A QUANTITATIVE APPROACH TO THE FEEDLOT REPLACEMENT DECISION*

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If all feeder cattle were identical and if all relative prices were constant the feedlot manager would still have an important and difficult decision to make. The decision involves selecting the time at which to replace a pen of cattle on feed with a new pen of feeder cattle such that profit is maximized, over time, to the feeding operation as a whole. Of course, all cattle are far from identical and prices, even relative prices, are never constant. The decision that is not simple with identical cattle and constant prices becomes most difficult with consideration of different types of replacement cattle and varying prices.

The need for study in this area was emphasized during the 1971 annual Cattle Feeder's Seminar on the Oklahoma State University Campus.

> In the final analysis we should recognize that for any group of cattle similar in sex, breed, type, grade, age and weight there is an optimum feeding regime, in terms of type(s) of ration(s) and length(s) of feeding period(s). One of the challenges of the cattle feeder, and research, is to accurately relate the cattle and the feeding regime to obtain maximum profit [11, p. 5L].

This article attempts to apply existing knowledge to the problem as well as to suggest a productive orientation for future research.

THE REPLACEMENT PROBLEM

Faris [4] shows that for short production periods (less than one year) the correct time to replace the present production lot is in accordance with the following criterion: Replace when the positive and decreasing marginal net revenue per unit of time for the present group is equal to the maximum of the expected average net revenue for the replacement group.

Several modifications of the Faris formulation have since been suggested. Chisholm suggests Faris does not always account for both opportunity cost and time preference for income and consequently, does not account for all the relevant opportunity costs of the resources tied up in the production process [1]. Perrin, in a recent article, attempts to further clarify the issue of replacement decisions. Among other suggested modifications, he considers the issue of replacement with technologically improved assets [10].

The theory underlying the making of replacement decisions continues to evolve. Modifications such as those suggested by Chisholm and Perrin do not appear to be crucially important in considering the replacement decision for cattle. The time period is too short for a reformulation of opportunity costs as suggested by Chisholm to exert significant influence on the replacement decision. Perrin's suggested modification relates to the issue of replacing with cattle which are different (in age, weight, herd background, etc.) from the cattle currently being fed. Such differences must be accounted for in formulating the expected average net revenue function for the "replacement" cattle. For these reasons and because of its appealing simplicity, the criterion presented by Faris will be used in this article. Work in the *application* of the currently available theory may well help to guide any subsequent reformulation of the theory as it applies to cattle feeding.

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The Growth Process

The input-output relationship of interest to the cattle feeder is one of converting feeder cattle, feed, and feedlot facility into a saleable product. A problem arises immediately in that the final saleable product is lean meat, but most feeders sell live cattle. The total input-output relationship is a composite of individual input-output relationships for lean, fat, and bone. This growth process is not fully understood; however, certain propositions have become widely accepted both among researchers and industry participants. Some of these propositions which will be useful in this paper are:

1. Every beef animal has a genetically inherent growth curve of sigmoid shape relating accumulated live weight to time. Carcass weight and carcass components are also often depicted as having sigmoid growth curves [12, p. 1F].
2. Some animals mature earlier or reach a given point on their growth curve at an earlier age than others [5, p. 6; 12, pp. 3-4F].
3. Muscle growth matures at an earlier age than fat. As an animal approaches maturity, a larger percentage of the increases in weight is composed of fat. This relationship is important in determining the quality grade and cutability for a given age, feeding period and weight [5, p. 6].
4. Feed conversion (lb. of feed/lb. of grain) usually improves as either feed intake and/or rate of gain increases [12, p. 20F].

Without sophisticated decision methods the above complex and interrelated factors can be considered only on a subjective basis.

The Gompertz Curve

Recent studies have shown that the postnatal growth of mammals and their parts can be described by a combination of a special case of the Gompertz function and a linear function of weight already attained due to the Gompertz function. Both empirical and biological evidence are given for the appropriateness of the function [6, 7, 8]. The linear effect is thought to be small until relatively late maturity has been reached. Therefore, since fed beef animals are typically slaughtered at less than two years of age, it is assumed that the accumulated live weight over time of beef animals can be described by a Gompertz function of the form

$$W_t = W_o\, e^{\frac{A_o}{\alpha}(1 - e^{-\alpha t})}, \tag{1}$$

where:

W_t = Weight at period t in pounds

W_o = A parameter, referring to weight at period t = o, the initial weight for the period of study

A_o = A parameter, the initial specific rate of growth and which is ~~unique~~ to t_o and W_o

α = A parameter, the rate of exponential decay of the specific growth rate and which is constant for all values of W_o and t_o

t = A variable, time (in days for this study).

Several properties of the Gompertz curve make it especially useful in a feedlot replacement model. Among the more important are the following:

1. Any two Gompertz curves employing the same units in t and W_t can be normalized such that they may be compared on a normalized time scale [7, p. 38].
2. The initial time and weight, t_o and W_o, are arbitrary and independent of α. Thus, (t_o, W) may be at birth, weaning, or entry into the feedlot. [7, p. 38].
3. The asymptote W_∞, or mature weight, is $W_\infty = W_o\, e^{\frac{A_o}{\alpha}}$ [6, p. 35].
4. Empirical estimation of the Gompertz curve is possible by iterative non-linear methods or by ordinary least squares through transformation [6, p. 30; 7, p. 235].

One possible transformation of the Gompertz function, which facilitates estimation via least squares, is as follows:

From (1)

$$\frac{d\,W_t}{dt} = W_o\, e^{\frac{A_o}{\alpha}(1 - e^{-\alpha t})}\, A_o\, e^{-\alpha t},$$

$$\frac{1}{W_t}\,\frac{d\,W_t}{dt} = \frac{1}{W_o\, e^{\frac{A_o}{\alpha}(1 - e^{-\alpha t})}}$$

$$\cdot\; W_o\, e^{\frac{A_o}{\alpha}(1 - e^{-\alpha t})}\, A_o\, e^{-\alpha t},$$

$$\ln\left[\frac{1}{W_t}\,\frac{d\,W_t}{dt}\right] = \ln A_o - \alpha t$$

By fitting the least squares model

$$Y = b_o + b_1 X$$

where $Y = \ln \left[\frac{1}{W_t} \frac{\Delta W}{\Delta t}\right]$, and

$X = t$,

the following equations can be generated:

$\hat{b}_o = \ln A_o, \hat{b}_1 = -\alpha$.

It follows, then, that estimates of A_o and α take the following form:

$\hat{A}_o = \text{antilog}\, \hat{b}_o$,

$\hat{\alpha} = -\hat{b}_1$.

By using these estimates and W_o, the weight corresponding to the first observation along the weight scale, the physical growth process can be typified via an appropriate member of the family of Gompertz function.

Cost Relationships

Cost equations are based on the net energy system introduced by Lofgreen and Garrett [9]. For steers

(2) $NE_{gt} = (.05272(g) + .0684(g)^2)(W_t^{.75})$,

(3) $NE_{mt} = .077\, W_t^{.75}$.

where:

- NE_{gt} = Net energy required for gain in megcal, per day
- NE_{mt} = Net energy required for maintenance in megcal, per day
- g_t = Daily gain in Kg. per day
- W_t = Body weight in Kg.

These estimates are for "average" steers and will be in error for steers with growth curves that are quite different from average.

The following assumptions are made in the construction of cost curves as a function of time for fed cattle:

1. Feeders entering the feedlot are on their growth curve, i.e., do not have potential for compensatory gain.
2. A balanced least-cost ration is fed containing at least the required NE_g and NE_m.
3. The cost of ration fed can be represented by a price per megcal. of energy for gain multiplied by gain requirements plus a cost per megcal. of energy for maintenance multiplied by maintenance requirements.
4. Non-feed costs are a constant value per head per day.

Costs can then be determined from the growth curve and energy requirements as follows:

- TC_t = Accumulated cost to day t (total cost).
- AC_t = $\frac{TC_t}{t}$ (average cost).
- MC_t = Addition to total cost in day t (marginal cost).
- P_{neg} = Price per megcal. of energy for gain.
- P_{nem} = Price per megcal. of energy for maintenance.
- P_f = Price of feeder cattle per pound.
- F = Fixed costs.

Adopting the above notation and employing previously defined concepts, the following relationships emerge:

From (1), (2), and (3) we get

$$g_t = \frac{dW_t}{dt} = W_o\, e^{\frac{A_o}{\alpha}(1 - e^{-\alpha t})} A_o\, e^{-\alpha t},$$

$$NE_{gt} = \left[.5272\left[\frac{g_t}{2.2}\right] + .0684\left[\frac{g_t}{2.2}\right]^2\right]\left[\frac{W_t}{2.2}\right]^{.75},$$

$$NE_{mt} = .007\left[\frac{W_t}{2.2}\right],$$

$$MC_t = NE_{gt} \cdot P_{neg} + NE_{mt} \cdot P_{nem} + F, \text{ and}$$

$$TC = W_o \cdot P_f + \sum_{t=t_o}^{T} MC_t.$$

Revenue Relationships

This paper will consider only decisions involving liveweight sales of fed beef. As such, the primary determinants of the value of a live beef animal are weight and grade. For purposes of exposition, price for each grade of slaughter cattle and the feeder animal will be held constant. The necessary per unit revenue functions are developed using the notation and relationships below:

- Pc_t = Price per lb. of choice grade slaughter steers at time t.
- Pg_t = Price per lb. of good grade slaughter steers at time t.
- Ps_t = Weighted price of mixed good and choice slaughter steers at time t with the implicit simplifying assumption that the ratio W_t/W_∞ represents the proportion of the lot grading choice at time t.

Quality grade typically moves through good to choice as the feeding period progresses.

MR_t = Addition to total revenue per head from feeding day t (marginal revenue).

TR_t = Total revenue per head for sale on day t.

$$Ps_t = Pg_t + \frac{W_t}{W_\infty}(Pc_t - Pg_t).$$

$$MR_t = g_t \cdot Ps_t = \frac{dW_t}{dt} \cdot Ps_t.$$

$$TR_t = W_t \cdot Ps_t.$$

The revenue functions were constructed on the basis of live sales. More investigation is needed to determine whether the parameters for carcass and/or lean-meat functional relationships can be accurately estimated from the growth function for the live animal. The tendency for early maturity of lean relative to fat suggests replacement points would occur earlier if production of lean meat were to be used instead of liveweight of the cattle.

THE REPLACEMENT MODEL

Net revenue curves follow directly from cost and revenue curves. Let

MNR_t =Marginal Net Revenue per head per day,
ANR_t =Average Net Revenue per head per day,
TNR_t =Total Net Revenue per head per day,

then

$MNR_t = MR_t - MC_t$,
$TNR_t = TR_t - TC_t$, and
$ANR_t = TNR/t$.

These represent net revenue curves for one initial group of cattle, label it group I. A similar set could be constructed for a second potential replacement group, label it group II. Then one wishes to know the time for which MNR^I = Maximum ANR^{II}. The implication, of course, is that most of the interdependent effects of the several factors such as sex, breed, type, grade and age can be accounted for by the Gompertz growth curve.

Empirical Example

Estimates of growth parameters were made from two different data sets. The first set was original data on 118 steers. After 105 days on full feed, 20 steers were randomly selected at approximately 10-day intervals for slaughter. There were only 18 head in the final group. No birth weights, weaning weights or other weight data prior to entry into the feedlot were available, so t_o for this set of steers corresponds with the beginning of the feeding period.

Estimates for a second group of 100 steers were made from secondary data [13, p. 30]. Weights were taken on samples of 10 head at 30-day intervals. These cattle had relatively low rates of gain, apparently due to restricted energy intake, and the estimates may not indicate the full potential of the cattle. No data were available prior to feedlot entry so again t_o corresponds to the time the cattle entered the lot. The more widely and more evenly spaced observations on this group yielded estimates of the parameters which were significant at the .05 level. The data from the first group were concentrated in a small segment of the time continuum and did not yield estimates significant at the .05 level. The apparent wide variation in average weights of the random 20-head lots undoubtedly contributed to the lower significance of growth parameter estimates from the 118-head set.

Table 1 and Table 2 tabulate the various bits of pertinent information. Shown are time, Marginal Net Revenue per head at day t, Average Net Revenue per head per day for the feeding period, gain per head in day t, attained weight, and Total Net Revenue per head.

Table 1 gives data for the group of 118 head and Table 2 for the group of 100 head tabulated at varying intervals. To illustrate replacement decisions, assume that group I is now on feed and consider the optimum decision from two alternatives. First, consider replacing group I with feeder cattle that are identical to group I. Replacement should occur when MNR_t^I = max ANR^I which is equivalent to $MNR_t^I = ANR_t^I$. This occurs at t = 104 where both MNR_t^I and ANR_t^I equal \$ .18 per head per day.

Group I should be replaced by cattle identical to group I after 104 days with all prices constant over time. Now consider the replacement of group I by group II. Maximum ANR^{II} = \$ .095 and decreasing MNR_t = \$ .095 when t^I = 142. Thus, with given parameters and prices the optimum replacement of group I by cattle of the type in group II is at 142 days.

Of course, if both sets of feeder cattle were available at the given prices $P_f^I = .38$, $P_f^{II} = .34$ the first group would be the most profitable replacement group since $ANR_t^I > ANR_t^{II}$. In general, one should make the replacement decision considering feeder cattle generating the greatest anticipated ANR.

Current State of the Arts

In the "real world" feeder cattle do not come with an attached tag stating their growth parameters

TABLE 1

SIMULATED REPLACEMENT SET FOR 118 STEERS (GROUP I)

$\hat{b}_o = -5.516$	$A_o = .00402$	$P_{nem} = \$ .02$	$P_f = \$ .34$
$\hat{b}_1 = -.0069$	$\alpha = .0069$	$P_{neg} = \$ .04$	$Pg_t = \$ .3038$
est. st. err. of $\hat{b}_1 = .0064$	$W_o = 797$ lbs.	$F = \$ .15$	$Pc_t = \$ .3262$

$$Ps_t = .3038 + \frac{W_t}{W_\infty}(.3262 - .3038)$$

t	MNR	ANR	g	W_t	TNR
30	$ .39	- $ .16	2.92	888	4.67
60	.30	.11	2.60	971	6.96
90	.22	.17	2.28	1045	15.98
100	.20	.184	2.18	1067	18.43
104	.18	.188	2.14	1075	19.33
110	.17	.187	2.08	1088	20.61
120	.14	.187	1.98	1108	22.53
142	.09	.182	1.76	1149	25.90
150	.07	.17	1.69	1163	26.85
180	.01	.16	1.44	1210	29.15

TABLE 2

SIMULATED REPLACEMENT SET FOR 100 STEERS (GROUP II)

$\hat{b}_o = -5.379$	$\hat{A}_o = .00416$	$P_{nem} = \$ .02$	$P_f = \$ .38$
$\hat{b}_1 = -.00512$*	$\hat{\alpha} = .00512$	$P_{neg} = \$ .04$	$Pg_t = \$ .3038$/lb.
est. st. err. of $\hat{b}_1 = .00183$	$W_o = 467.7$	$F = \$ .15$	$Pc_t = \$ .3262$/lb.

$$Ps_t = .3038 + \frac{W_t}{W_\infty}(.3262 - .3038)$$

t	MNR	ANR	g	W_t	TNR
30	$ .33	- $ .68	2.10	531	$ - 20.44
60	.29	- .18	2.01	593	- 10.51
90	.25	- .02	1.90	652	- 1.77
120	.20	.05	1.77	707	5.68
150	.16	.08	1.62	758	11.79
180	.11	.09	1.48	804	16.57
194	.095	.095	1.41	824	18.36

*Significant at .05 level

and age. But, many are bought as "reputation" cattle with information on the feedlot and carcass performance of other cattle from the same herd available to the feeder. Heritability (the extent to which variation in successive generations is predictable in terms of genetic control [3, p. 491] is known to be high for growth traits. Examples of these traits and their respective heritabilities are:

birth weight 50%, weaning weight 30%, weight at 15 months 90%, and rate of gain in feedlot 80% [3, p. 490].

Each of these traits is a function of the growth parameters W_O, A_O, and α which agrees with Laird's hypothesis [7, p. 245] that the growth parameters are "genetically programmed." If feeder cattle were purchased with knowledge of birth weight, weaning weight, and growth traits or parameters of parents or siblings, then estimates of lot parameters could be made initially and updated according to actual feedlot gain and/or feed consumption. This reduces the need for costly repeated weighings.

The applicability of any replacement model will remain a function of the available information. However, improved decision models will encourage improved record-keeping and information transmission.

LIMITATIONS

To be able to place high confidence in the estimates and the use of the Gompertz curve several conditions should be met or closely approximated. First, it is desirable that birth weight and weaning weight as well as other early observations on weight be included. Second, observations on time and weight should be relatively large in number and spaced throughout the growth period. Third, there should be no outstanding environmental factors that would affect the growth of cattle in question; e.g., early feeding of high-energy ration, restricted energy intake, or severe weather.

CONCLUSIONS

The replacement decision is a complex one for the feedlot manager. With increasing sophistication in other phases of feedlot management, however, the replacement decision is of increasing relative importance.

Growth curves of the form of the Gompertz function typify the physical growth process of the beef animal in the feedlot. With appropriate information on age, recurring observations on weight and other background information on feeder animals, the parameters of the Gompertz function can be estimated using traditional estimation procedures. The growth function can then be combined with cost and revenue data to develop an empirically-based replacement model which is consistent with accepted theory on the replacement decision.

There are data limitations. However, the developed model was empirically tested and shows results consistent with a priori expectations. With further development and testing, the model shows promise of moving the replacement decision ahead to a state of advancement consistent with realized levels of sophistication in the areas of nutrition, least-cost ration formulation and other production-oriented management practices.

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