AGRICULTURAL OUTPUT SUPPLY AND INPUT DEMAND RELATIONSHIPS WITH ENDOGENOUS LAND RENTS

Arlyn R. Maligaya and Fred C. White

Abstract

This study analyzed supply functions for agricultural output and demand functions for factors of production for Georgia. These relationships were derived with duality theory from a normalized quadratic profit function. Land has been included in other duality studies as a fixed factor as opposed to an endogenous factor. In this study, the system of supply and demand equations was augmented with a supply equation for land, which allowed land rents to be an endogenous factor. Consequently, it was possible to measure the impact of output and input prices on land rents.

Key words: agricultural output supply, input demand, agricultural technology, land rents, static duality.

In order to measure the interdependencies among outputs and the differential effects of various outputs on factor demands, several researchers have taken a multiple-input, multiple-output approach to duality theory (Ray; Weaver; Shumway). The dual cost function used by Ray does not allow for the endogeneity of output levels, and hence it cannot measure certain important cross-effects among inputs and outputs. The multiple-input, multiple-output profit function approach which has been used by Weaver, Shumway, and Shumway and Alexander overcomes these limitations by allowing for the endogeneity of output levels.

The effect of output and input prices on the demand for land has not been taken into account by any of these previous studies using duality theory. Typically the land resource has either been ignored or treated as a fixed factor in these studies. The reason for neglecting the land resource is that its price does not behave as other inputs. In the case of other inputs, it is assumed that producers can purchase any desired quantity at the going market price. Such a possibility exists for inputs with a perfectly elastic supply. However, the supply of land for agricultural purposes is expected to be price inelastic. Hence, the supply of land needs to be taken into account in measuring the impact of output and input prices on the demand for all inputs including land.

The overall objective of this study is to estimate a system of output supply and input demand equations for Georgia agriculture while recognizing the endogeneity of land rents. This study examines the structure of agricultural production in Georgia using multiple-output, multiple-input technology. Specific objectives are: to estimate supply functions for crops and for livestock and poultry and to estimate demand functions for hired labor, land, machinery, and materials. Because a unique aspect of the study is the endogenous nature of land rents, both the supply and the demand for land are considered.

THEORETICAL FRAMEWORK

While the goal of the study is to use a dual profit function to obtain a system of output supply and input demand equations, the land resource creates some special difficulties. Considering an upward sloping supply curve for land, the land prices (rents) do not satisfy the property of being given or exogenous to the decision makers. Hence, dual results are not in general valid for considering land. The difficulties associated with the land resource are overcome by incorporating land as a quasi-fixed resource into a normalized restricted profit function.

Although the model developed in this study is unique, it can be directly linked to previous literature. For example, Huffman considered land as a fixed factor in a similar model. Following Nadiri, Huffman also discussed shadow-value equations for fixed factors. He used this relationship to calculate shadow values of fixed inputs with mean values of variables rather than using the relationship in the econometric model, as this study does. Applebaum used an

Arlyn R. Maligaya is a Research Specialist, Department of Agricultural Economics, University of Kentucky; and Fred C. White is a Professor, Department of Agricultural Economics, University of Georgia, Athens.

approach close to the one in this study. He estimated a cost function in which quantity was a fixed factor. Using the shadow-value equation for output, he estimated an inverse supply equation, and he augmented his model with an output demand equation. In this study, land is considered to be a quasi-fixed factor. A shadow-value equation representing the demand for land is derived and estimated as a part of the theoretical model.

Consider the dual restricted profit function π^* which is a function of exogenous competitive input and output prices and quasi-fixed inputs:

$$(1) \quad \pi^* = f(P, W, Z) \,,$$

where π^* is maximum profit associated with the vector of competitive output prices P, the vector of competitive input prices W, and the vector of exogenous factors Z.

From this function, the input demand and output supply equations and the shadow-value equation for the quasi-fixed factors can be derived using Hotelling's Lemma. The partial derivative of the dual restricted profit function with respect to the ith input price (w_i) yields the negative ordinary demand function:

$$(2) \quad -x_i^* = \partial \pi^* / \partial w_i \,,$$

where x_i^* is the optimal quantity of the ith input. The partial derivative of the profit function with respect to the output price (p_j) yields the output supply function:

$$(3) \quad y_j^* = \partial \pi^* / \partial p_j \,,$$

where y_j^* is the optimal quantity of the jth output. The partial derivative of the profit function with respect to the quasi-fixed factor (z_k) yields the shadow-value equation:

$$(4) \quad \lambda_k = \partial \pi^* / \partial z_k \,,$$

where λ_k is the shadow price of the kth quasi-fixed factor. It is further assumed that the shadow price for land can be measured by land rental rates. Substituting land rent (r_k) for the shadow price in equation (4) yields the inverse demand equation for land. Land rent is now considered endogenous to the model, and the remaining conditions necessary to define a valid profit function are intact.

For a profit function to be considered theoretically valid, it has to meet the necessary regularity conditions. To meet these conditions, the profit function must be continuous,

convex, linearly homogeneous, monotonic in prices (increasing in output prices and decreasing in input prices) and monotonic increasing and concave in quasi-fixed inputs. For purposes of empirical application the function must be twice differentiable.

EMPIRICAL MODEL

Assuming competitive behavior, exogenous prices of outputs, and nonland variable inputs, the dual restricted profit function is modeled using the normalized quadratic form (Lau, 1976; Lerttamrab; Shumway and Alexander):

$$(5) \quad \pi^{*\prime} = b_o + \sum_{i=2}^{m} b_i p_i' + \sum_{i=m+1}^{n} c_i z_i + 0.5 \left(\sum_{i=2}^{m} \sum_{j=2}^{m} b_{ij} p_i' p_j' \right.$$
$$\left. + \sum_{i=m+1}^{n} \sum_{j=m+1}^{n} c_{ij} z_i z_j + \sum_{i=2}^{m} \sum_{j=m+1}^{n} d_{ij} p_i' z_j \right),$$

where $\pi^{*\prime}$ is profit divided by the price of netput 1 (p_l); the p_i's are normalized prices, $p_i' = p_i / p_l$, $i = 2, \dots m$; the z_i's are other exogenous variables, $i = m+1, \dots n$; and the b's, c's, and d's are parameters to be estimated.

Using Hotelling's Lemma, the first-order derivatives of equation (5) with respect to normalized prices of variable inputs and outputs are the input demand and output supply equations respectively:

$$(6) \quad q_i = b_i + \sum_{j=2}^{m} b_{ij} p_j' + \sum_{j=m+1}^{n} d_{ij} z_j, \quad i = 2, \dots, m \,,$$

where q_i is netput with positive output quantities and negative input quantities.

The demand equation for q_1 can be derived by taking the first derivative of the unnormalized profit function with respect to the numeraire price. It is quadratic in prices and other exogenous variables:

$$(7) \quad q_1 = b_o + \sum_{i=m+1}^{n} c_i z_i - 0.5 \left(\sum_{i=2}^{m} \sum_{j=2}^{m} \right) b_{ij} p_i' p_j'$$
$$+ \left(\sum_{i=m+1}^{n} \sum_{j=m+1}^{n} c_{ij} z_i z_j \right).$$

The inverse demand for the quasi-fixed factor land is obtained by differentiating the profit function with respect to land (z_n) to obtain the shadow-value equation. The shadow price is measured by the land rent (p_n') Hence, the inverse demand equation for land is:

$$(8) \quad p_n' = c_n + \sum_{j=m+1}^{n} c_{jn} z_j + \sum_{i=1}^{m} d_{in} p_i' \,.$$

A supply function for farmland is incorporated into the model. Residential, industrial, and related uses are assumed to be responsible for many land-use changes. Farming and forestry are residual claimants on the land resource. Therefore, such factors as population and per capita income are hypothesized to influence the quantity of land available for farming and forestry uses. To account for competition of forestry with farming, forest product prices are included in the supply equation for farmland. A linear supply equation for farmland is formulated as:

$$(9) \quad z_n = g_o + g_1 p_n' + \sum_{j=1}^{s} g_j h_j \, ,$$

where z_n is acreage of farmland, g_i's are parameters to be estimated, and h_j's are exogenous variables including population, per capita income, and pulpwood prices.

Equations (6) through (9) form the empirical model which is to be estimated as a system of equations after disturbance terms are appended to the equations.

DATA

The profit, output supply, input demand, and land supply and demand relationships were estimated using aggregate data for Georgia agriculture. The time series data used in the estimation were annual observations for the years 1950 through 1985. The model included two output categories (crops; livestock and poultry) and four input categories (land, hired labor, machinery, and materials).

Exogenous variables included expected product prices, current variable input prices, quantity of family labor, lagged government payments, a dummy variable for the 1983 Payment-in-Kind (PIK) program, and a time variable. Three-year moving average lagged prices represented expected prices for livestock and poultry. The endogenous variables are quantities of outputs and inputs.

Aggregate price indices for the input and output variables were calculated using the Tornqvist-Theil index, a discrete approximation to a Divisia index (Diewert). The base period for these indices was 1977. Aggregate quantity indices were computed by dividing aggregate revenue and expenditures by the aggregate price indices.

Data on revenue, cash expenses, and prices were obtained from *Georgia Agricultural Facts* (Georgia Crop Reporting Service). Pulpwood prices are also obtained from this source through 1979, but more recent pulpwood prices

were obtained from Cubbage and Davis. Land rents from 1960 to 1985 were obtained from Robison et al. and *Agricultural Land Values and Markets* (USDA). However, land rent data for the 1950–1959 period had to be extrapolated on the basis of farmland values using rent-to-value ratios from Robison et al. Land values were obtained from *Farm Real Estate: Historical Series Data 1950-1985* (USDA).

The machinery category included repairs and operation of capital items, interest, taxes, depreciation, and other consumption of farm capital. The materials category included feed, feeder-livestock, seed, fertilizer and lime, and miscellaneous. Prices for those items in both categories were the corresponding U.S. index of prices paid by farmers obtained from *Agricultural Statistics* (USDA).

The fixed factors (z's) included labor and government program variables. The quantity of family labor was computed following Evenson et al. The family labor estimate was based on surveys of the Statistical Reporting Service, USDA, published in *Farm Labor*.

A government payments variable was included to capture the effects of government intervention in agricultural production. Recognizing the simultaneous relationship between current government payments and output, the variable used was government payments lagged one period to reflect the expected value of government payments. Data on government payments were obtained from *Georgia Agricultural Facts*. Another exogenous variable used in the estimation was a dummy variable for 1983 which was the year for the implementation of the Payment-in-Kind program. This variable was an intercept and slope shifter in the profit function. Time was also included to measure the effects of technological change. The values of the time variable were 1950 = 1, 1952 = 2, . . . , 1985 = 36. These variables were included among the z's in equation (5).

A supply equation for farmland was included so that acreage of farmland, as well as land rents, could be considered as an endogenous variable. Land available for agricultural production was considered to be competitive with commercial forestry, so pulpwood prices were included in this supply equation. Land available for agricultural production is also affected by state-level population and per capita income. The data for population and per capita income were obtained from *U.S. Statistical Abstracts* (U.S. Department of Commerce).

ESTIMATION PROCEDURE

Six equations were derived from the normalized quadratic profit function. These were the supply equations for crops and livestock and poultry and the demand equations for land, hired labor, machinery, and materials. The price of materials was used to normalize all the other prices. The complete system of six stacked supply and demand equations and the supply equation for farmland were estimated using iterative three-stage least squares in the Statistical Analysis System (SAS) package. The profit function is not included in the system of estimating equations since all of its parameters are identified in the system. Linear homogeneity was imposed on the quadratic profit function by normalization. Conditions for symmetry are imposed on the models with the constraints $d_{ij} = d_{ji}$ for every $i \neq j$. As a result of the restrictions across equations, the degrees of freedom are based on the number of observations multiplied by the number of equations in the duality system. Errors were assumed to be independent, normally distributed with mean zero and variance $\partial_{ij}I$.

For convexity of the profit function to hold, the Hessian implied by the estimated d_{ij} matrix must be positive semi-definite. Monotonicity is checked by calculating the predicted values of the supply and demand equations. If at every observation the supply is positive and the demand is negative, then the necessary monotonicity conditions are met. Multicollinearity is detected through the use of condition indices. A condition index greater than 30 suggests the presence of moderate to strong collinearity. In the final estimation model, the interaction term between quantity of land and the dummy variable was excluded because of severe collinearity problems.

RESULTS

From the normalized profit function, demand and supply equations were estimated simultaneously with the supply equation for land. All of the eigenvalues computed from the Hessian for prices were positive, indicating a positive definite matrix. Hence, convexity held for this study. Given the quadratic form, this curvature property is global in nature. Monotonicity was not violated at any observation point of the demand and supply equations. Tests for serial correlation indicated no statistically significant problem in this regard. Multicollinearity was measured using condition indices. Results indicated that it is present be-

tween crop price and family labor, and among livestock and poultry price, hired labor wage, machinery price, time, and quantity of land. These condition indices ranged from 39 to 152, which indicates moderate collinearity.

Parameter estimates for the demand and supply equations are reported in Table 1. More than half of the 55 parameter estimates of the system of equations were statistically significant at the 0.10 level. The weighted R^2 for the system was 0.96, indicating a good fit. Consistent with economic theory, the respective own-price coefficients were negative for the demand for hired labor and machinery and positive for the supply of livestock and poultry, crops, and land.

The coefficient on lagged farmland in the supply equation for farmland was 0.799, indicating an annual adjustment rate of 0.201 or 20.1 percent. This result is similar to previous work based on dynamic duality theory. Taylor and Monson indicated that the annual rate of adjustment for farmland in the Southeast is 18 percent toward the equilibrium value. Others including Vasavada and Chambers and Alexander also found that farmland could be characterized as a quasi-fixed factor, indicating that this factor needs to be handled differently from most other factors.

The coefficient of the population and pulpwood price indicated a negative relationship with the supply of land, although not statistically significant at the 0.10 level. Similarly, disposable per capita income is negatively related to the supply of agricultural land.

Own- and cross-price elasticities of supply and demand equations are reported in Table 2. Except for land, all the own-price demand elasticities are negative and inelastic. The demand elasticity for land is, however, not statistically significant. The own-price elasticity of land in the supply equation was 0.0749 (not shown in the table). Hence, quantity supplied of land is not very responsive to land rents.

Own-price elasticities reported in this study are of similar magnitudes to those of previous studies. The current estimates of demand elasticities were generally more elastic than previous estimates (Taylor and Monson; Shumway and Alexander). The own-price elasticity for crop production was 0.50 in this study compared to 0.12–0.23 for commodity grouping in Shumway and Alexander. The own-price elasticity for livestock and poultry was 0.27 in this study and 0.15 in Shumway and Alexander.

The cross-price elasticities of crop production with respect to livestock and poultry prices and livestock and poultry production

TABLE 1. PARAMETER ESTIMATES OF INPUT DEMAND AND OUTPUT SUPPLY EQUATIONS, GEORGIA, 1950–1985

Parameter	Demand Equations			Supply Equations		
	Hired Labor	Land[a]	Machinery	Livestock and Poultry	Crops	Land
Intercept	69.403 (0.949)	0.098 (0.185)	298.327* (2.789)	621.491* (4.396)	-931.468* (-3.246)	6.275* (1.808)
Livestock and Poultry	-28.731 (-0.558)	0.937* (2.767)	210.674* (3.728)	180.738* (1.789)	-140.813* (-1.788)	
Crops	14.865 (0.357)	-0.205 (-1.122)	17.185 (0.329)	-140.813* (-1.788)	353.846* (2.383)	
Hired Labor	-99.574* (-2.518)	-0.133 (-0.610)	44.344 (1.437)	28.731 (0.558)	-14.865 (-0.357)	
Machinery	44.344 (1.437)	-0.054 (-0.273)	-397.457* (-7.979)	-210.674* (-3.728)	-17.185 (-0.329)	
Farmland	0.133 (0.610)	0.019 (0.791)	0.054 (0.273)	0.937* (2.767)	-0.205 (-1.122)	0.799*[b] (8.486)
Time	3.188 (1.318)	0.012 (0.821)	15.963* (4.969)	28.859* (6.772)	54.382* (7.761)	
Government Payments	-0.432* (-2.225)	0.001 (1.405)	0.869* (3.253)	1.490* (3.838)	0.006 (0.006)	
Family Labor	0.767* (4.117)	-0.003* (-3.582)	-0.002 (-0.007)	-1.383* (-3.581)	3.683* (4.173)	
Dummy	52.351* (2.525)		-35.385 (-1.165)	31.097 (0.770)	-208.745* (-1.773)	
Land Rent						1.462* (2.027)
Population						-0.001 (-1.451)
Disposable Per Capita Income						-0.0001 (-0.288)
Pulpwood Price						-0.741 (-1.127)

Weighted R^2 for system = 0.96

Mean values for 1950–1985 as used in the regression analysis are as follows: Quantity indexes are livestock and poultry—913.7, crops—835.2, hired labor—172.0, machinery—584.3, materials—909.9, and land—18.5. Price indexes are livestock and poultry—0.8, crops—0.8, hired labor—0.7, machinery—0.7, materials—0.7, and land—0.7. Government payments are 39.5. Family labor is 162.8.

t-values are in parentheses.

*Significant at 0.10 level.

[a]Inverse demand function.

[b]Lagged one year.

with respect to crop prices showed a complementary relationship. The small magnitude of the cross-price elasticities between input prices and output indicates that the outputs are not very responsive to input price changes. Land, however, is responsive to changes in livestock and poultry prices. The input demand elasticities imply that hired labor substituted for machinery and materials. Land complemented materials and substituted for hired labor and machinery. Moreover, substitutability was found between materials and machinery. The main differences between these results and previous estimates relate to the relationships between land and other inputs. The higher cross-price elasticities in this study compared to Taylor and Monson, for example, can be attributed to the unique formulation of land's supply and demand in this study.

For the normalized profit function, nonjointness in production exists if all mixed partial derivatives between output prices are zero (Lau, 1972, and Shumway). Nonjointness was tested using the simple t-test. If the cross-price coefficient of a commodity is significant, then there is jointness. Results indicated that there is jointness in production. Jointness is probably due to allocatable fixed inputs such as land.

Technological change was found to be machinery using and labor using. The demand for machinery exhibited the largest technical increase among the inputs. Technical change is labor using although at a low significance level. The supply of crops and livestock and poultry increased over time. Between the two

outputs, crops showed a larger technical shift than livestock and poultry.

The regression results further indicate that an increase in the quantity of family labor would increase the quantity demanded of hired labor. Hence, family labor and hired labor are complements rather than substitutes. The common assumption of substitutability may not hold because family and hired labor perform different activities within the farm. Administrative and managerial activities are an important component of family labor, while hired labor is mostly oriented to simpler, manual work (Lopez).

The dummy variable for the 1983 Payment-in-Kind (PIK) program showed the expected negative sign in the crops equation. The PIK program is an acreage-diversion program aimed at reducing both the production and stocks of commodities, particularly grains. Government payments (lagged) significantly reduced the demand for hired labor, increased the demand for machinery, and increased the supply of livestock and poultry.

CONCLUSIONS

This study analyzed agricultural production in Georgia using a normalized restricted profit function augmented with a supply curve for land. Georgia agriculture was found to be characterized by machinery-using and labor-using technical change. Own-price elasticities of the supply and demand functions were found to be inelastic. The cross-price elasticities for outputs indicated a complementary relationship between crops and livestock and poultry. Among the inputs, substitutability was found

TABLE 2. SHORT-RUN ELASTICITIES OF OUTPUT SUPPLY AND INPUT DEMAND FOR GEORGIA AGRICULTURE[a]

| Output or Input | Elasticity with Respect to the Price of | | | | |
	Livestock and Poultry	Crops	Materials	Hired Labor	Machinery
Livestock and Poultry	0.2674	-0.2068	0.0951	0.0273	-0.2162
Crops	-0.2132	0.4966	-0.2404	-0.0146	-0.0178
Materials	-0.1031	0.2486	-0.2615	0.0578	0.0583
Hired Labor	-0.2191	0.1058	0.4529	-0.5949	0.2755
Land	-3.2235	0.6578	-0.8574	0.3504	0.1509
Machinery	0.4166	0.0318	0.1068	0.0622	-0.5984

[a]The elasticities were calculated at mean levels for 1950–1985.

between the following input pairs: land and hired labor, land and machinery, materials and machinery, machinery and hired labor, and hired labor and materials. Land complemented materials.

Estimating separate demand and supply equation for land allowed the estimation of demand and supply elasticities for land. If the supply equation for land had not been estimated, the price elasticity in the demand equation would have included a conglomerate of demand and supply effects. Such an ambiguity would not reveal the true relationship between land rents and quantity of land. Using both a supply and demand equation helps explain the factors which affect land rents. A

reduction in the price of machinery reduces the demand for land and hence land rents. Urban influences were shown to reduce the supply of land and hence increase land rents.

Considering the importance of land rents, it seems appropriate to model both supply and demand relationships. However, it is recognized that other factors of production may also need to be analyzed in a similar fashion. In some cases an argument could be made that farmers compete for scarce labor within the state so that labor may also have an upward sloping supply curve. An area for further research would be to apply the concepts developed in this paper to other factors of production.

REFERENCES

Alexander, V. J. *A Dynamic Analysis of U.S. Agricultural Production.* Ph.D. Dissertation, University of Georgia, Athens, Georgia, 1988.

Applebaum, E. "Testing Price Taking Behavior." *J. Econometrics*, 9(1978):283–94.

Cubbage, F. W., and J. W. Davis. "Historical and Regional Stumpage Price Trends in Georgia." *Forest Products J.*, 36(1986):33–39.

Diewert, W. E. "Superlative Index Numbers and Consistency in Aggregation." *Econometrica*, 46(1978):883–900.

Evenson, R. E., D. Landau, and D. Ballou. "Agricultural Productivity Measures for U.S. States, 1950–1982." In *Evaluating Agricultural Research and Productivity.* Miscellaneous Publication 52-1987, Minnesota Agr. Exp. Sta., University of Minnesota, 1987.

Georgia Crop Reporting Service. *Georgia Agricultural Facts.* Various issues.

Huffman, W. E. "Research Bias Effects for Input and Output Decisions: An Application to U.S. Cash-Grain Farms." *Evaluating Agricultural Research and Productivity.* Miscellaneous Publication 52-1987, University of Minnesota, 1987.

Lau, L. J. "Profit Functions of Technologies with Multiple Inputs and Outputs." *Rev. Econ. and Statist.*, 54(1972):281–89.

Lau, L. J. "A Characterization of the Normalized Restricted Profit Function." *J. Econ. Theory*, 12(1976):131–63.

Lerttamrab, P. "Liquidity and Credit Constraints: Their Effect on Farm Household Economic Behavior: A Case Study of Northern Thailand." Ph.D. Dissertation, Stanford University, 1976.

Lopez, R. E. "Estimating Substitution and Expansion Effects Using a Profit Function Framework." *Amer. J. Agr. Econ.*, (1984):358–67.

Nadiri, M. I. "Producers Theory." *Handbook of Mathematical Economics.* Vol. II. Ed. K. J. Arrow and M. D. Intriligator. New York: North-Holland Pub. Co., 1982.

Ray, S. "A Translog Cost Function Analysis of U.S. Agriculture, 1939–1977." *Amer. J. Agr. Econ.*, 64(1982):490–98.

Robison, L. J., D. A. Lins, and R. VenKataraman. "Cash Rents and Land Values in U.S. Agriculture." *Amer. J. Agr. Econ.*, 67(1985):794–805.

Shumway, C. R. "Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops." *Amer. J. Agr. Econ.*, 65(1983):748–60.

Shumway, C. R., and W. P. Alexander. "Agricultural Product Supplies and Input Demands: Regional Comparisons." *Amer. J. Agr. Econ.*, 70 (1988):153–61.

Taylor, G., and M. Monson. "Dynamic Factor Demands for Aggregate United States Agriculture." *So. J. Agr. Econ.*, 17(1985):1–9.

U.S. Department of Agriculture. *Agricultural Land Values and Markets.* August 1985.

U.S. Department of Agriculture. *Agricultural Statistics.* Various issues.

U.S. Department of Agriculture. *Farm Labor.* Various issues.

U.S. Department of Agriculture. *Farm Real Estate: Historical Series Data, 1950–85*. December 1985.

U.S. Department of Commerce. *Statistical Abstract of the United States*. Various issues.

Vasavada, U., and R. G. Chambers. "Investment in U.S. Agriculture." *Amer. J. Agr. Econ.*, 68(1986):950–60.

Weaver, R. "Multiple Input, Multiple Output Production Choices and Technology in the U.S. Wheat Region." *Amer. J. Agr. Econ.*, 65(1983):45–56.