AN E-V ANALYSIS OF PRICING ALTERNATIVES FOR LONG-TERM MARKETING CONTRACTS*

Steven T. Buccola and Ben C. French

Increased use of marketing contracts by agricultural firms has stimulated a modest amount of literature in which the principles of decision theory are applied to the contracting problem. Much of this literature has focused on farmers' choices between cash and futures market positions [7]. Others have modeled the influence of annual open market and fixed forward price options on farm growth objectives [1, 3]. Little or no attention has been paid to expressed interest, especially among processors, for suitable long-term (multi-annual) contract price formulae and for a theoretical framework through which to evaluate them. This paper attempts to provide this service in special regard to the tomato and tomato paste contracting problems of a U.S. fruit and vegetable processing cooperative.

ALTERNATIVE PRICING ARRANGEMENTS

The modeled cooperative processor has committed a specific tonnage of bulk tomato paste for ten-year contract sale to a distributor who reprocesses the paste into tomato sauce. In the model variation considered here, 25 percent of anticipated raw tomatoes needed to service the paste contract is expected to be purchased from non-member growers. Non-member tomato purchase contracts are presently signed by the acre on an annual basis, but the cooperative wishes to consider ten-year contracts that would, apart from yield fluctuations, secure the integrity of the ten-year paste contract.[1] Alternative price formulae designed for both paste sale and tomato purchase contracts are: (a) a specifically defined market price, (b) seller's variable production cost times a markup ("cost-plus") and (c) buyer's revenue from resale times a markdown ("sales-minus").

Under market price contracts both buyer and seller may avoid opportunity losses; that is, short term situations in which the open market offers more advantageous terms than does the contract. However, if long-term contracting or vertical integration is widespread in an industry, market prices reflect such a thin proportion of trade that little opportunity is permitted for additional trade at those prices. Besides, market prices are often highly volatile, a drawback for security conscious business firms.

Cost-plus prices are advantageous to sellers because they guarantee sellers a fixed or fixed rate of gross margin. Variable costs of farm production are usually more stable than market prices of farm output, so that cost-plus prices for farm products are more predictable than associated market prices.

Sales-minus prices constitute a seller's share of buyer's resale revenue; buyers are guaranteed a fixed or fixed rate of gross return over the sales-minus priced input. Behavior of sales-minus prices depends on prevailing market conditions for the resold good and buyer's sales strategies and aggressiveness relative to that good. At present, little long term contracting is encountered in U.S. paste markets. The majority of paste is sold at spot market prices, but cost-plus contracts are beginning to attract interest. Some raw tomatoes are sold on three to five year market price

Steven T. Buccola is Assistant Professor of Agricultural Economics, Virginia Polytechnic Institute and State University; and Ben C. French is Professor of Agricultural Economics, University of California, Davis.

*This paper is based on a research project funded by the Farmer Cooperative Service, U.S. Department of Agriculture. The authors wish to acknowledge assistance of Jack Armstrong, Assistant Administrator of FCS.

[1] Contract alternatives were not considered for member growers since it is assumed all members receive a share of cooperative net operating margin.

contracts, and processing cooperatives often purchase vegetables on sales-minus agreements called secondary pools. Long term contracts employing variants of the above formulae will likely expand in both paste and tomato markets in the future.

MODEL CONSTRUCTION

Portfolio Efficiency Criteria

Work by Markowitz [5] and others has demonstrated that a portfolio of contract options possesses greater advantages for risk averting firms than does reliance on any single option, provided expected returns of each option are sufficiently close. In the absence of quantitative measures of firm money utility, research work is reduced to identifying "efficient" portfolios. The most general method of isolating contract portfolios that would never be employed by a risk averse, expected utility maximizing firm is to compare cumulative distribution functions of earnings under each portfolio and reject those functions whose underlying areas are greater than the minimum area (second degree stochastic dominance). If probabilities of each contract price are approximately normally distributed, or if the modeled firm has approximately quadratic money utility within a specified range of wealth, consideration of moments higher than the variance is unnecessary and the more familiar E-V analysis[2] produces results identical to stochastic dominance tests [4]. It was unnecessary to invoke the theoretically troublesome quadratic utility since, as shown below, the normality assumption was found acceptable in the present circumstance. E-V analysis was then selected over stochastic dominance on the basis of its substantially greater programming ease.

Paste and tomato contracting problems were separately studied from the point of view of nonmember grower, cooperative and distributor/reprocessor. In each case, alternative portfolios were evaluated by E-V analysis and by parametric expected utility maximizations [2]. Only E-V results and only the cooperative's case are reported here. To cast the contracting problem in an E-V framework it was necessary to: (a) specify the cooperative's net margin function[3] including all pricing options, (b) develop mean and variance formulae for net margins, (c) estimate means, variances and covariances of alternative price formulae, (d) substitute these moments into the mean and variance formulae and minimize net margin variance at selected mean values.

Net Margin Function

The cooperative's net margin function, which does not include a valuation of raw product delivered from member growers, is provided in Table 1. Lines 1, 10 and 12 contain revenue and costs of non-tomato-paste activities; lines 2-5, formulae defining operation of market price, cost-plus and sales-minus prices as they apply to tomato paste contract sales; lines 6-8, formulae defining these prices as they apply to nonmember tomato purchases; and lines 9 and 11, other variable and fixed costs of contract tomato paste operations.

A contract to trade a specified tonnage of goods is expected to have a different impact on profit variability than a contract to trade random output of a specified number of acres. Both can be modeled in Table 1. Note that the basis of all dollar calculations for paste sales and tomato purchases is to multiply acres of tomatoes by a price or cost variable per ton raw product and by tomato yields per acre. If tomato paste is sold by the ton, sales receipts do not fluctuate with tomato yields; these yields are usually represented by an average yield, which is a constant. If paste is sold by its equivalent raw product acre-yield, sales receipts vary randomly both with per ton price and with per acre tomato yields; in this case yields are represented by a random variable. It is usual for raw tomatoes to be purchased on an acre-yield basis. Where paste is sold on forward contract per ton, the cooperative may contract the number of tomato acres required under expected yields to produce this tonnage. Deviations from expected yields are thus a source of cooperative net margin variability.[4]

Cooperative revenue in the sales-minus purchase option may include revenues from all processing activities or those from paste contract sales only. In either event, contracting growers' crop payments are affected by the contract portfolio the cooperative adopts for its paste sales. This presents modeling problems. If, for example, in a sales-minus purchase contract the cooperative wishes to pay growers a share of its paste sale revenue earned under the optimal paste sale portfolio, lines 2-5 must be

[2] E-V analysis refers to identification of that set of strategies which provide minimum variance of return for selected fixed expected returns.

[3] Cooperative and non-cooperative business firms often consider other objectives than net margin. However, such alternatives are not molded in the present study.

[4] Processors forward contracting all product sales on a tonnage basis and purchasing raw product on an acreage basis must keep an inventory sufficient to carry "normal" pack shortfalls and in which to place "normal" pack excess. An alternative to such inventories, which may carry high maintenance costs, is to per-ton contract a sufficiently small portion of sales that pack shortfalls can be compensated by diversion from non-contract to contract sales.

TABLE 1. COOPERATIVE NET MARGIN FUNCTION

Line number	Formula	Meaning
1	$+REVC_{npst}$	nonpaste revenue
2	$+V_1(AAC)(1/x)(MP^t_{pst} \cdot Y^a_{tom})$	market price paste revenue
3	$+V_2(AAC)(m)(MP^t_{tom} \cdot Y^a_{tom})$	cost-plus paste revenue
4	$+V_2(AAC)(m)(1/x)(NTVCC^t_{pst} \cdot Y^a_{tom})$	cost-plus paste revenue
5	$+V_3(AAC)(n)(1/xy)(MP^t_{sce} \cdot Y^a_{tom})$	sales-minus paste revenue
6	$-R_1(AAC)(MP^t_{tom} \cdot Y^a_{tom})$	market price tomato cost
7	$-R_2(AAC)(k)(VCF^a_{tom})$	cost-plus tomato cost
8	$-R_3(AAC)(\ell)(REVC)$	sales-minus tomato cost
9	$-(AAC)(1/x)(NTVCC^t_{pst} \cdot Y^a_{tom})$	nontomato variable cost of paste production
10	$-VCC_{npst}$	variable cost of nonpaste production
11	$-FCC_{pst}$	fixed cost of paste production
12	$-FCC_{npst}$	fixed cost of nonpaste production

Term	Definition
V_1, V_2, V_3	Nonrandom variables (proportions) by which the cooperative chooses a portfolio of sales contract options.
R_1, R_2, R_3	Nonrandom variables (proportions) by which the cooperative chooses a portfolio of purchase contract options.
AAC	The acreage which, at expected yields per acre, the cooperative calculates will be required to just meet target tomato paste production.
Y^a_{tom}	Tomato yields in tons per acre.
MP^t_{tom}, MP^t_{pst}, MP^t_{sce}	Per ton market prices of: processing tomatoes at farmgate, tomato paste at paste plant, and tomato sauce at sauce plant respectively.
VCF^a_{tom}	Variable (cash) costs to produce an acre of processing tomatoes, Central Valley, California.
$NTVCC^t_{pst}$	Nontomato variable (cash) costs to produce a ton of bulk tomato paste, including tomato transport to cannery.
$REVC_{npst}$	Revenue earned by the cooperative in its nonpaste processing operations.
REVC	Revenue earned by the cooperative from all processing operations, or all paste processing operations.
VCC_{npst}	Variable (cash) costs allocated by the cooperative to its nonpaste processing operations.
FCC_{npst}, FCC_{pst}	Fixed costs allocated by the cooperative to its nonpaste and paste processing operations respectively.
$m > 1$	Markup for cost-plus paste sales.
$k > 1$	Markup for cost-plus tomato purchases.
$0 < n < 1$	Markdown for sales-minus paste sales.
$0 < \ell < 1$	Markdown for sales-minus tomato purchases.
y	Tons of tomato paste required to produce one ton of tomato sauce.
x	Tons of processing tomatoes required to produce one ton of tomato paste.

substituted for cooperative revenue (REVC) in line 8. Since lines 2-5 are linear in V and line 8 (with REVC) is also linear in V, the transformed expression in line 8 would be quadratic in V. The net margin function would become quadratic and net margin variance quartic, so that E-V optimizations would become exceedingly complex. To preserve a linear net margin function and hence quadratic E-V objective function, it is necessary in the sales-minus purchase option to employ a paste sales portfolio that is not a function of V and hence that is not necessarily optimal. In the present case it was assumed for this purpose that all paste is sold under market price contract and none under cost-plus or sales-minus.

Mean, Variance Formulae

All price, revenue and variable cost terms in Table 1 are random variables and remaining terms, except the V, R decision variables, are parametrically alterable constants. Once means, variances and covariances of the random variables are known, mean and variance of net margin can be determined from formulae expressing moments of a linear combination of random variables. One may then measure the effect on total mean and variance of programmed changes in sales and purchase portfolio proportions V, R.

Estimates of Probability Moments

An important problem encountered in estimating probability moments of individual variables is that these moments should represent not historical but prediction probabilities for the ten-year planning horizon. Sources of future uncertainty arise from two sources: uncertainty over which trend line a random variable will follow, and uncertainty caused by random movements expected to occur about whichever trend develops. Because random prices and costs are encountered by the firm in each year of the planning horizon, and the long-term contract decision has to be made in the first year, the appropriate variable measure is a present value sum of anticipated annual uncertainties. To reflect these requirements, a simulation model of a random variable X in year t, representing any random variable listed in Table 1 footnotes, was constructed of the form:

$$X_t = (K + Bt + E_t)/(1 + i)^t \qquad (1)$$

where

K = variable's current value
B = annual trend
E = error about trend, and
i = discount rate

Both B and E_t are random. Distributions B are, in this study, estimated from the firm's subjective probabilities assigned to alternative future trends. Distributions E_t are estimated from historical values about the historical trend, under the assumption that variance around whatever future trend develops is at least approximated by variance around the historical trend. From simulated values of X_t, mean and variance are calculated for each year t and these summed to provide prediction probability moments appropriate for a long-term decision. Prediction covariances are estimated by combining prediction standard deviations with correlation coefficients computed from historical series, where these series are adjusted to induce expected future trend.

An advantage of the simulation model is that random values are generated from which chi-square tests of alternative distribution forms can be designed. In the present case, it was assumed that trend errors E_t are normally distributed with zero means. Subjective trend probabilities B were discrete and did not conform to theoretical distributions; some were moderately asymmetric. However the hypothesis that resultant combined variables X_t are normally distributed was rejected only in the instance of grower variable costs (VCF^a_{tom}).

Efficient Portfolio Solution

The completely specified E-V model is not reported here due to the lengthy set of covariance data involved in the variance expression. Seven cooperative E-V curves were estimated by a quadratic programming routine, each representing a different set of assumptions regarding cost-plus markups m and k and sales-minus markdowns n and l[5]: tonnage or acreage basis paste sales; revenue bases for the sales-minus tomato purchase contracts; optimism of price forecasts; and use restrictions on selected contract formulae. Three of these curves are listed in Table 2 and graphed in Figures 1 through 3. Solid lines indicate efficient mean, variance tradeoffs and dotted lines indicate mean, coefficient of variation tradeoffs.[6] Assumptions under which each set of curves is constructed are given in Table 2 footnotes. Moments measured on axes represent ten-year sums of net margin.

SELECTED MODEL RESULTS

After an initial negatively sloped range that risk averse decision-makers would ignore, E-V curve #1 remains positive but is remarkably flat. The corresponding coefficient of variation curve behaves similarly. Since additional net margin expectation is purchased with very low increments of risk, only highly risk averse decision makers would avoid the profit maximum strategy. This strategy calls for market price paste sales and cost-plus tomato purchases. Exceedingly risk averting coops would ignore the market price sales option and evenly divide sales between cost-plus and sales-minus; their purchases would mostly be made at market prices.

In frontier set #2, the impact of a slightly higher cost-plus sales markup m and lower sales-minus sales markdown n is dramatic. The range of net margin choices increases 600 percent and both curves develop bowl shapes. Cost-plus replaces market price as the high mean profit sales option. Interesting changes are also noted in the tomato purchase portfolios, where market price increases its proportions in the mid-mean range at the expense of cost-plus. Because no changes were made in purchase side parameters, this effect must be due to covariances between revenue and cost terms in the net margin function. Specifically, 65 percent of tomato paste production costs are accounted for by tomato market price, so that sales side cost-plus option V_2 and purchase side market price option R_1 are related by a negative sign in the net margin function. Thus as cost-plus sales increase in portfolio importance due to a rise in the cost-plus markup, risk averters are motivated to increase the proportion of market price tomato purchases as well. Presence of covariances means that sales and purchase contract portfolios are interdependent.

Efficiency set #3 is included to demonstrate what happens when a firm restricts its own access to alternative price formulae. In this case, to guard against market opportunity losses, market price paste sales may not fall below 60 percent of total contract sales. Since market price sales are the high risk option, this constraint removes the lower portion of the E-V curve. The remaining portion is steeply sloped. Moderately to strongly risk averse utility indifference curves would become tangent at the risk minimizing corner solution but would be less steeply sloped than the E-V curve at this point. Hence, in this case the goal of avoiding market opportunity losses is inconsistent with the goal of maximizing expected utility of realized or accounting earnings.

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[5] Values m, n, k, ℓ employed here were selected to generate "meaningful" ranges of expected net margin for each frontier set and have no claim to special importance. An important study objective is to discover the sensitivity of E-V shape and range to changes in these price parameters.

[6] A random variable χ with mean m_χ and variance s_χ^2 has a coefficient of variation s_χ/m_χ. Coefficient of variation or its square, the relative variance, is often used to measure risk as a proportion of income level.

TABLE 2. E-V AND COEFFICIENT OF VARIATION FRONTIERS INDICATING EFFICIENT PORTFOLIOS OF MARKET PRICE, COST-PLUS AND SALES-MINUS CONTRACTS FOR COOPERATIVE TOMATO PASTE SALES AND RAW TOMATO PURCHASES

Efficiency set no. a/	Moments and Coefficients of Net Margin: Mean b/	Variance b/	Standard deviation b/	Coefficient of variation	Tomato Paste Sales c/: Market price (V_1)	Cost-plus (V_2)	Sales-minus (V_3)	Tomato Purchases c/: Market price (R_1)	Cost-plus (R_2)	Sales-minus (R_3)	Number of iterations d/
	million dollars	billion dollars	million dollars		proportions of portfolio			proportions of portfolio			
	339.620	316,344	17.786	.0524	0	0	1.000	0	0	.250	2
	341.330	229,683	15.155	.0444	0	.448	.552	.105	0	.145	16
1	343.049	218,305	14.775	.0431	0	.519	.481	.229	.021	0	16
	344.768	228,562	15.118	.0438	.009	.485	.506	.116	.134	0	28
	346.000	234,707	15.320	.0443	.064	.439	.497	.045	.205	0	31
	349.920	239,022	15.460	.0442	1.000	0	0	0	.250	0	2
	301.768	316,468	17.789	.0589	0	0	1.000	0	0	.250	2
	312.000	243,655	15.609	.0500	0	.174	.826	.112	0	.138	13
	325.480	212,032	14.561	.0447	0	.404	.596	.250	0	0	2
2	337.340	223,015	14.933	.0443	0	.627	.373	.250	0	0	2
	349.000	270,347	16.442	.0471	.052	.805	.143	.250	0	0	16
	356.000	308,866	17.574	.0494	.300	.668	.032	0	.250	0	14
	361.000	343,139	18.524	.0513	0	1.000	0	0	.250	0	2
	288.50	340,239	18.445	.0639	.623	.081	.296	0	0	.250	14
	289.60	345,230	18.580	.0642	.635	.073	.292	0	0	.250	14
3	293.80	397,480	19.937	.0678	.764	0	.236	0	.050	.200	13
	295.00	411,834	20.294	.0687	.804	0	.196	0	.084	.166	13
	298.00	450,621	21.228	.0712	.904	0	.096	0	.167	.083	13
	302.80	549,712	23.446	.0774	1.000	0	0	.166	0	.084	2

[a]For set #1, m = 1.63, n = .25, k = 1.25, ℓ = .08; paste contracts are signed on an acreage basis and sales-minus purchase options provide a share of total cooperative revenue. Set #2 differs only in that m = 1.70, n = .22. In set #3, m = 1.30, n = .22, k = 1.30, ℓ = .40; paste contracts are signed on a tonnage basis and sales-minus purchase options provide a share of contract paste revenue only. In set #3 only, $V_2 + V_3 \leq .40$.

[b]The mean, variance and standard deviation shown here refer to sum of profits over the 10-year planning horizon.

[c]Values listed under V_1, V_2, V_3 are percentages of 53,559.31 tons (or 12,680 acre-equivalents) of tomato paste contracted for sale. Values listed under R_1, R_2, R_3 are percentages of 12,680 acres of raw tomatoes contracted for purchase from nonmembers. (Nonmember purchases are 25% of total.)

[d]These are the iterations (number of linear subprogramming problems) required to reach a minimum-variance portfolio at each constrained-mean point.

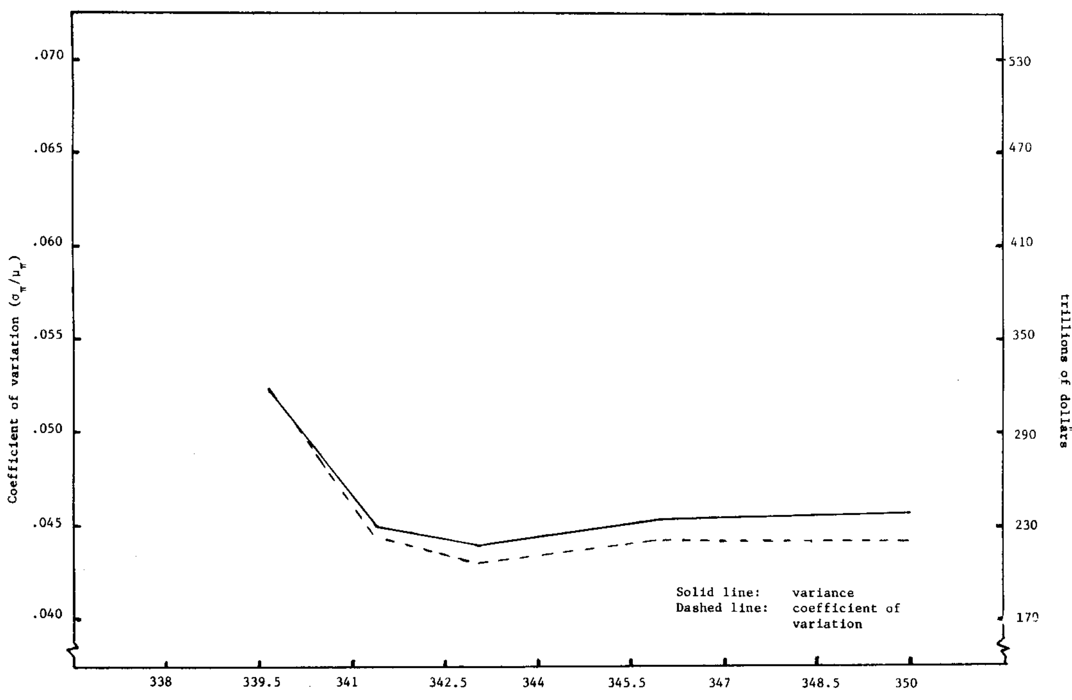

FIGURE 1. COOPERATIVE EFFICIENCY FRONTIER SET #1

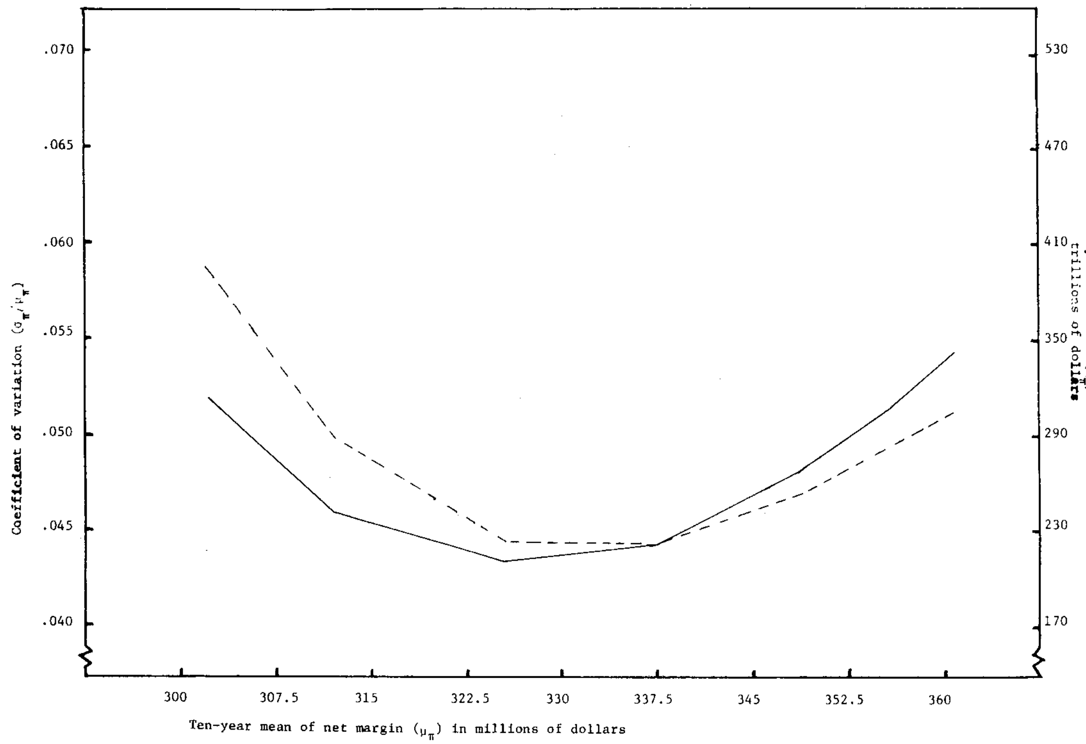

FIGURE 2. COOPERATIVE EFFICIENCY FRONTIER SET #2

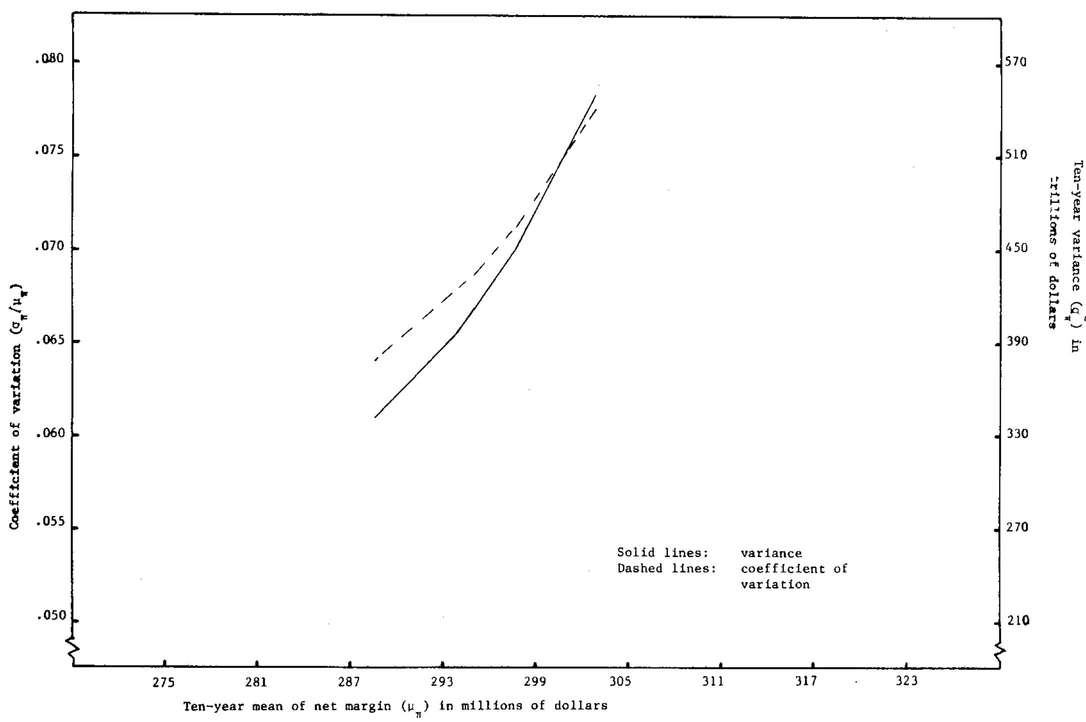

FIGURE 3. COOPERATIVE EFFICIENCY FRONTIER SET #3

OBSERVATIONS AND CONCLUSIONS

One E-V analysis drawback is that decision-makers have no way to select a portfolio they can be sure will maximize expected utility. The best a firm can do is apply a rule of thumb that is intuitively meaningful and associated with acceptable past experiences. However, variances have low intuitive value because they are expressed in different units than means and are generally extraordinarily large. A more meaningful measurement is the coefficient of variation curve which reflects changes in relative risk as net margin expectation increases. Useful rules of thumb might limit relative risk or its positive rate of change. Selection rules relating to coefficients of variation may be entirely inconsistent with those relating to variances. In efficiency sets #1 and #2, for example, portions of rising variance are associated with declining relative risk.

An important result of the E-V studies, for distributor and grower as well as cooperative, is that apparently minor changes in contract formulae can have significant impact on the content of efficient portfolios and their promised returns and risk. An efficiency curve analysis would therefore be a valuable guide to firms engaged in planning and negotiating contract terms. Special care must be taken in this regard to design a complete and accurate net returns function. It is usual for portfolio studies to examine a set of business activities in isolation from the decision maker's total profit and loss picture. This practice would involve significant inaccuracy in the present contracting problem since satisfaction from purchase and sales activities are not independent. Furthermore, alternative contract possibilities should not be evaluated apart from noncontract revenue or variable costs since covariances between contract and noncontract activities, and the level of noncontract business itself, affect desirability of contract portfolios.

Further work should be devoted to structuring methods whereby decision-makers can conveniently combine historical information with prognostications of future conditions to arrive at reliable prediction probability distributions. Historical probabilities alone are inadequate guides to decisions commiting oneself to future uncertainties. E-V results not reported here were highly sensitive to changes in employed probability distributions. In contrast, Porter and Gaumnitz [6] have shown that E-V and stochastic dominance tests produce similar results in all but the minimum variance region. Hence, emphasis on probability formulation should have greater impact on accuracy of long-term decisions than use of theoretically more powerful but less programmable stochastic dominance.

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