Post innovation profits for the private firm in the mixed duopoly are equal across states ($W_{LP}[x_D^*] = W_{WP}[x_S^*]$).
3. Private firm's profits in the mixed duopoly are always greater in the winning state than in the losing state	3. Public welfare is always greater in the winning state than in the losing state.
4. Relative to the pre-innovation profits the winning firm in a pure duopoly gains less than the winning private firm in the mixed duopoly, such that $W'_{WP}[x_D^*] > W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*] \rvert < \lvert W_{WP}[x_D^*] - W_P \rvert$.	4. Relative to the pre-innovation profits the winning firm in a pure duopoly gains more than the winning private firm in the mixed duopoly, such that $W'_{WP}[x_D^*] < W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*] \rvert > \lvert W_{WP}[x_D^*] - W_P \rvert$.
5. Relative to the pre-innovation profits the winning firm in a pure duopoly gains more than the welfare gains of the winning public firm in the mixed duopoly, such that $W'_{WS}[x_D^*] < W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*] \rvert < \lvert W_{WS}[x_D^*] - W_S \rvert$.	5. Relative to the pre-innovation profits the winning firm in a pure duopoly gains less than the welfare gains of the winning public firm in the mixed duopoly, such that $W'_{WS}[x_D^*] > W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*] \rvert > \lvert W_{WS}[x_D^*] - W_S \rvert$.
6. $W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) > W'_{WD}[x_D^*](2h[x_D^*])$	6. $W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) < W'_{WD}[x_D^*](2h[x_D^*])$
7. $W'_{WS}[x_D^*](h[x_D^*] + h[x_P^*]) < W'_{WD}[x_D^*](2h[x_D^*]) + 1$	7. $W'_{WS}[x_D^*](h[x_D^*] + h[x_P^*]) > W'_{WD}[x_D^*](2h[x_D^*])$
8. $x_P^* h'[x_D^*] < h[x_P^*]$	

Table III.3b. Sufficient conditions for ranking pure duopoly and mixed duopoly research effort

$x_D > x_S, x_P$ (implies $2x_D < x_S + x_P$)	$x_D < x_S, x_P$ (implies $2x_D < x_S + x_P$)
1. The profits from winning are strickly greater than that from losing in the duopoly case. ($W_{LD}[x_D^*] < W_{WD}[x_D^*]$) 2. The profits/welfare from winning or losing in a mixed duopoly are equal. $W_{LS}[x_P^*] = W_{WS}[x_D^*]$ and $W_{LP}[x_S^*] = W_{WP}[x_D^*]$ 3. The change in profits (welfare) for the private (public) firm in the mixed duopoly is less than that for the doupolist. That is $W'_{WS}[x_D^*] < W'_{WD}[x_D^*]$ and $W'_{WP}[x_D^*] < W'_{WD}[x_D^*]$. 4. $\begin{bmatrix} h[x_D^*](1+W'_{WP}[x_D^*](h[x_D^*]+h[x_S^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*])) \end{bmatrix} < h[x_S^*]$ 5. $\begin{bmatrix} 1+W'_{WP}[x_D^*](h[x_D^*]+h[x_P^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*]) \end{bmatrix} > \frac{h[x_P^*] - x_P^* h'[x_D^*]}{h[x_D^*]}$	1. The profits from winning or losing in the duopoly case are equal ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$) 2. The profits/welfare from winning are strickly greater than losing in a mixed duopoly. $W_{LS}[x_P^*] < W_{WS}[x_D^*]$ and $W_{LP}[x_S^*] < W_{WP}[x_D^*]$ 3. The change in profits (welfare) for the private (public) firm in the mixed duopoly is greater than that for the doupolist. That is $W'_{WS}[x_D^*] > W'_{WD}[x_D^*]$ and $W'_{WP}[x_D^*] > W'_{WD}[x_D^*]$. 4. $\begin{bmatrix} h[x_D^*](1+W'_{WP}[x_D^*](h[x_D^*]+h[x_S^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*])) \end{bmatrix} > h[x_S^*]$