FACTOR DEMANDS OF LOUISIANA RICE PRODUCERS: AN ECONOMETRIC INVESTIGATION: COMMENT

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In the December 1988 issue of the *Southern Journal of Agricultural Economics*, Mclean-Meyinsse and Okunade (hereafter MO) applied a cost function to aggregate Louisiana rice producers data in order to examine the substitutability conditions among the inputs of rice production. MO approximated the cost function with a generalized Leontief functional specification. The purpose of this comment is to show that there are some inconsistencies between the theoretical model and the empirical results of MO's article. In the following three paragraphs, a brief presentation of duality in its relation to functional forms will be given. Although this may seem redundant and repetitive to some extent, it is unavoidable since its purpose is to make clear where and why the inconsistencies arise. Be that as it may, I shall briefly specify the model first, and then point out some of the inconsistencies as they relate to MO's study.

Let the typical Louisiana rice grower minimize the cost of producing a fixed level of output, $Q \in \mathbb{R}_{+}$, by choosing the vector of variable inputs, $X \in \mathbb{R}^{n}_{+}$. Technological relationships are assumed to be embodied in a production function $Q = F(X)$, satisfying: $F(0) = 0$, $F(X)$ is quasi-concave, smooth, and nondecreasing in X. Further, let $W \in \mathbb{R}^{n}_{++}$ denote the vector of input prices. Then, there exists a cost function:

$$(1)\ C(W,Q) = \min_{X} \left(W'X\!: Q = F(X)\right).$$

C (W,Q) satisfies: nonnegativity and nondecreasingness in W and Q; concavity and positive linear homogeneity in W. Given that firms are price takers, and assuming that the input requirement set, V(Q), is strictly convex, Shephard's lemma, $X^{*}(W,Q) = \nabla_{w}C(W,Q)$, defines the cost of minimizing input vector, where ∇ denotes the vector differentiation operator.

Several functional specifications can approximate (1). Two of them, members of the family of quadratic functional forms, are the generalized Leontief (Diewert) and the transcendental logarithmic (Christensen et al.). The generalized Leontief (GL) is defined as:

$$(2)\ C = Q\sum_{i}\sum_{j}\beta_{ij}\left(W_iW_j\right)^{1/2} + Q^2\sum_{i}\beta_iW_i + Q\,t\sum_{i}\phi_iW_i,$$

where β_i, β_{ij}, and ϕ_i denote parameters to be estimated, while t denotes time trend. Applying Shephard's lemma to (2) results in the following factor demands:

$$(3)\ X_i^{*} = Q\sum_{j}\beta_{ij}\left(W_j / W_i\right)^{1/2} + \beta_iQ^2 + \phi_i t\,Q.$$

Notice that (2) is homogeneous of degree one in input prices regardless of the values of the parameters. This can be verified by noting that $C(\lambda W,Q) = \lambda C(W,Q)$, $\lambda \in \mathbb{R}_{++}$. Sometimes (3) is expressed in input-output ratio form.

The second approximation of interest is the transcendental logarithmic cost function (TL). It is defined as:

$$(4)\ \ln C = \gamma_0 + \ln Q + \sum_{i}\gamma_i\ln W_i + (1/2)\sum_{i}\sum_{j}\gamma_{ij}\ln W_i\ln W_j + \sum_{i}\phi_i t\ln W_i,$$

where γ_0, γ_i, γ_{ij}, and ϕ_i denote parameters to be estimated. For the sake of simplicity, the constant returns to scale restriction has been imposed on (4). Linear homogeneity restrictions in input prices require:

$$(5)\ \sum_{i}\gamma_i = 1, \sum_{i}\gamma_{ij} = \sum_{j}\gamma_{ij} = \sum_{i}\sum_{j}\gamma_{ij} = 0, \sum_{i}\phi_i = 0.$$

Applying Shephard's lemma to (4) results in the following cost share equations:

$$(6)\ S_i^{*} = \gamma_i + \sum_{i}\gamma_{ij}\ln W_j + \phi_i t,\quad S_i^{*} = \frac{X_i^{*}W_i}{\sum_i X_i^{*}W_i}.$$

Demand elasticities can be readily derived from the estimated parameters and the data. These are defined as:

$$(7)\ \eta_{ij} = \frac{\partial X_i}{\partial W_j}\bullet\frac{W_j}{X_i}\ .$$

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For efficiency reasons, the cost function and the factor demands (or cost share equations) are estimated together. Further, the existence of an error term satisfying all classical statistical properties is implicitly assumed. If the GL specification is used, then (2) and (3) are estimated together with the symmetry restriction imposed. If the TL specification is used, then (4) and (6) are estimated together with the restrictions specified in (5) and symmetry imposed. Suppose we have five inputs, as in MO's study; then, the GL case requires the estimation of a system of six equations; the TL case, however, requires the estimation of a system of five equations. This is so because the dependent variables of the cost share equations add up to unity, $\sum_{i=1}^{5} S_i^* = 1$; the variance-covariance matrix of the error term then becomes singular, thus non-invertible. To circumvent that problem, one equation is dropped prior to estimation. Note that because of (5), all parameters of interest along with their standard errors can be recovered.

MO estimated a GL cost function (i.e., equations (2) and (4) in MO's article). They write: "The factor cost share of each input . . .was the dependent variable in the estimation of the input demand functions given by equation (4) . . . " (p. 130). It looks like they have estimated (2) and (6) (these numbers correspond to this note). If that is the case, they have estimated a mixture of GL and TL.

Further, Table 1 (p. 132) is designed to report elasticity estimates of the derived demand equations calculated according to (7) as defined above (equivalently (6) as defined in MO's article). A cursory inspection of the reported elasticities reveals that, first, $\sum_i \eta_{ij} = \sum_j \eta_{ij} = 0$, and, second, $\eta_{ij} = \eta_{ji}$, contrary to the claim in equation (6) that $\eta_{ij} \neq \eta_{ji}$ in general. In any case, these estimates of the demand elasticities indicate that the implied functional form is not flexible at all, despite MO's claim on p. 128: "These unrestrictive models have been proven superior to the celebrated. . .." On the other hand, if these numbers report parameter estimates, they probably come from a TL system in which case the homogeneity restriction defined in (5) above applies. However, no mention regarding the TL functional form has been made, except in the introduction. If MO used the TL specification, then the tests as described in their "Methodology and Hypothesis" section are no longer applicable, to say the least.

Another point of interest, and misunderstanding at the same time, is the estimation method. MO applied 3SLS to determine the sensitivity of parameter estimates to the omission of a specific redundant factor share equation. The reason for applying 3SLS is unclear; at the outset no endogenous variables appear on the right-hand side of the equations to be estimated, hence no simultaneity bias. After all, an iterated SUR would give unbiased, consistent, and efficient estimates regardless of the equation omitted.

The above points show that there are clear-cut inconsistencies between the theoretical model and the empirical results. That makes MO's conclusions wrong and thus misleading when related to policy matters.

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