

A NEGATIVE-COST APPROACH TO THE FORMULATION OF A TRANSHIPMENT PROBLEM*

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The transshipment problem formulation has been and is still being used extensively by researchers to solve spatial equilibrium and plant location problems. Hurt and Tramel [2], King and Logan [4], Rhody [7], and Judge et.al. [3] have all treated the subject of alternative formulations of transshipment problems. This paper (1) proposes an alternative formulation of these problems using a negative-cost technique and, (2) suggests a matrix reduction scheme which will reduce computational time for some problems.

THE KING-LOGAN TRANSHIPMENT MODEL

King and Logan used a three region, two-stage formulation of the transshipment model [4, p.98] which was reformulated by Hurt and Tramel [2, p. 764]. As a point of departure, the same sample problem will be used to present the negative-cost formulation. This should enable the reader to more readily determine differences between formulations and decide which formulation best serves his particular needs. Table 1 presents the matrix format for the basic problem. Table 2 is the matrix of costs, supplies, and requirements. Costs in the submatrix A are those reported by King and Logan for live animal shipments. Submatrix B has no relevance to the problem in this formulation and sufficiently high costs have been inserted to prevent entries. In submatrix C, all elements except the main diagonal have been given sufficiently high costs to prevent their entry in the final solution. The main diagonal of submatrix C is composed of the cost of processing for that respective plant with the variance that processing cost is in-

gressed with a *negative* sign. Costs in submatrix D are those reported by King and Logan for transporting processed meat from the region of slaughter of the region of demand. Live animal supplies, processing capacities and requirements are indicated in the border totals.¹

Regular and negative-cost formulations of the transshipment-plant location model can be expressed in mathematical terms as follows:

Regular Formulation:

$$\text{Minimize: } \sum_i \sum_j T_{ij} X_{ij} + \sum_i H_i S^i + \sum_i \sum_j t_{ij} L_{ij}$$

Negative-Cost Formulation:

$$\text{Minimize: } \sum_i \sum_j T_{ij} X_{ij} - \sum_i H_i U^i + \sum_i \sum_j t_{ij} L_{ij}$$

Both models are subject to the following constraints:

$$\sum_j L_{ij} = S_i$$

(live animal shipments from region i equals supply in region i) $\sum_j X_{ij} = S^i$

(meat shipments from region i equals animals slaughtered in region i)

$$S^i = S_i - \sum_j (L_{ij} - L_{ji})$$

(animals slaughtered in region i equals supply in region i adjusted for live animal shipments)

$$\sum_i X_{ij} = D_j$$

(total meat shipments to region j equals demand in region j)

$$\sum_i S_i = \sum_j D_j$$

(supplies must equal demands)

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¹ For a discussion of the relevant theory see [1], [4], and [5].

Table 1. A NEGATIVE-COST FORMULATION OF A THREE REGION SLAUGHTER PLANT LOCATION MODEL: MATRIX FORMAT ^a

	Processing plant (region)			Consuming region			S_i
	1	2	3	1	2	3	
Producing region							
1	A			B			Raw product supply
2	Raw product shipments						
3							
Processing plant (region)							
1	C			D			Processing capacity
2	Excess capacity			Final product shipment from region of processing to demand region			
3							
R_j	Processing capacity			Final product demand			

^a Submatrix A provides for shipment of live animals from each producing region to each slaughter plant, submatrix B has no relevance to the problem, submatrix C permits excess slaughtering capacity, and submatrix D provides for shipment of meat from each slaughter location to each region of final demand. Total slaughtered in each area is the difference between processing capacity and excess capacity.

$$0 \leq S_i, S_i^i, D_j, L_{ij}, X_{ij}$$

(non-negativity constraints: there can be no negative supplies, slaughters, demands, or shipments)

Where:

- X_{ij} = meat shipment from region i to region j
- L_{ij} = live shipment from region i to region j
- S_i^i = slaughter of cattle in region i
- T_{ij} = meat transfer cost from region i to region j

t_{ij} = animal transfer cost from region i to region j

H_i = slaughter cost per head in region i

α = dressing percentage

S_i = supply of slaughter cattle in region i, adjusted for dressing percent, α

D_j = demand for meat in region j

U_i = unused slaughter capacity in region i

Table 2. A NEGATIVE-COST FORMULATION OF A THREE REGION SLAUGHTER PLANT LOCATION MODEL: MATRIX OF COSTS, SUPPLIES, AND REQUIREMENTS

	Processing plant (region)			Consuming region			S_i
	1	2	3	1	2	3	
Producing region			A			B	
1	0	2	3	*	*	*	15
2	2	0	4	*	*	*	8
3	3	4	0	*	*	*	5
Processing plant (region)			C			D	
1	-5	*	*	0	1	2	30
2	*	-7	*	1	0	3	30
3	*	*	-8	2	3	0	30
R_j	30	30	30	7	14	7	

*Denotes a cost sufficiently high to preclude entry in the minimum cost solution.

A constant can be added to or subtracted from all elements in a column or row without changing the optimum least-cost distribution patterns. It is with this license that the negative-cost formulation is created. Technically, this is essentially the same as the Hurt-Tramel formulation except that processing cost for a given plant has been subtracted from all elements of the column or row where it had originally been incorporated. This results in the matrix presented in Table 2. Thus, the negative-cost formulation simply says we are maximizing savings for non-use of processing capacity rather than minimizing processing costs—which is equivalent.

The usual transportation procedure was used

to solve the problem. The solution is presented in Table 3 and is the same as the solution previously reported [2],[4].

There are three justifications for using this proposed alternative formulation: (1) It is inherently simpler to formulate the cost matrix, as processing costs do not have to be incorporated with transfer costs in either submatrix A or D. Processing costs are simply inserted as negative values on the main diagonal of submatrix C. (2) The adjustment of the cost matrix after each iteration is greatly simplified when an iterative procedure such as the one utilized by King and Logan is employed. It is not necessary to reconstruct the entire transfer cost submatrix — which-

Table 3. A NEGATIVE-COST FORMULATION OF A THREE REGION SLAUGHTER PLANT LOCATION MODEL: MINIMUM COST SOLUTION

	Processing plant (region)			Consuming region			S_i
	1	2	3	1	2	3	
Producing region			A			B	
1	15	0	0	0	0	0	15
2	0	8	0	0	0	0	8
3	0	0	5	0	0	0	5
Processing plant (region)			C			D	
1	15	0	0	7	6	2	30
2	0	22	0	0	8	0	30
3	0	0	25	0	0	5	30
R_j	30	30	30	7	14	7	

ever is used, A or D. It is necessary only that the new processing costs be inserted on the main diagonal of submatrix C. (3) A matrix reduction technique that will be presented later in this paper provides the third justification.

It might be argued that simplification is not necessary with a matrix generator; but while matrices can be constructed with matrix generators, as a practical matter, researchers working with smaller problems often manually construct their problems. This avoids computing center turn-around time and in some cases eliminates the need for assistance from computer programmers (not all economic researchers are competent programmers and, while canned transportation algorithms are available, such is not the case for the

matrix generator needed for a specific problem). Despite the attributes of matrix generators, experience has shown that it is still desirable to reduce a problem's complexity to a minimum.

No problem is incurred when the negative-cost formulation is used in a limited plant capacities case as presented by Hurt and Tramel [2].

A MULTI-REGION, MULTI-PLANT, MULTI-PROCESSING, MULTI-PRODUCT TRANSHIPMENT FORMULATION

A negative-cost model for multi-product commodity space and multi-product plants (processing both final and intermediate products) has been formulated [8] and is equivalent to the problem

of this type formulated by Hurt and Tramel [2]. The authors feel that negative-cost formulation of transshipment problems is useful for many applications. However, depending upon the number of plants producing more than one product, the Hurt-Tramel formulation may be superior for transshipment problems involving multi-product plants. This view is based on the fact that matrix row space will be increased by the number of multi-product plants (the rank of the matrix will not necessarily be increased by this number). Due to space requirements, a sample multi-region, multi-plant, multi-processing, multi-product transshipment problem and comparison is omitted here.

MATRIX REDUCTION

Despite the rapidity with which modern digital computers handle computations, large regional spatial equilibrium problems can run into a substantial amount of computer time and money. One such problem processed by an IBM 360/40 computer in the Computing Center at Mississippi State University required slightly over eight hours for a single solution using the usual transportation algorithm [10].

When approximating optimum plant location, using an iterative technique such that used by King and Logan [4], there are at least three techniques which can be utilized to save computer time and expense.

First, by using the negative-cost formulation, revision of processing costs to reflect economies of scale is simplified. It is not necessary to revise an entire submatrix dealing with both processing and transfer cost—it is only necessary to revise the main diagonal of the submatrix corresponding to that processing activity.

Secondly, since solution time can increase geometrically rather than linearly as size of a matrix increases, it is imperative that matrix size be minimized. As noted by King and Logan [4], once a plant location has been excluded from a solution, diseconomies of scale reflected in the processing cost associated with it thereafter preclude its being included in future solutions. This can be taken one step further. The row and column associated with this processing plant can be deleted entirely from the matrix. In the minimum cost solution, all $X_{ij} = 0$, where i minus the number of suppliers equals j represent unused processing capacity. In cases where this is equal to orig-

inal capacity, the corresponding row and column can be removed without changing future solutions. Usually, this significantly reduces computational time. However, it would not be necessary to delete all inactive processing. If, in the learned opinion of the researcher, a plant should be left in for further consideration, this could be done. The economic feasibility of utilizing matrix reduction is dependent upon matrix size. Experience has shown that the feasibility of matrix reduction is questionable for matrices less than 100×100 and where less than three processing plants are deleted on an iteration.

A technique utilized to increase efficiency when using an iterative transshipment procedure involves using the solution for each iteration as a starting basis for the next. This often-used technique will work even if matrix reduction is employed because: (1) for a feasible solution (whether optimum or not), $m + n - 1$ active routes are required, (2) there are two active routes associated with an unused processing plant even if one of these is effectively zero, and (3) when a processing plant is deleted, the corresponding row and column is deleted as well as the two active routes reflected for this plant, thus still leaving $m + n - 1$ active routes and a feasible solution.

SUMMARY

With today's modern digital computers it is possible to formulate and solve increasingly larger and more complex spatial equilibrium problems. Increases in computer speed and efficiency, for the most part, more than offset increases in per-hour computer costs—resulting in a net decrease in computer cost for most operations. Even so, additional gains can be realized from devising, evolving and utilizing simpler and/or more efficient software for the research problems we analyze. This paper proposes two possible ways to increase efficiency when the transportation model is used to analyze plant location.²

First, a simplified method of formulating a transshipment-plant location model is presented. This formulation eliminates the necessity of combining processing cost with either the charge for trans-locating commodities between raw material producers and processors or the transfer charge for processor to consumer shipments. This charge is simply inserted as a negative value on the main diagonal of the processor excess capacity sub-

² The authors do not claim that these proposals are all entirely innovative.

matrix. (This method, while feasible, is not recommended for models involving multi-product plants.)

The second suggestion to increase efficiency involves reduction of matrix size. It is known that, when using an iterative procedure such as the one used by King and Logan [4], once a plant has eliminated in an optimum solution, it will not return in a later iteration. It follows that there is often no need to retain those rows and columns associated with inactive plants. Their removal,

while decreasing computational time, will not decrease accuracy or change the solution. (In the opinion of the authors such a procedure would be economically beneficial only with matrices with rank ≥ 100 .)

Given the limited funds any research agency has available, it is imperative that problem formulations and solution algorithms be simplified and made more efficient. The proposals presented above adhere to this philosophy.

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