Assessment of Market Risk in Hog Production using Value-at-Risk and Extreme Value Theory

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Paper prepared for the Annual Meeting of the American Agricultural Economics Association
2002 in Long Beach, CA

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Keywords: Value-at-Risk, Extreme-Value-Theory, Risk in Hog Production

May 2002

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1 Introduction

In the last quarter in 2000 German hog and cattle producers have been exposed to tremendous price fluctuations due to the BSE crisis and the foot and mouth disease. Many farmers suffered severe liquidity problems and the EU was forced to stabilize markets by massive intervention. These events in conjunction with the anticipation that price volatility on agricultural markets will rise according to the intended liberalization of the CAP increased the demand for indicators showing the risk exposition of farms. A concept discussed in this context is Value-at-Risk (VaR). VaR has been established as a standard tool in financial institutions to depict the downside risk of a market portfolio. It measures the maximum loss of the portfolio value that will occur over some period at some specific confidence level due to risky market factors (Jorion 1997). Though VaR has been primarily designed for the needs in financial institutions it also has been successfully applied in agriculture (Manfredo and Leuthold 1999). However, some well known problems have to be overcome when utilizing VaR. First, the time horizon in agricultural applications will in general be longer than in financial applications and hence the question of extrapolating, let's say, a week-to-week volatility forecast arises. The usual way to achieve this is to use the square-root-rule. Unfortunately this method presumes iid returns and little is known about its properties if returns are not independent (for instance if they follow a GARCH process or a mean reverting process). Secondly, common VaR models have difficulties in estimating the left tail of the return distribution in particular if long time series of historical prices are not available. However, in the case of the livestock crisis described above the prediction of extreme events is of particular interest. Diebold et al. (1998) suggest the use of Extreme Value Theory (EVT) to improve the estimation of extreme quantiles.

The objective of this paper is to investigate the performance of different VaR models in the context of risk assessment in hog production. Potential pitfalls of traditional VaR models are pinpointed and proposals to solve them are analyzed. After a brief description these methods are used to calculate the VaR of the hog finishing margin under German market conditions. In particular we apply EVT to our data and compare the results with historical simulation and the variance-covariance method. Hill's estimator is used to determine the tail index of the extreme distribution of the gross margin in hog finishing and farrow production. It turns out that the results are sensitive with respect to the choice of the sample fraction. To overcome this problem we adopt a bootstrap method proposed by Danielsson et al. (1999).

2 Value-at-Risk

2.1 Definition

Briefly stated VaR measures the maximum expected loss over a given time period at a given confidence level that may arise from uncertain market factors. Call W the value of an asset or a portfolio of assets and $V = W_{t_1} - W_{t_0}$ the random change (revenue) of this value during the period $h = Dt = t_1 - t_0$, then VaR is defined as follows:

$$\text{VaR} = E(V) - V^* \tag{1}$$

E(V) means the expectation of V and the critical revenue V^* is defined by:

$$\int_{-\infty}^{V^*} f(v)dv = \text{Prob}(v \leq V^*) = p \tag{2}$$

Using the identity $V = W_{t_0} \cdot X$ with $X = \ln(W_{t_1} / W_{t_0})$ VaR can also be expressed in terms of the critical return *X**:

$$\text{VaR} = W_{t_0}(E(X) - X^*) \tag{3}$$

E(X) and *X** are defined analogous to *E(V)* and *V**. From (2) it is obvious, that the calculation of VaR boils down to finding the *p*-quantile of the random variable *V*, i.e. the profit-and-loss-distribution. Alternative methods exist to achieve this. A brief summary is given in the next section.

2.2 Methods of VaR calculation

The literature offers three standard procedures for VaR estimation, namely the variance-covariance-method (VCM), Monte Carlo simulation (MS) and historical simulation (HS), all showing specific advantages and disadvantages. A detailed treatment of these methods can be found in Jorion (1997) and Dowd (1998). This paper only provides some basics thereby focusing on deficiencies that we try to overcome later.

Variance-Covariance-Method

The VCM (also called parametric approach or delta-normal method) determines VaR directly as a function of the volatility of the portfolio return s. If normality of the returns is assumed, VaR can be determined as:

$$\text{VaR} = W_{t_0} \cdot c \cdot s \cdot \sqrt{h}\ . \tag{4}$$

Herein c denotes the p-quantile of the standard normal distribution. $\sqrt{h}$ is a scaling factor that adapts the time horizon of the volatility to the length of the holding period h. The problems that may arise when using such a scaling factor are discussed in section 2.4. In the case of a portfolio that consists of n assets the volatility of the portfolio return is calculated according to:

$$\sigma_p = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_i \cdot w_j \cdot \sigma_{ij} \right)^{0.5} \qquad (5)$$

w_i and w_j are the weights of assets i and j and σ_{ij} is the covariance of their returns.

An apparent advantage of the VCM is its ease of computation. If the normality assumption holds, VaR figures can be simply translated across different holding periods and confidence levels. Moreover, time-varying volatility measures can be incorporated and what-if-analyses are easy to conduct. On the other hand, the normality assumption is frequently criticized. There is empirical evidence that return distributions are fat tailed and in that case the VCM will underestimate the VaR for high confidence levels. Further problems occur, if the portfolio return depends in a nonlinear way on the underlying risk factors, which is typically the case with options.

Monte Carlo simulation

With this method the entire distribution of the value change of the portfolio is generated and VaR is measured as an appropriate quantile from this relative frequency distribution. The simulation involves the following steps:

- Selection of distributions for the changes of the relevant market factors (e.g. commodity prices) and estimation of the appropriate parameters, in particular variances and correlations
- simulation of random paths for the market factors
- evaluation of the portfolio for the desired forecast horizon ("mark-to-market")
- calculation of the gains or losses related to the current portfolio value
- repetition of the three aforementioned steps until a sufficient accuracy is gained
- ordering of the value changes in ascending order and determination of the frequency distribution

The main advantage of the Monte Carlo simulation is the ability to handle different return distributions. Harmful are the high costs of computation in the case of complex portfolios.

Historical simulation

Historical simulation (HS) resembles the Monte Carlo simulation regarding the iteration steps. The difference is that the value changes of the portfolio are not simulated by means of a random number generator, but directly calculated from observed historical data. That means, VaR estimates are derived from the empirical profit-and-loss-distribution. Hence no explicit assumption about the return distribution is required here. However, this procedure implicitly assumes a constant (stable) distribution of the market factors. A general problem arises from the fact that the empirical distribution function, while being relatively smooth around the mean, shows discrete jumps in the tails due to the small number of extreme sample values. The higher the desired confidence level is, the more uncertain the estimation of the corresponding quantile becomes. Accordingly, VaR estimates based on HS react sensitively to modifications of the data sample. The probability of events that are worse than the sample minimum cannot be predicted per definitionem. The extreme value theory, described in section 3, offers an opportunity to avoid these problems.

2.3 Modeling the return distribution

If a parametric approach to VaR estimation is utilized the question arises, which distribution function fits best to the observed changes of the market factors. As mentioned above it is widely recognized in the literature that empirical return distributions of financial assets are characterized by fat tails. With respect to modeling the underlying stochastic process two consequences can be deduced (Jorion 1997, p. 166 f.): either one uses a leptocurtic distribution, e.g. a *t*-distribution, or one resorts to a model with stochastic volatility. Of course both approaches can be combined. The observation of volatility clusters in high frequency (i.e. daily) data rows favours the use of models with stochastic volatilities. The change of phases of relatively small and relatively high fluctuations of returns can be captured with GARCH models. Yang and Brorsen (1992) demonstrate that GARCH models are not only relevant for financial applications, but also appropriate to describe the development of daily spot market prices of agricultural commodities. Herein a stochastic process of the form

$$X_t = m_t + s_t e_t \qquad (6)$$

is assumed for the returns. e_t are iid rvs. In most applications normal or *t*-distributions for the disturbance variable e_t are presumed. In a GARCH(1,1) process the variance s_t^2 develops according to

$$s_{t+1}^2 = w^2 + dX_t^2 + bs_t^2 \qquad (7)$$

with $w = g\bar{s}^2 > 0, \quad d \geq 0, \quad b \geq 0, \quad d + b < 1$

$\bar{s}^2$ is a long-term average value of the variance, from which the current variance can deviate in accordance with (7). Obviously the use of models with stochastic volatility implies a permanent updating of the variances and thus the VaR forecasts.

While conditional models are superior for short term forecasts, their value vanishes with increasing time horizon. Christoffersen and Diebold (2000) argue that the recent history of data series has little to tell about the probability of events occurring far in the future. This applies especially to the prediction of rare events like disasters, which are assumed to be stochastically independent. Therefore Danielsson and de Vries (2000) recommend to derive predictions about extreme events from unconditional distributions.

2.4 Long-Term-Value-at-Risk

Much of this paper is motivated by the supposition that the relevant VaR horizon in agricultural applications in general will be longer than in a financial context, where one-day or few-day forecasts dominate. Moreover, the desired horizon will often be larger than the frequency of the data. For example, a farmer having in mind the length of a production cycle wishes to determine the VaR for three or six months on the basis of weekly price data. Basically two methods exist to calculate long-term-VaRs: either one measures the value changes that occur during the entire holding period, that means, the VaR is estimated on the basis of three or six month's returns. Alternatively, a short-term VaR is extrapolated to the desired holding period (time scaling). The first procedure is applicable independently of the return distribution. However, it has the serious drawback that the number of observations is reduced strongly. If, for instance, weekly data are available over a period of 10 years a six-month-VaR is based on 20 observations only, since the measurement periods should be non-overlapping. The second method avoids this problem. In practice the time-scaling is conducted by means of the square-root-rule.

$$\mathrm{VaR}(h) = \mathrm{VaR}(1) \cdot \sqrt{h} \qquad (8)$$

VaR(*1*) and VaR(*h*) denote the one-period-VaR and the *h*-period-VaR, respectively. Diebold et al. (1997) point out that the correctness of the square-root-rule relies on three conditions. First, the structure of the considered portfolio may not change in the course of time. Secondly, the returns must be identically and independently distributed, and thirdly, they must be normally distributed. Section 3.1 discusses the consequences of non-normality for time aggregation. At this point we ask what would happen if the iid assumption is not fulfilled. Though a general answer to this question is not available, Drost and Nijman (1993) provide a formula for the correct time aggregation of a GARCH process. For the GARCH(1,1)-process described above the h -period-volatilities can be determined from the one-period-volatilities as follows:

$$s^2_{t+1}(h) = w(h)^2 + d(h) X^2_t(h) + b(h) s^2_t(h) \qquad (9)$$

$$\text{with } w(h) = hw \frac{1-(d+b)^h}{1-(d+b)}$$

$$d(h) = (d+b)^h - b(h)$$

$$\text{and } |b(h)| < 1 \text{ as solution of the quadratic equation}$$

$$\frac{b(h)}{1+b^2(h)} = \frac{a-(d+b)^h - b}{a(1+(d+b)^{2h}) - 2b}$$

The coefficients *a* und *b* are defined as:

$$a = h(1-b)^2 + 2h(h-1)\frac{(1-d-b)^2(1-2db-b^2)}{(k-1)(1-(d+b)^2)}$$
$$+4\frac{(h-1-h(d+b)+(d+b)^h)(d-db(d+b))}{1-(d+b)^2}$$
$$b = (d-db(d+b))\frac{1-(d+b)^{2h}}{1-(d+b)^2}$$

k denotes the kurtosis of the return distribution.

Comparing (9) with (8) reveals systematic differences, which become larger with increasing *h*. If *h* goes to infinity, *d* and *b* in (9) converge to zero and hence the stochastic terms vanish, whereas the first deterministic term increases. That means that the average levels of the *h*-period-volatility coincide in both cases, but the square-root-rule magnifies the fluctuations of the volatility, while they actually become smaller with increasing time horizon. Diebold et al. (1997) illustrate the magnitude of the difference of both methods of volatility forecasting by means of simulation experiments. Similar calculations in the context of our application are presented in section 4.

3 Extreme-Value-Theory

From the discussion in section 2.2 some pitfalls of traditional methods of VaR estimation became obvious, in particular if the prediction of very rare events is desired and leptocurtic distributions are involved. Now we turn to the Extreme-Value-Theory (EVT) in order to improve the estimation of extreme quantiles[1]. EVT provides statistical tools to estimate the tails of probability distributions. Some basic concepts are briefly addressed below. A much more comprehensive treatment can be found in Embrechts et al. (1997).

3.1 Basic concepts

A main objective of the EVT is to make inferences about sample extrema (maxima or minima)[2]. In this context the so called Generalized Extreme Value distribution (GEV) plays a central role. Using the Fisher-Tipplet theorem it can be shown that for a broad class of distributions the normalized sample maxima converge towards the Generalized Extreme Value distribution with increasing sample size. If $X_1, X_2, \ldots, X_n$ are iid random variables from an unknown distribution *F*, and let a_n und b_n be appropriate normalization coefficients, then for the sample maxima $M_n = max(X_1, X_2, \cdots, X_n)$ holds:

1 We emphasize that EVT is not the only method to cope with extreme events and fat tailedness. For example, Li (1999) uses a semiparametric approach to VaR estimation, which takes into account skewness and kurtosis of the return distribution in addition to the variance. Moreover, stress testing is a rather widespread technique that may be used as a complement to traditional VaR methods. It gauges the vulnerability of a portfolio under extreme hypothetical scenarios

2 Embrechts et al. (1997, p. 364) express the objective of EVT vividly as „mission improbable: how to predict the unpredictable“.

$$p\,lim\left(\frac{M_n - b_n}{a_n} \leq x\right) = H(x) \qquad (10)$$

plim means the limit of a probability for $n \longrightarrow \infty$ and *H*(x) denotes the GEV, which is defined as follows:

$$H(x) = \begin{cases} exp\left(-(1 + xx)^{-1/x}\right) & if\, x \neq 0 \\ exp\left(-e^{x}\right) & if\, x = 0 \end{cases} \qquad (11)$$

The GEV includes three extreme value distributions as special cases, the Frechet-distribution $(x > 0)$, the Weibull-distribution $(x < 0)$, and the Gumbel-distribution $(x = 0)$. Depending on the parameter x a distribution *F* is classified as fat tailed $(x > 0)$, thin tailed $(x = 0)$ and short tailed $(x < 0)$. In the present context the focus is on the first class of distributions, which includes for example the t-distribution and the Pareto-distribution, but not the normal distribution. Embrechts et al. (1997, p. 131) prove that the sample maxima of a distribution exhibiting fat tails converges towards the Frechet-distribution $F(x) = exp\left(x^{a}\right)$, if the following condition is satisfied:

$$1 - F(x) = x^{-1/x} L(x) \qquad (12)$$

(12) requires that the tails of the distribution *F* behave like a power function. *L*(x) is a slowly varying function and $a = 1/x$ is the tail index of the distribution. The smaller a is, the thicker are the tails. Moreover, (12) indicates that inferences about extreme quantiles of a possibly unknown distribution of *F* can be made as soon as the tail index a and the function *L* have been determined. Section 3.2 describes an estimation procedure for a.

The results of the EVT are also relevant for the aforementioned task of converting short-term VaRs into long-term VaRs. Assume $P(|X| > x) = Cx^{-a}$ applies to a single-period return *X* for large *x*, then due to the linear additivity of the tail risks of fat tailed distributions for a *h*-period return can be deduced (Danielsson and de Vries 2000):

$$P(X_1 + X_2 + \cdots + X_h > x) = hCx^{-a} \qquad (13)$$

For a multi-period-VaR forecast of fat tailed return distribution follows under the iid assumption:

$$\text{VaR}(h) = \text{VaR}(1)h^{1/a} \qquad (14)$$

If the returns have finite variances then $a > 2$ and thus a smaller scaling factor applies than postulated by the square-root-rule (Danielsson et al. 1998). Obviously the square-root-rule is not only questionable if the iid assumption is violated, but also if the return distribution is leptocurtic.

3.2 Estimation of the tail index

Several methods exist to estimate the tail index of a fat tail distribution from empirical data. The most popular is the Hill estimator (Diebold et al. 1998). To implement this procedure the observed losses X are arranged in ascending order: $X_1 > X_2 > \cdots > X_k > \cdots X_n$. The tail index $a = 1/x$ then can be estimated as follows

$$\hat{a}(k) = \left(\frac{1}{k} \sum_{i=1}^{k} \ln X_i - \ln X_{k+1} \right)^{-1} \tag{15}$$

The function L(x) in (12) is usually approximated by a constant C. An estimator for C is (Embrechts et al. 1997, p. 334):

$$\hat{C}_k = \frac{k}{n} X_{k+1}^{\hat{a}} \tag{16}$$

This leads to the following estimator for the tail probabilities and the p-quantile:

$$\hat{F}(x) = p = \frac{k}{n} \left(\frac{X_{k+1}}{x} \right)^{\hat{a}}, x > X_{k+1} \quad \text{i.e.} \tag{17}$$

$$\hat{x}_p = F^{-1}(x) = X_{k+1} \left(\frac{k}{np} \right)^{\frac{1}{\hat{a}}} \tag{18}$$

It can be shown that the Hill Estimator is consistent and asymptotic normal distributed. (Diebold et al. 1998).

The implementation of the estimation procedure requires to determine the threshold value X_k, i.e. the sample size k, on which the tail estimator is based. Unfortunately, the estimation results are strongly influenced by the choice of k. Moreover, a trade-off exists: the more data are included in the estimation of the tail index a, the smaller the variance becomes; however the bias increases at the same time, because the power function in (12) applies only to the tail of the distribution. In order to solve this problem, Danielsson et al. (2001) develop a bootstrap method for the determination of the sampling fraction k/n. The different steps of this iterative procedure are described below.

First resamples $N_{n_1}^* = \{X_1^*, \ldots, X_{n_1}^*\}$ of predetermined size $n_1 < n$ are drawn from the data set $N_n = \{X_1, \ldots, X_{n_1}\}$ with replacement. For any k_1 the asymptotic mean square error (AMSE) $Q(n_1, k_1)$ is calculated:

$$Q(n_1, k_1) = E\left[\left(M_{n_1}^*(k_1) - 2\left(x_{n_1}^*(k_1)\right)^2 \right)^2 | N_n \right] \quad \text{with} \tag{19}$$

$$M^*_{n_1}(k_1)=\frac{1}{k_1}\sum_{i=1}^{k_1}\left(\ln X^*_{n_1,i}-\ln X^*_{n_1,k_1+1}\right)^2 \text{ and} \tag{20}$$

$$x^*_{n_1}(k_1)=\frac{1}{k_1}\sum_{i=1}^{k_1}\ln X^*_{n_1,i}-\ln X^*_{n_1,k_1+1} \tag{21}$$

Then find $k^*_{1,0}(n_1)$, i.e. the value of k_1, which minimizes the AMSE (19):

$$k^*_{1,0}(n_1)=\operatorname{argmin} Q(n_1,k_1) \tag{22}$$

A second step completely analogous to the first one but with a smaller sample size $n_2=\left.(n_1)^2\right/n$ yields $k^*_{2,0}(n_2)$. Next calculate

$$\hat{k}_0(n)=\frac{\left(k^*_{1,0}(n_1)\right)^2}{k^*_{2,0}(n_2)}\left(\frac{\left(\ln k^*_{1,0}(n_1)\right)^2}{\left(2\ln n_1-\ln k^*_{1,0}(n_1)\right)^2}\right)^{\frac{\ln n_1-\ln k^*_{1,0}(n_1)}{\ln n_1}} \tag{23}$$

This allows to calculate the reciprocal tail index estimator:

$$x_n\left(\hat{k}_0\right)=\frac{1}{\hat{k}_0}\sum_{i=1}^{\hat{k}_0}\ln X^*_{n_1,i}-\ln X^*_{n_1,\hat{k}_0+1} \tag{24}$$

This estimation procedure for k depends on two parameters, the number of bootstrap resamples, l, and the sample size, n_1. The number of resamples is in general determined by the available computational facilities. The application presented in section 4 utilizes 10000 repetitions giving very stable results. Evaluation and optimization of n_1, necessitates a further step. Calculate the ratio

$$R(n_1)=\frac{\left(Q(n_1,k^*_{1,0})\right)^2}{Q(n_2,k^*_{2,0})} \tag{25}$$

and determine $n^*_1=\operatorname{argmin} R(n_1)$ numerically. If n^* differs from the initial choice n_1, the previous steps should be repeated. Remember that the quantile estimates derived from EVT are only valid for the tails of the profit-and-loss-distribution. To allow inferences about quantiles in the interior of the distribution, Danielsson and de Vries (2000) propose to link the tail estimator with the empirical distribution function at the threshold X_{k+1}. Thus the particular advantages of the EVT and the HS are combined.

4 Application to hog production

4.1 Model and data

Following Manfredo and Leuthold (1999), who investigate the market risks in US cattle feeding, we now use the VaR approach to quantify the market risk in hog production under German market conditions. Our target is the determination of a 12-week-VaR for three types of producers: firstly, a specialized farrow producer, secondly, a farmer, who is specialized on hog finishing and purchases feeder farrows, and thirdly a farmer, who feeds his self-produced farrows. We assume that prices of farrows and hogs are not fixed by forward contracts; inputs and products are rather bought and sold at current spot market prices. The gross margin (cash flow) at time *t* associated with these production activities is defined as

$$CF_t = a \cdot P_t - \sum_{i=1}^{K} b_i Z_t \qquad (26)$$

Formally the gross margin can be considered as a portfolio consisting of a long-position (the product price *P*) and several short-positions (the factor prices Z_i). Thus (5) can be applied directly. The portfolio weights *a* and b_i now have to be interpreted as technical coefficients (slaughtering weight, fodder consumption etc.). Empirical investigations of Odening and Musshoff (2002) indicate that the market risk in hog production is mainly caused by the prices of farrows and hogs. Other items, e.g. fodder costs, have an impact on the level of the gross margins, but they do not contribute to the fluctuations of the cash flow and therefore we decided to ignore them. Due to this the VaR calculation simplifies considerably in the present case. In what follows we display the VaRs for the farrow prices (the perspective of the specialized farrow producer), for the hog prices (the perspective of the joint production of farrows and hogs) and for the residual of revenues from hog sales and expenses for feeder farrows (the perspective of the specialized hog finishing farm). The weights of feeder farrows and fattened hogs are assumed to be 20 kg and 80 kg, respectively.

Note, that in a strict sense we do not display a Value-at-Risk, but rather a Cash-Flow-at-Risk (CFaR) (Dowd 1998, p. 239 f.)[3]. Despite the formal analogy of both concepts one should have in mind the differences when it comes to an economic interpretation of the figures: VaR quantifies the loss of value of an asset, whereas CFaR addresses a flow of money. The knowledge of a CFaR is presumably valuable in the context of a risk-oriented medium-term financial planning. However, conclusions about the financial endangerment of the farm should be drawn carefully, since the initial cash flow level as well as the duration of the cash flow drop should be taken into account. Experience shows that specialized livestock farms are able to endure losses, if such a period does not persist too long and appropriate profits have been earned before.

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[3] Nevertheless we continue to speak of VaR (in a broader sense) below.

The time series of hog prices and farrow prices, which form the basis for our computations, consist of weekly quotations for East Germany spanning the period from January 1994 until October 2001[4]. Prices are measured in euro per kg live weight and slaughtering weight for farrows and hogs, respectively and refer to an average commercial quality. The original data series are presented in figure A1 in the appendix. Note, that in what follows not the prices themselves, but price changes are considered.[5]

4.2 Empirical results

In line with the discussion in section 2.2 the first step in VaR calculation is to clarify, what kind of distributions underlie the market factors, i.e. hog prices, farrow prices and the feeding margin. This task breaks down into two questions: Firstly, should a conditional or an unconditional model be used and secondly, are the respective distributions fat tailed or thin tailed? To answer the first question a Lagrange Multiplier test to test the presence of conditional heteroscedasticity is employed (Greene 2000, p. 808). This test rejects the null hypothesis of the homoscedasticity for the weekly hog prices changes and for the weekly changes of the hog finishing margin. Thereupon a GARCH(1,1) model for all three time series is estimated[6].

The estimated parameter values are summarized in table 1.

Table 1: Parameters of the GARCH (1,1)-Models

Parameter	farrows	hogs	margin
w	0.000875** (5.71)	0.000727** (3.79)	0.862557* (1.81)
d	0.710047** (6.24)	0.443897** (4,22)	0.164101** (4.24)
b	0.172849** (4.26)	0.276940** (2.34)	0.762881** (12.21)

* level of significance 95%   ** level of significance 99%; t-values in parentheses

All estimated parameters are significant. The standardized residuals $\hat{e}_t / s_t$ indicate no autocorrelations on a 1% level of significance. This applies also to the squared standardized residuals with exception of the farrow price series. Thus the inclusion of further lags into the GARCH model does not appear necessary. Inserting the parameters

[4] The data have been made available to us by the German Price Reporting Agency (Zentrale Markt- und Preisberichtstelle, Berlin).

[5] In financial applications it is common to analyze log returns instead of absolute changes. Their advantage is to be independent of the price level. However, problems occur if values become negative. While this is impossible for prices it may happen with the hog finishing margin.

[6] We refrain from estimating a Bi-GARCH-model for the farrow prices and pig prices to estimate the volatiliy and the VaR of the hog finishing margin. Instead, a univariate GARCH model for the margin is estimated. This corresponds to the procedure, that is used later for the EVT application. It takes into account that EVT at its present stage is only applicable to univariate distributions.

in table 1 into (7) yields 1-week-volatility-forecasts. Next, the 1-week-volatility-forecasts are projected on a 12-week-horizon: this is conducted with the square-root-rule (8) and alternatively with the Drost-Nijman-formula (9). The results for the volatility of the hog price changes are represented in figure 1. The corresponding results for the volatility forecasts of the farrow prices and the feeding margin look very similar (see figure A2 in the appendix).

Figure 1: Temporal aggregation of volatilities (GARCH 1,1)

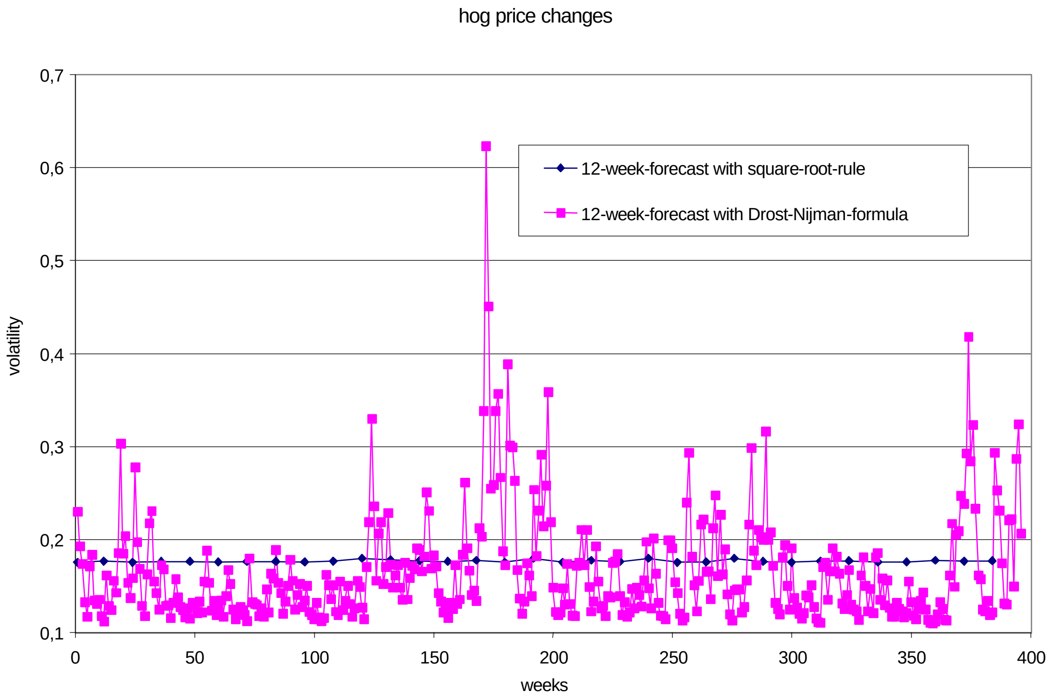


Figure 1 confirms the theoretical considerations in section 2.4. The square-root-rule cannot be regarded as a suitable approximation for a correct time aggregation of the volatility in GARCH models. The actual fluctuations of the volatility are substantially smaller than shown by multiplication with the factor $\sqrt{12}$. That means that VaR forecasts, that are based on this methodology, lead to a permanently overestimation and an underestimation of the true 12-week-VaRs. The correctly determined fluctuation of the 12-week-volatility appears so small that – in accordance with the argumentation of Danielsson and De Vries (2000) – the subsequent application of EVT is based on unconditional distributions, regardless of the measurement of conditional heteroscedasticity in weekly price changes.

We turn to the question whether the considered time series are fat tailed or not. At a first instance this issue can be inspected by QQ-plots, which compare the quantiles of an empirical distribution and a theoretical reference distribution. If the data points are approximately located on a straight line, it can be assumed that the observed data follow the reference distribution. In figure 2 the normal distribution is chosen as a reference distribution.

Figure 2: QQ-plots for farrows, hogs, finishing margin (1 week)

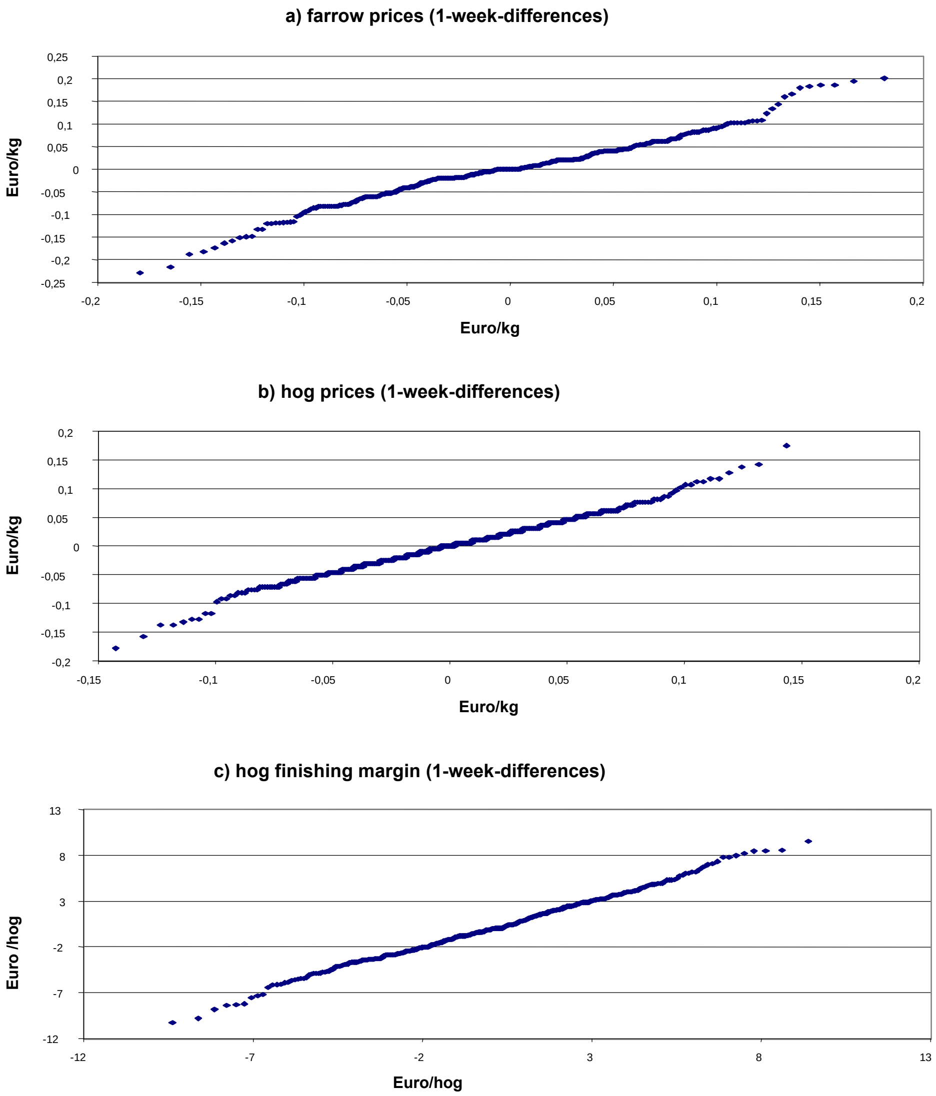


Figure 2 indicates a positive excess for the weekly changes of farrow prices and hog prices whereas the interpretation of the QQ-plot of the feeding margin is less clear. The realization of a Kolmogorov-Smirnoff goodness of fit test supports the conjecture that the analyzed series are not normally distributed. The null hypothesis is rejected on a 5% level for all three distributions. Finally, the Jarque-Bera-test, which summarizes deviations from the normal distribution with respect to skewness and kurtosis, provides further evidence about the non-normality of the distribution. The critical value of the test statistic is 9.2 on a 1% level of significance and is exceeded by the corresponding empirical values of the farrow prices (55.4), the pig prices (55.1) and the feeding margin (23.5). Thus the test results provide evidence that all distributions are fat tailed and justify to estimate an extreme value distribution.

Application of the estimation procedure presented in section 3.2 is straightforward in principle, but the treatment of the hog finishing margin deserves a further comment. Two stochastic variables, the hog prices and the farrow prices, are involved in this case and the question arises, how the EVT, which is designed for the estimation of univariate distributions, can be adapted. Danielsson and De Vries (2000) describe two different approaches to apply EVT to a portfolio, namely post fitting and presampling. Post fitting is similar to Historical simulation insofar as the returns of the different portfolio components are aggregated to give an univariate series of portfolio returns to which EVT can be applied. Correlations need not to be estimated explicitly but they are implicitly assumed to be constant. Presampling, in contrast, is a multivariate approach. A tail estimation is carried out for each portfolio component and samples are drawn from the fitted distributions. A vector of portfolio returns is then calculated taking into account the sample covariance. Obviously post fitting is computationally much simpler and is preferred here.

In order to motivate the aforementioned bootstrap procedure to determine the sample fraction for the tail estimation, we present Hill-estimators based on different values of *k* (figure 3). Apparently the result strongly depends on the number of extreme values which are included into the estimation. The extreme value distributions depicted in figure 4 are already based on the optimized number of extreme values. The respective figures are 6 for the farrow prices, 9 for the pig prices and 3 for the finishing margin. The results of VCM and HS are also depicted for comparative purposes. The estimated tail indices of the extreme value distributions for the 1-week-differences of farrow prices and pig prices are 5.37 and 4.08, respectively. Due to the positive correlation of the price changes of hogs and farrows the fluctuations of the hog finishing margin are smaller than those of the two single series. This is reflected by a relatively large tail index of 7.23 and corresponds also to the previous QQ-plots.

Figure 3: Tail estimators for different sample fractions (1 week)

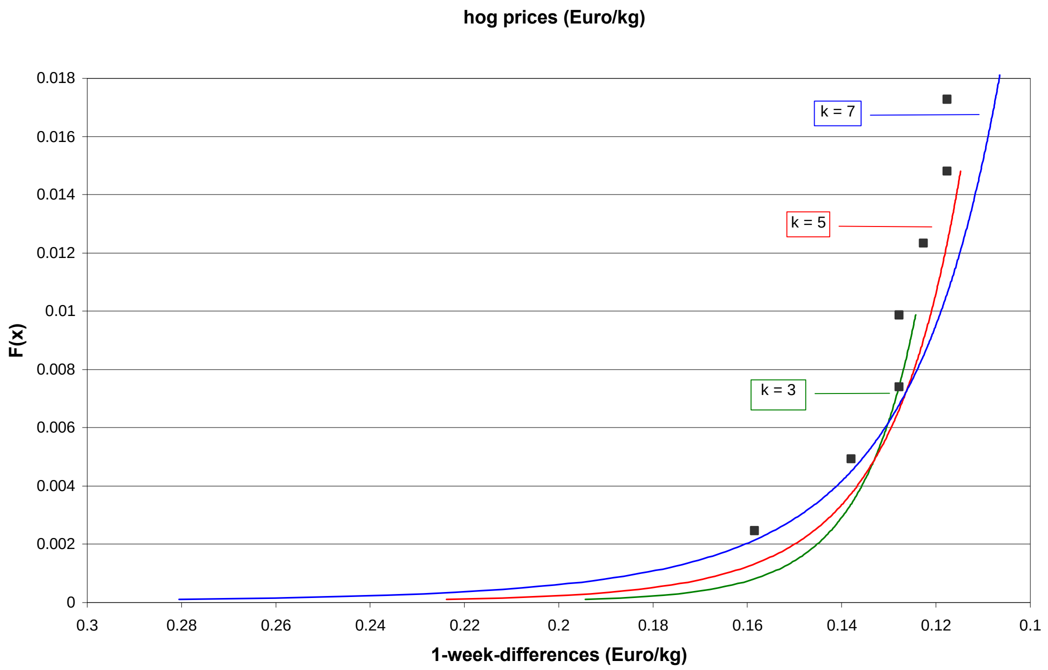
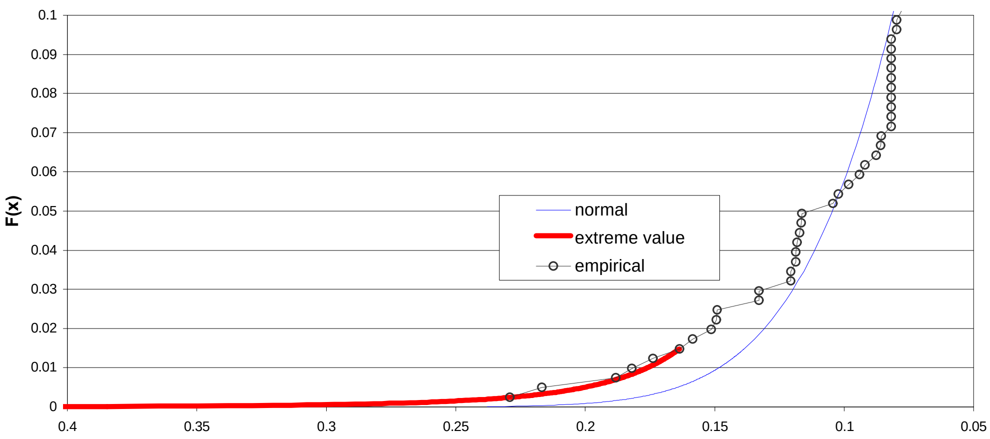
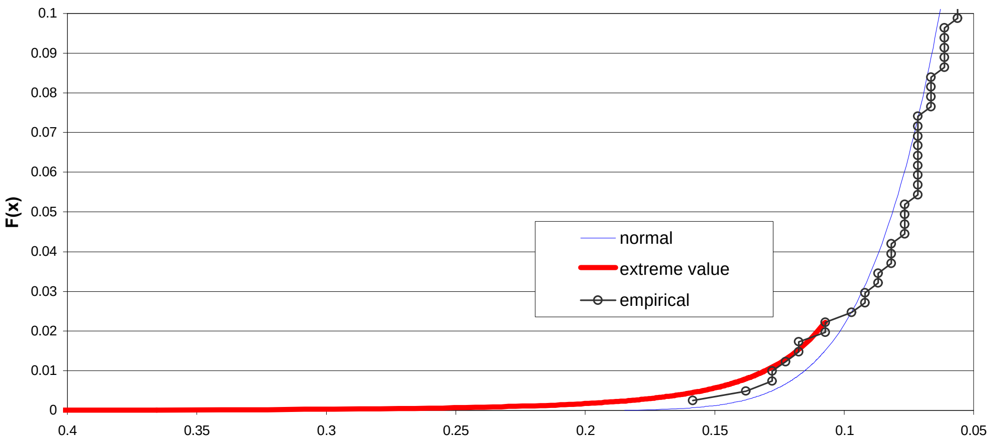


Figure 4: Comparison of normal distribution, empirical distribution and extreme value distribution

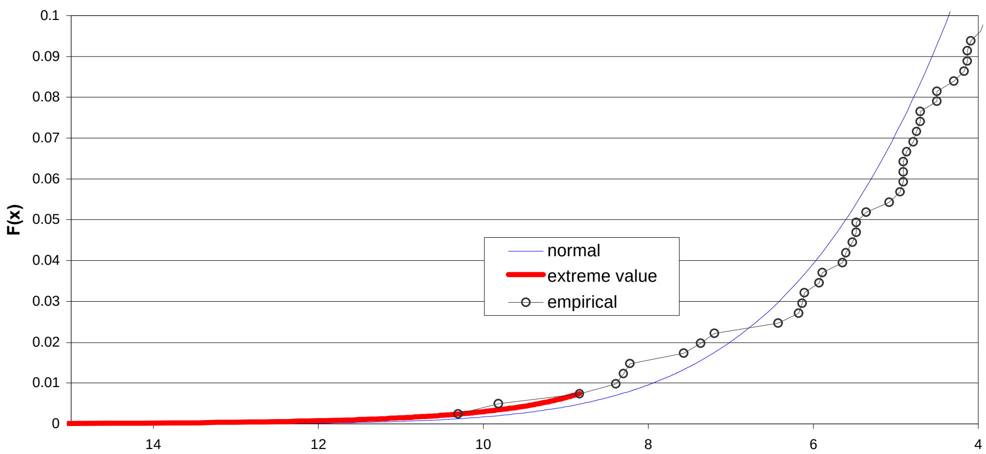
a) farrow prices (1-week-differences, Euro/kg)



b) hog prices (1-week-differences, Euro/kg)



c) hog finishing margin (1-week-differences, Euro/hog)



The last step consists of extrapolating the 1-week-VaRs derived from figure 4 to the target horizon of 12 weeks. In the case of HS and VCM this is done with the square-root-rule, i.e. via multiplication with the factor 3.464. In contrast the quantiles of the extreme value distribution are projected with the alpha-root-rule, i.e. using the respective tail indices á. Table 2 contains the results for different confidence levels. To allow a better comparison the values of the extreme value distributions are also depicted for a 95% confidence level although they already lie to the right of the order statistics X_{k+1} and thus should be taken from HS according to the proposal of Danielsson & De Vries (2000).

Table 2: 1- and 12-week-VaRs for the three time series and for different confidence levels (95%, 99%, 99,9%)

	farrows			**hogs**			**finishing margin**		
confidence level	95.00%	99.00%	99.90%	95.00%	99.00%	99.90%	95.00%	99.00%	99.90%
		Euro			Euro			Euro	
EVT									
1 week	0.130	0.176	0.270	0.088	0.131	0.230	6.786	8.476	11.653
SE	0.012	0,005	0.085	0.006	0.009	0.058	1.034	0.203	1.862
12 week	0.207	0,280	0.429	0.162	0.240	0.422	9.567	11.950	16.429
HS									
1 week	0.104	0.182	-	0.077	0.128	-	5.358	8.303	-
SE	0.439	1.001	-	0.877	0.995	-	0.366	0.501	-
12 week	0.361	0.631	-	0.266	0.443	-	18.562	28.764	-
VCM									
1 week	0.105	0.148	0.197	0.081	0.115	0.153	5.607	7.947	10.571
SE	0.004	0.005	0.007	0.003	0.004	0.005	0.199	0.281	0.373
12 week	0.362	0.514	0.684	0.282	0.400	0.532	19.422	27.531	36.620

Compared to the EVT estimator the VCM shows an underestimation of VaR for a short term forecast. The underestimation, which increases with the confidence level, is a result of the assumed normality of the VCM and the actual leptocurtosis of the distributions. The 1-week-VaR of the VCM for the farrows (hogs and margin) amounts to 0.197 Euro (0.153 and 10.571) on the 99.9% level. The respective figure of the EVT is 0,27 Euro (0.230 and 11.653). The difference should be related to the average price of 1.938 Euro (1.399 and 73.192).

HS and EVT differ only slightly on the 99% level, i.e. the distribution functions of the EVT and the HS intersect in that region (see figure 4). The VaR of the farrows derived from HS is with 0.182 Euro even higher than that the EVT with 0.176 Euro. On the 99.9% level quantiles can not be determined with HS, because losses of this size did not occur during the observation period.

Things appear completely different for the 12-week-VaRs. HS and the VCM overestimate the medium-term VaRs relative to EVT. For example, the 95-percent quantile for the farrows (hogs and margin), derived from the extreme value distribution amounts to 0.207 Euro (0.162 and 9.567) while the VCM and HS display values of 0.362 Euro (0.282 and 19.422) and 0.361 Euro (0.266 and 18.562), respectively. The short term underestimation of the VaRs by HS and VCM is overcompensated by a too conservative time scaling via the square-root-rule.[7] This bias becomes larger with increasing time horizon.

Table 2 further reports asymptotic standard errors of the estimated quantiles. The VCM seemingly shows the smallest estimation error. Some caution is necessary when interpreting the figures. The standard error of the VCM is calculated according to:

$$\mathrm{SE}(\hat{x}_p) = s\,(2n)^{-1/2}\,c_p \qquad (27)$$

$\hat{x}_p$ denotes the estimated p-quantil, c_p is the *p*-quantil of the standard normal distribution and *n* denotes the numbers of observations. However, using (27) is only correct in case of normally distributed rvs. Since the normality assumption was rejected by the data, the displayed standard errors are incorrect as well. Calculation of the standard errors of the HS and the EVT is based on the expressions given in Jorion (1998 p. 99) and Danielsson and de Vries (1997). The figures in table 2 highlight the aforementioned pitfall of HS when it comes to an estimation of extreme quantiles. The standard errors of HS are relatively large for the given sample size of 405 observations. In this respect EVT offers a better alternative.

Usually some kind of validation is conducted subsequent to the VaR estimation. This is usually done by an out-of-sample-prediction (backtesting). For that purpose the sample period is divided into an estimation period and a forecast period. Comparing theoretically expected and actually observed VaR overshoots occurring within the forecast period allows to validate competing models statistically. However, such a validation is not possible in this application due to the relatively short observation period of the price series. An overshoot of a 99%-VaR would occur only once during 100 periods. In our application such an event is expected to happen once within 100·12 weeks, i.e. once within 23 years. We conjecture that validation of VaR models will in general be difficult, if the usual short term horizon in financial applications is extended largely. These difficulties are intensified by the fact that an EVT estimation requires excessive data.

[7] Mc Neil and Frey (2000) criticize the forecast applied here along $h^{1/a}$ and favor a two-stage procedure, which considers conditional heteroscedasticity in a first stage via GARCH estimation and applies EVT to the residuals of the conditional estimation model in a second stage.

5 Summary and conclusions

The previous section exemplifies that the EVT can be applied to problems in agribusiness, what is basically not surprising. It has to be qualified now if and when such an application appears necessary and should be recommended. In order to do so costs and benefits of EVT have to be pondered. Undeniably, computational burden of EVT increases compared to VCM or to HS. The reason for this is not the tail estimation itself, but the bootstrap procedure, which turned out to be necessary for the determination of an optimal sample fraction. However, this disadvantage is weakened, since a tail index estimation will be executed less frequently compared with short-term financial applications, where a permanent updating of VaR forecasts is required when new price information drops in. Regarding the information gain of a EVT based VaR calculation three points have to be emphasized:

1. Short-term VaR is underestimated in particular by the VCM when the return distributions are leptocurtic.
2. Using the alpha-root-rule instead of the common square-root-rule leads to a substantially smaller VaR for longer forecast horizons.
3. The accuracy of the estimation increases compared to HS.

To summarize, the benefits of displaying extreme quantiles depend on the specific problem. Apparently, the informational needs concerning risk differ largely e.g. between a hog producer, a broker trading with hog futures and an insurance company insuring against animal diseases. In some cases the inclusion of additional sources of risk seems more important than to push the confidence level of VaR from 95% to 99.9%. For example, the production risks emanating from foot and mouth disease or BSE for a individual producer, are not echoed by aggregated market prices. However, *if* a calculation of extreme quantiles (e.g. 99% or higher) appears desirable then EVT should be used as a supplement. Additional cost of computation are overruled by a higher accuracy of the tail estimates as well as by significant differences in the temporal aggregation of VaR whenever leptocurtic distributions are involved.

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7 Appendix

Figure A 1: Price time series

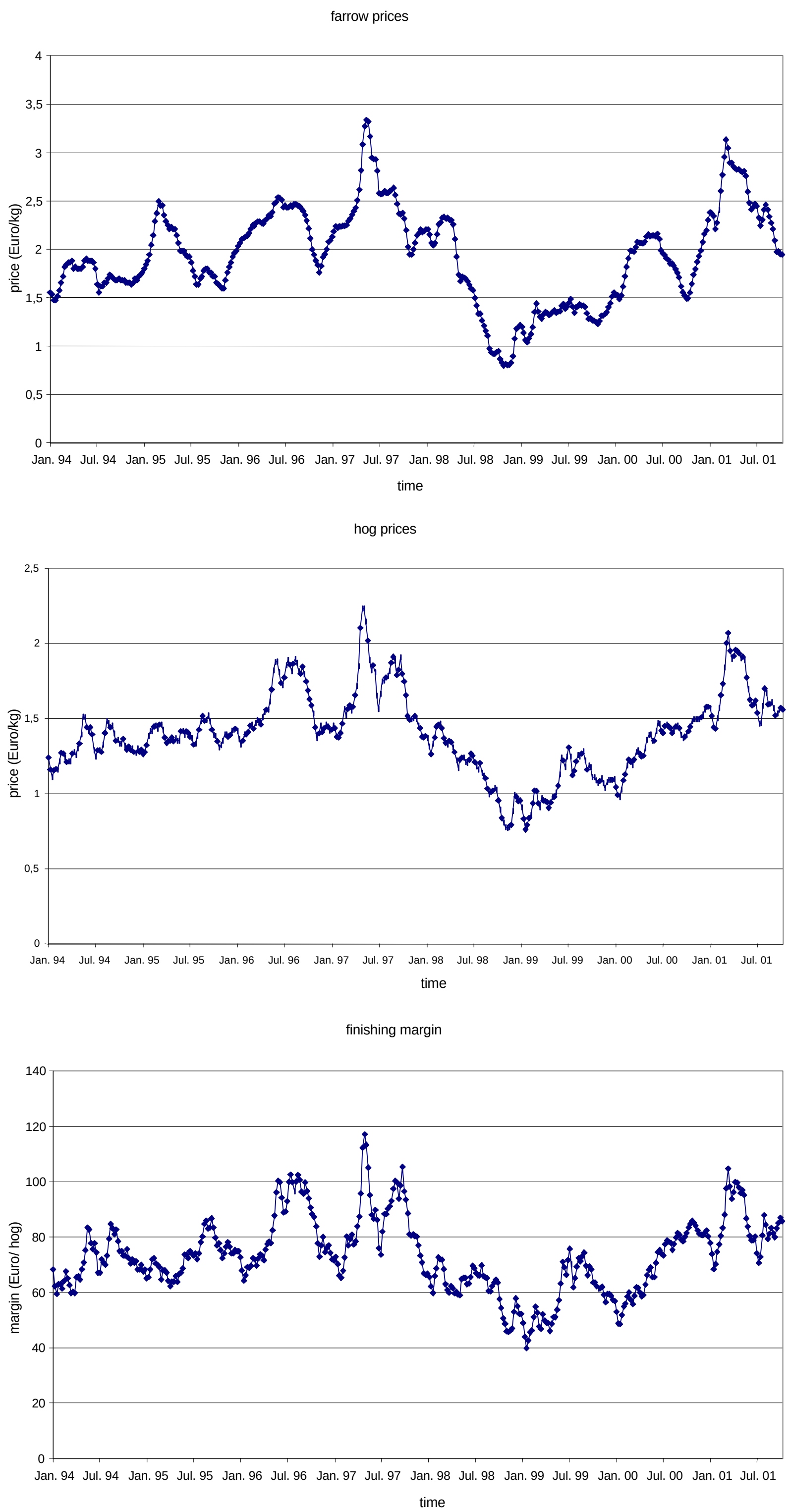

Figure A 2: Temporal aggregation of the volatilities of the GARCH models