MARKOV INTERTEMPORAL LAND USE SIMULATION MODEL

Bruce O. Burnham

The simulation model discussed in this paper evolved from problems encountered in estimating future United States cropland availability as part of the OBERS agricultural projection system.[1] Available literature describing land use changes indicate that land resource economists have not been concerned with projecting future patterns of land use implied by historic observations [2, 3, 4].

Some research has involved selection of optimum cropping patterns for agricultural cropland subject to alternative flood plain management policies [7]. However, the broader application of such models between sectors (agriculture, industrial, urban, etc.), in the main, has been ignored. Because of "historical bias" there has not been a concerted effort to develop analytical capabilities for use in evaluating the future implications of alternative regional and/or national policies designed to alter trends in land use shifts.

As land becomes increasingly dear and competition among alternative uses intensifies, shifts in land use may be affected by further institutional and environmental constraints. Thus, it is imperative that resource economists develop quantitative capabilities which can be implemented to evaluate the impact of alternative national and/or regional land use policies. This capability can be greatly facilitated by adapting quantitative techniques developed and applied in other fields of economics to the natural resource research environment.

This paper applies a methodology that attempts to narrow the gap between historical observation and the dynamics of future land use. If the processes underlying land use shifts are quantified, then future adjustments in land use can be projected. Further, specifications of the underlying shifts in land use makes it possible for resource economists to provide policy makers with economic intelligence concerning the variables that influence future use of the land resource, or alternatively, suggest changes in economic, social, or institutional variables to insure that desired future land use is realized. Quantification of the inter-land use relationships is accomplished by development of a Markov chain land use simulation model.

METHODOLOGICAL FRAMEWORK

The Markov land use simulation model depicts the intertemporal land use shifts and, most importantly, provides a framework for analyzing alternative means of attaining desired future use of land. Application of finite Markov Chain processes in economic research has traditionally been in industrial organization and wage and income distribution [1, 5, 6, 9, 10]. The Markov chain model is closely related to dynamic distributed lag models and consists of two major components: the "transition matrix" and the "transition probability matrix." The Markov process assumes that a variable can be segmented into various states (groups) and secondly, that shifts between states can be specified for some observed period in the past and subsequently summarized in the transition matrix. Given the groups and movements

Bruce O. Burnham is agricultural economist with the National Resource Economic Division, Economic Research Service, USDA.

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[1] OBERS is the acronym that denotes the Water Resources Council national projections developed by the Office of Business Economics (OBE now Bureau of Economic Analysis of the U.S. Department of Commerce) for the nonfarm sector and by the Economic Research Service (ERS, U.S. Department of Agriculture) for the farm sector.

between groups over a historical time frame, it is then possible to estimate the probability of shifts from each group to all other groups. Probabilities are summarized in matrix form and collectively referred to as the transition probability matrix. The probability matrix is subsequently used to project the structure of the variable at alternative future time points, ultimately culminating in the equilibrium distribution of the variable among groups.

Equilibrium configuration is that organization of the variable among groups so that the magnitude of movements out of one group is exactly equal to the movements into that group. Thus, equilibrium couched in the Markovian analysis is dynamic, implying that forces tending to effect movement into a group are precisely offset by factors encouraging movement out of the group.

Transition Matrix [T]

[T] – represents the transition matrix that summarizes the shift from each land use group to all other land use groups between two observed points in time.

The potential applicability of Markov chains is dictated solely on the availability of shift data necessary in developing a transition matrix.[2] A review of publications summarizing historical land use shifts provided a set of data that could be used directly in developing a transition matrix [4]. The categories (groups) of land use in the transition matrix include:

Land Use Groups [3]

L_1 . . . . cropland
L_2 . . . . grassland
L_3 . . . . transition
L_4 . . . . forest
L_5 . . . . urban
L_6 . . . . other

The land use transition matrix [T] (Table 1) provides an exhaustive accounting of acreage movements between alternative land uses in the Southern Mississippi Alluvial Valley (SMAV) between 1950 and 1969.[4] The beginning and ending total land acreage in the SMAV is 24,079,000. The acreage figures in each cell depict the source, destination, and magnitude of land use shifts. For example, the figure in row 2, column 1 (t_{21}), indicates that 219,000 acres of grassland were converted to cropland between 1950 and 1969. Similarly, (t_{12}) indicates that 186,000 acres of cropland shifted to grassland. The diagonal elements indicate the acreage initially in the land use group which remained in that category throughout the period.

Table 1. LAND USE TRANSITION MATRIX [T] FOR THE SOUTHERN MISSISSIPPI ALLUVIAL VALLEY, 1950 TO 1969*

From row i : to column j :	Cropland: (L_1) :	Grassland: (L_2) :	Transition: (L_3) :	Forest : (L_4) :	Urban : (L_5) :	Other : (L_6) :	1950 Total
			(1,000 acres)				
Cropland (L_1):	9,601	186	93	20	46	17	9,963
Grassland (L_2):	219	686	22	20	9	2	958
Transition (L_3):	61	13	24	28	2	1	129
Forest (L_4):	3,818	209	18	7,386	28	61	11,520
Urban (L_5):	1	0	0	1	362	1	365
Other (L_6):	10	1	0	2	0	1,131	1,144
1969 Total	13,710	1,095	157	7,457	447	1,213	24,079

*SOURCE [4], Table 4, p. 8.

[2] It should be noted that procedures have been developed for estimating transition probabilities when only final occurrence data are available, see [8]. A first order Markov process assumes transition probabilities are stationary; this assumption may be too restrictive in the case of land use shifts (see concluding comments). Hallberg [5] has used multiple regression techniques to identify factors causing transition probabilities to shift over time.

[3] Cropland includes (1) fields identifiable by tone, texture, and shape as planted or being prepared for crops, (2) other fields characterized by sharp corners and boundaries, and lack of large vegetation, and (3) areas recently cleared. Grassland consists of open areas generally maintained but lacking evidence of recent tillage. Transition is characterized by irregularly distributed brush and small trees, indefinite boundaries and corners, and uneven tone. Land classified as transitional appears photographically as open land, showing a tendency to revert to forest. Forest includes areas predominately covered with trees, including fairly young stands. Urban is comprised of land used for urban places, farmsteads, airports, factories, mining operations, etc. Other uses include rural highways and roads, small streams and ponds, drainage ditches, sand bars, swamps, and miscellaneous other areas [4].

[4] For a description of the SMAV boundary, see [4].

Table 2. **TRANSITION PROBABILITY MATRIX [P] FOR THE SOUTHERN MISSISSIPPI ALLUVIAL VALLEY, 1950 TO 1969**[a]

From row i : to column j :	Cropland: (L_1) :	Grassland: (L_2) :	Transition: (L_3) :	Forest : (L_4) :	Urban : (L_5 :	Other (L_6)
Cropland (L_1):	.9637	.0187	.0093	.0020	.0046	.0017
Grassland (L_2):	.2286	.7161	.0230	.0209	.0094	.0021
Transition (L_3):	.4729	.1008	.1860	.2171	.0155	.0078
Forest (L_4):	.3314	.0181	.0016	.6411	.0024	.0053
Urban (L_5):	.0027	0	0	.0027	.9918	.0027
Other (L_6):	.0087	.0009	0	.0017	0	.9886

[a]All entries were derived using equation (1).

Transition Probability Matrix [P]

[P] – represents the transition probability matrix that defines the probability of land shifting from one land use to another. [P] is of the same order as [T].

The transition probability matrix (Table 2) is derived directly from the transition matrix (Table 1). Cells (P_{ij}) of the SMAV transition probability matrix [P] indicate the percentage of land use L_i that will be in land use L_j after 20 years, should the 1950-69 history repeat itself.

The P_{ij}'s are calculated using the following notation (where the sum of P_{ij}'s for any row equals unity):

$$(1) \quad P_{ij} = \frac{t_{ij}}{L_i^{1950}}$$

where

P_{ij} denotes the probability of acreage in land use i shifting to land use j over a 20-year period; t_{ij} is the observed acreage shift from land use i to land use j between 1950 and 1969 (cells of the [T]); and L_i^{1950} equals total acres devoted to land use i in 1950 (row totals of the [T]).

For example, P_{11} = .9637 (Table 2) implies that 96 percent of the cropland in the base period will remain as cropland 20 years hence. Similarly, P_{41} indicates that 33 percent of forest land in the base period will shift to cropland 20 years hence.

SIMULATED LAND USE IN THE SMAV

Under the Markov framework the time interval between projection points is fully determined by the temporal relationship between the two observed points. Just as the historical land use change for the SMAV was specified over a 20-year period, so will the projections. Thus, the simulation runs will project land use shifts between 1969 to 1988, 1988 to 2007, etc. However, total acreage in a particular land use can be approximated for inter-projection years by interpolation. Cells of the projected [T], denoted t_{ij}^{t+19}, are developed as an intermediate step in the matrix manipulation process:

$$(2) \quad t_{ij}^{x+19} = L_j^{x} \cdot P_{ij}$$

where

t_{ij}^{x+19} denotes the projected acreage shifting from land use i to land use j over a 20-year period; P_{ij} is the probability of acreage in land use i shifting to land use j over a 20-year period (Table 2); and L_j^x represents total acres devoted to each land use in the base period. Subsequently, via matrix theory, the projected [T] collapses to a projected land use vector $(L)^{x+19}$:

$$(3) \quad (L)^{x+19} = \sum_{i=1}^{6} t_{ij}^{x+19} = (L)^{x} \cdot [P]$$

where

$(L)^x$ is a row vector that specifies total acres in each land use group during the base period and [P] is the transition probability matrix (Table 2).

Thus, the matrix process involves the summation of each column in the projected [T] resulting in the projected 1988 land use configuration (Table 3). The column totals comprise the row vector $(L)^{1988}$ which is subsequently entered in equation (3) as the basis for projecting $(L)^{2007}$. Cells of projected intermediate transition matrices are derived from each successive interval, using the matrix

Table 3. SUMMARY OF LAND USE IN THE SOUTHERN MISSISSIPPI ALLUVIAL VALLEY, 1950, 1969, 1988, 2007, AND 2026.

Year	Cropland	Grassland	Transition	Forest	Urban	Other	Total
	(1,000 acres)						
1950	9,963	958	129	11,520	365	1,144	24,079
1969	13,710	1,095	157	7,457	447	1,213	24,079
1988	16,019	1,192	194	4,870	538	1,266	24,079
2007	17,427	1,263	220	3,224	635	1,310	24,079
2026	18,269	1,311	237	2,180	732	1,350	24,079
Equilibrium land use	11,240	825	151	279	7,621	3,963	24,079

manipulation process, until an equilibrium land use configuration is specified.[5] The equilibrium configuration is a long-run concept. As such, equilibrium may not be very meaningful in terms of the absolute projected acreages; however, the potential outcome does provide valuable insights about the tendencies inherent in the transition probabilities.

Table 3 is a summarization of the historical and projected acreage devoted to each land use group in the Southern Mississippi Alluvial Valley as derived from the 1950-69 transition probabilities.

ANALYTICAL CAPABILITY OF THE MODEL

This portion of the paper recasts the regional cart-and-horse syndrome. In addition to forecasting the future outcome of historic economic, geographic, and/or social processes, why not specify one or several spatial futures a region desires and then analyze the implications of attaining the desired ends?

The Markov land use model does provide an analytical tool for studying alternative policies designed to attain specific land use futures.

Hypothetically, let us assume that a "SMAV Regional Planning Commission" was attempting to evaluate three broad objectives with respect to the future use of land in its region. Specifically, let us suppose the three objectives being considered in developing a comprehensive land use plan are:

1. Retention of the natural forest in the region;
2. Development of grassland at a more rapid rate to accommodate increased grazing; and
3. A reduction in the rate of land urbanization.

Four alternatives were specified as possible means of attaining the objectives under consideration. Each alternative was analyzed in the Markov land use simulation model through adjustments in specific cells of [P] (Table 2). The 1988 distribution of land by use for each simulation is summarized in Table 4.

Alternative 1 reflects the desire to retain more of the region's natural forest. A review of the [P] indicates that retention of forested land could be most directly effected by restraining the shift of land from forest to cropland. Therefore, Alternative 1 assumes a reduced rate of forest shifting to cropland; specifically, the probability of forest shifting to cropland (P_{41}) was reduced from .3314 to .1657, exactly one-half the (P_{41}) value derived from the 1950-69 observations. The reduction in (P_{41}) was added to (P_{44}), thus, forest retained .8068 of its base acreage each 20-year interval, compared with .6411 based on the 1950-69 trend simulation. Compared with the basic results in Table 3, a reduction in the cropland acreage and an increase in forest acreage resulted; however, the equilibrium configuration was

[5] Stochastic matrices such as the Markov probability matrix [P] tend to converge all rows to a unique vector when raised in power, thereby facilitating the determination of a unique equilibrium configuration. Let (k), a row vector, denote the equilibrium configuration, then;

$$(k)' = [P' - I]^{*-1} \cdot (v)$$

where

P' is the transpose of the transition probability matrix; I represents an identity matrix of the same order as P; $[P' - I]^*$ is the transposed transition probability matrix minus the identity matrix which has the last row replaced by a row containing all one's; $[P' - I]^{*-1}$ is the inverse of $[P' - I]^*$ matrix; (v) is a column vector with all zero elements except the last element, which is one; then, the transpose of (k)' leaves the equilibrium configuration (k) row vector.

Table 4. **SUMMARY OF SIMULATED 1988 LAND USE IN THE SMAV UNDER ALTERNATIVE ASSUMPTIONS**

Land Use Group	Current Trend Alternative	Percentage Change from the Current Trend Alternative to: Alt. 1	Alt. 2	Alt. 3	Alt. 4
	(1,000 acres)	(Percentage change)			
Cropland	16,019	-7.7	-15.4	-10.8	+ 0.4
Grassland	1,192	0	0	+62.6	0.8
Transition	194	0	0	0	0
Forest	4,870	+25.4	+50.7	+20.1	+ 0.4
Urban	538	0	0	0	-16.9
Other	1,266	0	0	0	0

not significantly different from the basic model's estimate.

Alternative 2, to a greater degree, reflects the land policy objective intended to preserve the natural forest. Under this alternative, forest was not allowed to shift to cropland; (P_{41}) was reduced from its 1950-69 probability of .3314 to 0.0; correspondingly, (P_{44}) was increased from .6411 to .9725. As a result, the acreage projected to be in forest increased substantially as contrasted to the basic projections; however, cropland acreage was projected to remain virtually constant, slightly above 13.3 million acres, through 2026. The 1988 distribution of land by use was considerably different from the current trend solution (Table 4). The equilibrium land use configuration was substantially different, particularly for cropland, under this alternative than in either Alternative 1 or the current trend projection equilibrium.

Paralleling the second broad planning criteria, Alternative 3 simulations reflect the hypothetical "SMAV Regional Planning Commission's" intent to stress more rapid grassland development in the future than was experienced in the past. It was decided that the transition probabilities would be adjusted to reflect an increased rate of shift from forest to grassland, as well as retention of a higher proportion of forest, thereby reducing the proportion of forest shifting to cropland. Specifically, the probability of forest shifting to cropland (P_{41}) was adjusted downward to .1000 from .3314; forest shifting to grassland (P_{42}) was adjusted upward from .0181 to .1181; and the retention of natural forest (P_{44}) was expanded from .6411 to .7725. As expected, the adjusted transition probabilities had the effect of substantially expanding grassland acreage; in fact in 1988, grassland acreage under Alternative 3 assumptions was 62.6 percent above the Current Trend Alternative estimate (Table 4). By 2026, this model produced grassland acreage estimates nearly double the level estimated using the 1950-69 current trend transition probabilities; however, the long-run equilibrium acreage was similar.

Alternative 4 reflects the objective of a reduced rate of urbanization by eliminating shifts from cropland, grassland, and forest to urban uses. The transition probabilities of cropland, grassland, and forest shifting to urban uses were reduced to zero and retained in their respective land uses. The land use configuration implied by Alternative 4 simulations was quite interesting. For example, urbanized acreage was in a state of dynamic equilibrium throughout the projected time frame (the 3,000-acre shift from urban to other land use groups was directly replenished by a 3,000-acre shift to the urban group each time period). Second, the restriction on the rate of movement from cropland to urban use greatly increased the volume of cropland at each projection point (Table 4). From Alternative 4 simulations and referring to Table 2, it can be concluded that if the basic assumption of Alternative 4 was similarly adopted for other land use groups, cropland would have increased dramatically and the affected group would have remained virtually in Markovian equilibrium throughout the projected time frame. Finally, summary figures of Alternative 4 simulations indicated that cropland, grassland, and transition land uses would approach their equilibrium levels by 2026.

One final note, the source [4] of the initial data used in this analysis estimated that an additional 5.2 million acres of land in the SMAV were suitable for cropland development; this placed an upper limit of 18.9 million acres of cropland in the SMAV. The Current Trend, Alternative 1, 2, 3 and 4 simulation runs projected cropland acreage below the potential, both in 2026 and at equilibrium.

CONCLUSIONS

As evidenced in this analysis, the Markovian framework can be adapted to project the future implications of past land use trends, provided appropriately specified data are available. Remote sensing and aerial photography are ideal methods for pinpointing regional land use shifts between two points in time. The Markov land use simulation model masks the causative variables; however, the model delineates the intertemporal land use shifts and, most importantly, provides a framework for analyzing alternative institutional policies designed to attain specific land use futures.

Further research needs include: (1) determine the extent to which land use transition probabilities change over time; (2) specify economic factors and relationships that cause transition probabilities to change over time; and (3) determine viable future environmental/institutional land use alternatives, and evaluate their impact on the transition probabilities and land use over time.

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