Agricultural production under uncertainty: A model of optimal intraseasonal management

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Abstract

This paper develops a dynamic model of crop production under uncertainty with intraseasonal input choices. Crop production involves multiple stages, including at least seeding, post emergence fertilizer/pesticide application and harvesting. If the farmer receives new information about the output and/or price during the stages, he may wish to adjust the input use at each stage in response to the future possible information. Whether future information leads to higher or lower input use at earlier stages depends on the production function, in particular whether inputs at different stages are substitutes or complements in the production function.

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Agricultural crop production takes place over an extended growing period during which a farmer must undertake multiple decisions before obtaining a saleable output. The production process is subject to production or yield uncertainty, as unpredictable growing conditions impact transformation relationships between inputs and outputs, and price uncertainty caused by random price fluctuations during the lag between input decisions and output realization. A model that captures the key features of real world agriculture production should incorporate the following characteristics:

1. Production takes place over an extended period of time and involves a sequence of multiple management decisions.
2. Production is influenced by price and yield uncertainty where most inputs are allocated before output prices are known.
3. Production and market uncertainty is resolved during the production process providing updated information that can be used to assist management decisions.

Static models of agricultural production, while tractably convenient, cannot incorporate the three features listed above and consequently overlook important behavioral elements of agricultural production under uncertainty. Empirical studies derived from static models of factor demand and output supply functions face similar limitations (see the extensive discussion in Just and Pope, *Handbook of Agricultural Economics*). For example, in a dynamic setting management decisions today must consider interactions with future management actions. Factor demands and output supplies will in general depend on a vector of production and market signals realized at different stages of a growing season.

This paper develops a dynamic model of crop production under uncertainty involving three management actions: planting, a post-emergence pesticide application and harvest. Each action incorporates currently available information about production and market shocks, and allows for future actions that will be made conditional on future information. Each stage's inputs such as labor and materials produce interim "products," subject to random production shocks. These products in turn become one of the inputs in the next stage. Given the "product" from the earlier stage, the farmer maximizes the expected payoff in choosing the inputs, incorporating future management actions in later stages.

In a sense, production at an earlier stage creates an "option" for the farmer to respond to new information in later stages. And, current management actions affect the value of the future management option. For example, it may be optimal to plant a crop even though market conditions at planting time suggest output prices that are below average production costs. Market uncertainty means that prices can rise as the growing season progresses. By incurring the cost of planting a crop, the farmer purchases an option to respond to future market developments. If future prices indeed increase, additional inputs can be applied to a planted crop, yielding a positive return. In contrast, the traditional static approach ignores the option value of planting, and predicts an expected price at which the crop is planted to be significantly higher than is predicted here.

On the other hand, excessive input use at early stages can also reduce the flexibility of future responses to unfavorable shocks. For instance, a crop may be subject to a mid-season pest infestation. If the infestation turns out to be severe and results in a total crop failure, all earlier investments in the crop may be lost. Anticipating this possibility, the farmer may have incentive to reduce his inputs in earlier stages, particularly if he can compensate for this lower input use by more intensive farming at later stages, i.e., after the pest risk has been resolved.

After developing the general model, we analyze two special cases where the inputs allocated at different stages can either be substitutes or complements in production. We use the cases to illustrate the role of future information and intraseasonal linkage in determining the optimal decisions.

1 Model Setup

This section develops a dynamic model of crop production. For concreteness, the model is developed in the context of field crop management problem involving three sequential decisions or input allocations by a crop manager. The first input allocation involves a planting decision in which the manager selects the quantity of seed to apply. It is clear that planting a crop in practice involves more than simply selecting the quantity of seed to apply. Our focus on a single scalar input simplifies the model and presentation of the basic intuition. The second input choice is a post-emergence pesticide application and the third input decision is a choice of inputs allocate to harvesting the crop. Let $x_j \in \Re_+$ denote the quantity of input allocated at management *event* $j = 0$, 1, and 2. Input prices, w_j are assumed to be known and constant.

Management events are assumed to occur at fixed times during the growing season, and are separated in time by a period in which the crop grows. The assumption that input allocations occur at fixed points in time is also made to simplify the model. The choice of input timing is itself an important element of the management process in agriculture, and in general, the option to delay a management decision in order to acquire new information will represent a valuable real option (see for example, Saphores, 2000). Extensions that consider input timing and multiple real options are reserved for future work.[1] For our purposes, planting occurs at management event $j = 0$, is followed by the pesticide application at $j = 1$, which is followed by harvest at $j = 2$. Because no growth occurs prior to planting, two growth stages are assumed. The first growth stage takes place between management events $j = 0$ and $j = 1$ and the second between management events $j = 1$ and $j = 2$. A chronology of the production problem is illustrated in Figure 1.

Growing conditions and output prices are uncertain. Consider market uncertainty first. Producers form expectations about future prices using all available information on current market conditions. For example, active futures markets such as the Chicago Board of Trade provide a continuous signal about the value of the crop that is being grown, i.e., the market conditions expected to prevail at harvest time. The implication for producers is that price expectations can change throughout the production period as new information arrives.

Let p denote the market price for the final output. Producers do not observe p until harvest time, $j = 2$. Producers have expectations over p based on market signals s_j, $j = 0, 1$. Let $\phi(p|s_0)$ denote the conditional distribution of the market price at $j = 0$. A new market signal

[1] The inputs under consideration may be interpreted broadly to include input quantities and more generally the *effort* that is allocated by the manager. We capture the essential features of the input timing, variety and quantity decisions through our interpretation of the input x_j. In particular, the input includes additional *effort* required to implement the optimal timing of each input application. Additional effort, as indicated by larger x_j, is expected to increase the productivity of a given quantity and quality of input.

arrives between management events. For example, at the time of the post-emergence pesticide application the conditional density of the output price is $\phi(p|s_1)$, where s_1 is the market signal that is available at $j = 1$. Random market signals may be correlated.

Production uncertainty is caused by random weather and pest conditions that affect crop growth throughout the growth period. Let θ denote a random variable that affects crop growth. Prior to planting, there is no growth although random precipitation, temperature, and sunlight impact the condition of the soil. θ_0 thus reflects the suitability of the soil for planting, with higher values indicating more favorable conditions. The first-growth-stage shock θ_1 may reflect the extent of a pest infestation coupled with the accumulation of favorable growing conditions that have occurred during the first growing stage. And θ_2 may reflect days without rain prior to harvest thus affecting the moisture content of the harvestable biomass. Let $\varphi(\theta)$ denote the known density function for θ. We assume that random growing conditions are serially independent.

There are two state variables in our model; cumulative crop growth or biomass, and the market signal. Crop growth is influenced by the actions of the manager, and random weather. We assume that market developments are independent of the actions of the manager and biomass.

Crop growth is governed by a transition equation. Denote the state of the biomass as z_j. Realized biomass is functionally related to allocated inputs and random growing conditions:

$$z_1 = f(x_0, \theta_1|\theta_0) \tag{1}$$

and,

$$z_2 = g(x_1, \theta_2|z_1). \tag{2}$$

The salable yield follows $y = h(x_3, z_2)$, where $h(.)$ describes the transformation relationship between harvesting inputs and saleable output given the biomass at $j = 2$ (see Figure 1).

We assume that the manager is risk neutral and has the objective of maximizing the expected returns from the production process, $E\left[py - \sum_j w_j x_j\right]$. The manager exploits all available information at each decision event. Let I_j denote the information set that is relevant for selecting input x_j. At planting time the manager observes market signal s_0 and growing conditions θ_0 and

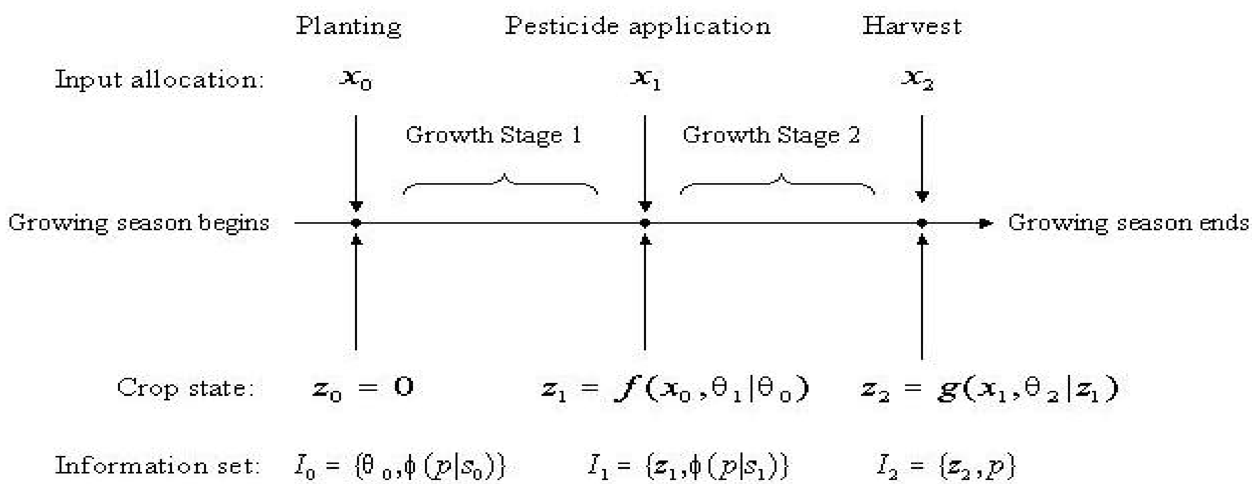

Figure 1: Chronology of the Crop Management Problem.

thus $I_0 = \{\theta_{0,}\phi(p|s_0)\}$. At the time of the post-emergence pesticide application the manager has acquired a new market signal s_1, and observes the effects of random growing conditions θ_1 and past management actions, which are summarized by the biomass z_1. The relevant information set at management event 1 is $I_1 = \{z_1, \phi(p|s_1)\}$. Similarly, the relevant information set at management event 2, harvest time, is $I_2 = \{z_2, p\}$.

2 Analysis

In developing the intuition of the model, it is helpful to distinguish observable crop biomass levels from future biomass. For this purpose, we will represent realized crop biomass levels using the variable z_j and unrealized biomass using the production function representations, i.e., the right hand side of equations (1) and (2). Following this convention, the pre-planting expected payoff from growing the crop can be expanded as

$$E_0\left[ph(x_2, g(x_1, \theta_2|f(x_0, \theta_1|\theta_0)))\right] - w_0x_0 - w_1x_1 - w_2x_2\right], \tag{3}$$

where E_0 is the expectation over price and yield at management event 0.

At this point it is instructive to contrast the dynamic framework with a static model of production under uncertainty. The static counterpart to the crop management problem assumes the manager simultaneously selects inputs x_j prior to the realization of price and production uncertainty. The problem can be specified as

$$\max_{x_0,x_1,x_2} \int\int pF(x_0, x_1, x_2, \theta)\phi(p|s_0)\varphi(\theta)dsd\theta - \textstyle\sum_j w_jx_j, \tag{4}$$

where $F()$ is a static production function that maps (x_0, x_1, x_2,θ) to output y. The model of equation (4) must assume that all inputs are allocated simultaneously and under uncertainty. The manager's information set is $I_0 = \{\theta_{0,}\phi(p|s_0)\}$ and is not updated as production and market uncertainty unravels.

In contrast, the optimization problem in equation (3) is sequential: at each stage, the input decision is based on the state of the growing conditions and the available information about the market price. Consider the last stage, when the farmer observes the crop biomass and has perfect information about price p, thus $I_2 = \{z_2, p\}$. The optimization problem is

$$v^2(p, z_2) \equiv \max_{x_2} ph(x_2, z_2) - w_2 x_2, \tag{5}$$

where $v^2(p, z_2)$ measures the payoff from harvesting the crop, given realized p and z_2. If $x_2 > 0$, the optimal allocation of harvest inputs is obtained from a first order condition $\nabla_{x_2} ph(x_2, z_2) = w_2$, where ∇ denotes partial differentiation with respect to the subscripted argument. Let $x_2^*(p, z_2) \equiv \arg\max_{x_2} ph(x_2, z_2) - w_2 x_2$. It should be emphasized that the allocation of x_2 is made with full information about price and biomass, whereas in the static model, all input allocations are made under price and production uncertainty.

Consider the benefit of selecting x_2 with crop biomass and price observed. At $j = 2$, the manager has an option but not an obligation to pay an exercise price $w_2 x_2^*$ in order to receive an asset that has value $ph(x_2^*, z_2)$ (function arguments are dropped where no confusion can arise). It is clear that the manager will exercise the option only if $v^2(p, z_2) > 0$. In other words, with some positive probability, realized price and biomass may require abandoning the crop, or plowing it back into the field thus avoiding additional losses (expenditures) from harvesting. Favorable realizations of p and z_2, on the other hand, can be exploited by allocating additional harvest inputs; if $h(., z_2)$ is an increasing and concave function of x_2, the optimal $x_2^*(p, z_2)$ will be non-decreasing in p. Optimally responding to the information that is available at harvest time allows the manager to avoid losses associated with low yields and/or weak markets while taking advantage of favorable yield and market conditions. The management option implies that $v^2(p, z_2)$ is truncated at zero, and skewed positive.

Next consider the pesticide application decision. The manager's information set is $I_1 = \{z_1, \phi(p|s_1)\}$. The pesticide application is made under price and production uncertainty, but with knowledge that x_2 will be chosen optimally at $j = 2$ and the payoff from harvesting the crop is $v^2(p, z_2)$. The optimization problem is

$$v^1(s_1, z_1) \equiv \max_{x_1} E_1 \left[v^2(p, z_2) - w_1 x_1 \right], \tag{6}$$

where E_1 is an expectations operator that is conditional on information set I_1. Applying the envelope theorem to (5), yields

$$\nabla_{z_2} v^2(p, z_2) = \nabla_{z_2} ph(x_2^*(p, z_2), z_2). \tag{7}$$

Assuming an interior solution for the choice of x_1, and noting that $z_2 = g(x_1, \theta_2|z_1)$, the first order condition for the problem in equation (6) is

$$E_1 \left[\nabla_g v^2(p, g(x_1, \theta_2|z_1)) \cdot \nabla_{x_1} g(x_1, \theta_2|z_1)\right] = w_1 \tag{8}$$

or, after substituting in (7),

$$E_1 \left[\nabla_g ph(x_2^*(p, g), g) \cdot \nabla_{x_1} g(x_1, \theta_2|z_1)\right] = w_1 \tag{9}$$

or, more explicitly,

$$\iint \left[\nabla_g ph(x_2^*(p, g), g) \cdot \nabla_{x_1} g(x_1, \theta_2|z_1)\right] \phi(p|s_1)\varphi(\theta) ds d\theta = w_1. \tag{10}$$

Let $x_1^*(s_1, z_1) = \arg\max_{x_1} v^1(s_1, z_1)$.

In general, management "flexibility" in later stages of the production process will affect earlier input allocations. Equation (9) illustrates. At the time of the pesticide application, the manager has knowledge that $x_2^*(p, z_2)$ will be selected at $j = 2$. Let $\tilde{x}_2$ denote the *expected* allocation of x_2 given expected price and harvestable biomass; at $j = 1$ the manager forms a price expectation conditional on s_1 and knows the expected growth given current biomass $z_2 = \int g(x_1, \theta_2, z_1)\varphi(\theta)d\theta$, and can thus solve $\tilde{x}_2(z_1) = \arg\max_{x_2} E_1 ph(x_2, g) - w_2 x_2$. Further, let $\tilde{x}_1$ denote the optimal pesticide quantity under the assumption that $\tilde{x}_2(z_1)$ will be allocated at $j = 2$. That is, $\tilde{x}_1$ satisfies

$$E_1 \left[p \nabla_g h(\tilde{x}_2(z_1), g) \nabla_{x_1} g(x_1, \theta_2|z_1)\right] = w_1 \tag{11}$$

which is more explicitly written as

$$\text{(12)} \qquad \iint \left[\nabla_g ph(\tilde{x}_2(z_1), g(x_1, \theta_2|z_1)) \cdot \nabla_{x_1} g(x_1, \theta_2|z_1)\right] \phi(p|s_1)\varphi(\theta) ds d\theta = w_1.$$

Now consider the effects of management flexibility on input choices. In a dynamic framework, the choice of x_1 will in general affect the crop biomass distribution at $j = 2$, through $z_2 = g(x_1, \theta_2|z_1)$, and thus the value of the management option $v^2(p, z_2)$. An interesting question is whether $v^2(p, z_2)$ is more valuable when x_1 is low or when x_1 is high. The answer to this question will depend, possibly in complex ways, on the production technology and random prices and growing conditions.

Suppose that the mid-season biomass z_1 is low, and that market indications, s_1, suggest weak demand. In this case, the value of $v^2(p, z_2)$ is likely to be low. If in addition, the manager is committed to allocating $\tilde{x}_2(z_1)$ at $j = 2$, the expected payoff from allocating x_1 at $j = 1$ may not justify the expenditures on this input. However, by allocating x_1 at $j = 1$, the manager changes the conditional distribution of the biomass through $z_2 = g(x_1, \theta_2|z_1)$. And, with some probability, weather conditions during the second growing stage will be favorable, in which case, the crop biomass at harvest time can exceed expectations. Similarly, with some probability favorable market developments may lead to a realized price that exceeds expectations. Finally, with some joint probability, both growing and market conditions will exceed expectations that are held at $j = 1$. By allocating x_1 at $j = 1$ the crop manager maintains the option to optimally respond to these future developments if they arise. This suggests that $x_1^*(s_1, z_1)$ will exceed $\tilde{x}_1(z_1)$.

It is conceivable that the substitution possibilities that underlie the production technology could have an opposing effect on the optimal x_1. Suppose that the low mid-season biomass and weak demand are such that with some probability the crop will be abandoned at harvest time, in which case $v^2(p, z_2) = 0$. The cost of allocating x_1 at $j = 1$ represents an irreversible investment in the crop. It makes sense to wait for and utilize the new information that will be available at $j = 2$. For example, suppose that the structure of h allows a large degree of substitutability between x_2 and z_2. The manager thus has the option to allocate a larger x_2 if growing and/or market conditions improve. The opportunity to acquire additional information has value that should be included as a cost of allocating x_1.

3 Special Examples

In this section, we consider two examples with only price uncertainties to illustrate how intraseasonal adjustment can lead to more or less input use in early stages of the production process. Introducing production uncertainties will complicate the model and, in some cases, prevents an analytical solution. But the intuition from price uncertainty only carries over to the more general case. In both cases, we consider versions of the von Liebig production function (see Paris, 1992 for additional discussion).

3.1 Additive Inputs: Stages as Perfect Substitutes

Suppose the production function at the three stages are given by

$$\begin{aligned} &\text{Stage 0:} \qquad z_0 = \theta_0 x_0 \\ &\text{Stage 1:} \qquad z_1 = z_0 + \theta_1 x_1 \\ &\text{Stage 2:} \qquad z_2 = \min\{z_1 + \theta_2 x_2, \bar{z}\}, \end{aligned} \tag{13}$$

where $\bar{z}$ is the exogenously given von Liebig bound on the output level. The bound represents a natural restriction imposed by the size of the land, the kind of seed, or other conditions that the farmer cannot control. Thus the final output is given by

$$Y = \min\left\{\sum_{i=0}^{2} \theta_i x_i, \bar{z}\right\}. \tag{14}$$

This is a form of von Liebig production function, and inputs at the three stages are perfect substitutes. Up to the limit $\bar{z}$, the farmer can increase inputs at either of the three stages to raise the output level.

Consider the benchmark case where there is no future information about the output price p. Since we assume that there is no production uncertainty (i.e., θ_i, $i = 0, 1, 2$, are known), this case is equivalent to choosing x_i, $i = 0, 1, 2$, at period 0:

$$\max_{x_0, x_1, x_2} \bar{p} \min\left\{\sum_{i=0}^{2} \theta_i x_i, \bar{z}\right\} - \sum_{i=0}^{2} w_i x_i, \tag{15}$$

where $\bar{p}$ is the expected price. Since the farmer can increase input at any of the three stages to boost the output, he will choose the one that gives the highest "benefit-cost ratio." Let $\bar{\imath} = \{i = 0, 1, 2 : w_i/\theta_i = \min\{w_j/\theta_j, j = 0, 1, 2\}\}$ be the stage where the input has the lowest cost/benefit ratio. We can verify that in the optimal solution, $x_j = 0$ if $j \neq \bar{\imath}$, $x_{\bar{\imath}} = \bar{z}/\theta_{\bar{\imath}}$ if $\bar{p} \geq w_{\bar{\imath}}/\theta_{\bar{\imath}}$ and $x_{\bar{\imath}} = 0$ otherwise. That is, the farmer uses the maximum level of inputs at the stage that is the most efficient in increasing his profit, if, based on the current information, the expected profit from production is positive. Otherwise, if the expected profit is negative, the farmer produces nothing.

With intraseasonal adjustment, suppose the farmer receives signal s_i at stage i, $i = 0, 1, 2$, about the price level. Consider first stage 2. Given s_2 and z_1, the farmer's decision is

$$\max_{x_2} \bar{p}(s_2) \min \{z_1 + \theta_2 x_2, \bar{z}\} - w_2 x_2, \tag{16}$$

where $\bar{p}(s_2)$ is the expected price level given signal s_2. In the optimal solution, $z_1 \leq \bar{z}$: otherwise, the farmer could have saved input costs while maintaining his output level. Clearly, the optimal decision is

$$x_2(z_1, s_2) = \begin{cases} \dfrac{\bar{z} - z_1}{\theta_2} & \text{if } \bar{p}(s_2)\theta_2 \geq w_2 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

The payoff at this stage given s_2 is

$$V_2(z_1, s_2) = \begin{cases} \left(\bar{p}(s_2) - \dfrac{w_2}{\theta_2}\right) \bar{z} + \dfrac{w_2}{\theta_2} z_1 & \text{if } \bar{p}(s_2) \geq w_2/\theta_2 \\ \bar{p}(s_2) z_1 & \text{otherwise} \end{cases} \tag{18}$$

At stage 1, the farmer observes signal s_1 and knows the distribution of signals s_2. The distribution of s_2 may or may not depend on the value of s_1, and for generality, we denote the cumulative distribution function of $\bar{p}(s_2)$ given s_1 as $F_{\bar{p}}(\bar{p}(s_2)|s_1)$: signal s_2 is independent of s_1 if $dF_{\bar{p}}/ds_1 = 0$. Further, let $\bar{\bar{p}}_2^- = \int_0^{w_2/\theta_2} \bar{p}(s_2) dF_{\bar{p}}(\bar{p}(s_2|s_1))$ and let $\bar{\bar{p}}_2^+ = \int_{w_2/\theta_2}^{\infty} \bar{p}(s_2) dF_{\bar{p}}(\bar{p}(s_2|s_1))$ be the expected price levels conditional on all possible "bad" (i.e., $\bar{p}(s_2) < w_2/\theta_2$) or "good" (i.e., $\bar{p}(s_2) > w_2/\theta_2$) stage two signals, given information at stage one. Then we know

$$E_{s_2|s_1} V_2(z_1, s_2) = \left(\bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)) \frac{w_2}{\theta_2} \right) z_1 + \left(\bar{\bar{p}}_2^+ - (1 - F_{\bar{p}}(w_2/\theta_2)) \frac{w_2}{\theta_2} \right) \bar{z}.$$

The stage one decision, given s_1 and z_0, is

$$\max_{x_1} E_{s_2|s_1} V_2(\min\{z_0 + \theta_1 x_1, \bar{z}\}, s_2) - w_1 x_1, \tag{19}$$

where we have imposed the condition that $z_1 \leq \bar{z}$ through the function $\min\{\cdot\}$. The optimal decision is given by

$$x_1(z_0, s_1) = \begin{cases} \dfrac{\bar{z} - z_0}{\theta_1} & \text{if } \bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)w_2/\theta_2 \geq w_1/\theta_1 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

To compare $x_1(z_0, s_1)$ with the benchmark case, note that if $\frac{w_2}{\theta_2} < \frac{w_1}{\theta_1}$, $\bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)w_2/\theta_2 < \bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)w_1/\theta_1 < w_1/\theta_1$. Thus $x_1(z_0, s_1) = 0$ if input at stage two is more efficient than that at stage one. However, the reverse is not true: even if $\frac{w_2}{\theta_2} > \frac{w_1}{\theta_1}$, it is still possible that $\bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)w_2/\theta_2 < w_1/\theta_1$ and thus $x_1 = 0$. The farmer may be willing to wait until stage two to raise his output even if the stage two input is less efficient than the stage one input. He does so in order to utilize the new information that will be available at stage two. Further, $\bar{\bar{p}}_2^- + (1 - F_{\bar{p}}(w_2/\theta_2)w_2/\theta_2 < \bar{p}$: even if $\bar{p} > w_1/\theta_1$, it is still possible that $x_1 = 0$: even if the expected price overcomes the marginal cost, the farmer may still wish to choose no input at stage one and wait to stage two to utilize the new information.

Similarly we can show that for x_0 to be positive, or for $x_0 = \bar{z}/\theta_0$, the farmer demands more than $w_0/\theta_0 = w_{\bar{i}}/\theta_{\bar{i}}$ and $\bar{p} > w_0/\theta_0$. In fact, the condition for $x_0 > 0$ is more strict than that for $x_1 > 0$.

The intuition is that more information about p arrives at each production stage. Since the inputs are perfect substitutes, whenever the farmer chooses to "lock in" the output $\bar{z}$ at an earlier stage, he kills the option of deciding whether or not to lock in the output in the future with better information. Thus, to induce him to decide in earlier stages, the payoff, or advantage over deciding in later stages, in terms of $\bar{p} - w_i/\theta_i$ and w_i/θ_i relative to the ratio in later stages, must be able to overcome the option value of delaying for better information. With this production process, the farmer reacts "strongly" (weakly) to negative (positive) price information in earlier stages of production, and less (more) strongly in later stages.

3.2 Leontief Inputs: Stages as Perfect Complements

Now consider a different production function:

$$\begin{aligned} &\text{Stage 0:} \quad && z_0 = \min\{\theta_0 x_0, \tilde{z}\} \\ &\text{Stage 1:} \quad && z_1 = \min\{\theta_1 x_1, z_0\} \\ &\text{Stage 2:} \quad && z_2 = \min\{\theta_2 x_2, z_1\}, \end{aligned} \tag{21}$$

where $\tilde{z}$ is the von Liebig bound. Thus the final output is given by

$$Y = \min\{\theta_0 x_0, \theta_1 x_1, \theta_2 x_2, \tilde{z}\}. \tag{22}$$

According to this specification, input choice at each stage determines to what extent the farmer can increase the output at other stages. The inputs are perfect complements: the production function is in fact Leontief, subject to the von Liebig bound $\tilde{z}$.

Again, we first consider the benchmark case without any future information about p. The farmer's decision at period 0 is

$$\max_{x_0, x_1, x_2} \bar{p} \min\{\theta_i x_i, i = 0, 1, 2, \tilde{z}\} - \sum_{i=0}^{2} w_i x_i. \tag{23}$$

Suppose the expected price is such that the farmer decides to produce. Given that the technology is linear, he will produce up to the full potential $\tilde{z}$. Then clearly the choices will be $x_i = \tilde{z}/\theta_i$ and the expected payoff is $\bar{p}\tilde{z} - \sum_i \tilde{z} w_i/\theta_i$. The farmer should produce if and only if $\bar{p} \geq \sum_i w_i/\theta_i$. Thus the optimal solution is given by

$$x_i = \begin{cases} \dfrac{\tilde{z}}{\theta_i} & \text{if } \bar{p} \geq \displaystyle\sum_{i=0}^{2} w_i/\theta_i \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

Consider now the case of intraseasonal adjustment. Again, at stage two, given z_1 and s_2, the farmer's decision problem is

$$\max_{x_2} \bar{p}(s_2) \min \{\theta_2 x_2, z_1\} - w_2 x_2. \tag{25}$$

The optimal solution is

$$x_2(z_1, s_2) = \begin{cases} \dfrac{z_1}{\theta_2} & \text{if } \bar{p}(s_2) \geq w_2/\theta_2 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

and the expected payoff is

$$W_2(z_1, s_2) = \begin{cases} \left(\bar{p}(s_2) - \dfrac{w_2}{\theta_2}\right) z_1 & \text{if } \bar{p}(s_2) \geq w_2/\theta_2 \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

At stage one, given s_1 and z_0, the farmer's decision is

$$\max_{x_1} E_{s_2|s_1} W_2(\min\{\theta_1 x_1, z_0\}, s_2) - w_1 x_1, \tag{28}$$

where $E_{s_2} W_2(z_1, s_2) = \left(\bar{\bar{p}}_2^+ - (1 - F_{\bar{p}}(w_2/\theta_2))w_2/\theta_2\right) z_1 \equiv \Delta_2 z_1$. Then the optimal choice of x_1 is

$$x_1(z_0, s_1) = \begin{cases} \dfrac{z_0}{\theta_1} & \text{if } \Delta_2 \geq w_1/\theta_1 \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

Now we analyze the condition for $x_1 > 0$: $\Delta_2 - w_1/\theta_1 \geq 0$. Note that $\Delta_2 - w_1/\theta_1 = \int_{w_2/\theta_2}^{\infty} (\bar{p}(s_2) - w_2/\theta_2) dF_{\bar{p}} - w_1/\theta_1 > \int_0^{\infty} (\bar{p}(s_2) - w_2/\theta_2) dF_{\bar{p}} - w_1/\theta_1 = \bar{p} - w_1/\theta_1 - w_2/\theta_2$. In the benchmark case without learning, given z_0, the condition for $x_1 > 0$ is $\bar{p} > w_1/\theta_1 + w_2/\theta_2$. Thus, with intraseasonal adjustment, the farmer is more likely to choose $x_1 > 0$: even if $\bar{p} < w_1/\theta_1 + w_2/\theta_2$, he may still choose to have $x_1 > 0$. Similarly, we can show that the condition for $x_0 = \tilde{z}/\theta_0$ is less strict than $\bar{p} > \sum_i w_i/\theta_i$.

The intuition for the higher willingness to raise the input in earlier stages is that, by increasing

$\theta_i x_i$, the farmer creates the option of being able to respond to future information about the output price. If the farmer responds to low price signals in earlier stages by choosing zero input, he cannot increase his production in later stages even if new signals predict higher prices. Thus, to induce the farmer to stop production in earlier stages, the expected price must be sufficiently low so that the option value of responding to future price information cannot overcome the expected profit loss in earlier stages.

4 Conclusion

The ability to respond to new information in the production process, rather than only at the beginning of a growing season, implies that intraseasonal adjustment can be an important factor in determining how the output responds to weather and price shocks. We constructed a stylized model to study this situation, and found that the output response depends on the production function, in particular whether the input use at different stages are complements or substitutes. The major intuition is that the farmer has incentive to adjust his input use at earlier stages to preserve the option of being able to respond to future new information.

Our model is a simple stylized representation of a complex management problem. For example, labor, machinery, fuel, the quality of seed, and the timing of the planting must be considered when sowing a crop. Our assumptions of a single decisions variable, seed quantity, and a fixed planting time allows us to present the important intuition of the model.

In order to illustrate the magnitude of the importance of intraseasonal adjustment, we plan to conduct a case study of a representative farm in Iowa, using the farm budget data. Using this example, we will illustrate the magnitude of the option values, and how the input demand and output supply responses to the final price can be different from the static models. We will also study how the responses depend on the correlation of the signals overtime. Finally, we will discuss the new paradigm based on our approach for data collection, empirical analysis and ultimately our understanding of agricultural production under uncertainty.

5 References

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