DEMAND INTERRELATIONSHIPS AMONG FRUIT BEVERAGES

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Abstract

In this study, both the Rotterdam model and the double logarithmic model were used to estimate the demand parameters for fruit beverages. The results show that: (1) under the conditions of block-independence and predetermined price changes, the Slutsky matrix for fruit beverages is symmetric and negative definite; (2) own-price elasticity estimates from both models are about the same; and (3) income elasticity estimates and cross-product relationships from the two models are not compatible.

Key words: Rotterdam model, fruit beverages, elasticities.

Economists have attempted to estimate demand relationships using empirical data, but these do not always conform to the rigid definition specified by economic theory. Since interest often focuses on the income and own-price elasticities for each good, it is natural to desire models which allow independent measurement of these relationships while providing plausible assumptions about the less essential responses. One of the most common and attractive simplifications is to attempt to characterize behavior in terms of two responses, income elasticity and own-price elasticity, allowing assumptions, rather than direct measurement, to determine appropriate values for the cross-responses (Wetmore et al., Brandow, George and King, Bieri and de Janvry, Heien).

Most of the empirical studies of consumer demand are based on a classification of the consumer's market basket in terms of relatively broad commodity groups. Although this is frequently sufficient when the analysis is macro-oriented, it will also occur that the interest is confined to detail within one single group (Chavas; Tsoa et al.; Lamm; Theil, 1976). The commodities of such a group are usually specific substitutes. In addition, most of the previous studies for specific commodity groups fail to recognize the connections between the commodity group of interest and all other goods and services which are available to the consumer. Therefore, the major purposes of this study are to estimate the structure of a conditional demand model under the assumptions of block independence preference (which is a special case of strong separability) and predetermined price changes; and to test the hypotheses of symmetry and negativity of the Slutsky matrix of this conditional demand system for fruit beverages.

A function often used for empirical analysis of consumer demand is the double logarithmic function, which is inconsistent with standard utility theory assumptions. Many researchers use the double logarithmic demand function because of superior fit, ease of estimation and the ready interpretation of the estimated parameters (Myers, Myers and Liverpool, Ward and Tilley). Also, since demand parameters are often estimated from market data, it has been argued that the double logarithmic function in some sense approximates aggregated individual maximizing behavior. Therefore, another purpose of this study is to compare the estimates from both the Rotterdam and double logarithmic models in terms of price and income elasticities and the symmetry property of the Slutsky matrix as suggested by standard utility theory.

MODELS AND ESTIMATION METHODS

This study is cast within the same analytical framework of the Rotterdam model developed by Theil (1965) and Barten (1968, 1969). The Theil-Barten approach to estimation of parameters of the demand equations' infinite changes is:

$$(1)\quad w^{*}_{it}\, Dq_{it} = \mu_i\, Dq_t + \sum_j v_{ij}\, (DP_{jt} - \sum_k \mu_k\, DP_{kt}) + \varepsilon_{it},\ i = 1, \ldots, n,$$

where:

$w^{*}_{it} = (w_{i,t-1} + w_{it})/2$ and w_{it} is the expenditure proportion of beverage i during time period t;

$Dq_{it} = \ln(q_{it}/q_{i,t-1})$ and q_{it} is the demand for beverage i during time period t;

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μ_i = $P_i(\partial q_i/\partial m)$ is the marginal budget share of the ith beverage, P_1 is the nominal price of beverage i, and m is nominal income;

$Dq_t = \sum_i w^*_{it} Dq_{it}$;

$DP_{jt} = \ln(P_{jt}/P_{j,t-1})$;

v_{ij} = $\lambda P_i P_j u^{ij}/m$ is the coefficient of the relative price j (Theil, 1971, pp. 575-78); and u^{ij} is the (i,j)−th element of the inverse of the Hessian matrix identified with the second-order conditions of the consumer optimization problem, and λ is the marginal utility of income; and

ε_{it} = the demand disturbance, which is regarded as the random effect of all variables other than income and prices. It is assumed that it has zero expectation and that the variances and contemporaneous covariances are constant over time, while other covariances are zero (Theil, 1971, p. 333).

Note that dividing v_{ij} by w_i gives implied or compensated price elasticities of demand, and the parameters μ_i and v_{ij} satisfy:

(2) $\sum_i \mu_i = 1$ (Engel aggregation)

(3) $v_{ij} = v_{ji}$ (Symmetry)

(4) $\sum_j v_{ij} = \varphi\mu_i$ (Homogeneity)

The term φ is the income flexibility parameter, i,e., $(\lambda/m)/(\partial\lambda/\partial m)$.

In order to use the above model, a decision about whether to estimate a complete demand system or to make assumptions about the relationships between fruit beverages and other commodity groups and estimate a subset of the complete demand system must be made. Heien's study showed that orange juice and fruit form an individual group and substitution effects between this group and other food and non-food items were negligible. In addition, the substitution effects between orange juice and fruit items were small.[1] These results suggest that the demand for fruit based beverages is probably independent to the demand for other commodity groups, and the assumption of block-independence preference suggested by Theil can be used in the estimation of the demand interrelationships between fruit beverages.

Under the assumption of block-independence preferences, which is a special case of strong separability, the n commodities can be divided into G groups, $S_1, \ldots, S_G$, such that each commodity belongs to exactly one S_g, $g = 1, \ldots, G$, and the utility function can be written as the sum of G functions, each involving the quantities of only one group. In this case, equation (1) can be rewritten as (Theil, 1976, pp. 1-4):

$$(5)\quad w^*_{it} Dq_{it} = (\mu_i/M_g)\, w^*_{gt} Dq_{gt} + \sum_{j\in S_g} \pi^g_{ij} DP_{jt} + \varepsilon^g_{it};\ i=1, \ldots, n_g,$$

where:

$M_g = \sum_{i\in S_g} \mu_i$;

$w^*_{gt} = \sum_{i\in S_g} w^*_{it}$;

$Dq_{gt} = \sum_{i\in S_g} (w^*_{it}/w^*_{gt})\, Dq_{it}$;

π^g_{ij} = $(P_iP_j/m)(\partial q_i/\partial P_j)$, the compensated cross-price elasticity weighted by the ith expenditure proportion, for all i, $j\in S_g$; π^g_{ij} is the (i,j)−th element of the conditional Slutsky matrix $[\pi^g_{ij}]$; and

n_g = the number of commodities in S_g.

If a conditional demand model whose left-hand variables are adjusted by conditional budget shares, i.e., w^*_{it}/w^*_{gt}, is desired, both sides of equation (5) can be divided by w^*_{gt} as follows:

$$(6)\quad w^*_{it}/w^*_{gt} Dq_{it} = (\mu_i/M_g)\, Dq_{gt} + \sum_{j\in S_g} \pi^*_{ij} Dp_{jt} + \varepsilon^*_{it},\ i=1, \ldots, n_g;$$

where:

$\pi^*_{ij} = \pi^g_{ij}/w^*_{gt}$ and $\varepsilon^*_{it} = \varepsilon^g_{it}/w^*_{gt}$. Equation (6) represents a modified version of a conditional demand model represented by equation (5). Note that the definition of π^*_{ij} implies that the original coefficients, π^g_{ij}, vary proportionally with the total fruit beverage budget share.

Adding up restrictions require that the marginal propensities sum to unity and that the net effect of a price change on the budget be zero (Deaton and Muellbauer, p. 69), i.e.,

[1] The price elasticities estimated by Heien (p. 220) are as follows.

Item	Orange juice	Fresh fruit	Processed fruit
Orange juice	−.535	.189	.169
Fresh fruit	.303	−3.021	1.956
Processed fruit	−.022	−.167	−.589

$$\sum_{i \in S_g} \mu_i = 1 \text{ and } \sum_{i \in S_g} \pi^g_{ij} = 0.$$

Provided the data add up as they should, if ordinary least squares estimation is used, the parameter estimates will automatically satisfy these restrictions. Using

$$\sum_{i \in S_g} \pi^g_{ij} = 0 \text{ and } \sum_{i \in S_g} \pi^*_{ij} = 0,$$

equations (5) and (6) can be respectively rewritten as:

$$(7)\quad w^*_{it} Dq_{it} = (\mu_i/M_g) w^*_{gt} Dq_{gt} + \sum_{j \in S_g}^{n_g - 1} \pi^g_{ij}(Dp_{jt} - Dp_{n_g t}) + \varepsilon^g_{it}, \quad i = 1, \ldots, n_g - 1;$$

and

$$(8)\quad \frac{w^*_{it}}{w^*_{gt}} Dq_{it} = (\mu_i/M_g)\, Dq_{gt} + \sum_{j \in S_g}^{n_g - 1} \pi^*_{ij}\,(DP_{jt} - DP_{n_g t}) + \varepsilon^*_{it}, \quad i = 1, \ldots, n_g - 1.$$

Note that $\sum_{i \in S_g} \mu_i = 1$ is implicitly imposed when μ_{n_g} is estimated as: $1 - \sum_{i \in S_g}^{n_g - 1} \mu_i$.

The homogeneity constraints,

$$\sum_{j \in S_g} \pi^g_{ij} = 0 \text{ or } \sum_{j \in S_g} \pi^*_{ij} = 0,$$

as shown in equations (7) and (8), are imposed by deflating each price by the price of the n_g^{th} commodity group. The only remaining constraints are the symmetry constraints, $\pi^g_{ij} = \pi^g_{ji}$ or $\pi^*_{ij} = \pi^*_{ji}$, and the negative semidefiniteness of the matrix $[\pi_{ij}]$ with rank $n_g - 1$.

Without the symmetry constraints, equations (7) and (8) can be estimated by the ordinary least squares method (OLS). In order to test the symmetry constraints, the generalized least squares method (GLS) should be used.

Theil suggests an iterative procedure which first uses the average value shares, $\bar{w}_i/\bar{w}_g$, of each commodity to approximate the covariance matrix Ω, by say Ω_o, such that its elements are defined as:

$$\omega^o_{ij} = (\bar{w}_i/\bar{w}_g)\ (1 - \bar{w}_i/\bar{w}_g), \text{ if } i = j$$

and

$$\omega^o_{ij} = (\bar{w}_i/\bar{w}_g)\ (\bar{w}_j/\bar{w}_g), \text{ if } i \neq j.$$

A two-pass GLS procedure with symmetry constraints was used to obtain the demand parameters for fruit beverages as shown in equations (7) and (8). The first pass was used to obtain a second approximation of Ω and the second pass was used to obtain the demand parameters of interest. A F-statistics was computed to test the symmetry constraints (Theil, 1971, pp. 341-4).

The coefficient of multiple correlation is the most popular measure of goodness of fit, but its application is hampered by a lack of uniqueness when it is used for models consisting of several equations. In this study, information inaccuracy indices will be used to decide whether equation (7) or (8) is to be preferred. Theil argues that the information index has several advantages over multiple correlations computed for each demand equation separately. First, it refers to all n equations simultaneously, which avoids the possibility of a conflicting verdict for different equations. Second, it recognizes explicitly that demand theory is concerned with the allocation of total expenditure[2] in terms of expenditure proportions for individual commodities; whereas, multiple correlations disregard this feature. Third, the information inaccuracy can be computed for each observation separately; whereas, multiple correlation coefficients typically refer to the sample of all observations (Theil, 1975, pp. 168-73; Theil, 1976, p. 21).

DATA SOURCE

Most of the data used to estimate the fruit beverage demand relationships were purchased by the Florida Department of Citrus from NPD Research Inc. NPD data are generated via a consumer panel of approximately 7,500 families located throughout the United States. Thus, they represent a measure of actual purchases. In the reports prepared by NPD Research Inc. for the Florida Department of Citrus, information for household purchases for nine single fruit juices, five single fruit drinks, and multi-fruit juices and drinks is available. These data include monthly observations on consumer purchases, expenditures, and unit prices for 16 fruit beverages in different packages. Data from December 1977 through August 1982 were used.

In this study, multi-fruit juices and drinks are combined into one category. Grape, tomato, pineapple, lemon, prune, and other single fruit juices are grouped into one category. Grapefruit drink, apple drink, lemon drinks and ades, and other single fruit drinks are considered as other

[2]In this study, the information inaccuracy indices apply to expenditures for fruit beverages only.

fruit drinks, while orange drink is considered as a separate category. In total, there are seven beverage categories, i.e., multi-fruit juices and drinks, orange juice, grapefruit juice, apple juice, other fruit juice, orange drink, and other fruit drinks. The unconditional budget shares ($\bar{w}_i$) of these seven categories in the 57-month period are: multi-fruit juices and drinks, .00018; other fruit juices, .00025; orange juice, .00085; orange drink, .00012; grapefruit juice, .00001; other fruit drinks, .00014; and apple juice, .00018.

Observations on consumer disposable income are reported by the United States Department of Commerce. Since monthly observations were used, 12 dummy variables were added to the right-hand side of equations (7) and (8) to account for the monthly trend or the gradual shift in consumer preferences in the demand for fruit beverages in a particular month during 1977 through 1982.[3] Since the major purpose of this study is to make comparison of the price and income parameters, estimates for these dummy variables were not presented but were used to make estimates.

RESULTS

The least-squares method was used to estimate the conditional marginal shares and Slutsky coefficients related to equation (7) by regressing per capita w^*_{it} Dq_{it} on w^*_{gt} Dq_{gt}, the six deflated price log-changes for the above commodity classifications and 12 monthly dummy variables. Results are shown in the upper half of Table 1.

The test statistic for Slutsky symmetry takes a low value, .92, when applied to equation (5), which indicates that the symmetry restrictions cannot be rejected (the α = .05 value of F^6_{222} is 2.10). The symmetry-constrained estimates obtained from equation (5) are given in the lower half of Table 1. The Eigen values of the symmetric estimate of matrix $[\pi^g_{ij}]$ with symmetric constraints are 0, $-.6826 \times 10^{-5}$, $-.9767 \times 10^{-5}$, $-.1086 \times 10^{-4}$, $-.2183 \times 10^{-4}$, $-.3113 \times 10^{-4}$, and $-.8267 \times 10^{-4}$, which indicate that this matrix is negative semidefinite with rank 6.

The least-squares estimates of equation (6) are shown in the upper part of Table 2. Notice that the estimates of the π^*_{ij}'s are almost 10^4 larger than π^g_{ij} in Table 1. This is in agreement with the division by the total beverage budget share w^*_{gt}.

The test statistic for Slutsky symmetry for equation (6) also takes a low value of .93 which indicates that the symmetry restrictions cannot be rejected. The symmetry-constrained estimates are shown in the lower half of Table 2. The Eigen values of the Slutsky matrix $[\pi^*_{ij}]$ are 0, −.0426, −.0647, −.0700, −.1441, −.2045, and −.5486; hence, this matrix is negative semidefinite with rank 6.

Note that all diagonal, symmetry-constrained coefficient estimates are negative and statistically different from zero at conventional levels (tables 1 and 2). Most of the off-diagonal terms are not statistically different from zero. In addition, all off-diagonal terms which are statistically different from zero have expected positive signs, except the unexpected negative sign for the estimate for grapefruit juice and other single fruit juices.

All conditional marginal share estimates are statistically significant and the results indicate that orange juice has the largest marginal share, followed by other single fruit juice, multifruit juice and drink, other single fruit drinks, grapefruit juice, apple juice, and orange drinks, the latter having the smallest marginal share.

In order to decide which equation is to be preferred, average information inaccuracy indices are computed, Table 3. The information inaccuracy statistics for all beverages indicate that equation (6) has a better fit than equation (5). The results in Table 3 also indicate that orange juice, other fruit juice, and apple juice equations have a better fit when equation (6) is used, and these juice categories account for 74 percent of the beverage expenditures in this study. Therefore, equation (6) will be used in later discussions, which means that the conditional Slutsky coefficients are considered as varying proportionally to w^*_{gt}.

Least-squares estimates of a double logarithmic model are presented in Table 4.[4] Slutsky symmetry in terms of the relationship between the cross-elasticities can be expressed as (Tomek and Robinson, pp. 39-40):

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[3]Theil adds a constant term to the right-hand side of the demand equation. The value of this constant is the expectation of the quantity component of a budget share change under the condition that real income and real prices remain unchanged. One interpretation of such a constant term is that of a gradual and persistent shift in the consumer's preferences (Theil, 1975, p. 187 and pp. 205-7).

[4]In the double logarithmic model, all price, income and quantity variables were converted to natural logarithms. In addition, eleven dummy variables were included in the model to capture seasonal variations. The parameter estimates for price and income variables from this model are elasticities.

Previous studies by Myers, Myers and Liverpool, and Ward and Tilley used information provided by the Market Research Corporation of America, which is not compatible with the information provided by NPD Research, Inc. Hence, their model is reestimated with information provided by NPD Research Inc., for comparison purposes.

TABLE 1. ESTIMATES OF DEMAND PARAMETERS FOR ROTTERDAM MODEL (EQUATION (5)), UNITED STATES, DECEMBER 1977 THROUGH AUGUST 1982

Beverage	Conditional marginal share	Conditional Slutsky coefficients (π^{g}_{ij})[a]: Multi-fruit juice/drinks	Orange juice	Grapefruit juice	Apple juice	Other fruit juices	Orange drinks	Other fruit drinks
				Unconstrained estimates				
Multi-fruit juice drinks	.0821	−.2955	.0730	−.0388	.1508	.0275	.0938	−.0108
	(.0233)	(.0681)	(.0841)	(.0610)	(.0929)	(.0775)	(.0834)	(.0419)
Orange juice	.6337	.1694	−.6842	.1996	.0571	.0823	.0436	.1321
	(.0329)	(.0961)	(.1186)	(.0861)	(.0131)	(.0110)	(.1176)	(.0592)
Grapefruit juice	.0483	.0532	.0613	−.0866	−.0122	−.0324	−.0009	.0176
	(.0149)	(.0435)	(.0537)	(.0390)	(.0594)	(.0495)	(.0534)	(.0268)
Apple juice	0504	.0410	.1553	.0065	−.1386	−.0065	−.0399	−.0179
	(.0101)	(.0297)	(.0366)	(.0266)	(.0405)	(.0337)	(.0363)	(.0183)
Other fruit juices	.1045	.1212	.2305	−.0396	−.1306	−.1719	.0169	.0074
	(.0193)	(.0564)	(.0696)	(.0505)	(.0770)	(.0642)	(.0691)	(.0349)
Orange drinks	.0344	−.0382	.1100	−.0184	.0445	.0204	−.1102	−.0081
	(.0162)	(.0475)	(.0586)	(.0425)	(.0648)	(.0540)	(.0581)	(.0092)
Other fruit drinks	.0467	−.0511	.0542	−.0228	.0290	.0807	.0303	−.1203
	(.0152)	(.0444)	(.0547)	(.0397)	(.0605)	(.0504)	(.0543)	(.0273)
				Symmetry-constrained estimates[b]				
Multi-fruit juice drinks	.0833	−.2299	.0779	.0331	.0446	.0648	.0157	−.0062
	(.0220)	(.0570)	(.0635)	(.0314)	(.0247)	(.0386)	(.0376)	(.0964)
Orange juice	6308		−.6975	.0871	.1378	.1737	.1246	.0964
	(.0317)		(.0725)	(.0418)	(.0327)	(.0554)	(.0491)	(.0369)
Grapefruit juice	.0512			−.0975	.0120	−.0669	.0149	.0173
	(.0141)			(.0321)	(.0205)	(.0296)	(.0289)	(.0184)
Apple juice	.0484				−.1421	−.0031	−.0307	−.0185
	(.0099)				(.0321)	(.0274)	(.0270)	(.0157)
Other fruit juice	.0989					−.2203	.0147	.0371
	(.0183)					(.0543)	(.0394)	(.0247)
Orange drinks	.0360						−.1283	−.0109
	(.0156)						(.0532)	(.0235)
Other fruit drinks	.0514							−.1152
	(.0139)							(.0225)

[a]Standard errors are shown in parentheses. All coefficient and standard error estimates should be multiplied by 10^{-4}.
[b]Eigen values of the Slutsky matrix are 0, $-.6826 \times 10^{-5}$, $-.9767 \times 10^{5}$, $-.1086 \times 10^{-4}$, $-.2183 \times 10^{-4}$, $-.3113 \times 10^{-4}$, and $-.8267 \times 10^{-4}$.

TABLE 2. ESTIMATES OF DEMAND PARAMETERS FOR ROTTERDAM MODEL (EQUATION (6)), UNITED STATES, DECEMBER 1977 THROUGH AUGUST 1982

Beverage	Conditional marginal share	Modified Conditional Slutsky coefficients (π^{*}_{ij})[a]: Multi-fruit juice/drinks	Orange juice	Grapefruit juice	Apple juice	Other fruit juices	Orange drinks	Other fruit drinks
				Unconstrained estimates				
Multi-fruit juice drinks	.0821	−.1937	.0499	−.0270	.0985	.0169	.0631	−.0076
	(.0234)	(.0444)	(.0544)	(.0397)	(.0604)	(.0505)	(.0541)	(.0271)
Orange juice	.6327	.1130	−.4591	.1310	.0378	.0561	.0353	.0859
	(.0337)	(.0638)	(.0783)	(.0571)	(.0869)	(.0726)	(.0779)	(.0393)
Grapefruit juice	.0465	.0341	.0378	0.0558	−.0058	−.0224	.0001	.0119
	(.0150)	(.0284)	(.0348)	(.0254)	(.0387)	(.0323)	(.0347)	(.0174)
Apple juice	.0512	.0265	.1039	.0067	−.0933	−.0043	−.0265	−.0130
	(.0103)	(.0196)	(.0240)	(.0175)	(.0266)	(.0223)	(.0239)	(.0120)
Other fruit juice	.1050	.0801	.1523	−.0249	−.0854	−.1134	−.0141	.0055
	(.0197)	(.0373)	(.0457)	(.0333)	(.0507)	(.0424)	(.0454)	(.0230)
Orange drinks	.0335	−.0239	.0722	−.0126	.0295	.0128	−.0729	−.0051
	(.0164)	(.0310)	(.0380)	(.0277)	(.0422)	(.0353)	(.0378)	(.0189)
Other fruit drinks	.0490	−.0362	.0430	−.0173	.0187	.0543	.0150	−.0775
	(.0155)	(.0293)	(.0359)	(.0262)	(.0399)	(.0333)	(.0357)	(.0180)
				Symmetry-constrained estimates[b]				
Multi-fruit juice drinks	.0825	−.1506	.0513	.0207	.0288	.0429	.0126	−.0057
	(.0222)	(.0372)	(.0415)	(.0205)	(.0164)	(.0254)	(.0246)	(.0166)
Orange juice	.6308		−.4622	.0538	.0934	.1172	.0801	.0664
	(.0325)		(.0725)	(.0274)	(.0217)	(.0367)	(.0322)	(.0244)
Grapefruit juice	.0495			−.0628	.0100	−.0443	.0112	.0114
	(.0142)			(.0209)	(.0136)	(.0194)	(.0188)	(.0122)
Apple juice	.0493				−.0950	−.0035	−.0206	−.0131
	(.0102)				(.0214)	(.0182)	(.0179)	(.0104)
Other fruit juices	.1004					−.1449	.0077	.0249
	(.0187)					(.0357)	(.0258)	(.0163)
Orange drinks	.0346						−.0831	−.0079
	(.0157)						(.0348)	(.0153)
Other fruit drinks	.0529							−.0760
	(.0143)							(.0149)

[a]Standard errors are shown in parentheses.
[b]Eigen values of the Slutsky matrix are 0, −.0426, −.0647, −.0700, −.1441, −.2045 and −.5486.

$$e_{ij} = \frac{w_j}{w_i} e_{ji} + w_j (e_{jy} e_{iy})$$

where:

w_i = expenditure on i as a proportion of total expenditures,

e_{ij}, e_{ji} = cross elasticities, and

e_{iy}, e_{jy} = income elasticities.

Since a consumer's expenditure on fruit beverages is a small fraction of total income, the symmetry restrictions were simplified so that the last term on the right-hand side of the above equation was dropped. The test statistic for Slutsky symmetry takes a value of 3.72 when applied to the logarithmic model, which indicates that the symmetry restrictions cannot be accepted. Therefore, only unconstrained estimates were presented.

DISCUSSION

Income Elasticities

By dividing a conditional marginal share by the corresponding conditional budget share, the ratio of the income elasticity of the good to that of the group to which it belongs is obtained:

$$(9)\quad \frac{\mu_i/M_g}{W^*_{it}/w^*_{gt}} = \frac{\mu_i/W^*_{it}}{M_g/w^*_{gt}},\ i \in S_g.$$

Theil (1976, p. 22) calls this ratio the conditional income elasticity of the i^{th} commodity within the g^{th} group. Using the symmetry-constrained estimates of the conditional marginal shares of Table 2 and the average budget shares given in the previous section, one obtains estimates of the conditional income elasticities of the seven fruit beverage categories. The estimates are shown in the first column of the upper half of Table 5. The result indicates that, within the fruit beverage group, orange juice is a luxury and other fruit beverages are necessities. Conditional income elasticities are useful when the analyst is interested in the effect of a change in the consumption volume of the commodity group on a conditional budget share. Theil (1976, p. 23) showed that an increase in the demand for group g raises the associated conditional budget shares for those commodities that have conditional income elasticities larger than 1. This applies to orange juice only, not to the other fruit beverages in this study.

The unconditional income elasticity can be obtained by multiplying equation (9) by the income elasticity of the fruit beverage group. The George and King estimates of income elasticities for fruit and vegetables range from 0.20 for canned fruit and vegetables to 0.66 for frozen fruit. If the income elasticity of the fruit beverage group is between 0.20 and 0.66, the unconditional income elasticity estimates from equation (6) would be anywhere between 0.1 to about 0.9 (orange juice has the highest unconditional income elasticity among them). These estimates are consistent with the results presented by George and King.

The income elasticity estimates obtained from the logarithmic model range from −2.48 for apple juice to 2.33 for grapefruit juice, and most of them are not statistically different from zero. However, the large differences between different estimates may indicate that some of the estimated income effect may be due to other factors (such as gradual changes in consumer's taste over time)[5] which affect the demand for fruit beverages.

TABLE 3. INFORMATION INACCURACY STATISTICS FOR ROTTERDAM MODELS (EQUATIONS (5) AND (6)), UNITED STATES, DECEMBER 1977 THROUGH AUGUST 1982

Type of beverage	Equation (5)	Equation (6)
All beverages	.1419	.0540
Multi-fruit juice/drinks	.0118	.0141
Orange juice	.2434	.0109
Grapefruit juice	.0093	.0100
Apple juice	0038	.0026
Other fruit juices	.0129	.0078
Orange drinks	.0033	.0101
Other fruit drinks	.0065	.0081

Modified Price Coefficients and Implied Price Elasticities

The estimates for equation (6) can be used to obtain the modified price coefficients (v^*_{ij})

$$(10)\quad v^*_{ij} = \pi^*_{ij} + \frac{\Phi M_g}{W^*_{gt}} \bullet \frac{\mu_i}{M_g} \bullet \frac{\mu_j}{M_g},\ i,j \in S_g.$$

The right-hand side contains $\Phi M_g/w^*_{gt}$, which is the own-price elasticity for fruit beverage as a whole (Theil, 1976, p. 17). Since there isn't a price elasticity available, a rough estimate is used. In a consumer demand study, George and King estimated the own-price elasticity for beverages other than coffee and soup to be −.4387 and own-price elasticities for fresh fruit to be between −.6 and −.7. Heien estimated the own-price elasticity for orange juice to be −.535. In this study, it is assumed that:

$$(11)\quad \Phi M_g/w^*_{gt} = -.5.$$

When this numerical value and the symmetry-constrained estimates from Table 2 are substi-

[5] The linear time trend variable is highly correlated with income variable. Thus, it is difficult to separate time trend and income effects.

TABLE 4. ELASTICITY ESTIMATES FROM THE DOUBLE LOGARITHMIC MODEL[a], UNITED STATES, DECEMBER 1977 THROUGH AUGUST 1982

		Price						
Quantity	Income	Multi-fruit juice/drink	Orange juice	Grapefruit juice	Apple juice	Other fruit juices	Orange drink	Other fruit drinks
Multi-fruit	−1.6448	−1.9083	−.0130	.0263	.2964	.8252	.5735	.1940
juice/drink	(1.4417)	(.3850)	(.3291)	(.3260)	(.5204)	(.5684)	(.4357)	(.1985)
Orange juice	.9828	−.2387	−1.3555	.2827	.5477	.0135	.1884	−.1196
	(1.0483)	(.2800)	(.2393)	(.2371)	(.3784)	(.4133)	(.3168)	(.1444)
Grapefruit	2.3326	−.3325	.0039	−1.0653	−.4753	.2630	1.1151	−.6605
juice	(1.5939)	(.4257)	(.3639)	(.3604)	(.5753)	(.6283)	(.4817)	(.2195)
Apple juice	−2.4844	.6381	.1065	−.2418	−1.5178	−.4152	−1.1832	.9927
	(1.1237)	(.3001)	(.2565)	(.2541)	(.4056)	(.4430)	(.3396)	(.1548)
Other fruit	−.0838	−.1302	.4701	.1237	−.0785	−1.1301	.3899	−.2249
juices	(.9136)	(.2440)	(.2086)	(.2066)	(.3298)	(.3602)	(.2761)	(.1258)
Orange drinks	.9437	−.6650	.4979	.0058	.7643	.2506	−.0877	−.5884
	(1.1954)	(.3193)	(.2729)	(.2703)	(.4315)	(.4713)	(.3613)	(.1646)
Other fruit	.6129	−.0777	−.2464	.1095	.0153	.6641	−.3460	−.9728
drinks	(.9445)	(.2522)	(.2165)	(.2136)	(.3409)	(.3724)	(.2854)	(.1301)

[a]The F statistic for testing symmetry hypothesis equals $F^{7}_{259} = 3.7188$, which rejects the hypothesis. Quantity of beverage consumed is measured by quarts per capita; price and income variables are deflated by CPI. All quantity, price, and income variables were converted to common logarithms prior to estimation. Standard errors are shown in parentheses.

tuted into equation (10) and then divided by $\bar{w}_i/w_g$, implied price elasticities are obtained. These are presented in Table 5.

The implied own-price elasticities derived from the symmetry-constrained estimates in Table 2 indicate that the absolute value of the own-price elasticities for these seven beverage categories are either close to or greater than one. In addition, their magnitudes approximate estimates obtained from the double logarithmic model, except the one for orange drink.

Cross-price elasticity estimates from both the Rotterdam model and the double logarithmic model are small compared to their corresponding own-price elasticities, which is consistent with expectations. However, the cross-product relationships are different as estimated by the Rotterdam model and logarithmic model. A definition of substitutability and complementarity is provided by the sign of cross-substitution term of the Slutsky equation (10), v^*_{ij}. If v^*_{ij} is positive, then i and j are substitutes, and if it is negative they are complements (Henderson and Quandt, p. 37). The results presented in Table 5 indicate that most fruit beverages are either substitutes or independents except grapefruit juice and other fruit juices which appear to have a complementary relationship. Similar results were obtained from the double logarithmic model except that complementary relationships were found between multi-fruit juice/drinks and orange drinks, grapefruit juice and other fruit drinks, and orange drinks and other fruit drinks.[6]

TABLE 5. ELASTICITY ESTIMATES FROM THE ROTTERDAM MODEL[a], UNITED STATES, DECEMBER 1977 THROUGH AUGUST 1982

		Implied price elasticities $(v_{ij}/(\bar{w}_i/\bar{w}_g))$ Beverage (j)						
Beverage	Conditional income elasticity	Multi-fruit juice/drink	Orange juice	Grapefruit juice	Apple juice	Other juice	Orange drinks	Other fruit drinks
		GLS estimates						
Multi-fruit juice/drink	.8281	−1.5459	.2539	.1876	.2691	.3890	.1122	−.0800
		(−.1540)	(.0253)	(.0187)	(.0261)	(.0388)	(.0112)	(−.0080)
Orange juice	1.3526	.0542	−1.4178	.0819	.1669	.1834	.1483	.1068
			(−.6612)	(.0382)	(.0778)	(.0855)	(.0692)	(.0498)
Grapefruit juice	.9933	.3750	.7663	−1.2848	.1761	−.9393	.2085	.2014
				(−.0640)	(.0088)	(−.0468)	(.0104)	(.0100)
Apple juice	.4917	.2672	.7757	.0874	−.9593	−.0597	−.2138	−.1434
					(−.0963)	(−.0060)	(−.0215)	(−.0144)
Other fruit juices	.7244	.2796	.6170	−.3377	−.0432	−1.0817	.0430	.1607
						(0.1499)	(.0060)	(.0223)
Orange drinks	.5064	.1635	1.0118	.1520	−.3139	.0872	−1.2242	−.1295
							(−.0837)	(−.0089)
Other fruit drinks	.6874	−.1036	.6470	.1304	−.1869	.2895	−.1150	−1.0051
								(−.0773)

[a]Numbers in parentheses are v_{ij}'s.

[6]If v_{ij} is not significantly different from zero at $\alpha = .05$ level, then i and j are independent. Variance of equation (10) is computed under the assumption that π_{ij} and $\frac{\mu_i}{M_g}\ \frac{\mu_j}{M_g}$ are independent. The variance of $\frac{\mu_i}{M_g}\ \frac{\mu_j}{M_g}$ is obtained by using a formula provided by Mood et al. (p. 180). The resulting variance for equation (10) is almost identical to the variance of π_{ij} in Table 2; therefore, they are not presented. The Slutsky substitution term for the double logarithmic model may be expressed as (Wold and Jureen, pp. 103-4): $k_{ij} = (q_i/p_j)(e_{ij} + (p_jq_j/m)e_{iy})$.

Since consumer per capita expenditures per month for any given fruit beverage are a very small proportion of total per capita income, the second term on the right-hand side of the equation is very small, and because q_i/P_j is positive, the results indicate that all k_{ij}'s have the same signs as the e_{ij}'s presented in Table 4. In the double-logarithmic model, if both e_{ij} and e_{ji} are not significantly different from zero or they have opposite signs and both are significantly different from zero at $\alpha =$ .05 level, then i and j are independent (Myers and Liverpool, p. 33).

Since fruit beverages are consumed primarily for their nutritional content and their particular flavor characteristics, it would seem reasonable that each of the fruit beverages is a potential substitute for the other beverages within the group. If this argument is correct, then the Rotterdam model provides more reasonable estimates than the double logarithmic model.

CONCLUDING REMARKS

The Rotterdam model developed by Theil and Barten was used to estimate the demand interrelationships among fruit beverages. The results show that income elasticity and cross-product relationships estimated with the Rotterdam model are more consistent with expectations than those estimated with the double logarithmic model. The method used in this study should be of interest to researchers involved in estimating demand relationships within a commodity group.

Results from this study show that: (1) under the conditions of block-independence and price exogeneity, the Slutsky matrix for fruit beverage is symmetric and negative semi-definite, which is consistent with consumer demand theory; (2) own-price elasticity estimates from both the Rotterdam model and double logarithmic model are about the same; and (3) income elasticity estimates and cross-product relationships from the Rotterdam model and the double logarithmic model are not compatible.

The conditional income elasticities obtained from the Rotterdam model are positive and are thus consistent with expectations. These conditional income elasticity estimates indicate that if the demand for fruit beverage increases, the conditional budget share for orange juice would be increased. When the income elasticity estimate for fruit beverage group becomes available, the conditional income elasticity estimates presented in Table 5 can be used to derive the unconditional income elasticity estimates for the seven fruit beverages in this study.

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