

A METHODOLOGY FOR ESTIMATING INTEGRATED FORECASTING/DECISION MODEL PARAMETERS USING LINEAR PROGRAMMING

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Abstract

A linear programming algorithm is used to estimate the parameters of a wheat storage decision model. This approach allows objective functions other than minimization of error squared to be used. It is demonstrated that by using a profit maximization objective function, an improved wheat storage decision model can be developed.

Key words: decision model, parameter estimation, linear programming, wheat, storage, forecasting.

Traditional agricultural economic forecasting models are often used to aid producers in making management decisions. A classic example is the decision of whether to store or sell wheat. An econometric model is first used to forecast expected future wheat prices. The forecasted wheat price is then used in a storage decision model. Generally, the decision model framework consists of some form of budgeting activity where storage costs are compared to expected revenues as derived from the forecasted wheat price. If an adequate positive return to storage is indicated, the decision to store follows.

The procedure described in the preceding paragraph is typical of many integrated forecasting and decisionmaking processes. It implicitly assumes that the statistical criteria used in developing the forecast model, i.e., minimizing error squared, are consistent with and optimal for the subsequent use of the forecasts in a decisionmaking model. The parameter estimation process does not consider the impact of the forecasting errors upon the decisions made and the resulting

profits. In many decisionmaking cases, the sensitivity of the decision to changes in the forecasted value varies over the range of forecasts to be made. For example, in the wheat storage decision case, the accuracy of forecasts that generate expected returns near the break-even level are quite critical, while those that show large expected profits or losses need not be as accurate.

In developing models for decision making, a methodology is needed that is capable of considering the decision objective and placing more emphasis/weight on forecasting accuracy within critical ranges. Indeed, in the overall decisionmaking process, the model desired is not just a forecasting model, but an integrated forecasting/decision model. The objective of such a model is not to minimize any measure of forecasting error, but to maximize the benefits obtainable from a series of decisions, where the benefits will be realized with uncertainty at some future time. The parameters sought for an integrated forecasting/decision model are those that link a set of known variables to a set of prescribed decision alternatives in an optimal manner. Optimal, in a generalized decisionmaking case, will be defined here as maximization of the profits associated with the decisions. The model is an integrated forecasting/decision model in the sense that the outcome of the decision is implicitly forecasted in determining the optimal decision. This article will present and test a methodology for estimating optimal parameters for integrated forecasting/decision models. It will be shown that the desired parameters can be obtained using mixed integer linear programming methods.

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METHODOLOGICAL DEVELOPMENT

As an initial reference point, Sposito's demonstration of the use of linear programming to estimate parameters that minimize absolute error will be reviewed. Sposito has demonstrated the use of linear programming to estimate parameters of a model (equation). The objective function he demonstrates is similar to that of ordinary least squares, except that deviations from the observed value are measured in terms of absolute values instead of squared values. Letting e_{1i} denote positive deviations and e_{2i} negative deviations, X a matrix of independent variable values, Y a vector of dependent variable values, and b a vector of n parameters, any fitted equation can be represented for the i^{th} observation as:

$$(1) \sum_{j=1}^n X_{ij}b_j + e_{1i} + e_{2i} = Y_i.$$

Thus, the appropriate objective function and constraint equation to estimate the parameter set b , using linear programming becomes:

(2) Minimize

$$\sum_{i=1}^k (e_{1i} + e_{2i})$$

subject to

$$\sum_{j=1}^n X_{ij}b_j - e_{1i} + e_{2i} = Y_i \text{ for all } i$$

$$e_{1i}, e_{2i} \geq 0; b_j \text{ unrestricted in sign,}$$

where k is the number of observations. The parameter vector b is in essence a set of activity level solutions.

Sposito's specification has been modified to allow for both negative and positive parameters and displayed in tableau form in Figure 1, Tableau 1. The tableau contains one row for each observation. The first two columns (activities) provide for estimates of either a positive or negative intercept param-

eter. Likewise, the third and fourth columns (activities) provide for estimates of a slope parameter for the independent variable X . The technical coefficients of columns three and four are positive and negative values of the observed independent variables. The sum of the intercept and slope parameter activities is constrained to equal the dependent variable observation, Y , for each period. To the degree this is not feasible, slack activities representing absolute errors are permitted, but with a penalty to the objective function. Thus, the parameters found will minimize the absolute error of a linear equation for the data set.

Use of linear programming to estimate equation parameters as described above is in essence an alternative to using ordinary least squares. The procedure still focuses upon forecasting error rather than profit maximization from the decision process. However, the framework of linear programming provides the flexibility to specify many different objective functions. By augmenting the parameter activities with different types of activity sets, various alternative objective functions can be specified. Each objective function specified will result in a different set of optimal parameters with any given data set. The augmentation sought here is one that describes the profits and losses generated by the decisions prescribed by the model.

ESTIMATING DECISION MODEL PARAMETERS

The example decision model case to be considered is that of wheat storage. What is sought is an equation that predicts storage profits and losses from which storage decisions can be based. Only two decision alternatives will be considered, i.e., to either store or not store wheat from harvest until December. The decision to store will be assumed to occur if positive profits are predicted,

Tableau 1 - Minimize Absolute Error

| Objective value | 0 | 0 | 0 | 0 | 1 | 1 | ... | 1 | 1 | 1 | ... | 1 | |
|-----------------|----------|-----------|----------|----------|-------|-------|----------|-------|--------|--------|----------|----------|--------|
| Activity | α | $-\alpha$ | B | $-B$ | e_1 | e_2 | \dots | e_N | $-e_1$ | $-e_2$ | \dots | $-e_N$ | |
| | (1) | (2) | (3) | (4) | (5) | (6) | | (7) | (8) | (9) | | (10) | |
| Observation #1 | 1 | -1 | X_1 | $-X_1$ | 1 | | | | -1 | | | $=Y_1$ | |
| Observation #2 | 1 | -1 | X_2 | $-X_2$ | | 1 | | | | -1 | | $=Y_2$ | |
| \vdots | \vdots | \vdots | \vdots | \vdots | | | \ddots | | | | \ddots | \vdots | |
| \vdots | \vdots | \vdots | \vdots | \vdots | | | \ddots | | | | \ddots | \vdots | |
| Observation #N | 1 | -1 | X_N | $-X_N$ | | | | 1 | | | | -1 | $=Y_N$ |

Figure 1. Parameter Estimation Tableaus

otherwise the assumed decision is to sell at harvest. More alternatives could theoretically be considered (sell at other times, sell part of the crop, etc.), but these will not be in order to keep the illustration and comparisons simple. The matrix of activities which augment the parameter activities must be capable of describing all of the decision/payoff combinations. In this case, there are only four: (a) generate positive contributions to the objective function equal to actual storage profits in cases where the model's solution recommends storage and profits actually occurred; (b) generate a negative contribution to the objective function equal to actual storage loss in cases where the model's solution recommends storage and losses actually occurred; (c) generate no impact upon the objective function in cases where the model's solution recommends no storage and losses occurred; and (d) generate no impact upon the objective function when the model's solution recommends no storage and storage profits actually occur. In the latter case, an improper decision was made, but the objective function should not be penalized since no actual losses were encountered.

Tableau 2 in Figure 1 illustrates the mixed integer linear programming matrix developed to describe the wheat storage decision model. Columns 10, 14, and 18 represent integer activities. The X and Y values are defined similar to the X and Y values in Tableau 1. They are historical values. In this case, the Y variables are a time series of budgeted net returns to storing wheat to the month of December. The X values are independent variables whose definitions will be discussed presently.

The matrix contains three basic types of row operations. The first type is labeled as an observation row. Considering observation row 1 and its column intersections, the first six columns are activities to estimate the parameters of the model and are similar to the first four columns in Spósito's general matrix reported in Tableau 1. Two, rather than one, independent variables have been specified to show that multivariable models are feasible and to be consistent with an actual application to be developed later. Values to the right of column 6 describe the decision payoff structure. In the case of observation row 1, a year (case) in which returns are positive is described. Since the X and Y values are historical values, it is possible to define whether the observation in

question is for a profit or loss year. Thus, within column 7, the historical amount of return, Y_1 , is entered in the objective function row as a positive value and in observation row 1 as a negative value. Since observation row 1 is constrained to equal zero and forced solution row 1 forces activity 7 into solution, the sum of the activities in columns 1 through 6 (the prediction equation activities) is being forced toward a positive value to offset the negative profit value entered in activity 7. Hence, the decision/prediction equation will predict values with the desired sign. To the extent this equality is not satisfied, activity SO_1 in column 8 allows for over-estimation of the return level and activity SU_1 in column 9 allows for under-estimation.

The objective function is not penalized for over- or under-estimation of storage returns, except when returns are under-estimated so badly that negative returns to storage are predicted. Such a prediction would lead to an incorrect decision. Wrong decision row 1 monitors the error condition to determine if this has happened. If the level of SU_1 exceeds $Y_1 - .1$, then penalty activity W_1 in column 10 is forced into the solution. This activity is designated as an interger activity. It causes the objective function to be penalized by the storage profit amount Y_1 . This offsets the forced-in positive return and makes the net return equal to zero. Thus, the erroneous decision to not store when profits could have been realized ends up netting zero profit. Entry of penalty activity W_1 also releases the constraint upon the amount of under-estimation allowed by 100 units. Hence, the wrong decision will not be penalized twice, which would result in negative profits. One hundred was picked as an arbitrarily large number to avoid any double penalizing of wrong decisions.

The nature of the constraint and right-hand-side variable in wrong decision row 1, and all wrong decision rows in general, bears more elaboration. For the decision parameter estimation process to work correctly, the penalty activity W_1 must be invoked when SU_1 becomes equal to Y_1 . Otherwise, the observation row equality constraint can always be satisfied by setting all parameter activities in rows 1 through 6 equal to zero and then having activity SU_1 always equal to activity Y_1 . Under these conditions, the observation row equality constraint would always be satisfied without the W_1 penalty activity being invoked; hence, the objective function value

Tableau 2 - Maximize Storage Profit

| Tableau 2 - Maximize Storage Profit | | | | | | | | | | | | | | | | | | | |
|-------------------------------------|----------|-----------|----------|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|-------|----------|--------|--------|--------|------------------|
| Objective value | 0 | 0 | 0 | 0 | 0 | 0 | Y_1 | 0 | 0 | $-Y_1$ | 0 | 0 | 0 | Y_2 | \dots | Y_N | 0 | 0 | $-Y_N$ |
| Activity | α | $-\alpha$ | B_1 | $-B_1$ | B_2 | $-B_2$ | $-R_1$ | SO_1 | SU_1 | W_1 | $-R_2$ | SO_2 | SU_2 | W_2 | \dots | $-R_N$ | SO_N | SU_N | W_N |
| Observation #1 | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | | (15) | (16) | (17) | (18) |
| Forced solution #1 | 1 | -1 | $X1_1$ | $-X1_1$ | $X2_1$ | $-X2_1$ | $-Y_1$ | -1 | 1 | | | | | | | | | | $= 0$ |
| Wrong decision #1 | | | | | | | 1 | | | -100 | | | | | | | | | $= 1$ |
| | | | | | | | | | 1 | -100 | | | | | | | | | $\leq Y_1 - .1$ |
| Observation #2 | 1 | -1 | $X1_2$ | $-X1_2$ | $X2_2$ | $-X2_2$ | | | | | $-Y_2$ | 1 | -1 | | | | | | $= 0$ |
| Forced solution #2 | | | | | | | | | | | 1 | | | | | | | | $= 1$ |
| Wrong decision #2 | | | | | | | | | | | | 1 | -100 | | | | | | $\leq -Y_2 - .1$ |
| \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | \vdots | | | | | | | | | \vdots | | | | |
| Observation #N | 1 | -1 | $X1_N$ | $-X1_N$ | $X2_N$ | $-X2_N$ | | | | | | | | | | $-Y_N$ | -1 | 1 | $= 0$ |
| Forced solution #N | | | | | | | | | | | | | | | | 1 | | | $= 1$ |
| Wrong decision #N | | | | | | | | | | | | | | | | | 1 | -100 | $\leq Y_N - .1$ |

Figure 2. Parameter Estimation Tableaus

would be at a maximum. However, the decision model/equation derived would be useless since it would always predict zero profits. To prevent this solution result, the wrong decision row constraint needs to be specified as a strictly "less than" $|Y_1|$ constraint instead of a "less than" or "equal to" $|Y_1|$ constraint. But a "less than" constraint is not an available constraint option in linear programming. However, a simple modification can be made to convert a "less than or equal to" constraint into an approximate "less than" constraint. The modification is to subtract an arbitrary small value from $|Y_1|$. In this case, .1 was used. Thus, a value "less than or equal to" $|Y_1| - .1$ is effectively "less than" $|Y_1|$.

The rows labeled observation 2, forced solution 2, and wrong decision 2 are the same as the first three rows, but are for a year (case) in which losses were encountered on storage, i.e., $Y < 0$. Because losses were encountered, the decision payoff activities are specified differently. The correct decision return level activity in column 11 now has a zero value in the objective function row since the correct decision is to not store and the result is zero storage profit. A negative Y_2 is entered in column 11 for observation row 2. This actually results in a positive value since Y_2 itself is negative; i.e., $-(-Y_2)$ equals a positive value. Given that observation row 2 is constrained to equal zero and activity 11 is forced into solution, the sum of the activities in columns 1 through 6 (the prediction equation activities) is being forced toward a negative value. As was the case for observation row 1, to the extent the observation row equality is not able to be satisfied, activity SO_2 in column 12 allows for over-estimation of the actual loss (i.e. predicting too large a loss), and activity SU_2 in column 13 allows for under-estimation of the loss (i.e., predicting too small a loss). For example, a loss of 10 cents per bushel would be reflected by the entering of a positive 10 in column 11 of observation row 2; i.e., $-(-10)$. If the sum of the activities in columns 1 through 6 over-estimated the loss by 2 cents, they would sum to a -12 . Activity SO_2 would then have to equal 2 to satisfy the equality condition for observation row 2. On the other hand, if the sum of the activities in columns 1 through 6 under-estimated the loss by 2 cents and totaled a -8 , activity SU_2 would have to equal 2, con-

tributing a -2 to the equality condition of observation row 2.

Wrong decision row 2 monitors the magnitude of the under-estimation of the actual loss. If the level of SU_2 exceeds a $| -Y_2 | - .1$, which for the example considered is 9.9, (i.e., $10 - .1$), profits will be predicted when losses actually occurred. Such a prediction will lead to an incorrect decision and the loss of Y_2 cents per bushel. In this situation, wrong decision row 2 causes the integer penalty activity W_2 in column 14 to be brought into solution. Entry of activity W_2 causes the objective value to be penalized by the loss Y_2 . It also released the constraint upon the amount of under-estimation allowed by 100 units.

In general, three row operations and four columns are required for each observation considered. Activities 1 through 6, which estimate the model parameters, apply over all observations. The solution values for activities 1 through 6 will yield the prediction/decision equation which maximizes storage profits.

The solution values for the prediction/decision equation parameters may not be unique. More than one equation can lead to the same set of correct decisions and thus the same objective function value. The discrete nature of the decision process and profit/loss consequences leads to this situation. In the terms of the matrix developed in Tableau 2, this is to say that a number of alternative activity levels for columns 1 through 6 will result in the same set of integer activities being forced into solution for erroneous decisions. However, as the number of observations considered increases, the parameter range over which the same set of integer activities would come into solution is reduced. This problem is of no major concern as long as an equation which renders a maximum number of correct decisions is found.

Unique solutions for the parameter values can be assured by adding penalty values to the objective function for the degree of over- and under-estimation. This seems to be a logical action since improperly anticipating the magnitude of profit to be received would likely lead to some economic cost due to improper planning. Penalties for the absolute error in forecasting judged to be reflective of such costs have been imposed for one of the decision model applications which follow.

ALTERNATIVE MODEL DEVELOPMENT

Three integrated wheat price forecasting and wheat storage decision models will be developed and presented. The first model consists of a traditional econometric forecasting model and budgeting decision model combination. The second and third models consist of two alternative integrated forecasting/decision models whose parameters are estimated using mixed integer programming. For comparison purposes, all three models will be based upon the same data and function. The data used are for the period 1960 to 1979. It consists of four series describing the rate of return to wheat storage, annual wheat supply, annual wheat disappearance, and wheat carryover stock levels. The series for rate of return to wheat storage was calculated to be the return for storing wheat from June (the harvest month) to December. Over the period 1960-1979, December, on average, was the most profitable month to sell stored wheat. Returns to storage were calculated as:

$$(3) \text{ Storage return} = \text{December price} - \text{June price} - \text{Storage costs} - \text{Interest cost.}$$

Wheat prices used were the national average mid-month price received by farmers for all wheat. Storage costs were calculated as 1.5 cents per bushel per month. Interest costs, reflecting the opportunity cost of the value of the stored wheat, were calculated as 3 percent of the June harvest price, thus reflecting a 6 percent annual interest rate. These storage and interest costs were selected as typical of the average costs incurred over the 1960-1979 period. Both the storage costs and the interest rate likely rose over the period in question. No reliable storage cost series could be found to adjust storage costs. Given storage cost changes could not be objectively quantified and that the major thrust of this effort was to develop methodology, changes in interest rates were also ignored.

The Econometric Model

The econometric forecast model, specified and estimated, is as follows (values in parenthesis are t-values for the parameters):

$$(4) Y = -33.94 + 14.59X_1 - 1.07X_2$$

(3.6) (4.3) (2.2)

$$\text{Standard error} = 17.65; R^2 = .66,$$

where:

Y = rate of return to storing wheat until mid-December, i.e., storage returns divided by harvest price times one hundred;

X_1 = $[1.0/\log (\text{quantity available/disappearance})]$ where quantity available is total wheat production plus carryin stocks (natural logarithms are used); and

X_2 = change in wheat stocks during the year as a percent of quantity available, i.e., carryin stocks minus carryout stocks divided by quantity available and multiplied by one hundred.

The theoretical basis for the this model will not be elaborated upon since it is not the primary focus of this paper. The basic theory underlying the model is that of excess demand for storage as presented by Bressler and King. This theory would indicate that as the supply/demand ratio increased the returns to storage would decline; hence, the inverse of the log of this ratio would be expected to be positively correlated with the rate of return to storage. A negative sign is theorized for the change in stock level variable. Declining stocks are generally associated with rising price and, hence, increased returns to storage. Both estimated parameter signs are as expected and are statistically significant at the .025 level of confidence.

Linear Programming Model 1

The first integrated forecasting/decision model to be estimated is the one described previously and represented in Tableau 2. The objective function specified can be thought of as maximizing the cumulative profits from a two-alternative decision situation over the data period considered. The equation derived using this objective function is:

$$(5) Y = -4.276 + 1.287X_1 + .0001X_2.$$

The definitions of Y , X_1 , and X_2 are the same as for the econometric model.

Linear Programming Model 2

The second integrated forecasting/decision model to be specified is very similar to the first. The only change made is to the objective function specification. In the previous model,

no consideration of the accuracy of the profit and loss level forecasted, other than proper sign, was given. It would appear reasonable to assume that the producer would encounter some economic costs by improperly anticipating the magnitude of profits to be received in years he chose to store wheat. With this logic in mind, a value of $-.1$, reflecting a 10 percent penalty of profit, was entered in the objective function row of all SO_i and SU_i activities. Unlike the first linear programming model specification, this specification leads to a unique set of parameter solutions for each objective function value.

The equation derived using this approach is:

$$(6) Y = -10.698 + 3.224X_1 + .00009X_2.$$

Variable definitions are again the same as previously given for the econometric model. The parameter values obtained are different than those obtained for LP Model 1. However, they are much closer in magnitude to the parameters for LP Model 1 than to the parameters for the econometric model.

Application and Evaluation

Table 1 presents a summary of the prediction accuracies of the three models devel-

oped. As seen from the table, the prediction accuracy of the econometric model is far superior to that of the two linear programming integrated forecasting/decision models referred to as LP 1 and LP 2. This is as expected given the econometric model was estimated with the objective of minimizing forecasting error squared. Outside of the data range used for estimating the models, i.e., years 1980 through 1984, the error squared values are quite comparable. Also, as might be expected, LP Model 2 has a lower sum of errors squared than LP Model 1. This would be expected since LP Model 2 was penalized by a $-.1$ for profit prediction errors, while LP Model 1 was not.

Table 2 summarizes the storage profits generated from using each of the models to make storage decisions. The cumulative profit columns show that the two LP decision models are superior to the econometric model and an arbitrary "always-store" model. Despite the fact that the two LP decision models had different objective functions and parameters, they yield the same set of decisions and profits. This is the case because of the lack of uniqueness of the parameter solution values when only storage profits are considered in the objective functions. This fact was noted

TABLE 1. ACTUAL STORAGE PROFIT, PREDICTED STORAGE PROFIT, AND PREDICTION ERROR SQUARED FOR ECONOMETRIC AND DECISION MODEL FORECASTS, 1960-1984

| CROTON PRODUCTION ESTIMATION, 1960-1984 | | | | | | | |
|---|-------------------------------|--------------------------------|-------|-------|--|-----------|-----------|
| Year | Actual profit ^a | Predicted profit by model type | | | Prediction error squared by model type | | |
| | | Econometric | LP 1 | LP 2 | Econometric | LP 1 | LP 2 |
| -----cents/bushel----- | | | | | | | |
| 1960 | -5.33 | -18.40 | -2.52 | -6.36 | 170.93 | 7.88 | 1.07 |
| 1961 | 1.65 | -9.23 | -2.45 | -6.07 | 118.35 | 16.83 | 59.55 |
| 1962 | -6.02 | -6.89 | -2.44 | -6.02 | .76 | 12.75 | 0.00 |
| 1963 | -1.93 | 9.62 | -1.76 | -4.16 | 133.19 | .03 | 5.01 |
| 1964 | -10.14 | -3.75 | -1.52 | -3.80 | 40.86 | 74.37 | 40.18 |
| 1965 | -.66 | 20.25 | -.73 | -1.60 | 437.06 | .01 | .89 |
| 1966 | -7.40 | 22.36 | -.10 | -.10 | 885.84 | 53.33 | 53.33 |
| 1967 | -15.75 | -1.10 | -.77 | -2.01 | 214.62 | 224.49 | 188.71 |
| 1968 | -8.65 | -20.01 | -1.72 | -4.53 | 129.16 | 47.91 | 16.92 |
| 1969 | -3.82 | -29.17 | -3.82 | -9.56 | 642.62 | 0.00 | 32.96 |
| 1970 | -3.82 | 7.00 | -1.39 | -3.34 | 117.09 | 5.91 | .23 |
| 1971 | -17.38 | -12.65 | -1.70 | -4.38 | 22.37 | 245.71 | 169.08 |
| 1972 | 69.18 | 36.74 | .35 | 1.16 | 1,052.35 | 4,738.11 | 4,626.85 |
| 1973 | 90.00 | 69.67 | 3.69 | 9.46 | 413.31 | 7,449.76 | 6,487.33 |
| 1974 | 23.33 | 24.96 | 1.39 | 3.42 | 2.66 | 481.28 | 396.41 |
| 1975 | 10.70 | 4.89 | .10 | .10 | 33.71 | 112.36 | 112.36 |
| 1976 | -35.75 | -22.05 | -1.55 | -4.15 | 187.69 | 1,169.98 | 998.69 |
| 1977 | 14.24 | -4.97 | -1.50 | -3.79 | 369.06 | 247.81 | 325.01 |
| 1978 | .55 | 15.20 | -1.95 | -1.96 | 214.74 | 7,318.11 | 6.28 |
| 1979 | -1.93 | 8.61 | -.60 | -1.47 | 110.99 | 1.77 | .21 |
| 1980 ^b | 33.90 | -2.41 | -.97 | -2.53 | 1,318.42 | 1,215.92 | 1,327.14 |
| 1981 ^b | -10.10 | -4.91 | -.98 | -2.58 | 26.94 | 83.17 | 83.17 |
| 1982 ^b | -8.17 | 1.26 | -.92 | -2.33 | 88.93 | 52.56 | 34.10 |
| 1983 ^b | -21.44 | -13.64 | -1.64 | -4.08 | 60.84 | 392.04 | 301.37 |
| 1984 ^b | -27.14 | 2.52 | -1.34 | -3.35 | 879.72 | 665.64 | 565.96 |
| Sum | | | | | 7,672.23 | 17,301.58 | 15,832.81 |

^a Profit is calculated assuming storage until the month of December, a storage charge of 1.5 cents per bushel per month, and an interest rate of 6 percent applied to the June mid-month average price received by farmers for wheat as reported in the *Agricultural Prices Received* series reported by the USDA.

^b These years are outside the data used to estimate the models.

in the methodological development. Table 2, like LP Model 1, considers only storage profits and losses. The equation obtained for LP Model 2 is unique with regard to maximizing the objective function of Model 2. But it and the solutions equation for LP Model 1 are only two of a number of equations that would yield the same set of correct and incorrect decisions as defined and reported in Table 2.

The LP decision models are superior to the econometric model both within the data range used to estimate the models and outside of it. Within the estimation period, the LP decision models generate only three improper decisions while the econometric model makes seven wrong decisions. For the 5 years reported outside of the data estimation range, the LP models generate only one improper decision while the econometric model generates three improper decisions. The fact that the two LP models perform the same both within and outside the data period, despite being estimated with somewhat different objective functions, testifies to the robustness of the estimation approach.

The performance results reflected in Table 2 are as expected. Since the LP models were developed using measures of storage profit as their objective function, they would be expected to out-perform an econometric

model in this respect. The question to be posed at this point is what objective function should the model being estimated have. If the purpose for developing the model is to use it for decisionmaking, then some measure of the decisionmaker's objective function is more appropriate than the traditional econometric objective function of minimizing the sum of errors squared. The linear programming approach to parameter estimation permits a variety of objective function choices to be made that are not possible with traditional econometric models. A model which has been specified and estimated to maximize (or minimize) a certain objective function should always do so with greater ability than one specified for another purpose.

SUMMARY AND CONCLUSIONS

An alternative method of estimating integrated forecast/decision model parameters has been presented. The method makes use of linear programming as the estimation algorithm. This allows the objective function for the estimation process to be flexible. It is contended and demonstrated that this capability can be used to improve the profits derived from a wheat storage decision model. The fundamental reason this approach is able

TABLE 2. *EX POST* RETURNS TO STORAGE USING ALTERNATIVE FORECASTING/DECISION MODELS, 1960-1984

| Year | Always store | | Econometric model | | LP model 1 | | LP model 2 | |
|--------------------------|--------------|------------------|---------------------|------------------|-------------------|------------------|-------------------|------------------|
| | Single year | Cumulative total | Single year | Cumulative total | Single year | Cumulative total | Single year | Cumulative total |
| cents/bushel | | | | | | | | |
| 1960 | -5.33 | -5.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1961 | 1.65 | -3.68 | 0.00 ^b | 0.00 | 0.00 ^b | 0.00 | 0.00 ^b | 0.00 |
| 1962 | -6.02 | -9.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1963 | -1.93 | -11.63 | -1.93 ^b | -1.93 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1964 | -10.14 | -21.77 | 0.00 | -1.93 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1965 | -.66 | -22.43 | -.66 ^b | -2.59 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1966 | -7.40 | -29.83 | -7.40 ^b | -9.99 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1967 | -15.75 | -45.58 | 0.00 | -9.99 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1968 | -8.65 | -54.23 | 0.00 | -9.99 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1969 | -3.82 | -58.05 | 0.00 | -9.99 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1970 | -3.82 | -61.87 | -3.82 ^b | -13.81 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1971 | -17.38 | -79.25 | 0.00 | -13.81 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1972 | 69.18 | -10.07 | 69.18 | 55.37 | 69.18 | 69.18 | 69.18 | 69.18 |
| 1973 | 90.00 | 79.93 | 90.00 | 145.37 | 90.00 | 159.18 | 90.00 | 159.18 |
| 1974 | 23.33 | 103.26 | 23.33 | 168.70 | 23.33 | 182.51 | 23.33 | 182.51 |
| 1975 | 10.70 | 113.96 | 10.70 | 179.40 | 10.70 | 193.21 | 10.70 | 193.21 |
| 1976 | -35.75 | 78.21 | 0.00 | 179.40 | 0.00 | 193.21 | 0.00 | 193.21 |
| 1977 | 14.24 | 92.45 | 0.00 ^b | 179.40 | 0.00 ^b | 193.21 | 0.00 ^b | 193.21 |
| 1978 | .55 | 93.00 | .55 | 179.95 | 0.00 ^b | 193.21 | 0.00 ^b | 193.21 |
| 1979 | -1.93 | 91.07 | -1.93 ^b | 178.02 | 0.00 | 193.21 | 0.00 | 193.21 |
| 1980 ^a | 33.90 | 124.97 | 0.00 ^b | 178.02 | 0.00 ^b | 193.21 | 0.00 ^b | 193.21 |
| 1981 ^a | -10.10 | 114.87 | 0.00 | 178.02 | 0.00 | 193.21 | 0.00 | 193.21 |
| 1982 ^a | -8.17 | 106.70 | -8.17 ^b | 169.85 | 0.00 | 193.21 | 0.00 | 193.21 |
| 1983 ^a | -21.44 | 85.26 | 0.00 | 169.85 | 0.00 | 193.21 | 0.00 | 193.21 |
| 1984 ^a | -27.14 | 58.12 | -27.14 ^b | 142.71 | 0.00 | 193.21 | 0.00 | 193.21 |

^aThese years are outside the data used to estimate the models.

^bIncorrect decision.

to improve profits is because the parameter estimation objective function is specified in terms of profit maximization instead of forecast error minimization.

A simple two variable, two decision alternative wheat storage model was reported to demonstrate the methodology. The capacity of the linear programming solution process will allow much more complexity to be developed in the model structure and objective function. Additional variables could be added. Also, a rather large number of decision alternatives can be easily considered by adding activities descriptive of the payoffs for these decisions; i.e., in the wheat storage case, alternative storage period lengths, partial storage, etc. could be considered. The most limiting restriction in this regard would appear to be one of specification ability as opposed to the linear programming algorithm's solution power. A potentially fruitful specification of the objective function appears to be that of considering risk in the objective function. A "MOTAD" type objective function which considers the amount of profit variation as well as the magnitude of profits seems quite amenable to the methodology developed.

The fundamental strength of the linear programming approach to decision model parameter estimation is in its capability to consider objective functions that are unique to the decision purpose being considered. It could be argued that the same uniqueness could be achieved through various econometric methods such as a quadratic loss function (Fisher) or logit models as recently suggested by Spreen and Arnade. However, these models in general suffer from a lack of ability to describe the nature of the decision alternatives and associated payoffs to the degree possible with the linear programming

approach. A unique strength of linear programming in this regard is its ability to simultaneously consider continuous as well as discrete decision options through the use of mixed integer programming. Optimal control also provides an alternative approach to decision model parameter estimation and may be superior in certain dynamic and adaptive cases. However, in general, the power of the solution process for optimal control problems is much more restrictive than the linear programming approach developed. In addition, the linear programming approach is in general easier to implement and is familiar to a broader spectrum of the profession and other potential clientele.

Several disadvantages exist with the linear programming approach to decision model parameter estimation. Compared to the traditional econometric approach, it is more difficult to implement and it provides no established statistical measures or properties with which to evaluate the model. Another more controversial disadvantage may lie in the methodology's fundamental strength. The model and decision process are interdependent. Exploitation of the methodology's strength is dependent upon knowledge of, and an ability to quantify the decision process and objective. In some cases, this may not be meaningfully possible because of the complexity, subjectiveness, or proprietary nature of the decision process. Related to this aspect is the fact that the methodology produces a less generalized result than traditional forecasting models. Indeed, its strength is in being specific to the decision purpose. Despite these problems, it is believed that in a significant number of applications the methodology's potential to estimate model parameters with more efficient performance in terms of the decision objective sought makes it a useful tool.

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