INTERTEMPORAL ALLOCATION OF GROUND WATER

IN THE CENTRAL OGALLALA FORMATION:

An Application of a Multistage Sequential Decision Model

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A closed underground water supply whose annual recharge is insignificant relative to its annual withdrawal is a stock resource subject to eventual economic exhaustion. Furthermore, it is a common property resource because its users tap the same reservoir. Economists have expressed their concern over the intertemporal misallocation of such fugitive resources, arising from a possible divergence between social and private costs [5, 6, 7, 8, 9]. While the practical determination of the marginal social cost of a ground water stock at different points in time is a formidable task, economists have suggested methods of evaluating ground water as a stock resource [5, 7, 9]. The most complete and notable contribution is Burt's [4, 5] application of Bellman's multistage sequential decision model to generate optimal ground water withdrawal policies. In the multistage sequential decision model, the marginal social cost of a unit of ground water at any period is considered equal to the marginal social value of water as a stock resource in the following period [5, p. 634] as reflected by the net output function of the basin. Therefore, evaluating the effect of removing alternative quantities of ground water at different points in time on the net output of subsequent periods and selecting those rates of withdrawal that make the basin's net output a maximum for the planning horizon by the dynamic programming technique yields the optimal intertemporal allocation of the ground water resource. The purpose of this paper is to formulate the optimal intertemporal allocation of ground water in the Central Ogallala Formation as a multistage sequential decision process to test whether projected rates of basin-wide withdrawals represent a potential misallocation of the water resource over time.

THE STUDY AREA

The Central Ogallala Formation is an unconsolidated aquifer underlying approximately 17,500 square miles of the land area between the Arkansas River on the north and the South Canadian River on the south.[1] The aquifer contains about 369 million acre-feet of water. It supplies practically all of the water used for irrigation, industrial and municipal purposes in the area. Irrigation is by far the largest user of ground water. In 1965, an estimated 2.32 million acre-feet were pumped for irrigation in the study area. The estimated volume used for industrial and municipal purposes in the same period was 0.10 million acre-feet.

Irrigated acreage in the study area increased from 69,564 in 1950 to 731,077 acres in 1960. By 1965 it had risen to 1,524,879 acres. The severe droughts of the 1950's, coupled with post war technological advances in commercial fertilizers and irrigation, have provided much of the impetus for the rapid growth of irrigation in the area.

Concomitant to the growth in irrigated acreage, the quantity of water pumped increased sharply from 0.12 million acre-feet in 1950 to 1.3 million acre-feet in 1960 and to 2.9 million acre-feet in 1964. As average annual recharge to the aquifer is estimated to be 0.27 million acre-feet, the annual overdraft increased from 0.11 million acre-feet in 1954 to 2.7 million acre-feet in 1964. The availability of the stock water supply and irrigable land, coupled with projected economic conditions, will result in continued

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[1]The study area includes a portion of two counties in Southeastern Colorado, eight counties in Southwestern Kansas, the three Panhandle counties of Oklahoma and seven counties in the northern part of the Texas High Plains [1, pp. 4-6].

growth of irrigation and an increased rate of aquifer depletion.

The consequence of continued overdraft of the aquifer is increased per unit cost of recovering water from the aquifer and, *ceteris paribus,* reduced net returns per acre of irrigated crops. As long as the ground water cannot be made a flow resource, its utilization entails its inevitable depletion. The saturated thickness of the aquifer is not uniform throughout the area. Sooner or later it will be uneconomical to pump water for irrigation purposes in some portions of the study area. This implies resources once committed to irrigated crop production will have to revert to dryland farming. The adjustment from irrigation to dryland farming may result in serious primary and secondary reductions of income in the region. The reduction in primary income is a result of reduced net returns per acre under irrigation and reversion to dryland farming. The secondary reduction of income stems from reduced land prices and the economic slump created in the region through the multiplier effect of the reduction in demand for inputs and services that complement irrigated crop production. How severe the adjustments to the declining water table will be is, in part, determined by how fast the ground water is depleted, and, in part, by the actions taken to lessen its adverse effects. Consequently, a projection of ground water withdrawals and an evaluation of whether these use estimates reflect an optimal intertemporal allocation of the water supply is in the interest of all members of the community (land owners, farm operators, businessmen and policymakers).

THE ANALYTIC MODELS

A recursive linear programming model was used to project the rate of ground water utilization in the future. The underlying assumption of the model is that irrigators acting individually consider only current production period costs and returns in making water use decisions. Because the long term growth of irrigation in the area will result from the interplay of complex social, political, economic, and physical factors, an effort to predict "the rate" of development was not made. Instead, the linear programming model was used to project what appears to be the minimum rate of development that might be expected in the area (Model I). Then certain assumptions and constraints were altered to project what appears to be the maximum rate of development that might be expected (Model II).

Model I assumes the study area will continue to produce the same proportion of U.S. production as it did in the 1965-67 period. The USDA's national supply projections for the years 1980, 2000, and 2020 were multiplied by the proportions developed for the 1965-67 period to develop area production levels for the three years. Linear interpolation was used to make estimates for intermediate years. Model I maximizes the study area's net returns subject to meeting the projected *a priori* production goals of the period in question. An area with rapid irrigation development normally increases its share of U.S. production. For this reason, projections with Model I are considered an estimate of the minimum increase in irrigated acreage and water use that can be expected.

Model II permits the study area to produce more than its historic share of the projected U.S. production. The parameters of an exponential equation were estimated, based on the past rate of growth in irrigated acres and the maximum physical limit of irrigated acres in the study area [1, pp. 60-64]. The growth equation was solved for future years and the values included as an upper limit on the acreage that could be irrigated for the respective time periods. This upper limit on the rate of irrigation development was included to reflect the limits imposed by the number of well drillers, the effect of capital rationing, as well as sociological and other noneconomic factors.

The product prices assumed in the study were the "adjusted normalized prices" issued by the Water Resources Council. Thus, all the analyses utilized prices which minimize the direct price support effects or payments under government programs.

The solutions of Model I and Model II provide a projected minimum and maximum rate of irrigation development and, hence, a projected minimum and maximum rate of ground water withdrawal from the Central Ogallala Formation. The solutions to Model I met the projected production levels for all future periods as expected. The projected national production levels (and, hence, study area production levels) increase by modest amounts in future years. Thus, the irrigated acreages, projected by Model I, increased to only 1.63 million acres during the 1990-99 decade and declined thereafter. Model II was expected to irrigate the maximum permitted acreage in each time period. Interestingly, the increasing irrigation costs in some resource situations resulted in a less rapid rate of irrigation development after 1980 than projected by the exponential growth model. The irrigated acreage projected by Model II increased to a maximum of 3.36 million acres during the 1990-99 decade. The corresponding water use projections for Models I and II are presented and discussed later in this paper.[2]

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[2]Estimates of the quantities of crops produced, the pattern of irrigated production and the aggregate annual income for the 1965 to 2070 period are presented and discussed in [2, pp. 73-121].

Having projected two rates of irrigation development in Models I and II, the next step is to formulate the multistage sequential decision model. The purpose of applying this model is to determine the rates of ground water withdrawal that maximize the present value of net income for the area under the specific assumptions of Models I and II.

Formulation of the Optimum Intertemporal Allocation of Ground Water as a Sequential Decision Model

The optimal allocation of ground water over time in a closed aquifer with relatively little recharge is essentially a problem of choice among the various quantities of water to leave stored in the aquifer at different points in time. The decision of how much water to withdraw in any period, t, has a direct bearing on how much will be left in storage for the following period, t + 1. More important, the decision to withdraw a certain quantity of water not only determines the net income for period t, but also influences the per unit cost of water in subsequent periods. The problem is to find, for all periods, the rates of ground water withdrawal that will maximize the study area's present value of net income over the entire planning horizon.

The multistage sequential decision process consists of a series of stages joined so that the output of one stage becomes the input of the next stage. A typical stage consists of five components; namely, an input state, an output state, a decision variable, a stage return, and a stage transformation. In formulating the intertemporal allocation of ground water as a discrete multistage sequential decision process, define M discrete water storage levels S_i, i = 1, 2, ..., M, each level representing a state, and k discrete alternative rates of water withdrawal, W_k, k = 1, 2, ..., k. Define P_{ij}^k as the transition probability of the system in transforming from input state i to output state j via alternative decision k. Define R_{ij}^k as the net return accruing from alternative decision k being carried out and the system transiting from input state i to output state j. In reference to a particular stage n, n = 1, 2, ..., N, of an N stage system we have,

$S_i(n)$ = input state of the system in the nth stage,

$S_j(n)$ = output state of the system in the nth stage,

$W_i^k(n)$ = k^{th} alternative decision selected as optimal in the nth stage,

$$R_{ij}^k(n) = f_n[S_i(n), W_i^k(n)], \qquad (1)$$

the stage return which is a function of the input state

$S_i(n)$, and the alternative decision selected, $W_i^k(n)$,

and

$$S_i(n) = T_{(n-1)}[S_i(n-1), W_i^k(n-1)], \qquad (2)$$

the stage transformation function, which indicates the input state in the nth stage (or alternatively the output state of the $n-1^{th}$ stage) is a function T of the input state in the preceding stage and the optimal decision taken in that stage.

In general, for the N stage system there exists a sequence of stage returns given by the criterion function

$$F[S_i(1), S_i(2), ..., S_i(N), W_i^k(1), \qquad (3)$$

$$W_i^k(2), ..., W_i^k(N)].$$

The optimization problem is one of choosing the $W_i^k(n)$ at each stage, n, so as to maximize the criterion function F over all stages, one through N, for the entire planning horizon.

Applying the property of Markovian dependence permits one to decompose the criterion function F into a sum of separate individual stage returns. Given the initial state of the system is S_1 and making the appropriate substitutions results in (4) as the function to be maximized where β is the appropriate discounting formula [1, pp. 31-41].

$$f_N S_1(1) = \underset{W_1^k}{\text{Max}} \ R_1^k(1) + \beta \cdot \sum_{j=1}^{M} P_{ij}^k \cdot f_{N-1} S_j(2)$$

$$\qquad (4)$$

$$i = 1, 2, ..., M.$$

Relation (4) indicates the maximum present value of net income, with respect to ground water withdrawal, of an N stage process under an optimal policy is the maximum sum of the expected net returns accruing to the decision in stage one and the discounted expected net returns from the remaining N-1 stages, provided an optimal policy will be carried out in the remaining N-1 stages [4, p. 38]. The backward recursive solution of relation (4) by the dynamic programming technique yields the optimal withdrawals and the associated expected net returns for all possible input states $S_i(n)$ of the n = 1, 2, ..., N stage system.

The Input Data

Input data required to determine the optimal intertemporal allocation of ground water in the Central Ogallala Formation via a multistage sequential decision model are the formulation of (1) the quan-

tity of ground water in storage as a set of finite discrete input and output states, (2) a finite set of discrete alternative rates of withdrawal for each input state of the system, (3) a transition probability matrix that defines the probability associated with each alternative in the set, and (4) the net returns that accrue to each alternative in each state. The number of stages in the planning horizon and the appropriate discount rate also need to be determined.

Discrete input and output states and the sets of alternative withdrawal rates were formulated by experimentation [1, pp. 127-130]. Annual recharge, a random variable, is small relative to the alternative rates of withdrawal. Consequently, the multistage sequential decision process is formulated as a deterministic case and the elements of the probability matrix are either one or zero. The estimated annual demand of ground water for municipal and industrial purposes in the study averages about 0.21 million acre-feet. It can be assumed that the average annual recharge of 0.27 million acre-feet will satisfy this demand. Thus, the recharge component and the industrial and municipal water demand component can be omitted from the multistage sequential decision model. This implies that at any stage the input state of the system is transformed into an output state only by the magnitude of ground water withdrawal.

The maximum net return for each alternative rate of ground water withdrawal for each state (storage level) was computed by applying parametric procedures to Models I and II. The resulting net returns for Model I (assuming a maximum of approximately 1.5 million irrigated acres) provide the input needed for sequential decision Model A. The net returns generated by Model II (assuming a maximum of 3.0 million irrigated acres) provide the data required for sequential decision Model B [1, pp. 137-139].

The planning horizon selected is the 100-year period from 1970-2069 divided into ten 10-year intervals defining the ten stages of the system. This period is considered sufficiently long to produce convergence or stability in the optimal policies. The optimal rates of withdrawal selected for each state in each stage represent the sum of ten equal annual rates. However, the same cannot be said of the associated stage returns. As the stages represent a 10-year interval at different points in the planning horizon, the net return attributed to the first year of a given stage is not of the same value as that attributed to the tenth year. More important is making the stream of net returns of the tenth stage comparable with that of the first stage. A combination of annuity and present value formulae are used to achieve this end [1, p. 140].

The selection of an appropriate discount rate is important. Too low a discount rate may discourage present use of ground water. On the other hand, too high a discount rate may discourage saving ground water for future use. This study uses three discount rates, 0.00, 0.04, and 0.08, to test the sensitivity of the optimal solution.

EMPIRICAL RESULTS
AND THEIR POLICY IMPLICATIONS

The optimal rates of ground water withdrawal obtained from the solutions of the four models are presented in Table 1. In general, solutions of the optimal rates of ground water withdrawal under the assumptions of Model A indicate that, except in a few borderline cases due to the discreteness of the states, the optimal policies are the same for discount rates of 4 and 8 percent. The optimal water withdrawal for a zero discount rate is substantially less. This implies that the optimal policy is sensitive only to discount rates close to zero. If future returns are discounted at very low interest rates, the solution includes lower rates of ground water withdrawal so that a greater supply remains for future years. Discount rates of four percent or more require greater rates of ground water withdrawal to maximize the present value of the net return stream from the ten-stage planning horizon.

The optimal policies for Model B for discount rates of 4 and 8 percent differ only in 3 stages. In stages 1, 6 and 7, discounting by 8 percent results in higher rates of ground water withdrawal. The difference in the policies are two, one and four million acre-feet, respectively. When future returns are not discounted (a zero discount rate) the optimal solution includes much lower withdrawal rates during stages 1 through 5 and higher use rates during stages 6 through 10.

It is recalled that the solutions of the recursive linear programming Models I and II are regarded as a close approximation of the resulting intertemporal allocation of ground water in the Central Ogallala Formation if irrigators make individual decisions on their rate of water use on a shortrun basis. On the other hand, the solutions of the multistage sequential decision Models A and B represent a situation in which decisions on the intertemporal allocation of ground water in the study area are made by all irrigators acting in concert through a public agency, or through one or more water districts. Hence a comparison of the policies obtained from Model I and Model II with those of Model A and Model B, respectively, provides a clue as to whether irrigators acting individually will misallocate the ground water resource over time.

TABLE 1. RATES OF GROUND WATER WITHDRAWAL FROM SOLUTIONS OF MODELS I, A, II, AND B, IN MILLIONS OF ACRE-FEET, 1970-2069

Model Type	Stage									
	1 1970-79	2 1980-89	3 1990-99	4 2000-09	5 2010-19	6 2020-29	7 2030-39	8 2040-49	9 2050-59	10 2060-69
Recursive Linear Programming Model I	26.4	27.9	29.3	24.9	26.4	23.5	23.6	23.3	21.6	15.6
Multistage Sequential Decision Model A										
r = 0.00	29.0	29.0	26.0	26.0	25.5	21.5	21.5	21.5	16.5	12.5
r = 0.04	34.5	34.5	32.5	31.0	31.0	22.0	21.5	22.0	17.0	13.0
r = 0.08	35.5	34.5	32.5	31.0	31.0	22.0	22.0	22.0	16.5	13.0
Recursive Linear Programming Model II	46.1	47.9	58.1	48.0	43.4	30.5	18.2	15.2	15.8	13.8
Multistage Sequential Decision Model B										
r = 0.00	29.0	29.0	29.0	37.0	33.0	38.0	30.0	28.0	21.0	16.0
r = 0.04	65.0	65.0	51.0	46.0	37.0	24.0	16.0	8.0	4.0	4.0
r = 0.08	67.0	65.0	51.0	46.0	37.0	25.0	20.0	8.0	4.0	4.0

A comparison of the solutions of Model I and Model A (Table 1) indicates that if the growth of irrigation in the study area progresses as projected by Model I, the rate of ground water withdrawal is less than the rates suggested optimal by Model A in stages 1, 2 and 4 for all three discount rates. In stages 3 and 5 they are somewhat higher than the optimal rate with no discounting, but less than those with 4 and 8 percent discounting. Model I's rates are slightly higher than those of Model A in stages 6 through 10. However, summing the stage withdrawals indicates that total withdrawals for Model I are less than for Model A during the first 6 stages regardless of discount rate and are lower throughout the ten-stage planning horizon for discount rates of 4 and 8 percent. The comparison implies that the lower rate of irrigation development projected by Model I will not result in general uneconomic mining of the Central Ogallala Formation. This analysis suggests that policies such as spacing wells which minimize interference between neighboring wells are the only control measures that may be economically justified.

Model II's rate of ground water withdrawal is substantially lower than that suggested optimal by Model B in stages 1 and 2 for discount rates of 4 and 8 percent. However, Model II's rates are greater than those of Model B for both discount rates in the remaining 8 stages. One can conclude that if irrigation development occurs as projected by Model II's solutions the population of the area should be concerned

about uneconomic mining of ground water after 1990. The policy implication is that some control measures other than well spacing may be necessary if the extraction of ground water from the Central Ogallala Formation is to be limited to those rates which will maximize the study area's net income over a longer period of time.

CONCLUSIONS

The results of the study indicate that the misallocation of ground water is not a direct corollary of its being a common property stock resource. Whether such a resource will be intertemporally misallocated depends to a large degree on whether or not it is the most limiting factor of production at the margin. Factors such as (1) a high discount rate, (2) constraints on the quantity of crops produced due to market or government program conditions, and (3) limited availability of capital and labor that complement the expansion of irrigated production may sufficiently constrain expansion by individual operators so that the mining of ground water from a closed exhaustible aquifer does not result in automatic intertemporal misallocation. The imposition of use taxes or restrictive quotas without first establishing empirically that intertemporal misallocation will result may conserve the stock resource for the future, but at the loss of present income with greater value.

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