Hedging Cash Flows from Commodity Processing

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Agribusinesses make long-term plant-investment decisions based on discounted cash flow. It is therefore incongruous for an agribusiness firm to use cash flow as a plant-investment criterion and then to completely discard cash flow in favor of batch profits as an operating objective. This paper assumes that cash flow and its stability is important to commodity processors and examines methods for hedging cash flows under continuous processing. Its objectives are (a) to determine how standard hedging models should be modified to hedge cash flows, (b) to outline the differences between cash flow hedging and profit hedging, and (c) to determine the effectiveness of hedging in reducing cash flow variability. A cash flow hedging methodology is developed. This methodology is similar to that used for batch profit hedging. This methodology balances the daily cash flow destabilizing effect of futures positions against the periodic cash flow destabilizing effect of cash price changes. The resulting cash flow hedges are simulated for soybean processors. These hedges are less effective than batch profit hedging. The reduction in cash flow variance achieved through hedging, though small, is nonetheless statistically significant.

Keywords: Cash flow hedging, soybean processing, hedge ratio, hedging effectiveness.

Introduction

Consider the economic criteria that agribusinesses use to evaluate decisions such as to whether to build a new processing plant, to develop a new product, to enter a new geographic market, to buy a distribution facility, or to expand into a new line of business. While the usual assumption is that a firm behaves in a manner so as to maximize profit, the criteria appropriate to each of these decisions is not explicit profit maximization but instead positive discounted cash flow. Cash flow represents financial capital. It is used in lieu of profits because projects such as these involve payables and receivables that vary through time. The net present value computation permits the comparison of receipts and expenditures of financial capital that occur at different times. An alternative interpretation of positive discounted cash flow is that the rate of return on the capital invested in the project exceeds the cost of the capital. The implementation of projects with a rate of return that exceeds capital costs is consistent with long-run profit maximization. The importance of cash flow in the evaluation of these risky decisions is the point that we wish to carry forward into this investigation.

Compare the cash flow criterion with the traditional Johnson (1960), Stein (1961), and Anderson and Danthine (1980, 1981) hedging formulation for managing price risk. In this formulation, x_s represents an agent's required spot-market position and x_f represents the attendant futures position. In addition, let s_0 and f_0 represent current or initial spot and futures prices and let s_1 and f_1 represent the terminal spot and futures prices. With hedging, profit is $\pi = x_s (s_1 - s_0) + x_f (f_1 - f_0)$. The agent is assumed to select x_f in an attempt to maximize the utility of π in a mean-variance utility framework. If the agent is extremely risk averse or expects no change in the futures price, we get the well-know result that $x_f = -x_s \, Cov(f_1 - f_0, s_1 - s_0) / V(f_1 - f_0)$.

In its infancy, this formulation was used to represent a farmer who, at time 0, made his planting decision (thereby determining x_s) followed by his hedging decision. Alternatively, this formulation can represent a cattle feeder who places cattle on feed at time 0 with their sale anticipated at time 1. These cases exemplify batch production in that output is hedged and produced one batch at a time. Continuous production occupies the other end of the spectrum where inputs are periodically purchased and outputs are continuously produced and periodically sold. Batches can overlap under continuous production as inputs for the next batch are purchased before the products from the previous batch are sold. In such a case, the historical cost of inputs has less economic meaning than the opportunity cost of input replacement. Thus, the traditional hedging approach of valuing inputs at their historical cost has less appeal than valuing the inputs at their replacement cost. When attention focuses on current revenues and current input expenditures, then cash flow becomes the hedging target.

Concern with cash flow may seem trivial or misdirected given the standard assumption of profit maximization as the firm's objective. However, the following observations underscore its importance. First, the standard criterion for a firm's investment in a processing facility is discounted cash flow. Having constructed or purchased a facility based primarily on discounted cash flow, it seems inconsistent to discard this criterion and replace it with profit objectives. Second, periodic cash flow and batch profits converge when they are aggregated to annual accounting periods but annual profit maximization and stabilization objectives differ from processing-cycle cash flow maximization and stabilization objectives. However, the two objectives are not inconsistent with each other. Finally, agribusinesses hire financial managers. These managers are responsible for ensuring that cash is available to pay for inputs and that receivables for product are collected in a timely manner. Costs are incurred in the exercise of these duties and the stabilization of cash flows will lower these costs.

This paper deals with methods for hedging cash flows under conditions of continuous processing. Our specific objectives are (a) to determine how standard hedging models should be modified to hedge cash flows, (b) to outline the differences between cash flow hedging and profit hedging, and (c) to determine the effectiveness of flow hedging in reducing cash flow variability. The soybean-processing sector is used to represent continuous processing because (a) soybean crushing conforms to the continuous processing assumption, (b) soybean processing transformation coefficients are well known in that a 60 pound bushel of soybeans yields eleven pounds of soybean oil and 47 pounds of soybean meal, (c) the sector is economically important, and (d) cash and futures prices for soybeans and soybean products are available with a frequency that corresponds to continuous processing.

While the soybean crushing sector was chosen for study, it is important to note that other examples are also available. Cottonseed crushing and meatpacking are also characterized by continuous processing. In addition, some traditional agricultural production enterprises such as broiler production and hog feeding have moved toward continuous production as these enterprises have become more industrialized.

Literature Review

Modern hedging methods trace back to Johnson's (1960) and Stein's (1961) treatment of a commodity market position as part of a portfolio that may also contain futures position. This treatment is outlined above. Johnson and Stein derived the risk-minimizing hedge ratio, which is estimated as the slope in the regression of futures price changes over the portfolio's life against spot price changes over the portfolio's life. Hedging effectiveness defined as the proportionate price risk reduction due to hedging, is measured as the squared correlation between spot and futures price changes over the portfolio's life.

Anderson and Danthine (1980, 1981) generalized the Johnson and Stein approaches by including multiple futures contracts in the portfolio and by assuming mean-variance utility maximizing behavior by the agent. Their formulation provides for multi-contract hedging (Anderson and Danthine 1980) and cross hedging (Anderson and Danthine 1981). Risk-minimizing hedge ratios are obtained by assuming either infinite risk aversion or no expected speculative returns. These hedge ratios are estimated by the multiple regression parameters where the dependent variable is the change over the portfolio-holding period in the cash price of the commodity and the independent variables are changes over the portfolio-holding period in the price of futures contracts. Hedging effectiveness is estimated by the regression multiple correlation statistic.

Ederington (1979) found that for a wide variety of commodities, the Johnson portfolio-risk minimization approach is more effective than the one-unit futures to one-unit cash approach. Consequently, the Johnson, Stein, and Anderson and Danthine methods are typically employed in agricultural production and storage hedging. Some studies suggest that the simplest hedging models such as the constant-hedge ratio models proposed by Johnson, Stein, and Anderson and Danthine work best. Garcia, Roh and Leuthold (1995) find that time-varying hedge ratios "provide minimal gain to hedging in terms of mean return and reduction in variance over a constant conditional procedure." Collins (2000) reports that multivariate hedging models offer no statistically significant improvement over "naive equal and opposite hedges."

Both the time and product-form price dimensions are potentially hedgeable in soybean processing and several methods for hedging these dimensions have been proposed (Tzang and Leuthold 1990; Fackler and McNew 1993). In a *one-to-one hedge* (a.k.a. *equal and opposite*), each unit of cash market commitment is matched with a corresponding unit of futures market commitment. In a more general *risk-minimizing direct hedge*, each unit of cash market commitment is hedged with a risk-minimizing futures commitment in the same commodity. More general still is a *commodity-by-commodity cross hedge*, where each unit of cash market commitment is hedged with a risk-minimizing futures commitment in a different but related commodity. In a *multi-contract hedge*, each unit of cash market commitment is hedged with risk-minimizing commitments in several futures contracts. These futures contracts may differ by maturity, may specify the delivery of a different commodity (i.e., a cross-hedge), or may specify non-commodity financial instruments (currencies, securities, indices, or weather).

Other hedging strategies are defined in terms of the speculative soybean futures crush spread. In a *one-to-one crush hedge*, the processor is long one bushel in a soybean crush spread for each anticipated bushel to be processed. This strategy is identical to a one-to-one hedge if the soybean oil and the soybean meal are sold simultaneously. A generalization of the one-to-one crush hedge is the *proportional crush hedge* whereby the soybean processor employs a risk-minimizing crush spread that is proportional to the cash soybean market position.

Various studies have examined these soybean-crush-hedging strategies. Tzang and Leuthold (1990) use weekly prices from January 1983 through June 1988 to investigate multi- and single-contract soybean processing hedges over 1 through 15-week hedging horizons. Fackler and McNew (1993) use monthly prices to examine three soybean processing hedging strategies: multi-contract hedges, single-contract hedges, and proportional crush-spread hedges. The multi-contract approach has recently been extended to cross hedging in the cottonseed-processing sector (Dahlgran 2000; Rahman, Turner, and Costa 2001).

Some production hedges resemble processing hedges. These include the cattle feeding hedge using corn, feeder cattle, and live cattle futures (Leuthold and Mokler 1979; Shafer, Griffin and Johnson 1978), and the hog feeding hedge using live hog, soybean meal and corn futures (Kenyon and Clay 1987). The hedging methods in the studies mentioned thus far are of the Johnson, Anderson and Danthine type with the objective being the minimization of the variance of batch profits. Dahlgran (2004) demonstrated that when continuous processing is approximated with multiple batches, and when the traditional hedging approach is applied to each batch, annual aggregate profits are stabilized and each batch's profits are stabilized, but cash flow becomes more variable. With the exception of a study on cattle feeding done by Purcell and Rife, and Dahlgran's (2004) study of transaction frequency as a risk management strategy, cash flow hedging is largely unexplored and cash flow hedging strategies for commodity processors have yet to receive any attention.

In the next section we focus on the difference between cash flow hedging and batch profit hedging. We also derive a cash flow risk minimizing hedge ratio estimator. This estimator provides managers with a tool that can be used to manage another type of price risk. A comparative evaluation of profit hedging versus cash flow hedging will provide an understanding of the tradeoffs encountered in adopting each of these approaches.

Empirical Model

We model a process in which soybeans are purchased periodically and then gradually transformed into soybean meal and oil. The assumed transformation coefficients are 47 pounds of soybean meal and 11 pounds of soybean oil for each 60-pound bushel of soybeans processed. As soybeans are processed, soybean inventories decline and product inventories accumulate. Figure 1 represents these relationships. Input and output inventory cycles are of length θ and the cycles repeat n times over the course of a year as annual throughput of x is processed. Though figure 1 depicts inventories, these inventories are more generally considered as positions because a processor's contractual commitment to receive soybeans at some future time (a long position) is conceptually the same as having soybeans in inventory on the premises (also a long position).

Concurrent product sales are not assumed and product sales are not assumed to occur concurrently with soybean purchases. Figure 1 does not specify whether soybean meal or soybean oil is sold first in the cycle. Instead, the products are designated merely as a and b with the sale of product a occurring either before or concurrently with the sale of product b. The product sales delays after the exhaustion of input inventories are designated by δ. Thus δ_a designates the delay in the sale of product a and δ_b designates the delay in the sale of product b where $0 \leq \delta_a \leq \delta_b$. L designates the inter-cycle difference between the purchase of soybeans and the first sale of product containing those soybeans. In terms of batch profit, L (= $\theta + \delta_a$) represents the temporal separation between the pricing of soybeans and the pricing of the products produced with those soybeans. A designates an anticipatory period, which is used for planning for the next cycle.

When $\delta_a = \delta_b = 0$, the phases of the input and output inventory cycles match. Products are sold on the same day that soybeans for the next cycle are purchased. Daily cash flow variability is greatly reduced by this condition.

The prices applicable to the transactions are s_t, the per bushel price paid for soybeans purchased at time t, and $p_{a,t}$ and $p_{b,t}$ the respective prices received for products a and b sold at time t. These prices are consolidated in the column vector $\mathbf{S_t} = [\ s_t\ ,\ p_{a,t}\ ,\ p_{b,t}\ ]' = [s_t : \mathbf{P_t}\ ]'$.

The production coefficients are contained in the 3 x 1 vector $\mathbf{\Gamma}$ and are arranged to correspond to the price vector. $\mathbf{\Gamma}$ is defined as $\mathbf{\Gamma} = [\ -1\ ,\ \gamma_a\ ,\ \gamma_b\ ]'$ where γ_a and γ_b represent per bushel yields of products a and b. The production coefficients are also represented by the 2 x 1 vector $\boldsymbol{\gamma}$ where $\boldsymbol{\gamma} = [\gamma_a\ ,\ \gamma_b\ ]'$. For completeness, $\mathbf{\Gamma'} = [\ -1 : \boldsymbol{\gamma'}\ ]$.

Without hedging, profits anticipated at time t, generated by the batch that will be initiated at the end of the anticipatory period A are represented by π_t^u where

$$\pi_t^u = (x/n)\,[-s_{t+A} + \gamma_a p_{a,t+A+\theta+\delta_a} + \gamma_b p_{b,t+A+\theta+\delta_b}] \tag{1a}$$

This expression can be enhanced to separate the variables that are given at time t from those that have yet to be determined and also to designate different phases of the processing cycle. This rearrangement gives

$$\begin{aligned}\pi_t^u = (x/n)\ [\,-(s_t + \Delta_A s_{t+A}) + \gamma_a(p_{a,t} + \Delta_A p_{a,t+A} + \Delta_{\theta+\delta_a} p_{a,t+A+\theta+\delta_a}) +\\ \gamma_b(p_{b,t} + \Delta_A p_{b,t+A} + \Delta_{\theta+\delta_a} p_{b,t+A+\theta+\delta_a} + \Delta_{\delta_b-\delta_a} p_{b,t+A+\theta+\delta_b})]\end{aligned} \tag{1b}$$

where $\Delta_a X_t = X_t - X_{t-a}$. This expression indicates that the profit outcome depends on the current crushing margin ($-s_t + \gamma_a\ p_{a,t} + \gamma_b\ p_{b,t}$), the change in the crushing margin over the anticipatory period ($-\Delta_A s_{t+A} + \gamma_a\ \Delta_A p_{a,t+A} + \gamma_b\ \Delta_A p_{b,t+A}$), the change in product prices after the soybeans are purchased but before the products are sold ($\gamma_a \Delta_{\theta+\delta_a} p_{a,t+A+\theta+\delta_a} + \gamma_b \Delta_{\theta+\delta_a} p_{b,t+A+\theta+\delta_a}$), and the change in the price of product b during the period when only product b is held in inventory ($\gamma_b \Delta_{\delta_b-\delta_a} p_{b,t+A+\theta+\delta_b}$). This statement can be expressed more succinctly as

$$\pi_t^u = (x/n)\ [\,\mathbf{S'_t}\,\mathbf{\Gamma} + \mathbf{\Delta_A S'_{t+A}}\,\mathbf{\Gamma} + \mathbf{\Delta_{\theta+\delta_a} P'_{t+A+\theta+\delta_a}}\,\boldsymbol{\gamma} + \gamma_b \Delta_{\delta_b-\delta_a} p_{b,t+A+\theta+\delta_b}] \tag{1c}$$

When hedging is added, profit becomes

$$\pi_t^h = \pi_t^u + \mathbf{x'_{f,A}}\,\mathbf{\Delta_A F_{t+A}} + \mathbf{x'_{f,\theta+\delta_a}}\,\mathbf{\Delta_{\theta+\delta_a} F_{t+A+\theta+\delta_a}} + \mathbf{x'_{f,\delta_b-\delta_a}}\,\mathbf{\Delta_{\delta_b-\delta_a} F_{t+A+\theta+\delta_b}} \tag{1d}$$

where $x_{f,a}$ represents futures positions held during phase a. This formulation designates futures positions for the anticipatory period ($\mathbf{x_{f,A}}$), for the period when both products are stored ($\mathbf{x}_{\mathbf{f,A+\theta+\delta_a}}$), and for the period when only one product is stored ($\mathbf{x}_{\mathbf{f,\delta_b-\delta_a}}$). The price risks differ in each of these periods. In the anticipatory period (A), there is risk of change in input and output prices, in the transformation period ($\theta+\delta_a$) there is risk of change in both output prices and in the single product holding period (δ_b-δ_a), there is risk of change in only the single output price.

The firm's hedging objective is assumed to be price risk minimization where price risk is measured by the variance of profit. For the sake of notational convenience, let $L = \theta + \delta_a$ and $\delta = \delta_b - \delta_a$. Now

$$V(\pi_t^h \mid \Omega_t) = V(\pi_t^u \mid \Omega_t) + \mathbf{x'}_{\mathbf{f,A}} \, \mathbf{\Sigma}_{\mathbf{\Delta_A F,\Delta_A F}} \, \mathbf{x}_{\mathbf{f,A}} + \mathbf{x'}_{\mathbf{f,L}} \, \mathbf{\Sigma}_{\mathbf{\Delta_L F,\Delta_L F}} \, \mathbf{x}_{\mathbf{f,L}} + \mathbf{x'}_{\mathbf{f,\delta}} \, \mathbf{\Sigma}_{\mathbf{\Delta_\delta F,\Delta_\delta F}} \, \mathbf{x}_{\mathbf{f,\delta}}$$
$$+2\,(x/n)\,(\mathbf{x'}_{\mathbf{f,A}} \, \mathbf{\Sigma}_{\mathbf{\Delta_A F,\Delta_A S}} \, \mathbf{\Gamma} + \mathbf{x'}_{\mathbf{f,L}} \, \mathbf{\Sigma}_{\mathbf{\Delta_L F,\Delta_L P}} \, \boldsymbol{\gamma} + \mathbf{x'}_{\mathbf{f,\delta}} \, \mathbf{\Sigma}_{\mathbf{\Delta_\delta F,\Delta_\delta p_b}} \, \gamma_b) \tag{2}$$

where $\mathbf{\Sigma_{x,y}}$ represents the matrix of covariances between the variables in vector $\mathbf{x}$ and vector $\mathbf{y}$. Minimizing this variance with respect to $\mathbf{x_{f,A}}$, $\mathbf{x_{f,L}}$, and $\mathbf{x_{f,\delta}}$ gives the variance-minimizing futures positions for each of the periods, A, L, and δ.

$$\mathbf{x}^*_{\mathbf{f,A}} = -(x/n)\ (\mathbf{\Sigma}_{\mathbf{\Delta_A F,\Delta_A F}})^{-1}\, \mathbf{\Sigma}_{\mathbf{\Delta_A F,\Delta_A S}}\, \mathbf{\Gamma} \tag{3a}$$

$$\mathbf{x}^*_{\mathbf{f,L}} = -(x/n)\ (\mathbf{\Sigma}_{\mathbf{\Delta_L F,\Delta_L F}})^{-1}\, \mathbf{\Sigma}_{\mathbf{\Delta_L F,\Delta_L P}}\, \boldsymbol{\gamma} \tag{3b}$$

$$\mathbf{x}^*_{\mathbf{f,\delta}} = -(x/n)\ (\mathbf{\Sigma}_{\mathbf{\Delta_\delta F,\Delta_\delta F}})^{-1}\, \mathbf{\Sigma}_{\mathbf{\Delta_\delta F,\Delta_\delta p_b}}\, \gamma_b \tag{3c}$$

Hedge ratios are estimated as the coefficients in the regression models

$$\mathbf{\Gamma'}\ \mathbf{\Delta_A S_t} = \mathbf{\Delta_A F'_{At}}\ \boldsymbol{\beta}_\mathbf{A} + \varepsilon_{A,t} \tag{4a}$$

$$\boldsymbol{\gamma'}\ \mathbf{\Delta_L P_t} = \mathbf{\Delta_L F'_{L,t}}\ \boldsymbol{\beta}_\mathbf{L} + \varepsilon_{L,t} \tag{4b}$$

$$\gamma_b\ \Delta_\delta p_{b,t} = \mathbf{\Delta_\delta F'_{\beta,t}}\ \boldsymbol{\beta}_{\boldsymbol{\delta}} + \varepsilon_{\delta,t} \tag{4c}$$

These regression models correspond to the hedge horizons A, L and δ in figure 1. Hedge ratios for the anticipatory period are estimated by (4a). The hedging target in this period is the change in the crush margin over the anticipatory period ($\mathbf{\Gamma'\Delta_A S_t}$). Hedge ratios for the transformation period, the period between the purchase of the soybeans and the sale of the first product, are estimated by (4b). The hedge target in this period is the change in the value of both products over the transformation period ($\boldsymbol{\gamma'}\mathbf{\Delta_L P_t}$). Hedge ratios for the remaining product are estimated by (4c). The hedge target in this time period is the change in the value of the product ($\gamma_b\ \Delta_\delta p_{b,t}$). The multiplication by the transformation coefficient, γ_b, indicates that these hedge ratios are expressed per bushel of soybeans. This approach exemplifies current methods for hedging soybean processing.

We now examine anticipated cash flow and methods for hedging it. In the absence of hedging, cash flow results from soybean product sales and soybean purchases. Anticipated cash flow from a processing cycle that begins at time t is represented as

$$\phi_t^u = (x/n)\ [-s_{t+A} + \gamma_a p_{a,t+A+\delta_a} + \gamma_b p_{b,t+A+\delta_b}] \tag{5a}$$

Rearranging to isolate the current given prices from the unknown future spot prices gives

$$\phi_t^u = (x/n)\ [\mathbf{S'_t}\,\Gamma + \mathbf{\Delta_A S'_{t+A}}\,\mathbf{\Gamma} + \mathbf{\Delta_{\delta_a} P'_{t+A+\delta_a}}\,\gamma + \gamma_b \Delta_{\delta_b-\delta_a} p_{b,t+A+\delta_b}] \tag{5b}$$

This expression differs from (1c) in that changes in product prices over the processing cycle (θ) are excluded. This exclusion occurs because the embodiment of soybeans purchased in specific products is immaterial from the standpoint of cash flow.

With hedging, cash flows are generated by both spot market transactions and the daily revaluation of futures positions. These cash flows are reflected by the expression

$$\phi_t^h = \phi_t^u + \sum_{\tau=1}^{A} \mathbf{x'_{f,A}}\ \mathbf{\Delta F_{t+\tau}} + \sum_{\tau=1}^{\delta_a} \mathbf{x'_{f,\delta_a}}\ \mathbf{\Delta F_{t+A+\tau}} + \sum_{\tau=1}^{\delta_b-\delta_a} \mathbf{x'_{f,\delta}}\ \mathbf{\Delta F_{t+A+\delta_a+\tau}} \tag{5c}$$

where $\Delta F_t = F_t - F_{t-1}$. This expression differs from (1d) in that it recognizes cash flows from futures positions on each day that the position is held whereas (1d) recognizes only aggregate cash flow effect of the position at its termination. Another difference between (5c) and (1d) is that the third and fourth terms of (1d) include a transaction cycle (θ) that doesn't appear in the corresponding terms of (5c).

The variance of cash flow over the transaction cycle is

$$\begin{aligned} V(\phi_t^h \mid \Omega_t) = V(\phi_t^u \mid \Omega_t) + \sum_{\tau=1}^{A} \mathbf{x'_{f,A}}\ \mathbf{\Sigma_{\Delta F,\Delta F}}\ \mathbf{x_{f,A}} + \sum_{\tau=1}^{\delta_a} \mathbf{x'_{f,\delta_a}}\ \mathbf{\Sigma_{\Delta F,\Delta F}}\ \mathbf{x_{f,\delta_a}} + \sum_{\tau=1}^{\delta_b-\delta_a} \mathbf{x'_{f,\delta}}\ \mathbf{\Sigma_{\Delta F,\Delta F}}\ \mathbf{x_{f,\delta}} + \\ +2(x/n)\{ \sum_{\tau=1}^{A} \mathbf{x'_{f,A}}\ \mathbf{Cov(\Delta F_{t+\tau}, \Delta_A S'_{t+A})}\ \mathbf{\Gamma} + \sum_{\tau=1}^{\delta_a} \mathbf{x'_{f,\delta_a}}\ \mathbf{Cov(\Delta F_{t+A+\tau}, \Delta_{\delta_a} P'_{t+A+\delta_a})}\ \gamma \\ + \sum_{\tau=1}^{\delta_b-\delta_a} \mathbf{x'_{f,\delta}}\ \mathbf{Cov(\Delta F_{t+A+\delta_a+\tau}, \Delta_{\delta_b-\delta_a}} p_{b,t+A+\delta_b})\ \gamma_b \} \end{aligned} \tag{6}$$

This expression differs from (2) in that it recognizes the daily cash flows attributed to futures position resettlement. $V(\phi_t^u \mid \Omega_t)$ indicates risk without hedging, which depends on spot market positions and is unaffected by the selection of futures positions.

Minimizing the variance with respect to the futures positions for each of the three periods gives the normal equations

$$A\ \mathbf{\Sigma_{\Delta F,\Delta F}}\mathbf{\tilde{x}_{f,A}} + (x/n)\sum_{\tau=1}^{A} \mathrm{Cov}(\mathbf{\Delta F'_{t+\tau}}\mathbf{\Delta_A S_{t+A}})\mathbf{\Gamma} = 0 \tag{7a}$$

$$\delta_a\ \mathbf{\Sigma_{\Delta F,\Delta F}}\mathbf{\tilde{x}_{f,\delta_a}} + (x/n)\sum_{\tau=1}^{\delta_a} \mathrm{Cov}(\mathbf{\Delta F'_{t+A+\tau}}\mathbf{\Delta_{\delta_a} P_{t+A+\delta_a}})\gamma = 0 \tag{7b}$$

$$(\delta_b - \delta_a)\ \mathbf{\Sigma_{\Delta F,\Delta F}}\mathbf{\tilde{x}_{f,\delta}} + (x/n)\sum_{\tau=1}^{\delta_b-\delta_a} \mathrm{Cov}(\mathbf{\Delta F'_{t+A+\delta_a+\tau}}\Delta_\delta p_{b,t+A+\delta_b})\gamma_b = 0 \tag{7c}$$

where $\mathbf{\tilde{x}_{f,A}}, \mathbf{\tilde{x}_{f,\delta_b}},$ and $\mathbf{\tilde{x}_{f,\delta}}$ indicate the cash flow risk minimizing futures positions during each segment (A, δ_a and $\delta=\delta_b-\delta_a$) of the cash flow cycle.

The covariance terms in (7a) through (7d) can be further simplified. An example of this simplification is provided by (7a) where

$$\begin{aligned} \sum_{\tau=1}^{A} \mathrm{Cov}(\mathbf{\Delta F'_{t+\tau}}\ \mathbf{\Delta_A S_{t+A}})\mathbf{\Gamma} = \{\mathrm{Cov}[(\mathbf{F_{t+1}} - \mathbf{F_t})'\mathbf{\Delta_A S_{t+A}}] + \mathrm{Cov}[(\mathbf{F_{t+2}} - \mathbf{F_{t+1}})'\mathbf{\Delta_A S_{t+A}}] + \\ \mathrm{Cov}[(\mathbf{F_{t+3}} - \mathbf{F_{t+2}})'\mathbf{\Delta_A S_{t+A}}] + \dots + \mathrm{Cov}[(\mathbf{F_{t+A}} - \mathbf{F_{t+A-1}})'\mathbf{\Delta_A S_{t+A}}]\}\mathbf{\Gamma} \end{aligned} \tag{8a}$$

so $$\sum_{\tau=1}^{A} \mathrm{Cov}(\mathbf{\Delta F'}_{t+\tau}\, \mathbf{\Delta_A S_{t+A}})\, \mathbf{\Gamma} = \mathrm{Cov}[(\mathbf{F_{t+A}} - \mathbf{F_t})'\mathbf{\Delta_A S_{t+A}}]\, \mathbf{\Gamma} = \mathrm{Cov}(\mathbf{\Delta_A F'}_{t+A}\, \mathbf{\Delta_A S_{t+A}})\, \mathbf{\Gamma} \tag{8b}$$

Thus, the cash flow risk minimizing futures positions for each of the hedging periods is

$$\tilde{\mathbf{x}}_{\mathbf{f,A}} = -(x/n)\,(A\, \mathbf{\Sigma}_{\mathbf{\Delta F,\Delta F}})^{-1} \mathbf{\Sigma}_{\mathbf{\Delta_A F_{t+A}, \Delta_A S_{t+A}}} \mathbf{\Gamma} \tag{9a}$$

$$\tilde{\mathbf{x}}_{\mathbf{f,\delta_a}} = -(x/n)(\boldsymbol{\delta}_{\mathbf{a}}\, \mathbf{\Sigma}_{\mathbf{\Delta F,\Delta F}})^{-1} \mathbf{\Sigma}_{\mathbf{\Delta_{\delta_a} F_{t+A+\delta_a} \Delta_{\delta_a} P_{t+A+\delta_a}}} \boldsymbol{\gamma} \tag{9b}$$

$$\tilde{\mathbf{x}}_{\mathbf{f,\delta}} = -(x/n)(\delta\, \mathbf{\Sigma}_{\mathbf{\Delta F,\Delta F}})^{-1} \mathbf{\Sigma}_{\mathbf{\Delta_{\delta_b - \delta_a} F_{t+A+\delta_b} \Delta_\delta p_{b,t+A+\delta_b}}} \gamma_b \tag{9c}$$

These expressions are similar to (3a) through (3c), respectively, except that the futures-price-change variance-covariance matrix ($\mathbf{\Sigma_{\Delta F,\Delta F}}$) is for day to day changes rather than for changes over the hedging interval applicable to (3a) through (3c).

The hedge ratios in (9a) through (9c) can be determined from estimated moment matrices. However, for the sake of comparison, we seek to determine how regression analysis can be used to estimate these hedge ratios. Because of the similarity of the three equations, we can focus on (9a) knowing that the analysis of (9b) and (9c) will proceed similarly. To simplify, assume that the anticipatory period (A) is one transaction cycle. To incorporate standard regression notation, let $\mathbf{X = \Delta F}$ where $\mathbf{X}$ is NA x k with A representing the transaction cycle length, N representing the number of transaction cycles in the data set, and k the number of futures contracts considered for hedge vehicles. Let $\mathbf{Z = \Delta_A F}$ where $\mathbf{Z}$ is N x k and let $\mathbf{Y = \Delta_A S}$ where $\mathbf{Y}$ is N x 3. Then $\hat{\mathbf{\Sigma}}_{\mathbf{\Delta F,\Delta F}} = \mathbf{X'X}$ / NA, $\hat{\mathbf{\Sigma}}_{\mathbf{\Delta_A F, \Delta_A S}} = \mathbf{Z'Y}$ / N and

$$\hat{\tilde{\mathbf{x}}}_{\mathbf{f,A}} = -(x/n)\frac{1}{A}\left(\frac{\mathbf{X'X}}{NA}\right)^{-1}\left(\frac{\mathbf{Z'Y}}{N}\right)\mathbf{\Gamma} = -(x/n)\left(\mathbf{X'X}\right)^{-1} \mathbf{Z'Y\,\Gamma} \tag{10a}$$

But $\mathbf{Z = (I_N \otimes 1'_A)\, \Delta F = (I_N \otimes 1'_A)\, X}$ and $\mathbf{Y = (I_N \otimes 1'_A)\, \Delta S}$. Thus $\mathbf{Z'Y = X'\,(I_N \otimes 1'\,1)\, \Delta S}$ so

$$\hat{\tilde{\mathbf{x}}}_{\mathbf{f,A}} = (x/n)\left(\mathbf{X'X}\right)^{-1}\mathbf{X'(I_N \otimes 1 1')\,\Delta S\,\Gamma} \tag{10b}$$

Thus, the regression model that estimates cash flow risk minimizing hedge ratios is

$$\mathbf{(I_N \otimes 1\ 1')\ \Delta S\ \Gamma = \Delta F}\ \boldsymbol{\beta}_{\mathbf{A}} + \boldsymbol{\varepsilon} \tag{11a}$$

The explanatory variables in this formulation are daily futures price changes. The dependent variable is the change in the crushing margin, $\mathbf{\Delta S\ \Gamma}$, summed over the hedging period, $\mathbf{(I_N \otimes 1')\ \Delta S\ \Gamma}$, with each observation in a hedging period being the sum of the daily changes over the hedging period, $\mathbf{(I_N \otimes 1)(I_N \otimes 1')\ \Delta S\ \Gamma}$. We recognize at the outset that this formulation implies that cash flow risk reduction will be difficult to achieve through hedging.

Data Considerations

Market data to empirically test the analytical model in the previous section were obtained from BarChart.com. These data consist of daily observations of cash and futures prices for soybeans, soybean oil and soybean meal. The cash prices all apply to central Illinois. The data set also contains daily futures prices for all soybean, soybean oil, and soybean meal maturities traded on the Chicago Board of Trade between January 1990 and December 2004.

Product characteristics for both the cash commodity and futures contracts changed during the sample period. Early in the sample period, the soybean meal quality standard was 44 percent protein but this changed to 48 percent by the end of the period. The quality change took place in the futures market when the deliverable grade of soybean meal was changed from 44 percent to 48 percent beginning with the September 1992 contract. Comparison of cash prices in the data set to soybean meal prices published in the Wall Street Journal reveals that cash prices were for 44 percent protein soybean meal through November 17, 1992 but were for 48 percent protein thereafter. During a transition period from November 18, 1992 through December 26, 2001 the Wall Street Journal reported prices for both 44 percent and 48 percent protein soybean meal.

44 percent soybean meal prices were converted to the new 48 percent standard by the following procedure. Cash prices for both 44 percent and 48 percent soybean meal were collected for each Wednesday during the period when both prices were quoted. The relationship between the 44-percent and 48-percent soybean meal prices estimated with ordinary least squares is

$$\underset{}{S_{M48,t}} = \underset{(0.476)}{5.96} + \underset{(0.00257)}{1.0221}\, S_{M44,t} \qquad \text{Observations} = 476,\ R^2 = 0.997$$

where $S_{M48,t}$ is the 48 percent soybean meal cash price in period t, $S_{M44,t}$ is the 44 percent soybean meal cash price in period t, and standard errors are in parentheses. This relationship was then used to generate fitted values for 48-percent soybean meal cash prices prior to November 18, 1992 and to generate fitted values for 48-percent soybean meal futures prices that matured prior to September 1992. The fitted values were then used as proxies for the unobservable 48-percent cash and futures prices. The high regression R^2 assures that these fitted values are good proxies for the unobservable prices.

The model ignores calendar issues that constrain our analysis. Specifically, the model assumes that the transaction cycle parameters (A, θ, δ_a, and δ_b) can take any integer value but the empirical analysis must accommodate the idiosyncrasies of the business calendar. Specifically, our daily observations permit profit and cash flow computations for frequencies of one day or longer. However, transaction cycles of two days through one week will clash with the market's weekend closures making the cycle length ambiguous. For example, if $\theta+\delta_a$ is 3 calendar days, then prices will be unavailable on weekends, and if $\theta+\delta_a$ is set at three business days, then the observations become unevenly spaced in time because of weekends. Weekly or multi-weekly cycles will generally be consistent with the market cycle except when holidays fall on the observation day. Our approach will be to use daily, weekly, and multi-weekly cycles. Weekly and multi-week cycles will be assumed to start on Wednesdays. When a holiday falls on Wednesday, we will use Tuesday's prices but assume that those prices were observed on Wednesday to preserve evenly time-spaced observations.

The purpose of the empirical analysis is to determine the effectiveness of cash flow hedging and to compare the effectiveness of cash flow hedging to batch profit hedging. We make these observations over a range of hedge horizons that correspond to those studied by Tzang and Leuthold, and Fackler and McNew. For batch profit hedging, we will examine anticipatory periods (A) of 1, 7, 14, 21, 28, 42, 56 and 91 days, transformation periods ($L=\theta+\delta_a$) of 1, 7, 14, 21, and 28 days, and product sales timing differences (δ_b-δ_a) of 0, 1, 7, and 14 days. For comparison, we will examine cash flow hedging effectiveness for anticipatory periods (A) of 0, 1, 7, 14, 21, 28, 42, 56 and 91 days, input-purchase output-sales time lags (δ_a) of 0, 1, 7, 14, 21, and 28 days, and product sales time differences (δ_b-δ_a) of 0, 1, 7, and 14 days. These values correspond to processing and hedging strategies that fit into the business calendar.

The futures contracts used for hedging were selected according to the following rules. First, only contracts that permit the construction of a pure crushing spread, where the soybean, soybean oil, and soybean meal contracts have the same maturity, are used. This eliminates October and December soybean oil and soybean meal contracts because soybean contracts are not traded for these months. Likewise, November soybeans are eliminated because November soybean oil and soybean meal are not traded. This leaves the January, March, May, July, and September soybean, soybean oil and soybean meal contracts to be used in the hedging portfolio. Second, only contracts with at least seven days to maturity at the time of hedge closure were used.

With these two broad exclusions in place, the nearby futures maturity at the time of the final cash market transaction was used for our simulated hedges. Hedging in all three contracts, soybeans, oil and meal, is always allowed. To eliminate crush spreading with intertemporal features, the nearby maturity is defined relative to the last product sold. This means, for example, that if a batch of soybean meal is sold two weeks after the soybean oil and if soybeans were purchased for the batch six weeks before the soybean oil is sold, then the nearby maturity for all three futures contracts is defined relative to the soybean meal sale.

Empirical Results

Batch profits and cash flows for each cycle specified by the values of A, θ, δ_a and δ_b were computed. Regression models (4a) through (4c) were used to determine the batch-profit risk-minimizing hedge ratios and hedged batch profits are computed using these hedge ratios. Likewise, regression models (9a) through (9c) are used to determine the cash flow risk minimizing hedge ratios and hedged cash flows are computed using these hedge ratios. Table 1 reports the hedged and unhedged variances, and the effectiveness for various transaction cycles. The results are grouped to show the effect of increasing each parameter.

The crushing margin, $\mathbf{\Gamma' \Delta_A S_t}$, is the hedging objective in the anticipatory period. The batch profit hedging effectiveness during the anticipatory period is generally in the 0.2 to 0.3 range but appears to be sensitive to the hedge horizon. Specifically, table 1 indicates relatively low effectiveness estimates of 0.075 and 0.082 for seven-day and 28 day horizons, respectively, but other effectiveness estimates exceed these values. Longer hedge horizons appear to offer somewhat greater hedging effectiveness. Batch profit hedging effectiveness is significant at beyond the five-percent level for all anticipatory hedging horizons.

For a one-day anticipatory period, the cash flow hedging effectiveness is roughly equal to the batch profit hedging effectiveness. A comparison of the analytical models (3a) and (11a) with A=1 confirms that they should in fact be equal. The effectiveness estimates differ because of the additional observation available for the computation of cash flow. Beyond a one-day anticipatory period, the effectiveness of cash flow hedging diminishes to the point that hedging provides little cash flow stabilization. Even though the cash flow hedging effectiveness estimates for the anticipatory periods are small, they are statistically significant at beyond the five-percent level for all but the seven-day anticipatory horizon.

The revenue from the sale of both products, $\boldsymbol{\gamma}'\boldsymbol{\Delta}_L\mathbf{P}_t$, is the hedging objective in the transformation period. The effectiveness of hedging product revenues over the transformation period of roughly 0.80 exceeds the effectiveness of hedging the crushing margin in the anticipatory period. These effectiveness estimates are all highly significant. Cash flow hedging effectiveness estimates for the transformation period are not reported because cash flow risk is independent of the transformation period and instead depends on the timing difference between output sales from the current cycle and input purchases for the next cycle. These timing differences are captured by the model parameters δ_a and δ_b and the effect of these two parameters is reported in the final two sections of table 1. Because δ_a and δ_b must both be set to zero in order to capture the transaction cycle effect (θ), the corresponding cash flow hedging effectiveness over the transformation period has no meaning and is therefore not reported.

The hedging objective in the single-product-holding period is the sales revenue from that product $(\gamma_b \Delta_{\delta_b - \delta_a} p_{b,t})$. We have thus far avoided assigning specific products to the designations a and b. The penultimate section of table 1 shows the effectiveness of hedging batch profits and cash flows while holding only soybean meal inventories. The last section of table 1 shows the effectiveness of hedging batch profits and cash flows while holding only soybean oil inventories. These sections indicate generally that the longer the hedging horizons, the more effective batch profit hedges become while cash flow hedges become less effective. The effective estimates are all statistically significant at beyond the five-percent level. The results also indicate that hedging soybean oil inventories is more effective than hedging soybean meal inventories and that hedging batch profits is more effective than hedging cash flows. When δ_b - δ_a = 1 cash flow and batch profit hedges are equally effective and a comparison of (3c) and (9c) indicates that this should be the case.

Summary and Conclusions

The objectives of this study were to determine how to hedge cash flows from commodity processing, to compare the analytical solution for cash flow hedging to the traditional solution for batch profits hedging, and to obtain some empirical estimates of the effectiveness of cash flow hedging. In accomplishing these objectives, we first developed the empirical method for determining cash flow variability and for estimating cash-flow-risk-minimizing hedge ratios. The analytical solution for computing these ratios is similar to the method for computing traditional profit-risk-minimizing hedge ratios. The primary difference is that cash-flow-risk-minimizing hedge ratios balance the risk of cash flow destabilizing spot price changes against the cash flow destabilizing effects of futures price changes. The multiple regression result is that the cash-flow-risk minimizing hedge ratios are found by multiplying the inverse of the covariance matrix of daily futures price changes by the covariance matrix between spot and futures price changes over the hedging interval. Regression models that accommodate these unequal differencing intervals are presented.

In the empirical analysis cash-flow-risk-minimizing hedge ratios and profit-risk-minimizing hedge ratios were computed and used to compute profit and cash flow outcomes over the sample period. Hedging effectiveness was used to compare hedged and unhedged processing. Cash flow hedging resulted in a statistically significant reduction in cash flow variation but this reduction is proportionately less than the reduction batch profit variation that is afforded by hedging batch profits.

Much work remains to be done in this line of research. Questions that remain unanswered are how do the cash-flow-risk-minimizing hedge ratios perform out of sample? The estimation period of 1990 through 1999 leaves an out-of-sample period of 2000 through 2004 in which to address this issue. Second, we wish to use out-of-sample data to examine how the application of cash flow hedging impacts batch profits and how profit-risk hedging impacts cash flow.

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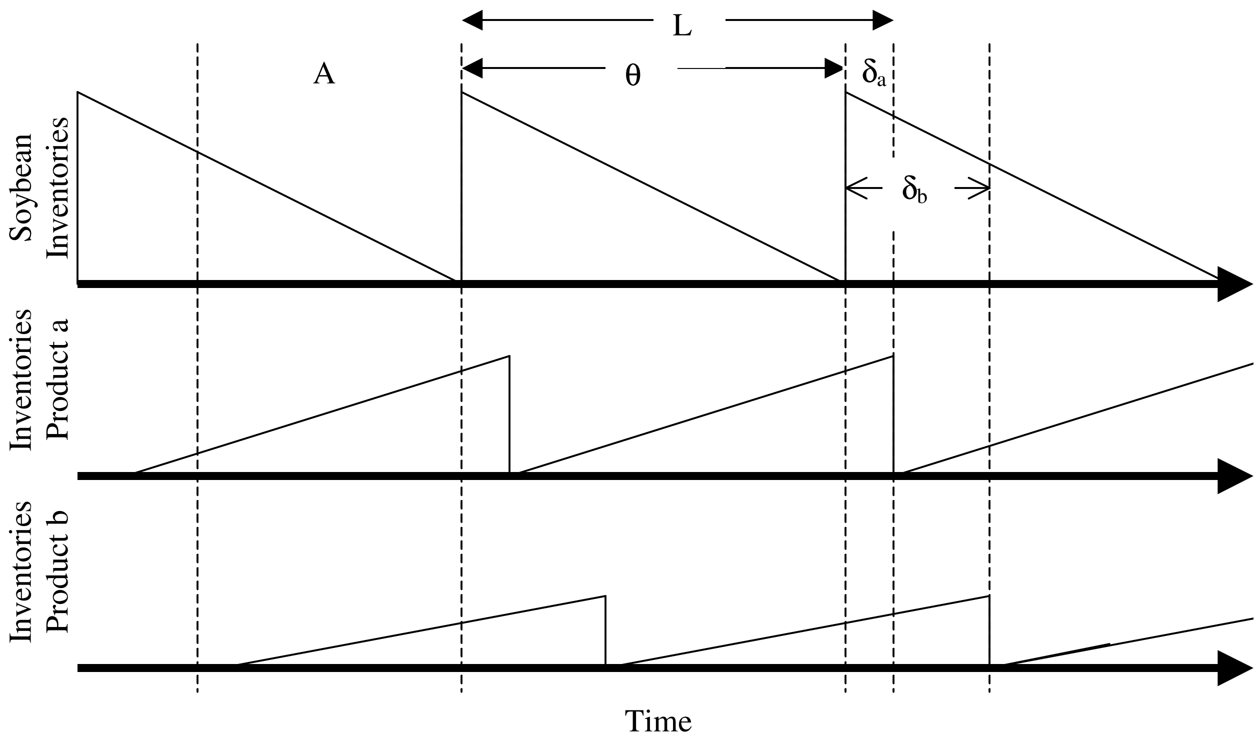

Figure 1. Inventory Levels Over the Processing/Transaction Cycle.

Table 1. In Sample (1990-1999) Hedge Ratio Estimation Results.

Transaction cycle	Batch Profit Hedges				Cash Flow Hedges			
$(A, \theta, \delta_m, \delta_o)$	N	V_u	V_h	Eff[a]	N	V_u	V_h	Eff[a]
Anticipatory Period Effect								
(1, 28, 0, 0)	126	23.82	19.01	0.221****	127	23.66	18.89	0.220****
(7, 28, 0, 0)	128	84.43	79.95	0.075**	639	84.56	84.06	0.011*
(14, 28, 0, 0)	127	108.23	91.62	0.173****	1269	108.31	106.42	0.020****
(21, 28, 0, 0)	127	210.76	170.73	0.209****	1904	210.86	208.26	0.014****
(28, 28, 0, 0)	128	296.59	278.71	0.082**	2559	296.70	296.09	0.003**
(42, 28, 0, 0)	127	413.94	326.20	0.231****	3776	415.62	413.62	0.006****
(56, 28, 0, 0)	128	429.18	337.70	0.232****	5036	430.56	429.15	0.004****
(91, 28, 0, 0)	128	957.02	704.91	0.281****	8128	961.68	958.90	0.003****
Transaction Cycle Effect								
(1, 1, 0, 0)	1979	69.92	16.49	0.765****				
(7, 7, 0, 0)	512	350.10	64.03	0.818****				
(14, 14, 0, 0)	255	637.04	128.95	0.800****				
(28, 28, 0, 0)	128	1182.24	216.99	0.821****				
Soymeal Inventory Holding Effect								
(28, 28, 1, 0)	123	31.30	9.18	0.714****	123	31.30	9.20	0.714****
(28, 28, 7, 0)	128	226.94	56.18	0.758****	640	226.94	187.91	0.176****
(28, 28, 14, 0)	127	433.42	109.74	0.753****	1270	433.42	403.64	0.071****
Soyoil Inventory Holding Effect								
(28, 28, 0, 1)	123	8.52	0.85	0.903****	123	8.52	0.85	0.903****
(28, 28, 0, 7)	128	45.21	4.28	0.907****	640	45.21	35.42	0.220****
(28, 28, 0, 14)	127	91.31	6.11	0.935****	1270	91.31	82.83	0.095****

Notes: Asterisks used to denote significance levels. * means $0.10 \geq Pr(>F) > 0.05$, ** means $0.05 \geq Pr(>F) > 0.01$, *** means $0.01 \geq Pr(>F) > 0.001$, and **** means $0.001 \geq Pr(>F)$.