U.S. AGGREGATE AGRICULTURAL PRODUCTION ELASTICITIES ESTIMATED BY AN ARIMA FACTOR SHARE ADJUSTMENT MODEL

C. Richard Shumway and Hovav Talpaz

In an effort to circumvent the multicollinearity problems associated with direct estimation of the aggregate agricultural production function, many economists have used indirect estimation procedures. Because in equilibrium the partial production elasticities of an industry composed of perfectly competitive firms are equal to their respective factor shares, the latter have been used as a means of estimating production elasticities. Most researchers have simply assumed that actual factor shares are equilibrium values (e.g., Griliches; Rosine and Helmberger). Substantive contributions recently have been made in explaining the process of factor share adjustment by changes in prices and technology over time (Binswanger; Lianos). However, except for the work nearly 15 years ago by Tyner and Tweeten (1965), agricultural economics literature is largely silent on the measurement of differences between actual and equilibrium factor shares. It is this issue with which we are primarily concerned in this article. Therefore, our point of departure is the work by Tyner and Tweeten.

Because there is no assurance that current economic equilibrium is ever actually achieved, Tyner and Tweeten imposed a less restrictive assumption than other economists — i.e., that producers are not necessarily ever in a perfectly competitive equilibrium. They assumed that producers adjust *toward* equilibrium in their factor shares following a geometric lag adjustment,

$$(1) \quad F_t - F_{t-1} = \gamma(F_t^* - F_{t-1}),$$

where F is actual factor share, F^* is equilibrium factor share, and γ is the proportion of adjustment accomplished during year t, $0 < \gamma \leqslant 1$. Their estimation equation was expressed in stochastic form as

$$(2) \quad F_t = \beta_o + \beta_1 F_{t-1} + e_t,$$

where $\beta_o = \gamma F_t^*$, $\beta_1 = 1-\gamma$, and e_t is a stochastic disturbance. Implied equilibrium factor share, $\beta_o/(1-\beta_1)$, was postulated to change from decade to decade by inclusion of dummy variables, d_k, on the intercept, i.e., $\beta_o = \beta_o' + \Sigma d_k D_k$, where k represents a decade index.

Although this work received considerable attention and was recognized as an important contribution to agricultural economics literature,[1] the theoretical justification for the conceptual model was largely *ad hoc.* A connection between the neoclassical theory of the firm and factor share disequilibrium was explored recently by Shumway, Talpaz, and Beattie. However, no rigorous theory of factor share disequilibrium has yet emerged.

Tyner and Tweeten's estimation model is restricted by, among other things, the two assumptions that equilibrium factor shares are constant for a decade and that only two variables, current and lagged factor shares, are necessary to define the equilibrium share. Although we do not purport to derive a theory of factor share disequilibrium either, we do report the development of an autoregressive integrated moving average (ARIMA) model of factor share adjustment that relaxes the two assumptions.

U.S. factor share data for eight inputs for the years 1910-1976 are used to estimate equilibrium factor shares by year for the period 1919-1976. Production function implications during this 58-year period are then explored.

A DYNAMIC EQUILIBRIUM FACTOR SHARE ADJUSTMENT APPROACH

Equilibrium factor shares are clearly functions of product and input prices and the production function. Many input prices are

C. Richard Shumway and Hovav Talpaz are Associate Professors of Agricultural Economics, Texas A&M University. Hovav Talpaz is concurrently at the Department of Statistics, the Volcani Center, Israel.

Texas Agricultural Experiment Station Technical Article No. 15123.

Thanks are due Bruce Beattie, Bruce Gardner, John Penson, Robert Taylor, and the anonymous *Journal* reviewers for their constructive comments on drafts of the article. The interest and assistance of Fred Tyner, Luther Tweeten, Don Durost, and Peter Chang are also gratefully acknowledged.

[1]A second article (Tyner and Tweeten, 1966) that explores the implication of these production parameter estimates was subsequently selected for inclusion in the *A.E.A. Readings in the Economics of Agriculture.*

partially determined outside the agricultural sector, i.e., in the environment of national and international economic systems. Although we do not consider the general structure of equilibrium factor share determination, we assume that equilibrium factor shares within the agricultural sector are a consequence of price signals which also stimulate a process of adjustments toward these equilibrium levels. We postulate that, with no change in price signals or technology and under perfectly competitive conditions, actual factor shares will converge in finite time to steady-state values[2] equal to the equilibrium factor shares (which are also the partial production elasticities). In the real world, though, the probability of full convergence is very small because of the continuously changing set of prices and technologies and the existence of fixed factors, risks, and uncertainties.

Hence, the idea behind the approach we describe is to observe the movements or behavior of the actual factor shares and project their future steady-state values. This model emanates from the same conceptualization of factor share disequilibrium as implicitly underlies the Tyner-Tweeten model but without the two questioned assumptions. The logic is that entrepreneurs are making decisions which steer the sector toward the equilibrium position given their perceptions of current prices and technology and their anticipations of future prices and technology. They are assumed to make a forecast of market and technological conditions, implicity forecast the set of equilibrium shares, and adjust their actual factor shares subject to resource constraints. If these behavioral assumptions are generally valid, the following procedure is an approximate formulation for dynamic estimation of equilibrium factor shares.

Approximation of Dynamic Equilibrium Factor Shares

In developing an estimation procedure for dynamic equilibrium factor shares, we begin with the following identity:

$$(3) \quad F_{i,t} \equiv F^*_{i,t} + V_{i,t}$$

where $V_{i,t}$ is the deviation of $F_{i,t}$ from its equilibrium, $F^*_{i,t}$, i is the factor share index, and t is time. By rearranging equation 3, we obtain an identity for the equilibrium share:

$$(4) \quad F^*_{i,t} \equiv F_{i,t} - V_{i,t}.$$

Because $F^*_{i,t}$ is the equilibrium point, it is reasonable to assume that $V_{i,t}$ is a random variable, not necessarily independent of $V_{i,t'}$, for $t' \neq t$.

As postulated heretofore, in a stable economic system adjustments are made to steer the system toward a steady-state equilibrium position. That is, entrepreneurs acting within the system make decisions, the effect of which is to move toward equilibrium; consequently, if current trends of the exogenous conditions remain, equilibrium would be attained in τ future periods. Accordingly, let

$$(5) \quad \hat{F}_{i,t+\tau} \cong F^*_{i,t}, \tau \geqslant 1$$

where $\hat{F}_{t+\tau}$ is the steady-state forecasted factor share τ periods into the future, and τ is the minimum number of time periods required for convergence within a prespecified tolerance level.[3] Hence, the problem of estimating equilibrium shares is converted into a problem of adaptive forecasting dependent on actual shares in periods t, ..., t−p.

Factor Share Forecasting Methodology

Methodology for such forecasts is based mainly on the pioneering works of Box and Jenkins on ARIMA models. These models are regarded as efficient and practical for forecasting (Makridakis and Wheelwright, p. 245). It is beyond the scope of this article to describe this methodology in detail; however, a brief outline follows (based on Box and Jenkins, chapters 1-7; a short version is available in Makridakis and Wheelwright, chapter 18).

For each factor share (dropping the subscript i), an ARIMA process of order (p, d, q) is given by

$$(6) \quad F'_t = \Phi_1 F'_{t-1} + \ldots + \Phi_p F'_{t-p} + a_t - \Theta_1 a_{t-1} - \ldots - \Theta_q a_{t-q}$$

and

$$(7) \quad F'_t \equiv \nabla^d F_t$$

where F_t is the nonstationary time series of actual factor shares;[4] ∇^d is the difference operator of order d (for example $F'_t \equiv \nabla^1 F_t = F_t - F_{t-1}$ is the first-order difference of the process F_t), and differencing is required to create a stationary series from the nonstationary time series F_t; Φ_l, $l = 1, \ldots, p$, are the p autore-

[2]A variable is in steady state when its first derivative with respect to time is arbitrarily small in absolute value, approaching zero as $t \to \infty$.

[3]For a stable system with first-order delay, convergence will be non-oscillatory. Higher order delays tend to produce oscillatory paths toward the equilibrium position (Box and Jenkins, pp. 174-86).

[4]A stationary time series is a process with a mean that is unchanged as a function of time or, more rigorously, the joint distribution is invariant with regard to any time displacement m, i.e., $\text{Prob}(F'_t, \ldots, F'_{t+k}) = \text{Prob}(F'_{t+m}, \ldots, F'_{t+m+k})$ (see Nelson, pp. 20-1).

gressive parameters to be estimated; a_{t-m}, m = 0, ..., q, are the (q+1) random deviations between the observed F'_{t-m} and their predicted values; Θ_m, m = 1, ..., q are the q moving average parameters to be estimated. The parameters p and q are chosen such that their respective minimum positive integer values satisfy the requirement that (a_t, a_{t-1}, ...) is a white noise series.[5] In summary, an ARIMA model is an efficient operation for reducing the deviations ($V_{i,t}$) in equation 4 to a series of completely unexplainable residuals with no pattern.

For the ARIMA process outlined, three basic steps are involved — model identification, model estimation, and forecasting. First, each series is made stationary by differencing d times. The ARIMA model is then identified, i.e., p and q for equation 6 are determined. The identification process (Box and Jenkins, chapter 6, or Makridakis and Wheelwright, pp. 247-51) begins by examining the autocorrelation and partial autocorrelation coefficient plots of $F'_{i,t}$ for each i. The presence of random effects in an identified ARIMA model may lead to alternative combinations of (p,q) for the i^{th} series. The model can be selected by applying the test for white noise on the a_t's. According to Box and Jenkins (pp. 290-4), the test is performed by computing

$$(8) \quad Q = (N-d) \cdot \sum_{k=1}^{K} r_k^2(a_t)$$

where N−d is the number of F'_t observations used to fit the i^{th} model and $r_k^2(a_t)$ are the first K (it is customary to choose K = 20) autocorrelations of a_t with a_{t-k}. Then Q is approximately χ^2 with (K−p−q) degrees of freedom. If the a_t series is white noise, Q is below χ^2 at a prespecified significance level (0.05 chosen here).

If the density function of the a_t's were known, ARIMA parameters could be estimated by maximum likelihood procedures. Fortunately, for moderate and large samples, it can be shown "... that the parameter estimates obtained by minimizing the sum of squares [Σa_t^2] ... will usually provide very close approximations to the maximum likelihood estimates" (Box and Jenkins, p. 213). This conclusion is important because it frees us from making any prior assumptions about the a_t's, particularly their density functions. Because of nonlinearities in obtaining the moving average parameters and calculation of the first p values of the autoregressive process, the least squares estimate requires a nonlinear procedure (Box and Jenkins, pp. 231-42). Once the ARIMA model is estimated and tested for model adequacy, the time series can be forecasted to any number of future periods.

DIFFERENCES BETWEEN TYNER-TWEETEN AND DYNAMIC EQUILIBRIUM FACTOR SHARE MODELS

It is instructive to summarize the main difference between the Tyner-Tweeten model and the ARIMA-based model. First, $F^*_{i,t}$ are considered constant over a decade in the Tyner-Tweeten model, whereas in the ARIMA-based model they are regarded as dynamic projections changing constantly but computed as frequently as the observations are sampled. Second, in the former model $F^*_{i,t}$ is assumed to be related only to $F_{i,t}$, $F_{i,t-1}$, and in some cases $F_{i,t-2}$. In the latter model, in addition to $F_{i,t}$, $F_{i,t-1}$, and $F_{i,t-2}$, earlier factor shares may also affect estimated $F^*_{i,t}$. Third, in terms of technical estimation, Tyner and Tweeten's model construction requires the assumption of *random, independent,* and *normally* distributed disturbances whereas the ARIMA model requires only the assumption of *random* disturbances.[6] In the ARIMA modeling, one proceeds to construct an autoregressive, moving average scheme such that the remaining errors are approximately independent and identically distributed with mean zero and variance σ^2.

It is important to note that the ARIMA model does not impose a restriction on the form of the disturbances' density function. This issue is crucially important because both models estimate $F^*_{i,t}$ on the basis of current and past values of F_i only. The ARIMA model partially salvages the contribution of the missing observable and unobservable structural terms by projecting the behavior of the disturbances accounting for the dynamic path of the missing variables.[7]

ESTIMATED EQUILIBRIUM FACTOR SHARES

In this section, dynamic equilibrium factor share estimates are reported for the years 1919-1976. Data for the analysis are from published (mainly *Farm Income Statistics* and *Farm Income Situation*) and unpublished USDA sources, as are Tyner-Tweeten's. Esti-

[5]A white noise sequence is a set of identically and independently distributed random variables (Nelson, p. 31).

[6]By independence it is required that $E(V_{i,t} \cdot V_{i,t+m}) = 0$, for m>0 in equation 4. Tyner and Tweeten implicitly made the assumption of normality (Kmenta, pp. 235-9) in order to perform t-tests on the autocorrelation coefficients and to select OLS versus ALS models based on the F-test (Tyner and Tweeten, 1965, p. 1466).

[7]This discussion is not offered as a criticism of the original Tyner-Tweeten work as the computerized ARIMA methodology postdates their study.

mates are developed for eight farm input categories on the basis of annual U.S. data from 1910 to 1976. (Tyner and Tweeten's data are for 1910 to 1961.) Our input categories are similar; we have eliminated one category by combining real estate taxes with other real estate expenses. A brief description of expense items included in each input category is given in the appendix.

For each of the eight factor share time series, covering the 1910-1976 period, an ARIMA model was identified, estimated, and forecasted.[8] The identified p, d, and q parameters (equations 6 and 7) were 2, 1, and 0, respectively, for each of the factor share series.[9] This means that a stationary series was achieved by first-order differencing. A second-order autoregressive model was selected to represent each of the differenced series. The forecast errors showed no particular pattern, i.e., they acted as independent random variables. Each series was estimated such that the forecasting error passed a chi-square test at the 5 percent significance level (Box and Jenkins, chapter 6). Each equilibrium factor share was then forecasted for each of 10 years into the future, from 1919 through 1976. However, convergence was achieved after no more than 5 years, providing an $\hat{F}_{i,t+\tau}$ ($\tau = 5$) for each $F^*_{i,t}$.[10]

Table 1 lists the estimated parameters for the undifferenced models. Note that for p = 2 and d = 1 there must be three autoregressive parameters (Box and Jenkins, pp. 101-3). This indicates that perhaps even Tyner-Tweeten's ALS models were not adequate, although, with their decade dummy variables, the comparison is not completely valid.

On the basis of Table 1, one should expect that the forecasted steady-state values, i.e., the estimated equilibrium shares, would be close to their respective actual factor shares. When no moving average terms exist, the autoregressive parameters are equivalent to the weighted averages of the lagged series. Because $\tilde{\Phi}_1$ is close to unity for most factor shares, we can expect to find that the equilibrium shares are close to their respective actual shares. Table 2 reports the estimated equilibrium shares and confirms this expectation.[11]

TABLE 1. ESTIMATED PARAMETERS FOR THE UNDIFFERENCED ARIMA MODEL[a]

Input Category	Estimated Autoregressive Parameters $\tilde{\Phi}_1$	$\tilde{\Phi}_2$	$\tilde{\Phi}_3$	Total Sum Square Error
Fertilizer and lime	0.83601	−0.04386	0.20785	.000021
Feed, seed, and livestock	0.48075	0.35575	0.16349	.000013
Labor	0.96078	0.04963	−0.01041	.001043
Machinery investment	1.15898	−0.21634	0.05736	.000114
Real estate	1.23219	−0.26773	0.03554	.001212
Machinery operating	1.02570	0.03129	−0.05699	.000040
Miscellaneous operating	1.10519	−0.15185	0.04666	.000040
Crop and livestock inventory	0.53516	0.67099	−0.20615	.000004

[a]An ARIMA (2,1,0) process is

$$F'_t = \Phi_1 F'_{t-1} + \Phi_2 F'_{t-2} + a_t.$$

Hence, by equations 6 and 7 the equivalent undifferenced process is

$$F_t = \tilde{\Phi}_1 F_{t-1} + \tilde{\Phi}_2 F_{t-2} + \tilde{\Phi}_3 F_{t-3} + a_t$$

where

$$\tilde{\Phi}_1 = 1 + \Phi_1,\ \tilde{\Phi}_2 = (\Phi_2 - \Phi_1),\ \text{and}\ \tilde{\Phi}_3 = -\Phi_2.$$

PRODUCTION FUNCTION IMPLICATIONS

The implied consequences on the aggregate agricultural production function of the estimated equilibrium factor shares are briefly explored. Because the estimated equilibrium shares are generally close to actual shares, the production function implications are generally similar to the implications that could be derived from the actual factor shares[12] and, for decade averages, from updated estimation of the Tyner-Tweeten model (see Shumway, Talpaz, and Beattie).

The sum of the estimated partial production elasticities (i.e., equilibrium factor shares) ranges from a low of 0.723 to a high of 1.316 for

[8]The IMSL package was employed in the modeling process. Subroutine FTAUTØ was used for identification of alternative models. Subroutine FTSIMP was used for estimation, testing for adequacy, and forecasting. Model redundancy was avoided (Box and Jenkins, pp. 248-50). The parameters are estimated over the entire time period and are then used to forecast factor shares within the time period.

[9]For the lowest d that makes the series stationary, the practice is to select the minimum (p,q) such that the test for white noise is satisfied. The autoregressive parameter, p, is greater than 1 (for d = 1). This fact may suggest that the implicit assumption of independently distributed disturbances in the Tyner-Tweeten model is violated.

[10]"Convergence" is used here to mean that $|\hat{F}_{t+\tau+1} - \hat{F}_{t+\tau}| < \varepsilon$ where ε is an arbitrarily small positive value.

[11]Fully 91 percent of actual annual factor shares are within a 90 percent confidence interval about the forecasted equilibrium, 94 percent are within a 95 percent confidence interval, and 80 percent are within a 75 percent interval.

[12]The estimated equilibrium shares are reported here for documentation of their similarity in general, but dissimilarity at times, to actual factor shares.

TABLE 2. ACTUAL FACTOR SHARES AND ARIMA EQUILIBRIUM ESTIMATES FOR U.S. AGRICULTURE, 1919-1976.

	Input Category																	
	Fertilizer & Lime		Feed, Seed & Livestock		Labor		Machinery Investment		Real Estate		Machinery Operating		Misc. Operating		Crop-Livestock Inventory		Sum	
Year	A[a]	E	A	E	A	E	A	E	A	E	A	E	A	E	A	E	A	E
1919	.021	.020	.026	.026	.361	.341	.044	.045	.225	.212	.019	.018	.046	.044	.052	.052	0.794	0.759
1920	.026	.021	.033	.026	.470	.360	.054	.043	.308	.228	.026	.020	.053	.046	.054	.051	1.025	0.796
1921	.026	.024	.031	.031	.473	.468	.081	.055	.443	.328	.037	.027	.060	.053	.054	.054	1.206	1.040
1922	.023	.025	.029	.031	.432	.474	.064	.083	.378	.473	.032	.038	.055	.061	.051	.055	1.064	1.240
1923	.023	.024	.031	.030	.434	.433	.060	.061	.345	.356	.030	.033	.050	.054	.049	.052	1.023	1.042
1924	.022	.024	.039	.030	.412	.434	.061	.061	.305	.340	.031	.030	.048	.050	.046	.049	0.963	1.017
1925	.023	.022	.033	.036	.389	.412	.057	.061	.279	.297	.034	.031	.046	.048	.045	.046	0.906	0.953
1926	.024	.023	.033	.035	.404	.389	.061	.057	.285	.274	.040	.034	.051	.046	.045	.045	0.943	0.903
1927	.022	.024	.034	.034	.394	.403	.064	.062	.275	.287	.040	.041	.050	.052	.043	.045	0.922	0.947
1928	.025	.022	.038	.034	.386	.394	.063	.064	.271	.272	.042	.041	.053	.050	.046	.044	0.923	0.921
1929	.023	.025	.035	.036	.376	.386	.062	.062	.261	.271	.043	.042	.054	.053	.046	.045	0.900	0.920
1930	.029	.023	.036	.036	.446	.376	.080	.062	.324	.259	.053	.044	.061	.054	.049	.046	1.078	0.900
1931	.025	.028	.028	.036	.463	.444	.096	.082	.372	.339	.057	.054	.071	.062	.046	.049	1.158	1.093
1932	.020	.025	.031	.031	.506	.463	.114	.096	.426	.381	.071	.058	.083	.071	.050	.047	1.303	1.174
1933	.019	.022	.032	.031	.445	.505	.091	.115	.349	.437	.066	.073	.072	.084	.049	.048	1.123	1.316
1934	.027	.020	.038	.031	.457	.447	.083	.088	.324	.327	.067	.066	.070	.071	.054	.050	1.121	1.101
1935	.021	.025	.030	.035	.375	.456	.062	.084	.236	.322	.052	.067	.056	.071	.044	.053	0.876	1.113
1936	.029	.021	.043	.033	.400	.378	.069	.060	.233	.216	.057	.051	.059	.055	.049	.046	0.939	0.860
1937	.025	.028	.038	.039	.364	.398	.063	.071	.192	.236	.053	.057	.053	.060	.040	.046	0.828	0.934
1938	.028	.025	.038	.039	.446	.365	.082	.062	.229	.182	.067	.053	.063	.053	.044	.042	0.997	0.821
1939	.030	.028	.047	.039	.448	.443	.079	.084	.219	.240	.066	.068	.063	.064	.044	.042	0.996	1.008
1940	.031	.029	.048	.044	.427	.448	.075	.078	.208	.215	.066	.067	.061	.063	.044	.045	0.960	0.988
1941	.026	.030	.043	.047	.388	.428	.061	.075	.161	.206	.057	.066	.051	.060	.043	.044	0.829	0.956
1942	.023	.027	.039	.045	.364	.389	.056	.060	.119	.150	.048	.056	.039	.050	.040	.043	0.728	0.820
1943	.023	.024	.043	.041	.398	.364	.049	.057	.108	.110	.049	.047	.038	.039	.043	.040	0.751	0.723
1944	.026	.023	.045	.042	.452	.397	.049	.048	.118	.107	.055	.049	.040	.039	.043	.042	0.828	0.746
1945	.028	.025	.045	.044	.460	.451	.046	.049	.125	.121	.055	.055	.039	.040	.042	.044	0.841	0.830
1946	.025	.027	.043	.045	.433	.460	.035	.046	.126	.126	.053	.056	.036	.038	.040	.042	0.791	0.841
1947	.025	.025	.048	.044	.416	.434	.040	.034	.129	.126	.058	.053	.039	.036	.042	.040	0.797	0.792
1948	.024	.025	.048	.046	.382	.416	.051	.041	.123	.130	.061	.058	.038	.040	.037	.041	0.765	0.798
1949	.031	.024	.053	.048	.426	.383	.080	.052	.151	.121	.078	.062	.048	.038	.040	.039	0.908	0.766
1950	.031	.029	.053	.051	.382	.424	.087	.083	.144	.158	.074	.080	.047	.049	.039	.039	0.858	0.913
1951	.029	.030	.057	.053	.343	.384	.088	.086	.142	.140	.071	.075	.048	.046	.041	.040	0.818	0.854
1952	.033	.030	.059	.055	.341	.343	.098	.088	.156	.141	.077	.070	.052	.048	.041	.040	0.856	0.817
1953	.036	.032	.055	.057	.358	.341	.112	.099	.175	.160	.086	.078	.057	.052	.041	.041	0.920	0.860
1954	.038	.035	.056	.056	.335	.357	.116	.113	.177	.179	.086	.087	.059	.058	.038	.040	0.905	0.926
1955	.038	.037	.056	.056	.342	.336	.119	.115	.192	.177	.091	.087	.065	.058	.038	.039	0.940	0.905
1956	.037	.037	.055	.056	.335	.341	.123	.119	.197	.196	.096	.091	.068	.065	.037	.038	0.949	0.944
1957	.036	.037	.055	.056	.335	.336	.126	.123	.210	.198	.099	.097	.070	.068	.037	.037	0.969	0.952
1958	.033	.037	.055	.055	.299	.335	.115	.126	.198	.213	.089	.099	.070	.070	.037	.037	0.896	0.972
1959	.038	.034	.060	.055	.311	.300	.123	.114	.223	.194	.093	.088	.083	.069	.038	.037	0.969	0.892
1960	.037	.037	.057	.058	.318	.310	.124	.125	.231	.230	.089	.093	.085	.084	.037	.038	0.978	0.974
1961	.039	.037	.058	.058	.317	.318	.122	.124	.235	.232	.085	.089	.086	.085	.037	.038	0.979	0.979
1962	.040	.039	.059	.058	.316	.317	.113	.122	.240	.235	.082	.085	.087	.086	.037	.037	0.979	0.978
1963	.044	.040	.058	.058	.317	.316	.117	.118	.247	.241	.080	.082	.088	.087	.036	.037	0.986	0.978
1964	.050	.042	.060	.058	.352	.317	.126	.117	.272	.249	.082	.079	.097	.088	.038	.036	1.076	0.987
1965	.048	.048	.057	.059	.328	.351	.120	.127	.263	.277	.075	.082	.092	.097	.035	.037	1.018	1.079
1966	.050	.048	.059	.058	.317	.329	.118	.118	.262	.260	.074	.075	.089	.091	.037	.036	1.005	1.015
1967	.054	.050	.060	.058	.326	.317	.128	.118	.278	.262	.077	.073	.097	.090	.035	.036	1.056	1.004
1968	.053	.053	.056	.059	.337	.326	.135	.129	.297	.282	.077	.077	.102	.098	.037	.036	1.093	1.060
1969	.047	.053	.057	.057	.327	.337	.131	.135	.294	.301	.072	.077	.097	.102	.037	.036	1.061	1.098
1970	.046	.049	.058	.057	.323	.328	.132	.130	.296	.292	.070	.071	.096	.096	.036	.037	1.058	1.060
1971	.047	.047	.060	.057	.300	.323	.130	.133	.297	.296	.067	.069	.100	.097	.033	.037	1.035	1.059
1972	.042	.047	.058	.059	.276	.301	.121	.130	.285	.297	.060	.067	.095	.100	.034	.034	0.971	1.035
1973	.036	.044	.057	.059	.210	.276	.091	.120	.225	.283	.046	.059	.072	.095	.032	.033	0.770	0.968
1974	.062	.039	.056	.058	.216	.212	.110	.088	.274	.211	.061	.044	.080	.070	.032	.033	0.893	0.754
1975	.068	.056	.050	.057	.223	.216	.131	.114	.309	.289	.070	.062	.090	.082	.031	.032	0.973	0.907
1976	.065	.063	.055	.052	.232	.223	.144	.132	.351	.315	.077	.072	.098	.091	.031	.031	1.053	0.980

[a]Codes: A is actual factor share; E is estimated equilibrium factor share (i.e., estimated partial production elasticity).

individual years, but most often is slightly below 1.0. The time series reflects a roller-coaster effect with declining amplitude over time and with a nearly constant mean. Thus, we conclude from examination of the 58-year estimated production function series that returns to scale at the end of the period are similar to returns at the beginning and at several intermediate points in time.

This conclusion, however, does not apply to the estimated production elasticities of individual inputs. The responsiveness of output to a single input, as given by the partial production elasticity, has increased substantially for several inputs. The largest relative increases are for machinery operating inputs, fertilizer and lime, and machinery investment. In comparison with 1921 levels, when estimated returns to scale were considerably closer to the mean level than either the 1919 or 1920 values, the estimated output responsiveness of each of these three inputs has increased approximately 150 percent. However, the pattern of increase has varied considerably among these inputs. Estimated output responsiveness of machinery operating inputs nearly quadrupled by 1958 and has since declined. The major increases occurred after 1950 for fertilizer and lime and prior to 1955 for machinery investment. Estimated output responsiveness has increased about two thirds for feed, seed, and livestock (with most of the increase prior to 1950) and for miscellaneous operating inputs (with major increases since 1950).

Estimated output responsiveness was significantly lower in 1976 than in 1921 for only two inputs, labor (with the entire decrease since 1950) and crop and livestock inventories. Responsiveness declined for both by about 50 percent. Estimated output responsiveness to real estate dropped markedly prior to 1950 but has since increased to its earlier levels.

The input having the largest estimated production elasticity over most of the period was labor; its elasticity was generally about 0.4 until 1950. By 1973, labor's elasticity had dropped to 0.276, placing it second in magnitude to real estate (0.283). In 1976, labor's elasticity was only 0.223 whereas real estate's elasticity had increased to 0.315. With the conventional grouping of inputs into three categories, land, labor, and capital, the capital input category would include all non-real estate and non-labor inputs. The combined elasticity of such inputs has increased markedly, from 0.244 in 1921 to 0.442 in 1976. Thus, capital inputs now have a higher estimated production elasticity than either real estate or labor.

SUMMARY AND CONCLUSIONS

An autoregressive factor share adjustment model is developed to permit fully dynamic estimation of equilibrium factor shares and consideration of the time path of adjustment in estimating equilibrium shares. The model is applied to annual data for eight U.S. agricultural input categories, 1910-1976.

The sum of the estimated equilibrium shares implies an aggregate production function with slightly diminishing returns to scale and little permanent change in returns to scale over the last half-century. The relative importance of individual inputs in output response, however, has changed dramatically. Labor's elasticity was cut in half and the combined capital input elasticity increased by 80 percent. Although real estate's production elasticity changed substantially during the period, it went full cycle — in 1976 it was at nearly the same level as 55 years earlier.

In retrospect, one must conclude that the empirical conclusions are roughly the same as one would draw from estimation via the Tyner-Tweeten model (see Shumway, Talpaz, and Beattie) or, perhaps more significantly, directly from the original factor share data. Estimated equilibrium shares closely parallel their respective actual factor shares. Consequently, although the Tyner-Tweeten model relaxes the questionable assumption that economic equilibrium prevails continuously in the agricultural sector and our application of the Box-Jenkins model relaxes two restrictive assumptions of the Tyner-Tweeten model, little has been gained empirically. Though seemingly negative in light of the substantial research investment already made, the recent conclusion of Shumway, Talpaz, and Beattie about production function estimation seems additionally germane. That is, "... if the factor share approach is followed, the least-cost research alternative of assuming instantaneous and complete adjustment, i.e., using actual factor shares, seems appropriate" (p. 564). The commonly imposed assumption in empirical economic research that observed factor shares can be treated as equilibrium shares in perfectly competitive industries has not been refuted.

APPENDIX

U.S. FARM INCOME AND EXPENDITURE DATA

Most data used in the article are from July 1977 *Farm Income Statistics* (USDA, 1977) and from July 1957 and July 1965 *Farm In-*

come Situation (USDA, 1957, 1965) with appropriate supplementation and adjustment. Some additional expense items that were not included in Tyner and Tweeten's data are included in several of the input categories. Details of data development and sources are available on request from the authors. Factor share for any category consists of actual expenditures on that category divided by farm income for the same year. A brief description of each data category follows.

Farm income is gross farm income net of government payments and rental value of farm dwellings.

Fertilizer and lime expenditures are current purchases.

Feed, seed, and livestock expenditures are adjusted to exclude interfarm sales. They basically reflect marketing charges paid to the non-farm sector.

Labor is current expenses for hired labor multiplied by the ratio of the annual average number of total and hired farm workers.

Machinery investment charges include depreciation, interest, and personal property taxes on machinery.

Real estate charges are the sum of (a) the value of farm real estate excluding dwellings multiplied by the farm real estate mortgage interest rate on outstanding loans, (b) service building depreciation, repairs, and operation, (c) accidental damage to service buildings and machinery, and (d) real estate taxes.

Machinery operating expenditures include repairs and operation of motor vehicles and machinery plus petroleum fuel and oil.

Miscellaneous operating expenditures are total miscellaneous farm operating expenses except interest on non-real estate debt.

Crop and livestock inventory expenditures include interest and personal property taxes on crop and livestock inventories.

REFERENCES

Binswanger, H. P. "A Cost Function Approach to the Measurement of Elasticities of Factor Demand and Elasticities of Substitution." *Amer. J. Agr. Econ.* 56(1974):377-86.

Box, G. E. P. and G. M. Jenkins. *Time Series Analysis: Forecasting and Control,* 2nd ed. San Francisco: Holden-Day, 1976.

Griliches, Z. "The Demand for Inputs in Agriculture and Derived Supply Elasticity." *J. Farm Econ.* 41(1959):309-22.

IMSL. *The IMSL Library.* Houston: International Mathematical and Statistical Libraries, 1977.

Kmenta, J. *Elements of Econometrics.* New York: Macmillan Co., 1971.

Lianos, T. P. "The Relative Share of Labor in United States Agriculture, 1949-1968." *Amer. J. Agr. Econ.* 53(1971):411-22.

Makridakis, S. and S. C. Wheelwright. *Interactive Forecasting Univariate and Multivariate Methods,* 2nd ed. San Francisco: Holden-Day, 1978.

Nelson, C. R. *Applied Time Series Analysis for Managerial Forecasting.* San Francisco: Holden-Day, 1973.

Rosine, J. and P. Helmberger. "A Neoclassical Analysis of the U.S. Farm Sector, 1948-1970." *Amer. J. Agr. Econ.* 56(1974):717-29.

Shumway, C. R., H. Talpaz, and B. R. Beattie. "The Factor Share Approach to Production Function 'Estimation': Actual or Estimated Equilibrium Shares?" *Amer. J. Agr. Econ.* 61(1979):561-4.

Tyner, F. H. and L. G. Tweeten. "A Methodology for Estimating Production Parameters." *J. Farm Econ.* 47(1965):1462-7.

Tyner, F. H. and L. G. Tweeten. "Optimum Resource Allocation in U.S. Agriculture." *J. Farm Econ.* 48(1966):613-31. Reprinted in *A.E.A. Readings in the Economics of Agriculture,* K. A. Fox and D. G. Johnson, eds., pp. 286-306. Homewood, Ill.: Richard D. Irwin, Inc., 1969.

U.S. Department of Agriculture. *Farm Income Statistics.* Economic Research Service Statistical Bulletin 576, July 1977.

U.S. Department of Agriculture. *Farm Income Situation.* Economic Research Service, July 1977, July 1965.