PROJECTION OF COTTON WAREHOUSE FACILITIES FOR OKLAHOMA: A MARKOV PROCESS

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The cotton industry's decline in competitive strength is a matter of increasing concern to industry members and public policymakers. There are many complex economic adjustments arising from changes in government policies, technological advancement, and changing market conditions at home and abroad. The impact of these factors, coupled with increased competition of synthetic fibers and foreign-grown cotton, has been felt by all segments of the industry. Industry experience and recent research results [1, 3] indicate that serious overcapacity has developed in warehousing facilities and, as a consequence, producers and consumers have suffered through increased per-bale storage and assembly costs.

In view of these factors, this paper addresses itself to projecting Oklahoma warehouse viability and determining sizes (bale capacity) of active firms. Firms surviving in the long run are assumed to take advantage of economies of scale. In support of the projections, cost of storage and assembly data are employed to construct an economic planning curve. It is hypothesized that the bale capacity at the minimum point on an envelope curve will represent capacity level of warehouses predicted to survive.

MARKOV PROCESS

Markov chains are employed as a predictive tool, assuming firm growth (changing bale capacity) is a first-order Markov process. The Markov tool provides long-run projections of firms operating in various size categories.

This process requires that any population of firms be classified into n different states, and that movement of firms between states over time be regarded as a stochastic process.[1] Once states have been defined, it is possible to estimate probabilities (P_{ij}) of firms moving from any state i (S_i) to any state j (S_j). These are expressed as a transition matrix, P, where:

$$[1]\quad \sum_{j=1}^{n} P_{ij} = 1, \text{ for all i,}$$

$$[2]\quad 0 \leqslant P_{ij} \leqslant 1 \text{ for all i and j.}$$

Each P_{ij} represents that fraction of firms starting in period t at position S_i and moving to S_j the following period, $t + 1$. The movement interval in the study, t to $t + 1$, is five years.

If all states are accessible, indicating a non-zero probability of moving from state i to state j in a finite number of time periods, then P defines a regular Markov chain. If any P_{ii} element in the transition matrix takes on a value 1.0, a special case of the Markov chain exists. This is known as an "absorbing chain." A Markov chain is absorbing if it has at least one absorbing state which can be entered from all non-absorbing states. Once a firm goes out of business it is mathematically absorbed, since the "out of business" size category is an absorbing state.

When a Markov chain is absorbing, some measures characterizing the system can be calculated. These are: (1) mean length of time in each transient state, (2) probability of absorption and (3) length of time to absorption [2]. A more exhaustive discussion of the Markov technique can be found in Kemeny and Snell [5].

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[1] In this study the states are the various bale capacities of warehouses.

EMPIRICAL RESULTS

Micro-data on movement (changes in bale capacity) of all firms under consideration are essential for calculating transition probabilities. Oklahoma warehouse movement from 1964 to 1973 was constructed using data abtained from the Fibers Group, Economic Research Service, U.S. Department of Agriculture [4]. A flow chart of firm movement between bale capacities from 1964 to 1973 was constructed. To allow sufficient time for firms to change size, two five-year intervals, 1964-1968 and 1969-1973, were selected.

The Markov states ($S_0 \ldots S_5$) are:

STATE	WAREHOUSE CAPACITY (BALES)
S_0	0 (out of business)
S_1	1-15,000
S_2	15,001-25,000
S_3	25,001-40,000
S_4	40,001-75,000
S_5	> 75,000

Given this information and warehouse flows, the following transition matrix was constructed for the period 1964 to 1968.

$$P = \begin{array}{cc} & \begin{array}{cccccc} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{array}{c} S_0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ .4 & .6 & 0 & 0 & 0 & 0 \\ .2 & 0 & .8 & 0 & 0 & 0 \\ .2 & 0 & 0 & .8 & 0 & 0 \\ .333 & 0 & 0 & 0 & .333 & .333 \\ 0 & 0 & 0 & & 0 & 1 \end{bmatrix} \end{array}$$

Similar calculations, using the 1969-73 base, were also made and used in later analysis.[2]

Elements of the transition probability matrix provide meaningful information not easily obtained from other types of projections. The principal diagonal elements of the transition matrix are generally large, indicating industry stability. The first element row S_0 indicates that a firm out of business in 1964 stays out of business in 1968. The S_{55} element indicates a firm in state S_5 in 1964 will remain in S_5 through 1968; thus, S_0 and S_5 are absorbing states. Row S_1 proportions indicate forty percent of warehouses having 1-15,000 bale capacity will cease operation during the 1964-1968 period. Sixty percent will remain.

Given the number of firms in 1964 in six size categories, $S_0, \ldots, S_5$; the number of firms in these categories in 1968 may be obtained. The new industry configuration-by-size category repre-

Table 1. PROJECTED AND ACTUAL NUMBER OF OKLAHOMA COTTON WAREHOUSES, 1973

State Size	Years	
	1973 Actual	1973 Projected
S_0	9	10.24
S_1	1	1.08
S_2	4	2.56
S_3	3	2.56
S_4	0	.11
S_5	1	1.45
Total	18	18.00

2 The transition matrix is available upon request.

sents total warehouses in each category in 1964 times the transition probabilities of moving between size categories. Projections for 1973 through 1998 are calculated, using successive powers of the transition matrix.

Using this technique to test the Markov model's projection ability, projections for 1973 were made. They compared favorably to the actual values, Table 1. Analysis indicated that by 1973, ten firms would go out of business. Actually, nine firms ceased operation. One was operating in the 1-15,000 size category in 1973, thus supporting the projection. Results obtained for states S_2 through S_5 supported the Markov tool's projection ability.

To analyze the absorption process, additional projections were made for each five-year interval to 1998 (Table 2). Warehouse capacity greater than 75,000 bales (S_5) stablized in 1983 at 1.5 warehouses.[3]

Projections based on 1969-1973 industry movement were developed and are shown in Table 2. As a basis for comparison, projections were made to 1998. The 1978 projection compared reasonably well with the previous one, with one exception. The projection for 15,001-25,000 capacity (S_2) for the first base period was 2.05, compared to 4.0 for the second. This resulted from S_2 being an absorbing state in the second base period.

Table 2. PROJECTIONS OF COTTON WAREHOUSES IN OKLAHOMA USING 1964-1968 AND 1969-1973 BASE PERIODS FOR 1978, 1983, 1988, 1993 AND 1998

	1964-1968 Base Period							1969-1973 Base Period						
Year	S_0	S_1	S_2	S_3	S_4	S_5	Total	S_0	S_1	S_2	S_3	S_4	S_5	Total
1978	11.74	.65	2.05	2.05	.03	1.48	18.00	12.14	.11	4.00	.75	.00	1.00	18.00
1983	13.62	.24	1.32	1.32	.00	1.50	18.00	12.59	.04	4.00	.37	.00	1.00	18.00
1988	14.73	.09	.84	.84	.00	1.50	18.00	12.81	.01	4.00	.18	.00	1.00	18.00
1993	15.43	.03	.52	.52	.00	1.50	18.00	12.91	.00	4.00	.09	.00	1.00	18.00
1998	15.86	.00	.32	.32	.00	1.50	18.00	13.00	.00	4.00	.00	.00	1.00	18.00

Since the second set of projections indicate four firms at absorption in S_2, and the first set indicate no active firms at absorption, a question arises as to which is more accurate. While the first set were accurate for actual 1973 firm configuration, the second may be more accurate for 1978 through 1998, because they were based on more recent industry movements.[4] Studies [1, 3] indicate firms of size 15,001-25,000 operate economically, thus supporting the latter projections.

Calculations regarding warehouse movement between categories were made. Specifically, these included mean number of years a firm spends in each non-absorbing size category, time before absorption, and probability of being absorbed. To estimate these values, it was necessary to calculate fundamental matrices from canonical forms of 1964-1968 and 1969-1973 transition matrices. These are a collection of mean stay times in each non-absorbing size category; however, since a five-year movement interval was selected, elements in the matrices must be multiplied times the scaler 5. The sum of the elements of each row represents mean length of time all firms remain in non-absorbing states. The probability of firms being absorbed was found by multiplying the fundamental matrix times probabilities of absorption found in the southwest corner of the canonical matrix. The fundamental matrix for the first time interval under study is:

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{bmatrix} S_1 & S_2 & S_3 & S_4 \\ 2.5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1.5 \end{bmatrix}$$

The matrix containing long run probabilities of absorption is:

$$\begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \end{array} \begin{bmatrix} S_0 & S_5 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ .5 & .5 \end{bmatrix}$$

[3] Integer values are appropriate for industry configuration; however, actual calculations are presented.

[4] A chi-square test for stationarity was preformed and the null hypothesis of stationarity could not be rejected at the 95% level. The calculated test statistic was 4.45 with 30 degrees of freedom.

The probability is 1.0 that S_1, S_2, and S_3 will go out of business in the long run, and 0.5 that S_4 will be either absorbed by S_0 or S_5.

Similar calculations were made using the 1968-73 base period. Results indicate all firms in size classes S_1 and S_3 will, in the long run, be absorbed by S_0. Length of time prior to absorption was 7.5 years for S_1 and 10 for S_3. As of 1973, there were no firms in S_4. Firm survival, then, could not be determined. In the long run, four firms are in S_2 and one in S_5.[5] Economic theory hypothesizes that firms surviving in the long run operate on the low point of industry long run average-cost curve. In view of this, Markov projections are interesting because firms surviving in the long run are not of unique size. Markov results show firms in size classes S_2 and S_5 will survive in the long run, suggesting that firms of either size have cost curves tangent to the falt portion of the long run average-cost curve.

To test this hypothesis, average cost functions for all warehouse sizes had to be developed. These were derived from total cost functions determined by combining costs of all services performed by warehouses. The functions utilized were those reported by USDA. [4].

The services performed and the associated total cost functions are:

Receiving:
$\hat{Y} = 10.367 + 0.785X_3$
$R^2 = 98.4, N = 18$

Storing-variable cost:
$\hat{Y} = -24.339 + 0.529X_1 + 1.652X_2$
$R^2 = 97.3, N = 18$

Storing-fixed cost:
$\hat{Y} = -16.207 + 0.938X_1$
$R^2 = 98.6, N = 18$

Breakout:
$\hat{Y} = 1.770 + 0.511X_3$
$R^2 = 82.4, N = 18$

Shipping:
$\hat{Y} = 9.577 + 0.308X_3$
$R^2 = 94.1, N = 18$

Compression:
$\hat{Y} = 18,693 + 1.159X_3$
$R^2 = 97.2, N = 18$

where: $\hat{Y}$ = Total cost of the associated service in thousands of dollars,
X_1 = Plant capacity in thousands of bales,
X_2 = Percent occupancy of warehouse,
and
X_3 = Bales received in thousand of bales.

Neither t-values nor standard errors of the regression coefficient were reported in [4]; however, it was reported that all coefficient were significant at the 99-percent level.[6] Summing the total cost of each warehouse service yields a function representing total warehouse cost. This can be expressed as:

$$\hat{Y} = -0.139 + 1.467X_1 + 1.652X_2 + 2.763X_3$$

where: $\hat{Y}$ = Total storage and assembly cost in thousands of dollars and independent variables are as earlier defined.

As shown, the above function, representing total warehouse cost, is composed of six components, each expressed in functional form. Increased cotton production density results in increased total assembly costs and in decreased average costs, as reflected by coefficients in each of the six subequations. For example, the greater the density of production, the greater the receipt level by a warehouse. When number of bales received (X_3) increases, a greater percentage occupancy (X_2) will result. Increases in either X_2 or X_3 will result in a total cost increase.[7]

The model was used to construct average cost curves for various size warehouses. Individual warehouse curves enveloped by the planning curve are presented in Figure 1. SAC 1 represents a firm of bale capacity 19,000, SAC's 2, 3, 4 and 5 representing firms of bale capacity 32,000, 50,000, 75,000 and 95,000. These were derived for warehouses corresponding to the Markov states presented earlier.[8]

Points A and B represent points of tangency between the relatively flat portion of the envelope curve and points on the 32,000 and 95,000 bale SAC curves, respectively. At these points, average cost of handling and storing per bale is lower than that resulting from other warehouse sizes. Warehouses represented by SAC_1, SAC_3 and SAC_4 have higher per bale costs than the 32,000 bale plant, SAC_2, has at 15,001-25,000 bales. SAC_2

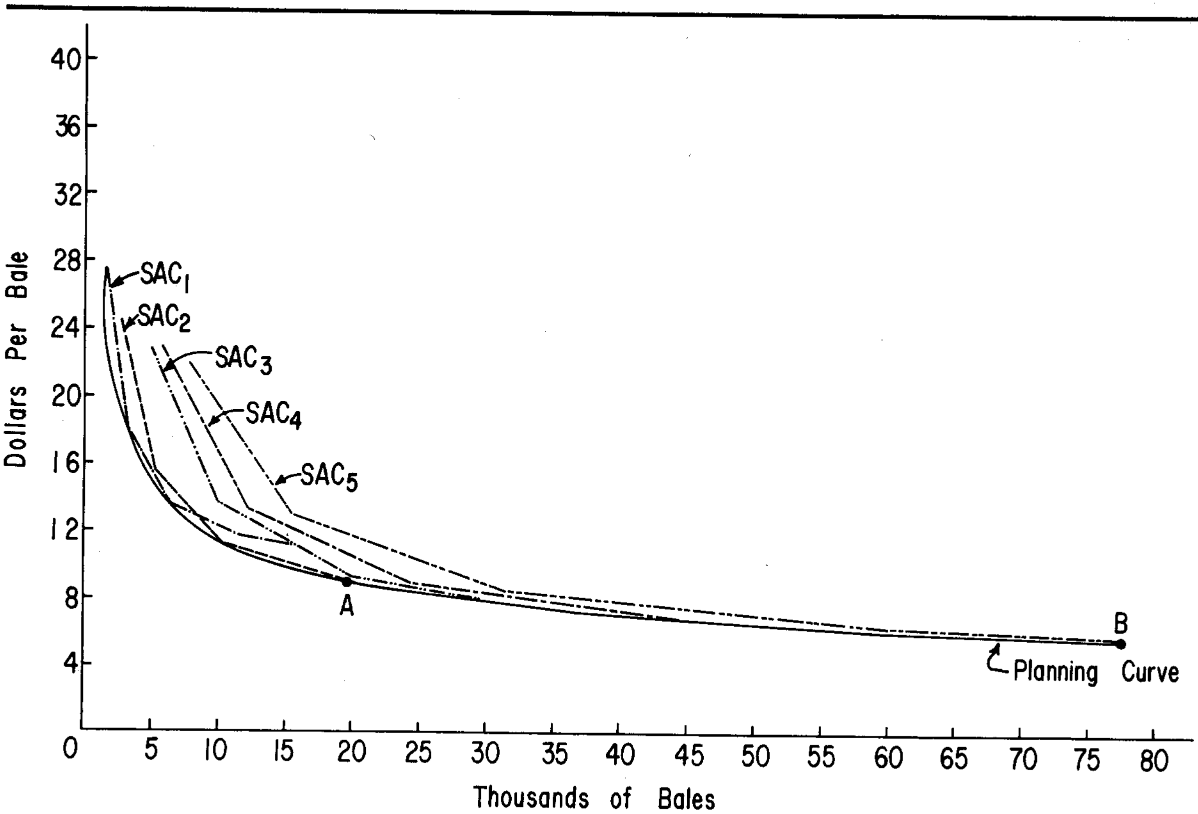
5 The long run projection is 1998 since convergence occurred.

6 The total cost models presented imply downward sloping average costs and constant marginal costs. It should be noted these models were the ones selected over several alternative models.

7 Assuming an oligopolistic industry setting and an increasing cost industry, warehouse rent can be defined as the difference between total revenue and total cost. The charge per bale made by warehouses for storing and handling cotton is relatively invariant with location in Oklahoma; hence, per unit revenue is equal for all warehouses. Costs, however, are dependent on bales received. As discussed, average costs per bale decline with increasing volume; hence, warehouses located in the high production density areas earn more rent than warehouses located in the less dense areas. Rent calculations, although possible, were not in the objectives of this study.

8 Warehouse sizes used to construct the SAC curves are twenty percent larger than warehouse sizes employed in the Markov analysis. This adjustment was necessary because warehouse receipts can be no greater than eighty percent of capacity [6].

Figure 1. AVERAGE COST OF STORING AND HANDLING COTTON IN OKLAHOMA WAREHOUSES



and SAC_5 correspond to Markov states S_2 and S_5, and are tangent to the flat portion of the envelope curve. Therefore, the economies of size hypothesis used to explain the existence of two warehouse sizes could not be rejected, thus supporting long run projections.

COMMENTS

Markov chains were employed to analyze long run viability of cotton warehouses in Oklahoma. The analysis indicated five firms, in two size categories, would survive.

Failure to obtain a unique size category suggested an interesting question. Why will different bale capacities exist? A function representing warehouse total cost was used to develop average costs of various size firms and an economy of size hypothesis was used to support the projections. The hypothesis could not be rejected.

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