## STORAGE UTILIZATION IN A DEFICIT REGION

## Wayne A. Boutwell and David E. Kenyon

During the past two decades corn production has increased in the South Atlantic region defined as Virginia, North Carolina, South Carolina, and Georgia but not as rapidly as total U. S. production. The region accounted for 6.7 percent of the U. S. corn production in 1950 compared with 3.7 percent in 1970. During the same period soybean production has increased in the South Atlantic relative to other areas, accounting for 5.3 percent of U . S. production in 1970 , up from 2.9 percent in 1950.

The major consumer of both corn and soybeans is the livestock industry. During the past twenty years this industry has expanded in the South Atlantic. In terms of grain consuming animal units (GCAU), the region accounted for 7.4 percent of U. S. production in 1953 compared with 9.1 percent in 1970. As a result the area is a deficit producer of both corn and soybeans, although with the relative increase of soybean production, the soybean deficit is expected to decrease.

With the South Atlantic being deficit in the production of both corn and soybeans, users of each must look to surplus markets at some point during the year for additional supply. Grain storage in the area therefore has a two-fold purpose; (1) that of holding grain produced in the region, and (2) that of holding grain purchased from surplus areas outside the region for consumption later in the year. In the latter case, the critical question is when should the deficit quantities be purchased. The effective cost to the region for storage of these quantities in the surplus market is determined by the price change
during the year. According to theory the price change between two time periods should reflect storage cost [2]. However during 1963-64 to 1969-70, corn price in Chicago increased more than storage cost in four years but in three years the change in price would not have covered the cost of storage. Chicago soybean prices, during the same period, increased more than storage cost during five of the years and in two of the years the fall price was the high for the year.

The purpose of this study was to provide decision makers in the corn-soybean sector of the grain economy in the South Atlantic with information, prior to harvest, needed to determine the utilization of storage capacity. To fulfill this purpose the objectives were: (1) to determine the amount of corn and soybeans required to make up the deficit, (2) the quarter or quarters each should be purchased, ${ }^{1}$ and (3) the surplus market from which the purchase should be made.

## THE MODEL

A competitive equilibrium model which included the dimensions of time, space, and products was specified to meet the above objectives. ${ }^{2}$ Consider the following statement of the model:

To Maximize


Wayne A. Boutwell is an agricultural economist with the Marketing Development Division, Economic Research Service, USDA, and David E. Kenyon is assistant professor of agricultural economics at Virginia Polytechnic Institute and State University.
${ }^{1}$ The analysis was on a crop year basis with the fall quarter (IV) including October, November, and December and each ensuing quarter consisting of three months.
${ }^{2}$ For a more detailed look at models of this nature including the necessary and sufficient conditions for a maximization see $[1,5,6,7]$.

$$
\begin{aligned}
& P_{j}^{k}(t) d\left(\sum_{t} Q_{j}^{k}(\bar{t}, t)\right)-\underset{t \tilde{t} k j}{\Sigma \Sigma \Sigma \Sigma S^{k}(t, \tilde{t}) Q_{j}^{k}(t, \tilde{t}) ; ~}
\end{aligned}
$$

Subject to

$$
\begin{equation*}
\underset{\text { t kj }}{\Sigma \Sigma \Sigma Q_{j}} \underset{(t+1, \tilde{t})}{ } \leq \mathrm{SC}(\mathrm{t}), \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{t} \mathrm{Q}_{\mathrm{O}}^{\mathrm{k}}(\mathrm{t}, 1)=\mathrm{X}^{\mathrm{k}}(1)-\mathrm{Q}_{0}^{\mathrm{k}}(\mathrm{~T}+1,1) \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
\underset{\mathrm{tj}}{\Sigma \Sigma \mathrm{Q}_{\mathrm{j}}^{\mathrm{k}}(\mathrm{~T}+1, \tilde{\mathrm{t}})+\left(\mathrm{Q}_{\mathrm{o}}{ }^{\mathrm{k}}(\mathrm{~T}+1,1)=\mathrm{I}^{\mathrm{k}}(\mathrm{~T}+1), ~\right.} \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{Q}_{\mathrm{j}}^{\mathrm{k}}(\mathrm{t}, \tilde{\mathrm{t}}), \mathrm{Q}_{\mathrm{j}}^{\mathrm{k}}(\overline{\mathrm{t}}, \mathrm{t}) \geq 0 ; \tag{5}
\end{equation*}
$$

where:
$\mathrm{j}=1, \quad-\quad \mathrm{N}$, where N is the number of markets;
$k=1, \quad-, L$, where $L$ is the number of products;
$t=1, \quad-, T$, where $T$ is the number of time periods;
$\overline{\mathrm{t}}=\mathrm{t}, \quad-\mathrm{T}$, where T is the number of time periods;
$\tilde{\mathrm{t}}=1 . \quad-\quad \mathrm{t}$, where t is a time period within T ;
$\mathrm{P}_{\mathrm{j}}^{\mathrm{k}}(\mathrm{t}) \quad-\quad$ Supply price of the $\mathrm{k}^{\mathrm{th}}$ product in the $j^{\text {th }}$ market;
$\mathrm{P}^{\mathrm{k}}(\mathrm{t}) \quad$ - Demand price of the $\mathrm{k}^{\mathrm{th}}$ product in the study region;
$\mathrm{Q}_{\mathrm{j}}^{\mathrm{k}}\left(\mathrm{t}, \mathrm{t}^{2}\right)$ - Quantity demanded of the $\mathrm{k}^{\text {th }}$ product from the $j^{\text {th }}$ market in the study region over the time period $\tilde{\mathrm{t}}$;
$\mathrm{Q}_{j}^{\mathrm{k}}(\overline{\mathrm{t}}, \mathrm{t})$ - Quantity supplied of the $\mathrm{k}^{\text {th }}$ product from the $\mathrm{j}^{\text {th }}$ market to the South Atlantic region over the period $\overline{\mathrm{t}}$;
$\mathrm{TR}_{\mathrm{j}}^{\mathrm{k}}(\mathrm{t})$ - Transportation rate per unit in period $\tilde{\mathrm{t}}^{\mathrm{T}}$ for product $k$ from $j^{\text {th }}$ market to the region;
$S^{k}\left(t, t^{2}\right)$ - Storage cost per unit for storing the $k^{t h}$ in the South Atlantic region over the period $\tilde{\mathrm{t}}^{\text {; }}$
$\mathrm{SC}(\mathrm{t})$ - Storage capacity in period t ;
$\mathrm{X}^{\mathrm{k}}(1)$ - Production plus beginning stocks for product $k$ in the first time period;
$\mathrm{I}^{\mathrm{k}}(\mathrm{T}+1)$ - Ending inventory for the $\mathrm{k}^{\text {th }}$ product;
$Q_{0}(t, 1)-\quad$ Quantity of product $k$ allocated to time period $t$ from the supply available within the region.

This model, as defined by equation 1 and constrained by equations $2-5$, allocates the beginning stocks plus production in the region to each of the four quarters and to ending inventory. Additional quantities needed to meet the demand may be acquired, through the model, from an outside source at the price in that market plus transportation to the South Atlantic region. The deficit may be bought and stored for consumption or the purchase may be made during the quarter in which it is consumed.

A matrix generator written in Fortran IV was used to automate the data input process. This program provides the researcher with the flexibility of adding time, space, and/or product dimensions to the problem with little effort. It also makes easier additional analysis resulting from changes in input data.

## STUDY SCOPE

The study region includes the states of Virginia, North Carolina, South Carolina, and Georgia. Grain could be purchased outside the region from Chicago, Toledo, and St. Louis for corn and Chicago, Toledo, and Illinois points for soybeans. Transportation rates were based on rail rates from each of these to Charlotte, North Carolina. Charlotte was selected as the basing point for the region through the use of iso-distance lines from each of the surplus markets.

## INPUT DATA

The general model uses as input data: (1) quarterly demand equations for corn and soybeans in the region, (2) prices in the surplus markets, (3) production and stocks in the region, (4) regional storage capacity, (5) transportation rates for corn and soybeans from each surplus market to the region, and (6) storage cost. Using these input data in conjunction with the model specified by equations $1-5$, a competitive equilibrium can be obtained. In order to provide decision makers with utilization information prior to harvest, quarterly prices in the surplus markets were predicted a year in advance and expected rather than actual production was used. The crop year 1969-70 was used to evaluate the model and to determine its value to the corn-soybean sector.

## Demand for Corn and Soybeans

Quarterly demand equations for corn and soybeans for feed were estimated and their intercepts adjusted to account for domestic non-feed
consumption and exports. The structural parameters for the feed demand relations were estimated by two stage least squares (TSLS) using quarterly time series data over the period 1963-64 through 1969-70. Zero-one dummy variables were used to test whether the demand level in each quarter was significantly different from the fall or base quarter, and the price index of livestock lagged one quarter was included as a shift variable in the equations. The resulting structural equations with the standard error of the parameters in parentheses are as follows:

$$
\begin{align*}
& \hat{\mathrm{Y}}_{\mathrm{lt}}=28.930-24.556 \mathrm{X}_{\mathrm{lt}}-12.991 \mathrm{D}_{2}  \tag{6}\\
& \\
& (13.340) \quad(3.133) \\
& - \\
& -23.624 \mathrm{D}_{3}+.290 \mathrm{Z}_{\mathrm{t}-1} \\
& (3.019) \quad(.039) \quad \mathrm{R}^{2}=.85
\end{align*}
$$

and

$$
\begin{array}{r}
\hat{\mathrm{Y}}_{2 \mathrm{t}}=4.390-5.040 \mathrm{X}_{2 \mathrm{t}}+1.378 \mathrm{D}_{1}+.841 \mathrm{D}_{2}  \tag{7}\\
(1.807) \quad(.476) \quad(.492) \\
+\underset{ }{\left(.078 \mathrm{Z}_{\mathrm{t}-1}\right.} \\
(.006) \quad \mathrm{R}^{2}=.85
\end{array}
$$

Where
$\hat{\mathrm{Y}}_{1 \mathrm{t}}$ - Quantity of corn fed in the South Atlantic in quarter $t$ in millions of bushels;
$\hat{Y}_{2 t}-$ Quantity of soybeans fed in the South Atlantic in quarter $t$ in millions of bushels;
$\mathrm{X}_{1 \mathrm{t}}$ - Weighted average price of corn received by farmers in the $t^{\text {th }}$ quarter in the South Atlantic in dollars per bushel;
$X_{2 t}$ - Weighted average price of soybeans received by farmers in the $t^{\text {th }}$ quarter in the South Atlantic in dollars per bushel;
$Z_{t-1}$ - Index of prices received by farmers for livestock and livestock products in quarter $\mathbf{t - 1}$;
$\left.\begin{array}{l}D_{1} \\ D_{2}\end{array}\right\}-0-1$ dummy variables for the winter,
$\mathrm{D}_{3}$ spring, and summer quarters respectively.

The dummy variables omitted from equations 6 and 7 were not significantly different from the fall quarters.

In addition to being consumed as feed, both corn and soybeans are used for domestic non-feed consumption and exports. These two uses were included by adding them to the intercepts of the feed demand equations. ${ }^{3}$

## Price Prediction

Quarterly prices of corn and soybeans in the surplus markets were predicted on September 30 for the ensuing four quarters. These predictions can be used: (1) to indicate which of the markets the area should look to for corn and/or soybeans during each of the four quarters, and (2) to help determine the purchase and storage pattern for the region.

Chicago prices were estimated first using such variables as estimated production, stocks, futures prices, and 0-1 dummy variables. Prices in the other markets were then regressed against the estimated Chicago prices. This process yielded price predictions for each of the surplus markets under consideration. The standard error of the estimates were less than $\$ .034$ per bushel for corn and $\$ .087$ per bushel for soybeans.

## Production and Stocks

Total quantity available at the beginning of the year within the region consists of beginning inventory plus production. This plus the quantity purchased from the surplus markets represents the supply used to meet the demand in each of the four quarters and to satisfy ending inventory. For this analysis estimated production on September 30 was used instead of actual production of corn and soybeans [8]. The 1969 estimated supplies for the region including ending stocks were 204.676 million bushels of corn and 60.823 million bushels of soybeans. For corn, this was 6.872 million bushels too high and for soybeans, 6.276 million bushels too low.

## Storage Capacity

Capacity of storage available for corn and soybeans was determined by adjusting total off farm capacity in the region by the level of utilization of other grains, and adding to this the quantity of farm storage. These other grains included wheat, rye, oats, barley, and sorghum. Their quarterly stocks in off farm storage were used as the basis for making the initial adjustment [11]. Farm capacity was not

[^0]available directly; however, it was estimated by finding the highest quarterly stock for each state in the area over the period from 1963-64 through 1969-70. The resulting storage capacity available for storing corn and soybeans in quarters IV, I, II, and III was $250.857,256.052,257.685$, and 261.027 million bushels, respectively.

## Transportation Rates

Rail is the major mode of transportation of grain received in the South and East. According to a survey of plants in 1964-65, 70 percent of grain received was by rail [9]. Therefore, rail rates were used as the cost of transporting corn and soybeans from the surplus markets to the South Atlantic. The rates were $\$ .3035, \$ .172$, and $\$ .267$ for corn from Chicago, St. Louis, and Toledo, respectively and $\$ .3675, \$ .185$, and $\$ .375$ for soybeans from Chicago, Illinois points, and Toledo, respectively [1].

## Storage Cost

Storage cost is an integral part of the input data required by a model of the nature used in this analysis. It represents the cost to the industry for a temporal transfer. The change in price in the various markets from one period to another represents the effective storage cost between regions. For a deficit supply area, such as the South Atlantic, the change in
price in the surplus markets is compared with the storage cost to determine the purchasing pattern during the year. Storage cost for the South Atlantic was composed of two parts. First, a cost of 13.65 cents per bushel per year was used as the cost for physical storage [10]. To this was added a seven percent opportunity cost for the capital tied up in stocks.

## RESULTS

Separable programming was used to obtain the solution to the multiple time-space-product model specified. ${ }^{4}$ The solution (using the input data above) determined the optimal purchase and temporal allocation of both corn and soybeans. The results are presented in Table 1. Since the solution was for crop year 1969-70 based on information prior to harvest, these results indicate the desired utilization of capacity in the region for that year. Table 1 indicates that 382.781 million bushels of corn and 100.535 million bushels of soybeans were needed to meet the demand in the region during the year. Of these totals, 187.995 million bushels of corn and 40.784 million bushels of soybeans were brought in from outside the region. The solution indicated that both corn and soybeans should have been purchased early and stored in the region for consumption later in the year.

Table 1. OPTIMAL PURCHASE, STORAGE, QUANTITY DEMANDED, AND THE PRICE EQUILIBRIUM FOR CORN AND SOYBEANS FOR THE SOUTH ATLANTIC REGION, QUARTERLY 1969-70,
MODEL II.

| Quarter | Price | Quantity |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Demanded | Purchased ${ }^{\text {a }}$ | Stored |
|  | (Dollars) |  | (1,000,000 Bushels) |  |
| Corn |  |  |  |  |
| IV 1969 | 1.41 | 111.220 | 83.481 | 176.937 |
| I 1970 | 1.47 | 110.317 |  | 66.620 |
| II 1970 | 1.54 | 84.122 | 104.514 | 87.012 |
| III 1970 | 1.60 | 77.122 |  | 9.890 |
| Soybeans |  |  |  |  |
| IV 1969 | 2.51 | 27.687 | 40.784 | 73.920 |
| 11970 | 2.59 | 27.803 |  | 46.117 |
| II 1970 | 2.67 | 24.742 |  | 21.375 |
| III 1970 | 2.75 | 20.303 |  | 1.072 |

[^1]${ }^{4}$ For a discussion of separable programming see [3] and for the algorithm used see [4].

It should be noted that the equilibrium price pattern for corn does not reflect a competitive equilibrium since the price change between quarters I and II is greater than storage cost. All of the storage capacity available for the storage of corn and soybeans was used in quarter IV forcing a non-optimal purchase of 104.514 million bushels of corn in quarter II.

To evaluate the performance of the model, the equilibrium purchase pattern and prices were compared to the purchase pattern and prices in 1969-70. Since import data for the region were not available, the purchase pattern for the year was estimated by adding the ending inventory each quarter to the quantity consumed and subtracting beginning inventory and production. ${ }^{5}$ This procedure indicated that $20,34,25$, and 21 percent of the deficit for corn and $21,6,32$, and 41 percent of the deficit for soybeans were purchased in quarters IV, I, -

II, and III, respectively. Based on $1969-70$ prices, this resulted in an average cost of $\$ 1.50$ per bushel for corn and $\$ 2.83$ per bushel for soybeans for the quantity purchased outside. This assumes that purchases made outside the region were from the least cost market each quarter in terms of price plus transportation cost. These were compared with the prices in Table 1 adjusted to account for differences in the quantity purchased. This resulted in an average cost of $\$ 1.47$ for corn and $\$ 2.58$ for soybeans. Thus the industry could have saved $\$ 0.03$ per bushel on corn and $\$ 0.25$ per bushel on soybeans purchased outside the region. Part of these price differences may have been offset through hedging and other contractual arrangements. However, these savings are a measure of the value to the industry for information on price movements prior to the time decisions had to be made on the $1969-70$ purchase and storage pattern.

[^2]
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[^0]:    ${ }^{3}$ Domestic use was based on per capita consumption multiplied by the estimated population plus the quantity used for seed. Quarterly exports were obtained from the U. S. Department of Agriculture, Fibers and Grains Branch, Washington, D. C. The total of these two for both corn and soybeans was small in comparison to that used for feed, representing approximately 17 percent for corn and 28 percent for soybeans of the total utilization in the region during 1969-70.

[^1]:    ${ }^{\text {a }}$ Corn was purchased in the St. Louis market and soybeans were purchased in the Illinois points market.

[^2]:    ${ }^{5}$ Quantity consumed quarterly was estimated by multiplying the U. S. consumption per animal unit each quarter times the number of animal units fed in the region during each quarter, and adding to this the domestic non-feed uses and exports. Grain consuming animal units were used in the estimations for corn and high protein grain consuming animal units were used for soybeans. For the procedure to determine the quarterly distribution of these animal units see [1].

