INCORPORATING GOVERNMENT PROGRAM PROVISIONS INTO A MEAN-VARIANCE FRAMEWORK

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Abstract

E-V studies traditionally have relied on historical data to calculate returns and variance. Historical data may not fully reflect current conditions, particularly when decisions involve government-supported crops. This paper presents a method for calculating mean and variance using subjectively-estimated data. The method is developed for both government-supported and non-program crops. Comparisons to alternative methods suggest the approach provides reasonable accuracy.

Key words: government farm program, mean-variance, simulation, subjective.

Numerous studies of the crop-mix decision have been conducted using quadratic programming mean-variance (E-V) models. It has been shown that E-V models correctly represent decisionmaker behavior if returns are normally distributed (Freund) or utility can be approximated by a quadratic function (Markowitz). The assumptions of quadratic utility have been challenged in numerous articles (e.g., Pratt; Arrow), and little evidence exists for suggesting returns are normally distributed (Buccola). Other techniques, such as stochastic dominance (Hadar and Russell) and target MOTAD (Tauer), have been identified as superior in considering decisions under risk.

A number of papers have defended E-V as a reasonable approximation of optimal decisions under risk. Porter and Gaumnitz found little difference between E-V and Second-degree Stochastic dominance efficient sets. Levy and Markowitz suggested the quadratic utility function can provide an excellent second-order approximation to more desirable functions. Meyer demonstrated that E-V provides the same ranking among different alternatives as stochastic dominance if all alternatives have similar distributions. The relative simplicity and reasonableness of results suggest E-V will continue in use for analysis of firm–level decisions.

Most E-V models designed to analyze the crop-mix decision have treated prices and/or yields as the only sources of uncertainty (e.g., Scott and Baker; Lin et al.; Stovall). In such studies, a set of historical prices and yields is used to calculate expected returns for each crop and the covariance matrix for risk relationships between crops, assuming all crops are sold in the open market.

The current status of agriculture suggests this simple approach, in many cases, may be outdated. Government programs have become much more important to farmers than they were historically. Although voluntary in nature, participation in programs for some crops is essential in some years to farm survival. But participation imposes a number of restrictions on acreage devoted to a program crop or set of crops. Therefore, an analysis of the crop-mix decision is likely incomplete unless it simultaneously considers the program participation decision. The participation decision in a programming model framework requires multiple activities be included for each crop, with one activity accounting for production outside the program and one or more activities representing production within the program.

Relatively few studies have incorporated government program provisions into analyses of crop-mix decisions (e.g., Musser and Stamoulis; Persuad and Mapp; Scott and Baker). In these studies, modified price distributions were created for each program crop. The price distributions consisted of the original historical

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price distributions, with historical prices replaced by loan rates when the latter were greater. The modified set of prices was multiplied by historical yield values to generate a gross income distribution. Deficiency payments were also added to each income value based on target price and proven yield levels. The modified income distribution was then used to calculate expected return and variance of return for the program participation activity (or activities). This approach presumed the historical income distributions accurately represented current or future distributions for crop prices and yields and for farm program provisions.

The changing economic environment in which farmers operate makes this approach outdated. Excess production and large carryover stocks of many commodities have depressed nominal (and real) prices to levels far below those observed during the previous 10–15-year period. Expectations are that stocks will remain at price-depressing levels for several years (Thompson). Loan rates and target prices have also fallen, although not as much as prices. Thus, the current price and government policy environment is quite different from that experienced during the 1970s and early 1980s. As a result, use of historical data to calculate current income distributions in and out of the government program may misrepresent actual benefits and costs of farm program participation.

Subjectively-estimated data are a reasonable alternative to historical data, given the current situation (Bessler). Subjective estimates made by experts can account for both historical trends and current events which may modify these trends. The subjective or Bayesian approach is not without its critics, however. Statisticians complain that subjective estimates will vary from individual to individual, thus violating a basic canon of empirical science—the open and "objective" treatment of results (Poirier, p. 122). Cognitive psychologists suggest that the heuristics used in making subjective judgments may lead to biases in results (Tversky and Kahreman). Nevertheless, use of subjectively-estimated data is generally recognized as preferable when analyzing individual's decisions (Anderson et al.). We argue it is also a preferable approach when current or future economic conditions differ markedly from what has occurred historically.

Obtaining subjective estimates of expected returns is a relatively easy task. However, few individuals have sufficient knowledge to subjectively estimate a covariance matrix for various crop production activities. An alternative to estimating the covariance matrix directly is to subjectively estimate price and yield distributions separately, then combine these distributions with a correlation matrix to obtain the covariance matrix. Although all estimation problems are not completely resolved, this latter approach could produce a more reasonable estimate of the covariance matrix.

Given the correlation matrix and price and yield distributions, one can use Monte-Carlo simulation techniques to generate a series of gross revenue values for several crops, as well as for different government program participation strategies for each crop. The resulting data can be used to calculate a covariance matrix. Simulation is not without its weaknesses, however. The simulation process generally introduces some error into the calculations because the simulated distributions are seldom a perfect representation of the original distributions. In addition, correlating random variables requires a Cholesky factorization of the correlation matrix. Factorization may not be possible for large near-singular correlation matrices because of rounding error.

The purpose of this paper is to suggest an alternative approach which can be used to calculate per acre expected returns and a corresponding covariance matrix when government programs influence the crop-mix decision. The expected returns vector and covariance matrix can then be incorporated into an E-V model to identify crop-mix/government-program-participation strategies that maximize utility. The approach permits use of either historical or subjective data (or some combination of the two), incorporates government program provisions, and can be used for any size of covariance matrix.

We begin our presentation by reviewing the paper by Bohrnstedt and Goldberger, which is used as a basis for our approach. After this review, we discuss the different 1985 Farm Bill provisions pertinent to the problem at hand. Generalized equations are developed for calculating per-acre income, mean, and variance values for government-program crops. These equations are used to calculate the returns vector and covariance matrix. After the equations are derived, an example problem is analyzed to compare the accuracy of the equation approach to that of the simulation approach.

OPEN MARKET INCOME, MEAN, AND VARIANCE

Bohrnstedt and Goldberger have suggested a procedure for estimating mean and variance for the product of two random variables. The procedure utilizes the statistical parameters of each random variable. In this case, price and yield are the random variables and represent the only sources of uncertainty influencing per-planted-acre farm income for a particular crop. Consider the situation in which a farmer does not participate in the government program for the crop (or that the crop does not have a government program). Expected per-acre gross revenue[1] is

$$(1)\quad E(R_F) = E[\tilde{P} \cdot \tilde{Y}] = \mu_P \mu_Y + \sigma_{PY},$$

where R_F is crop revenue in the open market, $\tilde{P}$ is the random variable price, $\tilde{Y}$ is the random variable yield per acre, μ_P is expected price, μ_Y is expected yield per acre, and σ_{PY} is covariance between price and yield. Variance for this bivariate income distribution is

$$(2)\quad Var(R_F) = E[\tilde{P} \cdot \tilde{Y} - E(\tilde{P} \cdot \tilde{Y})]^2, \text{ or}$$

$$(3)\quad Var(R_F) = \mu_Y^2 \sigma_P^2 + \mu_P^2 \sigma_Y^2 + E[(\tilde{P} - \mu_P)^2 \cdot (\tilde{Y} - \mu_Y)^2] + 2\mu_P \cdot E[(\tilde{P} - \mu_P) \cdot (\tilde{Y} - \mu_Y)^2] + 2\mu_Y \cdot E[(\tilde{Y} - \mu_Y) \cdot (\tilde{P} - \mu_P)^2] + 2\mu_P \mu_Y \sigma_{PY} - \sigma_{PY}^2,$$

where σ_P^2 is price variance and σ_Y^2 is yield variance. If $\tilde{P}$ and $\tilde{Y}$ are bivariate normally distributed, $E[(\tilde{P} - \mu_P)^2 \bullet (\tilde{Y} - \mu_Y)^2] = \sigma_P^2 \sigma_Y^2 + 2\sigma_{PY}^2$ and all third and higher moments are zero. The variance equation reduces to

$$(4)\quad Var(R_F) = \mu_Y^2 \sigma_P^2 + \mu_P^2 \sigma_Y^2 + 2\mu_P \mu_Y \sigma_{PY} + \sigma_P^2 \sigma_Y^2 + \sigma_{PY}^2.$$

When price and/or yield are not bivariate normally distributed, (4) represents an approximation of variance for gross revenue. The amount of error introduced into variance calculations by using (4) instead of (3) depends on the degree to which the price and/or yield distributions are non-normal, in combination with the magnitude of price and yield variance. Covariance of crop revenue between two crops (R_{F1} and R_{F2}) is

$$(5)\quad Cov(R_{F1}, R_{F2}) = \mu_{Y1}\mu_{Y2}\sigma_{P1P2} + \mu_{P1}\mu_{Y2}\sigma_{Y1P2} + \sigma_{Y1P2}\sigma_{P1Y2} + \sigma_{P1P2}\sigma_{Y1Y2} + \mu_{P1}\mu_{P2}\sigma_{Y1Y2} + \mu_{Y1}\mu_{P2}\sigma_{P1Y2},$$

where R_{F1} is R_F for crop one, R_{F2} is R_F for crop two, σ_{P1P2} is covariance between prices for crops one and two, with other covariances defined in a similar manner. Equation (5) collapses to (4) when $R_{F1} = R_{F2}$. Thus, equation (5) could be used to calculate each element of an n x n covariance matrix, where n is the number of crops included in the analysis. Previous studies using this approach include those by Tew and Boggess, Burt and Finley, and Boggess et al.

GOVERNMENT FARM PROGRAM IMPACTS ON MEAN AND VARIANCE OF RETURNS

Review of Program Provisions

There are a number of features in the current government program which modify the per-acre expected return and variance of program crops. The farm program, as defined by the 1985 Farm Bill, revolves around a target price and three types of loan rates (Glaser). If average market price during a particular segment of the marketing year falls below the target price, a deficiency payment is made to eligible farmers to offset the income shortfall. Payment is based on a historical average of crop yields (hereafter referred to as proven yield). Deficiency payments per unit of proven yield are calculated as the smaller of (a) the difference between target price and market price, or (b) the difference between target price and the formula loan rate. Total deficiency payments are limited to $50,000 annually per farmer.

Three types of loans defined by the 1985 Farm Bill are (a) the formula loan, (b) the adjusted loan, and (c) the marketing loan. The formula loan has been available to farmers in one form or another during most years since the 1930s. At harvest, the farmer may place the crop in the Commodity Credit Corporation (CCC) loan program and receive a prespecified loan value for the crop. If the farmer elects to sell the crop within the next nine months, the loan must be repaid plus accrued interest charges. Ownership of the crop is forfeited to the government to satisfy the loan debt, and no interest costs are incurred if the loan is not repaid within nine months. The formula loan rate represents a pseudo-price floor for the crop,[2] reducing income risk by eliminating the chance of receiving a price less than the effective rate.

Adjusted and marketing loans were created to reduce forfeitures and increase sales of commodities in storage. The Secretary of Agriculture is given authority to implement either (or both) of these loans for certain commodities. The Secretary may lower the formula loan as much as 20 percent to arrive at

[1]Costs are assumed constant in this part of the presentation, resulting in gross revenue and net revenue variance (and covariance) being the same.

[2]The actual price received when forfeiting may be somewhat less than the formula loan due to storage costs and any payment reductions resulting from the Gramm-Rudman-Hollings Deficit Reduction Bill (GRH).

the adjusted loan rate. The difference between formula and adjusted loans is then paid to the farmer as a second deficiency payment, if market price is less than the adjusted loan rate. This second deficiency payment (known as the Findley payment) is not subject to the $50,000 payment limit imposed on target price deficiency payments.

The marketing loan takes one of two forms. In one form, the market loan rate is calculated weekly and approximates world market price for the commodity. In the second form, the market loan is pre-set at some level below the formula or adjusted loan, whichever is lower.[3] In either case, the farmer may forfeit the crop to the CCC and receive the formula loan rate. He then has the option of buying back the crop at the marketing loan rate and reselling it at the prevailing market price. This option is elected if the market price is sufficiently above the marketing loan.

Farm program participation requires a farmer to plant within this base acreage for each crop. Base acreage is calculated for each program crop as the five-year average of planted and "considered-planted" acreage. Participation in the program often requires a farmer to idle a percentage of base acreage. In some cases, the government pays the farmer (in cash or in kind) for idling base acreage as an extra enticement to participate in the program. The acreage-idlement programs generally differ from crop to crop, causing expected returns and variance of returns per base acre to vary by crop. Because of these complicating factors, expected returns and variance of returns are calculated here based on an acre of planted cropland, rather than an acre of base acreage, to provide a more generic presentation.

Government Program Equations

Given this background, gross income per acre of planted cropland under the program (R_p), assuming both an adjusted loan rate and the second form of marketing loan are in effect, can be summarized as follows

$$R_P = \begin{cases} \tilde{P}\cdot\tilde{Y} & \text{when } P > T \\ \tilde{P}\cdot\tilde{Y} + G\cdot(T-\tilde{P}) & L < P \le T \\ \tilde{P}\cdot\tilde{Y} + G\cdot(L-\tilde{P}) + G\cdot(T-L) & A < P \le L \\ (A-M+\tilde{P})\cdot\tilde{Y} + G\cdot(L-A) + G\cdot(T-L) & M < P \le A \\ A\cdot\tilde{Y} + G\cdot(L-A) + G\cdot(T-L) & P \le M \end{cases}$$

where T is the target price, L is the formula loan, A is the adjusted loan, M is the marketing loan, and G is proven yield. This formulation presumes the farmer participates in the marketing loan program as long as market price exceeds market loan rate. If the adjusted loan is not in effect, A can be set equal to the formula loan. Similarly, if no marketing loan is in effect, M can be set equal to P.

In this formulation, only price and yield are random variables. It is assumed L, G, T, L, A, and M are known with certainty at the time the crop-mix decision is made. To facilitate collapsing R_p to a single equation, the following new random variables are defined:

$$\tilde{P}T = \begin{cases} T & \text{when } P > T \\ \tilde{P} & L < P \le T \\ L & P \le L, \end{cases}$$

$$\tilde{P}M = \begin{cases} \tilde{P} & \text{when } P > A \\ \tilde{P} + A - M & M < P \le A \\ A & P \le M, \end{cases}$$

and

$$\tilde{P}A = \begin{cases} L & \text{when } L < P \\ \tilde{P} & A < P \le L \\ A & P \le A. \end{cases}$$

The variables $\tilde{P}T$, $\tilde{P}M$, and $\tilde{P}A$ are not normally distributed unless (a) they are identical to the $\tilde{P}$ distribution, and (b) $\tilde{P}$ is normally distributed. The resulting gross revenue equation for farmers participating in the program is

(6) $R_P = \tilde{P}M\cdot\tilde{Y} + G\cdot(T-\tilde{P}T) + G\cdot(L-\tilde{P}A).$

The expected per-acre gross return is

(7) $E(R_P) = \mu_{PM}\mu_Y + \sigma_{PMY} + G(T-\mu_{PT}) + G(L-\mu_{PA}),$

where μ_{PM} is the mean of the random variable $\tilde{P}M$, σ_{PMY} is covariance between $\tilde{P}M$ and $\tilde{Y}$, μ_{PA} is the mean of the random variable $\tilde{P}A$, μ_{PT} is the mean of the random variable $\tilde{P}T$, and other variables are defined as before. Variance of per-acre gross returns is

(8) $$\begin{aligned} Var(R_P) = {} & \mu_{PM}^2\sigma_Y^2 + \mu_Y^2\sigma_{PM}^2 + 2\mu_Y\mu_{PM}\sigma_{PMY} + \sigma_{PMY}^2 + \sigma_{PM}^2\sigma_Y^2 \\ & + G^2\sigma_{PA}^2 + 2G^2\sigma_{PAPT} + G^2\sigma_{PT}^2 \\ & - 2G(\mu_Y\sigma_{PMPT} + \mu_{PM}\sigma_{YPT}) \\ & - 2G(\mu_Y\sigma_{PMPA} + \mu_{PM}\sigma_{YPA}), \end{aligned}$$

where σ_{PM}^2 is variance of $\tilde{P}M$, σ_{PM}^2 is variance of $\tilde{P}A$, σ_{PAPT} is covariance between $\tilde{P}A$ and $\tilde{P}T$,

[3] This second form is known as the "repayment level," rather than a marketing loan. The implementation for the repayment level is the same as for the marketing loan, except its method of calculation is different. Because the marketing loan and repayment level programs are so similar, both are referred to as marketing loans.

σ_{PMPT} is covariance between $\tilde{P}M$ and $\tilde{P}T$, σ_{PMPA} is covariance between $\tilde{P}M$ and $\tilde{P}A$, and σ_{YPA} is covariance between $\tilde{Y}$ and $\tilde{P}A$. Verbally, gross income variance under the 1985 Farm Bill is equal to variance under the loan program minus variance reduced because of the deficiency and Findley payments. Covariance between two program crops (R_{P1} and R_{P2}) becomes:

$$
\begin{aligned}
(9)\quad Cov(R_{P1}, R_{P2}) = {} & \mu_{PM1}\mu_{PM2}\sigma_{Y1Y2} + \mu_{Y1}\mu_{Y2}\sigma_{PM1PM2} + \mu_{Y1}\mu_{PM2}\sigma_{PM1Y2} \\
& + \mu_{PM1}\mu_{Y2}\sigma_{Y1PM2} + \sigma_{PM1PM2}\sigma_{Y1Y2} + \sigma_{PM1Y2}\sigma_{Y1PM2} \\
& + G_1G_2\sigma_{PA1PA2} + G_1G_2\sigma_{PT1PT2} + G_1G_2\sigma_{PA1PT2} \\
& + G_1G_2\sigma_{PT1PA2} \\
& - G_1(\mu_{Y2}\sigma_{PA1PM2} + \mu_{PM2}\sigma_{PA1Y2}) \\
& - G_2(\mu_{Y1}\sigma_{PM1PA2} + \mu_{PM1}\sigma_{Y1PA2}) \\
& - G_1(\mu_{Y2}\sigma_{PT1PM2} + \mu_{PM2}\sigma_{PT1Y2}) \\
& - G_2(\mu_{Y1}\sigma_{PM1PT2} + \mu_{PM1}\sigma_{Y1PT2}),
\end{aligned}
$$

where σ_{PM1PM2} is covariance between $\tilde{P}M_1$ and $\tilde{P}M_2$ and other variables are defined in similar fashion. Use of equations (7), (8), and (9) permits calculation of mean, variance, and covariance for multiple government-program crops being considered in an E-V model. It is significant to note, however, that the calculations are seldom as complex as presented here, because not all of the possible program provisions are actually in effect for a particular crop each year.

The effect of the 1985 Farm Bill on per-acre gross revenue, assuming the first form of the marketing loan is in effect (R_M), can be summarized as

$$
R_M = \begin{cases}
\tilde{P}\cdot\tilde{Y} & \text{when } P < T \\
\tilde{P}\cdot\tilde{Y} + G\cdot(T - \tilde{P}) & L < P \leq T \\
\tilde{P}\cdot\tilde{Y} + G\cdot(L - \tilde{P}) + G\cdot(T - L) & A < P \leq L \\
(A + D)\cdot\tilde{Y} + G\cdot(L - A) + G\cdot(T - L) & P \leq A,
\end{cases}
$$

where D is the difference between market price and market loan rate, with other variables as previously defined. Whether D is better handled as a random variable or a known parameter is not clear because of the newness of the marketing loan program. Mean, variance, and covariance can be calculated, however, by following a procedure similar to that used in calculating (7), (8), and (9).

The methodology presented here could also be applied to more complex calculations. Costs of production were assumed constant when calculating gross income mean, variance, and covariance. If costs were also considered uncertain, calculation of expected net return and variance of net return would be

$$(10)\ E[NR] = E[R] - E[C], \text{ and}$$

$$(11)\ Var[NR] = Var[R] + Var[C] - 2\bullet Cov[R,C],$$

where C is cost and the other variables are defined as before. This approach would be valid in calculating mean and variance for either R_F or R_P. The influence of a secondary crop product (such as cottonseed) on income mean and variance could also be included. Modeling the effects of crop insurance could be accomplished using this methodology, recognizing that insurance affects the yield distribution.

EXAMPLE PROBLEM

An empirical example is provided in this section to illustrate the accuracy of the equation approach in calculating returns and variance of returns for use in an E-V analysis. The example is based on data for an actual farm situation in the Coastal Bend Region of Texas. The farmer subjectively estimated price and yield distributions for all crops and provided information from his farm records for historical prices and yields on his two major crops (cotton and sorghum). In this example, only these two crops are considered. Note that the marketing loan for cotton and the adjusted loan for sorghum are different than the levels actually announced in 1986 so as to fall in the middle of their respective price distributions. This change in the loan levels tends to increase the error that can occur when using the equation approach.[4]

Three different options are available to the farmer when producing and marketing each crop. They are (a) non-participation in the farm program, with the crop being sold in the open market, (b) participation in the farm program, receiving all program benefits, and (c) participation in the program, receiving all but deficiency payments. Option (c) would occur once the farm has reached the deficiency payment limit, a common occurrence for this size of farm operation. The example problem, therefore, requires three activities for each crop, resulting in six expected returns and a 6x6 covariance matrix.[5]

The data were obtained from the farmer prior to the 1986 crop year but after most farm-

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[4]The equations provide exact estimates of gross revenue mean and variance when the price and yield distributions are normal or when price and yield have no variability. Placing loan levels in the middle of the price distributions results in a modified price distribution that is decidedly nonnormal but does have substantial variability. It seems reasonable to expect this situation to introduce substantial error into estimates of gross revenue mean and variance.

[5]The example was created presuming the farmer was not subject to the Findley payment limits ($200,000).

program provisions had been announced.[6] Localized target prices were $0.81/lb. for cotton and $5.45/cwt. for sorghum. Localized formula loan rates were $0.55/lb. for cotton and $4.38/cwt. for sorghum. An adjusted loan rate of $3.10/cwt for sorghum was assumed, as was a cotton marketing loan of $0.42/lb. Proven yields were 620 lbs./acre and 46 cwt./acre for cotton and sorghum, respectively.[7] The price and yield distributions for cotton and sorghum were estimated using the fixed interval method (Huber). The estimated distributions are as follows:[8]

| 0.05 | 0.15 | 0.2 | 0.2 | 0.15 | 0.10 | 0.10 | 0.05 |

300 400 500 600 700 800 900 1000 1100

Cotton Yield (lbs/acre)

| 0.05 | 0.10 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.05 | 0.05 |

20 25 30 35 40 45 50 55 60 65

Sorghum Yield (cwt/acre)

| 0.05 | 0.15 | 0.3 | 0.3 | 0.15 | 0.05 |

.36 .38 .40 .42 .44 .46 .48

Cotton Price ($/lb)

| 0.1 | 0.4 | 0.4 | 0.1 |

2.70 2.90 3.10 3.3 3.5

Sorghum Price ($/cwt)

The farmer estimated rather wide distributions for crop yields, reflecting the risky nature of non-irrigated crop production in the Coastal Bend Region. Yield distributions were assumed the same whether the farm was in or out of the program.[9] Both price distributions were rather tight, reflecting his belief that large stocks of both commodities would minimize price fluctuations. Both price distributions were normally distributed, but the yield distributions were skewed to the right. The correlation matrix (Table 1) was calculated using the farmer's historical price and yield data for 1975 to 1985.

A simple Monte-Carlo simulation model was constructed to generate 500 correlated prices and yields for cotton and sorghum. Per-acre gross returns for each crop when participating in the farm program were calculated based on the program provisions outlined previously. Gross returns when not participating in the program were calculated by multiplying price times yield for each crop.

TABLE 1. CORRELATION MATRIX BETWEEN COTTON AND SORGHUM PRICES AND YIELDS

	Cotton Yield	Sorghum Yield	Cotton Price	Sorghum Price
Cotton Yield	1.000			
Sorghum Yield	0.3571	1.0000		
Cotton Price	0.1496	0.3043	1.0000	
Sorghum Price	-0.0391	0.6826	0.6816	1.0000

Assume the randomly generated data represent actual observations of price and yield for cotton and sorghum. Under this assumption, the expected returns vector and covariance matrix calculated from the data represent the "true" statistical parameters for the data. As the previous discussion has already suggested, the means, variances, and correlations from the prices and yields could be used in the equations to approximate the "true" statistical parameters. The difference between the two sets of estimates would be the result of inaccuracies in the equation approach. This procedure should illustrate quite clearly the error introduced when using the equations to calculate expected return and covariance. A second comparison can then be made between simulation and equation approaches to identify error introduced by simulation when both rely on the original data.

Table 2 provides the simulated gross returns vector and covariance matrix for cotton and sorghum produced under different program participation options.

Table 3 is an estimate of the expected gross returns vector and covariance matrix using the

[6] Actual local loan rates were still not known when estimates were made. Therefore, historical differences between national and local loan rates were used to calculate localized loan rates.

[7] Some additional information pertinent to the calculations was ignored to simplify the example. This included income from cottonseed, crop-share rental arrangements, per-unit production costs, storage and interest costs, and government payment reductions caused by GRH.

[8] The yield distributions reported here are for cotton following sorghum and sorghum following cotton. Returns and covariance of returns differ for other rotational schemes. The values above each distribution represent the probabilities of yields or prices falling within the interval indicated.

[9] Program participation could result in a different yield distribution than nonparticipation. Participation can result in better acreage being planted and greater resource availability (if program participation requires idling land). Consequently, one might expect the yield distribution to have a higher mean and lower variance when the farm is in the program. Differences between yield under the program and outside the program depend on the particular farm involved and program participation requirements. Nevertheless, any difference could easily be incorporated into the equations presented in this paper.

TABLE 2. EXPECTED RETURNS AND COVARIANCE MATRIX FOR THE STUDY FARM USING SIMULATION APPROACH

	--Non-Participation--		-------Program Participation-------			
	Open Market		Loan Only		Loan & Target Price	
	Cotton	Sorghum	Cotton	Sorghum	Cotton	Sorghum
Expected Return[a] ($)	281.79	133.49	374.94	190.99	536.14	240.21
Covariance between:						
Open Market:						
Cotton	7165	1301	9140	1235	9140	1235
Sorghum	1301	1507	1553	1312	1553	1312
Loan Only:						
Cotton	9140	1553	11848	1526	11848	1526
Sorghum	1235	1312	1526	1164	1526	1164
Loan & Target Price:						
Cotton	9140	1553	11848	1526	11848	1526
Sorghum	1235	1312	1526	1164	1526	1164

[a]Returns and covariance of gross returns are per planted acre.

TABLE 3. EXPECTED RETURNS AND COVARIANCE MATRIX FOR THE STUDY FARM USING THE EQUATION APPROACH

	--Non-Participation--		-------Program Participation-------			
	Open Market		Loan Only		Loan & Target Price	
	Cotton	Sorghum	Cotton	Sorghum	Cotton	Sorghum
Expected Return[a] ($)	281.79	133.49	375.30	191.57	536.50	240.79
	(0.0)[b]	(0.0)	(0.10)	(0.30)	(0.07)	(0.24)
Covariance between:						
Open Market:						
Cotton	7155	1278	9114	1215	9114	1215
	(0.14)	(1.77)	(0.28)	(1.62)	(0.28)	(1.62)
Sorghum	1278	1500	1525	1295	1525	1295
	(1.77)	(0.46)	(1.8)	(1.3)	(1.8)	(1.30)
Loan Only:						
Cotton	9114	1525	11804	1507	11804	1507
	(2.28)	(1.8)	(0.36)	(1.26)	(0.36)	(1.26)
Sorghum	1215	1295	1507	1097	1507	1097
	(1.62)	(1.30)	(1.26)	(6.11)	(1.26)	(6.11)
Loan & Target Price:						
Cotton	9114	1525	11804	1507	11804	1507
	(0.28)	(1.8)	(0.36)	(1.26)	(0.36)	(1.26)
Sorghum	1215	1295	1507	1097	1507	1907
	(1.6)	(1.4)	(1.26)	(6.11)	(1.26)	(6.11)

[a]Returns and covariance of gross returns are per planted acre.
[b]Percent error from values in Table 2.

equation approach. Table 3 also includes in parentheses the percent difference between values in Table 2 and Table 3. The data used in calculating some of the Table 3 values are given in Appendix A. The covariance values for participating in the loan or the loan and target price were the same as under the loan only because the formula loans were higher than the price distributions, resulting in a constant deficiency payment.

Percentage differences in calculating expected return using the equation approach were extremely small (0.30 or less). The differences between simulated and equation-based covariance matrices were also less than 2 percent for all but sorghum variance under the program. In Appendix B, a comparison is made between the two approaches when both utilize the original data. This comparison suggests simulation generally introduces more error into the estimation of mean and variance than does the equation approach.

Again, it is important to note that the comparisons made here were under a worse-case scenario for these data. Use of the actual adjusted loan for sorghum ($3.55/cwt.) and market loan for cotton ($0.44/lb.) resulted in almost no estimation error.

A significant disadvantage of the equations is the need to calculate correlations between $\tilde{P}T$, $\tilde{P}M$, $\tilde{P}A$, and the standard variables ($\tilde{P}$ and $\tilde{Y}$). In some cases, an examination of the data may be sufficient to assign values to many of these correlations. For example, the price distribution for cotton was well below the formula loan, so $\sigma_{\tilde{Y}\tilde{P}T}$, $\sigma_{\tilde{P}M\tilde{P}T}$, $\sigma_{\tilde{P}A\tilde{P}T}$, and $\sigma^2_{\tilde{P}T}$ could all be set to zero. Simple simulations between two variables (such as $\tilde{P}M$ and $\tilde{Y}$) represent another option that can be used to create a realistic data set for purposes of calculating correlation.

SUMMARY AND CONCLUSIONS

Crop-mix decisions are increasingly becoming intertwined with government-program-participation decisions. Mean-variance models are one method by which these decisions can be analyzed for risk-averse decisionmakers. Incorporating government-program provisions into mean-variance calculations is a difficult task. Monte-Carlo simulation is one method; however, it cannot be used in all cases and may not be desirable to use in some cases. This paper presents an equation-based approach which, in many cases, closely approximates actual mean-variance values.

The presented example offers evidence the equation approach introduces little error into the expected returns vector and covariance matrix, and may be more accurate than a simulation approach. The accuracy of the equations, in fact, is a function of the price and yield distributions, as well as the government-program provisions. Estimation error is increased as the distributions widen and/or become more skewed. Error also increases as the non-recourse loan moves toward the center of the price distribution. Simulation may be preferred if (a) the correlation matrix can be factored, (b) the cost of using a simulation approach is not important, and (c) the inaccuracies introduced by simulation can be minimized or ignored. The availability of either approach, however, makes possible the analysis of virtually any crop-mix/government-program-participation problem using either subjective or objective data.

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APPENDIX A

TABLE A1. VALUES USED IN CALCULATING EXPECTED RETURNS AND VARIANCES REPORTED IN TABLE 3

Symbol	Cotton	Sorghum
μ_Y	668.56	42.48
μ_P	0.4202	3.113
μ_{PM}	0.56	3.176
μ_{PA}	0.55	3.176
μ_{PT}	0.55	4.38
G	620.0	46.0
T	0.81	5.45
L	0.55	4.38
A	N/A	3.10
M	0.42	N/A
σ_Y	189.92	10.70
σ_P	0.024	0.1708
σ_{YP}	0.884	1.2258
σ_{PM}	0.014	0.324
σ_{YPM}	0.523	0.849
σ_{PA}	0.0	0.103
σ_{YPA}	0.0	0.678
σ_{PT}	0.0	0.0
σ_{YPT}	0.0	0.0
σ_{PMPA}	0.0	0.011
σ_{PMPT}	0.0	0.0
σ_{PAPT}	0.0	0.0

Values were calculated from 500 randomly generated prices and yields based on distributions and correlation matrix reported in the text.

APPENDIX B
DISCUSSION OF SIMULATION PROCEDURE

A Monte-Carlo simulation procedure was used to test the accuracy of the equations developed in the paper. Some additional details about the simulation procedure may be desired by some readers. Also, because simulation represents an alternative to the equation approach, a comparison between the two may aid in identifying which produces more accurate results.

Table B1 contains the statistical properties for the four sets of correlated random deviates used in conjunction with table lookup functions to generate random prices and yields. The first two sets were used for random yields and the third and fourth sets were used for random prices. If each set of uniform correlated random deviates were to display perfect statistical properties, they would each have a mean of 0.5 and a variance of 0.0833. Set numbers one and three are the closest to the ideal, with mean and variance errors of less than 1 percent. The other sets have percentage errors that exceed most errors reported in Table 3 for the equation approach. Note also the simulated correlation values are somewhat different from their parent values in Table 1.

In Table 3, the accuracy of the equation approach was demonstrated by first generating a set of random prices and yields, followed by comparing the resulting income means, variances, and covariances to those approximated using the equations and the statistical properties of the simulated prices and yields. One might also test how accurately the simulation approach approximates the actual price and yield distributions through a reverse process. That is, first use the equations and actual price and yield statistical data to calculate the income means, variances, and covariances (Table B2), and then compare the results with the simulated values reported in Table 2.

Especially relevant in this comparison are the differences between the non-program participation values for the equation vs. simulated approaches. As can be noted when comparing the percent errors in Table B2 to those in Table 3, the simulation approach introduced more error into the calculation of income statistical parameters than did the equation approach. In fact, the percentage errors reported in Table B2 were generally twice as large as the errors reported in Table 3.

The random number generator used here is Algorithm B, a generator recommended by Knuth and used in the FLIPSIM V farm-level simulator (Richardson and Nixon). It should be noted that all comparisons were done using a microcomputer, with a 16-bit processor. Better statistical properties for the uniform correlated deviates might be obtained using a different random number generator or a different starting value (seed). Based on this analysis, however, the equation approach apparently performs better than the simulation approach for this data set.

TABLE B1. STATISTICAL PROPERTIES OF RANDOM NUMBERS GENERATED FOR SIMULATED PRICES AND YIELDS

	Random Number Set			
	#1	#2	#3	#4
Mean	0.4974	0.5179	0.5006	0.5209
	(0.52)	(3.58)	(0.12)	(4.18)
Variance	0.08568	0.08320	0.08352	0.08208
	(2.82)	(0.16)	(0.22)	(1.51)
Correlation Matrix:				
#1	1.0000	0.3972	0.1860	0.0037
#2	0.3972	1.0000	0.2635	0.6662
#3	0.1860	0.2635	1.000	0.6639
#4	0.0037	0.6662	0.6639	1.0000

TABLE B2. COVARIANCE MATRIX FOR FARM BASED ON ACTUAL DATA

	--Non-Participation--		-------Program Participation-------			
	Open Market		Loan Only		Loan & Target Price	
	Cotton	Sorghum	Cotton	Sorghum	Cotton	Sorghum
Covariance between						
Open Market:						
Cotton	6762	1125	8940	1092	8940	1092
	(5.62)	(13.53)	(2.19)	(11.58)	(3.98)	(10.49)
Sorghum	1125	1413	1462	1233	1462	1233
	(13.53)	(6.24)	(11.58)	(6.02)	(5.86)	(6.02)
Loan Only:						
Cotton	8940	1462	11826	1455	11826	1455
	(2.19)	(5.86)	(0.19)	(4.65)	(0.19)	(4.65)
Sorghum	1092	1233	1455	1096	1455	1096
	(11.58)	(6.02)	(4.65)	(5.84)	(4.65)	(5.84)
Loan & Target Price:						
Cotton	8940	1462	11826	1455	11826	1455
	(2.19)	(5.86)	(0.19)	(4.65)	(0.19)	(4.65)
Sorghum	1092	1233	1455	1096	1455	1096
	(11.58)	(6.02)	(4.65)	(5.84)	(4.65)	(5.84)