AGGREGATION ISSUES IN THE ESTIMATION OF MALMQUIST PRODUCTIVITY MEASURES

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ABSTRACT

The paper contributes by demonstrating the sensitivity of nonparametric programming productivity measures to the choice of model –time series versus panel models of Malmquist productivity, and to various levels of commodity aggregation compared to the traditional Tornqvist-Theil index approach employing U.S. state-level data from 1960-96. To illustrate the sensitive of nonparametric programming productivity measures, we compare the implicit shadow shares recovered from the dual values of the Malmquist productivity and total factor productivity methods to the observed shares of the Tornqvist-Theil index for U.S level data from 1948-1994.

JEL classification: O3, C6, Q1

Keywords: Tornqvist-Theil Index, Time series, and Panel models, Malmquist productivity and Malmquist total factor productivity programming, Share-weights.

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Since 1990s[1], the nonparametric programming approach has gained popularity due to its ability to impose little a prior functional form, can handle multiple output-input without the need of price data, and accommodate weak and strong disposability assumptions. However, nonparametric programming approach due to its piecewise linear approximation of the technology or theoretical frontier is conditioned by the level of commodity aggregation in the primal framework. These aggregation issues have been addressed in the literature (Blackorby, and Russell, 1999; Färe and Zelenyuk, 2003; and Simar, and Zelenyuk, 2003) with the use of dual input, output and netput prices. Theoretically, the use of dual price information would allow the aggregation of individual firms' to industry, and the aggregation of commodities or inputs to aggregate output or aggregate input. However, the aggregation issue in the primal framework without the explicit or implicit use of dual price is challenging.

This paper demonstrates the sensitivity of nonparametric programming productivity measures due to commodity and input aggregation in the primal framework. Specifically, we compare the productivity measures estimated by Malmquist productivity and Malmquist total factor productivity programming approach using time series and panel data[2] for various levels of

[1] Google scholar search of Malmquist productivity index resulted in 3,990 articles, with the addition of data envelopment analysis (DEA) reduced the number of articles to 2,550, with the addition of aggregation the number of articles were reduced to 1,260, with the addition of dual prices the articles reduced to 91 and finally with the additional primal framework there were no articles.

[2] The programming Malmquist productivity measures can be estimated for a single firm using time series data (identified with technical change), multiple firms using cross-sectional data (identified with technical efficiency), and multiple firms over time using panel data (identified as a product of technical change and technical efficiency).

commodity aggregation (single and multiple[3] technologies) employing U.S. state-level data from 1960-96.

Output and input based Malmquist productivity or Malmquist total factor productivity (Diewert, 1992 pp. 240 referred to it as Hick-Moorsteen approach) method is employed in the estimation of total factor productivity (TFP) measures. The input and or output based Malmquist productivity measures can be estimated employing the concept of input (scalar decrease in inputs for an output vector) or output (scalar increase in outputs for an input vector) distance function – for discussions see Fare, Grosskopf and Lovell. The Malmquist total factor productivity measure can be calculated as the ratio of Malmquist output index (scalar increase in outputs) over Malmquist input index (scalar decrease in inputs) – for discussion see Caves, Christensen and Diewert (1982a).

To illustrate the sensitivity of nonparametric programming productivity measures due to aggregation of commodities and inputs, we take a closer look at the share-weights[4] for a time-series model. Specifically, we compare the endogenous share-weights recovered from the dual values of the nonparametric linear programming constraints of the output/input Malmquist productivity and Malmquist total factor productivity programming method for various levels of commodity aggregation to the exogenous share-weights of the Tornqvist-Theil index employing U.S aggregate data from 1948-1994.

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[3] The importance and limitation associated with multiple outputs multiple input technology in the primal framework has been the subject of researchers. With the availability of price data, dual framework was preferred over primal technology as it reveals more information. These primal and dual technologies form the underlying assumptions of the index (Tornqvist index), non-parametric (data envelopment analysis) and parametric methods of productivity measures.

[4] Other relative issues –slack and disposability are important but beyond the scope of the paper and further we will not be dealing with non-marketable goods or assume weak disposability in estimating productivity measures.

NONPARAMETRIC PROGRAMMING APPRAOCH

For nonparametric programming approach, technology that transforms input vector $x_t = \left(x_{1t}, x_{2t}, ..., x_{it}\right)$ into output vector $y_t = \left(y_{1t}, y_{2t}, ..., y_{jt}\right)$ for state $k = 1, 2, ..., K(48)$ over time $t = 1(1960), 2, ..., T(2004)$ satisfying constant returns to scale can be represented by the output set as:

(1) $P\left(x_t^k\right) = \left\{ y_t^k : x_t^k \text{ can produce } y_t^k \right\}$

or input set as:

(2) $L\left(y_t^k\right) = \left\{ x_t^k : y_t^k \text{ is produced by } x_t^k \right\}$

and follow the properties described by Fare, including strong disposability of outputs and inputs, and constant returns to scale.

In a given year, t the concept of output set can be represented by output distance function for k firms as:

(3) $OD_t\left(x_t^k, y_t^k\right)^{-1} = \max\ \theta: \quad \theta\, y_t^k \in P\left(y_t^k\right)$

or input distance function for k firms as:

(4) $ID_t^k(y_t^k, x_t^k)^{-1} = \min\ \lambda: \quad \lambda\, x_t^k \in L(y_t^k)$

PANEL MALMQUIST PRODUCTIVITY

In a panel data series of observations on a multiple units (such as 48 states in the U.S), output based Malmquist productivity $\left(OMP_{t-1}^t\right)$ is defined as the geometric mean of four output distance functions based on current (t) and previous $(t-1)$ period technologies for k firms as:

(5) $OMP_{t-1}^t = \sqrt{\dfrac{OD^{t-1}(x_t^k, y_t^k)}{OD^{t-1}(x_{t-1}^k, y_{t-1}^k)} \dfrac{OD^t(x_t^k, y_t^k)}{OD^t(x_{t-1}^k, y_{t-1}^k)}}$

or input based Malmquist productivity $\left(IMP_{t-1}^{t}\right)$ as:

$$(6) \qquad IMP_{t-1}^{t} = \sqrt{\frac{ID^{t}(y_t^k, x_t^k)}{ID^{t}(y_{t-1}^k, x_{t-1}^k)} \; \frac{ID^{t-1}(y_t^k, x_t^k)}{ID^{t-1}(y_{t-1}^k, x_{t-1}^k)}}$$

Under constant return to scale technology, productivity improvements will result in values of greater than one while values less than one signify productivity declines.

For a given year, t the OMP_{t-1}^{t} [5] defined in equation (5) requires the estimation of two same-period (7a and 7b) distance functions:

$$(7a) \qquad OD^{t}\left(x_t^k, y_t^k\right)^{-1} = \max\left\{\theta: \; \theta\, y_t^k \in P\left(x_t^k\right)\right\}$$

$$(7b) \qquad OD^{t-1}\left(x_{t-1}^k, y_{t-1}^k\right)^{-1} = \max\left\{\theta: \; \theta\, y_{t-1}^k \in P\left(x_t^k\right)\right\}$$

and two mixed-period (7c and 76d) distance functions:

$$(7c) \qquad OD^{t}\left(x_{t-1}^k, y_{t-1}^k\right)^{-1} = \max\left\{\theta: \; \theta\, y_{t-1}^k \in P\left(x_t^k\right)\right\}$$

$$(7d) \qquad OD^{t-1}\left(x_t^k, y_t^k\right)^{-1} = \max\left\{\theta: \; \theta\, y_t^k \in P\left(x_{t-1}^k\right)\right\}$$

The same-period output based distance functions may be calculated as the solution to the linear programming problem

$$(8a) \quad OD^{t}\left(x_t^k, y_t^k\right)^{-1} = \max_{\theta, z} \; \theta \qquad s.t. \quad \theta\, y_{j,t}^k \le \sum_{k=1}^{K=48} z^k y_{j,t}^k, \quad \sum_{k=1}^{K=48} z^k x_{i,t}^k \le x_{i,t}^k, \quad z^k \ge 0$$

$$(8b) \quad OD^{t-1}\left(x_{t-1}^k, y_{t-1}^k\right)^{-1} = \max_{\theta, z} \; \theta \qquad s.t. \quad \theta\, y_{j,t-1}^k \le \sum_{k=1}^{K=48} z^k y_{j,t-1}^k, \quad \sum_{k=1}^{K=48} z^k x_{i,t-1}^k \le x_{i,t-1}^k, \quad z^k \ge 0$$

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[5] Similarly input Malmquist productivity measures can also be estimated, but under the assumption of constant returns to scale assumption the input Malmquist productivity measures are identical to output Malmquist productivity measures.

where the z's being the intensity variables with $z \geq 0$ identifying the constant return to scale boundaries of the reference set.

The mixed-period output based distance functions may be calculated as the solution to the linear programming problem

$$(8c)\ OD^{t}\left(x_{t-1}^{k}, y_{t-1}^{k}\right)^{-1} = \max_{\theta, z}\ \theta \qquad s.t.\quad \theta\, y_{j,t-1}^{k} \leq \sum_{k=1}^{K=48} z^{k} y_{j,t}^{k},\quad \sum_{k=1}^{K=48} z^{k} x_{i,t}^{k} \leq x_{i,t-1}^{k},\quad z^{k} \geq 0$$

$$(8d)\ OD^{t-1}\left(x_{t}^{k}, y_{t}^{k}\right)^{-1} = \max_{\theta, z}\ \theta \qquad s.t.\quad \theta\, y_{j,t}^{k} \leq \sum_{k=1}^{K=48} z^{k} y_{j,t-1}^{k},\quad \sum_{k=1}^{K=48} z^{k} x_{i,t-1}^{k} \leq x_{i,t}^{k},\quad z^{k} \geq 0$$

TIME SERIES MALMQUIST PRODUCTIVITY

In a time series of observations on a single economic unit (such as the state of North Dakota), a Malmquist output-based measure of productivity (OMP^{T}) in year *t* relative to the final year *T* can be represented as follows. Consider the multiple of year *t* output that is revealed to be possible relative to the set of all observations including year *T*, using the year *t* bundle of inputs. If outputs could be doubled (the multiple is 2.0), then the productivity at time *t* is the inverse of this multiple, or 0.5. This concept can be represented by an output or input distance function evaluated for any year *t* using reference production possibilities set *T* as:

$$(9a)\ OD^{T}\left(x_{t}, y_{t}\right)^{-1} = \max_{\theta, z}\ \theta \qquad s.t.\quad \theta\, y_{j,t} \leq zY_{j},\quad zX_{i} \leq x_{i,t},\quad z \geq 0$$

$$(9b)\ ID^{T}\left(y_{t}, x_{t}\right)^{-1} = \min_{\lambda, z}\ \lambda \qquad s.t.\quad y_{j,t} \leq zY_{j},\quad \lambda\, x_{i,t} \geq zX_{i},\quad z \geq 0$$

where $Y_{j} = \left(y_{j}^{1}, y_{j}^{1}, \ldots\ldots\ldots, y_{j}^{T}\right)$ *and* $X = \left(x_{i}^{1}, x_{i}^{2}, \ldots\ldots\ldots, x_{i}^{T}\right)$

The OMP^T measure for a single economic unit, between two time-periods *t* and *t+1*, given technology ***T***, is defined as:

$$(10) \qquad OMP^T = \sqrt{\frac{OD^T(x_t, y_t)}{OD^T(x_{t-1}, y_{t-1})}\frac{OD^T(x_t, y_t)}{OD^T(x_{t-1}, y_{t-1})}} \equiv \frac{OD^T(x_t, y_t)}{OD^T(x_{t-1}, y_{t-1})}$$

and input based Malmquist productivity IMP^T measure for a single economic unit, between two time-periods *t* and *t+1*, given technology ***T***, is defined as:

$$(11) \qquad IMP^T = \sqrt{\frac{ID^T(y_t, x_t)}{ID^T(y_{t-1}, x_{t-1})}\frac{ID^T(y_t, x_t)}{ID^T(y_{t-1}, x_{t-1})}} \equiv \frac{ID^T(y_t, x_t)}{ID^T(y_{t-1}, x_{t-1})}$$

TIME SERIES MALMQUIST TOTAL FACTOR PRODUCTIVITY

An alternative to the time-series output or input based Malmquist productivity, Malmquist total factor productivity $(MTFP^T)$, is defined as the ratio of Malmquist output index (MO) and Malmquist input index (MI). The Malmquist output index measure the scalar change in outputs assuming the inputs are given and constant over time. Similarly the Malmquist input index measures the scalar decrease in inputs assuming the outputs are given and constant over time. This concept of MO and MI can be represented by the modifying equation (9a and 9b) output and input distance functions evaluated for any year *t* for a single firm employing a reference production possibility set *T*

$$(12a) \quad OD^T\left(y_t\right)^{-1} = \max_{\theta, z} \ \theta \qquad \text{s.t.} \quad \theta y_{j,t} \leq zY_j, \quad z \geq 0$$

$$(12b) \quad ID^T\left(x_t\right)^{-1} = \min_{\lambda, z} \ \lambda \qquad \text{s.t.} \quad \lambda x_{i,t} \geq zX_i, \quad z \geq 0$$

where $Y_j = \left(y_j^1, y_j^1, ..., y_j^T\right)$ *and* $X = \left(x_i^1, x_i^2, ..., x_i^T\right)$

where the intensity variables $z \geq 0$ identifies the constant return to scale boundaries of the reference set.

The Malmquist total factor productivity for a single economic unit maintaining the index productivity notion is represented as:

$$(13) \qquad MTFP^{T} \equiv \frac{MO}{MI} = \frac{OD^{T}\left(y_{t}\right)^{-1}}{ID^{T}\left(x_{t}\right)^{-1}}$$

To illustrate the sensitivity of nonparametric program approach to the level of commodity aggregation, we examine the share-weights, recovered from the dual values implicit in the linear programming constraints. In the programming approach the share-weights are recovered from the dual values (τ) of the output (input) constraints of the OMP^{T} (IMP^{T}) in equation 9a (equation 9b) as well as the dual values recovered from the output (input) constraints in equation 12a of MO (equation 12b of MI) of $MTFP^{T}$.

The dual values of the linear programming input (equations 9b and 12b) and output (equations 9a and 12a) constraints are normalized to one, and are equivalent to the shares-weights. Following Shaik et al the nonparametric implicit output and input share-weights in terms of the dual values are represented as:

$$(14) \qquad RS_{j} \simeq \frac{\tau_{j}}{\sum_{j}\tau_{j}}$$

and

$$(15) \qquad CS_{i} \simeq \frac{\tau_{i}}{\sum_{i}\tau_{i}}$$

where RS_j and CS_i are the implicit output and input share-weights recovered from the linear programming constraint and τ are the dual values obtained from the output and input linear programming constraints.

U.S. AGRICULTURE DATA

Economic Research Service of U.S. Department of Agriculture complies and publishes annual indexes of output, input use, and total factor productivity for the aggregate U.S. farm sector and for the individual states utilizing the Tornqvist-Theil index. The U.S. state level data is available for the period 1960 to 1996. For each state, quantity indexes of total output, crop production, livestock, and indices of total input, capital, land, labor, and intermediate inputs are available. These quantity indexes are constructed as weighted sums of the rates of growth of the components, where the weights are the respective value (output or input) shares. As such, the indexes measure the annual rates of change in the output or input aggregate.

At the U.S. level, ERS publishes output and input quantity indexes and implicit prices data for the period 1948-1994 at a much disaggregate level. Hence we collapse the disaggregate variables to the variables available at the state-level. Specifically, we aggregated the available durable equipment, farm real estate and inventories into capital[6] and farm real estate leading to four inputs capital, land, labor and intermediate inputs.

The state wise annual growth rate of the variables' employed in the estimation of productivity for the period 1960-1996 is presented in Table 1. Annual growth rate is defined as

[6] Capital quantity index is computed as the share weight rate of change in durable equipment and inventories.

$\left[(X_{t+1}/X_t)^{1/T} - 1\right]*100$ where X is input or output variable and T is the time period. Within outputs, the average annual growth rate across all the states for crops is 1.568 followed by livestock with 1.233. Here the average annual productivity growth rate represents a simple arithmetic mean of the annual productivity growth rate across all states. In the input category, capital (-0.003), land (-0.943) and labor (-2.266) had a negative average annual growth rate across all the states compared to positive average annual growth rate of material inputs (0.735). The aggregate output, aggregate inputs and productivity indicated an average annual growth rate of 1.506, –0.402, and 1.916 respectively for U.S. agriculture sector over the time period 1960-96. However the productivity computed based on the average annual growth rate of output and input leads to average annual productivity growth rate of 1.908 indicating the averaging of state-wise annual productivity growth rates provide a true measure than the ratio of the average annual growth rate of output and input.

EMPIRICAL APPLICATION AND RESULTS

To illustrate the sensitivity of nonparametric programming to the level of aggregation, equation 5 (panel model) and equation 10 (time series model) is estimated for various levels of commodity and input aggregations using U.S. state-level data from 1960-1996 by SHAZAM. These nonparametric programming output based Malmquist productivity measures are compared to the Tornqvist-Theil index productivity measures.

The state-wise annual productivity growth rate[7] estimated for the period 1960-96 using output based Malmquist time series and panel models for various levels of aggregation are presented in Table 2. For aggregate technology i.e., $i=1$ and $j=1$, the output based Malmquist time series $OMP^{T}(i, j)$ and output based Malmquist panel $OMP_{t-1}^{t}(i, j)$ models estimated an annual growth rate of 1.916 identical to Tornqvist-Theil index measure. As indicated earlier the aggregate technology might be immune to the divergences in productivity measures as share-weights are not used in the estimation process.

The sensitivity of nonparametric Malmquist measures due to the level of aggregation using time-series and panel methods is clearly illustrated. Within nonparametric programming approach the three levels of aggregation estimated involves the use of four inputs and aggregate output, $OMP_{t-1}^{t}(4,1)$), aggregate input and two outputs, $OMP_{t-1}^{t}(1,2)$), and finally four inputs and two outputs, ($OMP_{t-1}^{t}(4,2)$ for the panel model. Similarly the three models, $OMP^{T}(4,1)$, $OMP^{T}(1,2)$ and $OMP^{T}(4,2)$ are also estimated using time series model. Results from the time series $OMP^{T}(4,1)$ and panel $OMP_{t-1}^{t}(4,1)$ models of nonparametric programming approach indicate an average annual productivity growth rate across all the states of 0.794 and 1.580 respectively. Similarly time series (panel) model for other levels of commodity aggregation i.e., $OMP^{T}(1,2)$ and $OMP^{T}(4,2)$ ($OMP_{t-1}^{t}(1,2)$ and $OMP_{t-1}^{t}(4,2)$) indicated an average annual productivity growth rate of 1.257 and 0.360 (1.597 and 1.412) respectively across all the states. Overall, results from Table 2 demonstrates the sensitivity of the nonparametric Malmquist productivity measures to the level of aggregation as well as the use of time-series versus the panel model compared to Tornqvist-Theil productivity or aggregate nonparametric programming productivity measures.

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[7] The detailed annual productivity measures computed using the time series programming and panel model programming approach could be obtained from the authors.

To illustrate the sensitivity of nonparametric productivity measures to the choice of method – output/input based Malmquist productivity or Malmquist total factor productivity and to commodity aggregation, we compare the endogenous share-weights recovered from the dual values of the nonparametric linear programming constraints of the output/input Malmquist productivity and Malmquist total factor productivity programming method for various levels of commodity aggregation. Also we compare the endogenous share-weights recovered from the nonparametric Malmquist programming approach to the exogenous share-weights of the Tornqvist-Theil index employing U.S aggregate data from 1948-1994.

The average input and output shares of the Tornqvist-Theil index approach, the Malmquist productivity programming approach and the Malmquist total factor productivity programming approach for various levels of disaggregation are presented in Table 3. Results from Table 3 indicate the average shadow shares[8] of the Malmquist productivity and the Malmquist total factor productivity programming approach are different from the exogenous observed market shares of the Tornqvist-Theil index approach for various levels of commodity disaggregation. For Tornqvist-Theil index approach, the average capital, farm real estate, farm labor and intermediate inputs shares are 0.111, 0.162, 0.265 and 0.463 respectively. Compared to the Tornqvist-Theil index approach, the average shadow input shares computed for the $IMP^{T}(1,4)$ ($IMP^{T}(2,4)$) of input based Malmquist productivity programming approach are 0.003, 0.023, 0.009 and 0.965 (0.004, 0.028, 0.020 and 0.948) for capital, farm real estate, farm labor and intermediate inputs respectively. However, the average capital, farm real estate, farm labor and intermediate inputs shadow shares computed from the input index ($MI^{T}(0,4)$) of the

[8] Due to the piecewise linear approximation of the programming approach for some inputs or outputs, the shares approximated from the linear programming constraints might attach zero or 100 percent weight. However the shares present in the Table 3 are averaged across the whole time period.

Malmquist total factor productivity programming approach is 0.017, 0.318, 0.354 and 0.312 respectively.

Similar differences between Tornqvist-Theil index observed output shares and the shadow output shares computed from $OMP^T(1,2)$ and $OMP^T(4,2)$ (of output based Malmquist productivity) and $MO^T(1,2)$ (of Malmquist total factor productivity) models are observed. The average output shares of crops and livestock computed for $OMP^T(1,2)$ (0.0.64 and 0.936) and $OMP^T(4,2)$ (0.085 and 0.915) models demonstrated skewed shadow shares compared to Tornqvist-Theil index (0.554 and 0.446). Similar skewed crop and livestock output shadow shares are recovered from the $MO^T(0,2)$ of the Malmquist total factor productivity programming approach.

To examine the effects of share-weights on the sensitivity of the nonparametric programming approach - Malmquist productivity and Malmquist total factor productivity measures, the U.S. annual productivity growth rates computed for various levels of commodity aggregation are presented in Table 4 along with the Tornqvist-Theil index productivity measures. The annual productivity growth rate for aggregate technology computed from the output-based Malmquist productivity ($OMP^T(1,1)$), input-based Malmquist productivity ($IMP^T(1,1)$) and Malmquist total factor productivity $MTFP^T(1,1)$ programming approach is identical to the Tornqvist-Theil index approach of 1.963. In general, for various levels of commodity aggregation the nonparametric programming approach identify annual productivity growth rate different from the Tornqvist-Theil index approach. Specially, the annual productivity growth rates of 0.486 ($OMP^T(4,1)$ or $IMP^T(1,4)$), 1.729 ($OMP^T(1,2)$ or $IMP^T(2,1)$) and 0.267 ($OMP^T(4,2)$ or $IMP^T(2,4)$) does not identify as much increase in productivity growth rate of

1.893 ($MTFP^{T}(4,1)$), 1.728 ($MTFP^{T}(1,2)$) and 1.659 ($MTFP^{T}(4,2)$) or more importantly to the annual productivity growth rate of 1.963 from the Tornqvist-Theil approach for the time period, 1948-94.

One of the main reasons for the difference in the productivity measures across models is the use of average share-weights to form the technology or the theoretical frontier. Unlike the index approach, the average share-weights or average shadow prices used in nonparametric programming approach is driven by the quantity data used in the estimation. For example with a four inputs-two outputs aggregation model, if the nonparametric programming approach allocates maximum share-weight on a single input with a huge positive rate-of-change then the productivity measures would be very low. Alternatively, if the nonparametric programming approach allocates maximum share-weight on a single input with a lowest rate-of-change then the productivity measures would be very high.

Based on Table 1, the rate of change in intermediate inputs for the entire period was a positive 0.735 compared to negative rate of change with the remaining three inputs – capital, land and labor. From Table 3, the exogenous average share-weight allocated to intermediate input by index approach was 0.463 compared to the endogenous average share-weight or average shadow price of 0.956 ($IMP^{T}(1,4)$) and 0.948 ($IMP^{T}(2,4)$) level of aggregation. Under these conditions the overall use of input to produce the given output is higher leading to lower productivity measures. This is because the nonparametric programming approach allocated highest average share-weight to intermediate input with highest positive rate-of-change leading to increased overall use of inputs.

In contrast, Table 1 indicates the average rate-of-change in the livestock and crop is 1.233 and 1.568 respectively. From Table 3, the exogenous average share-weight allocated to livestock and crop by index approach was 0.446 and 0.554 respectively. The endogenous average share-weight allocated to livestock and crop by index approach was 0.936 and 0.064 respectively for $IMP^T(1,4)$ level of aggregation. Similar trends in the average share-weights were indicated by $IMP^T(2,4)$ level of aggregation. Under these conditions the overall production of output is lower for a given use of input leading to lower productivity measures.

CONCLUSION

The paper examines the sensitivity of nonparametric programming productivity measures to the choice of model –time series and panel Malmquist productivity, and to commodity aggregation compared to the traditional Tornqvist-Theil index approach employing U.S. state-level data from 1960-96. The importance of share-weights in explaining the sensitivity of the nonparametric productivity measures is illustrated by comparing the implicit shadow shares recovered from the dual values of the linear programming constraints in the time series Malmquist productivity and Malmquist total factor productivity programming methods to the observed shares of the Tornqvist-Theil index employing U.S level data from 1948-1994.

The analysis at the U.S. state level indicate productivity measures estimated from the time series and panel models of Malmquist productivity programming approach are identical to the Tornqvist-Theil productivity measures for aggregate (single output single input) technology. Divergence in productivity measures is observed not only due to choice of method –Malmquist productivity and Malmquist total factor productivity methods and various levels of commodity and input aggregation, but also between the index and programming approach. Due to the piecewise linear approximation of the nonparametric programming approach, the shadow share-weights are skewed leading the difference in the productivity measures across methods, models and various levels of commodity aggregation.

The importance of the results reported in this paper will depend upon the researcher's objectives and availability of data. If prices are available utilizing the price information (as share-weights) in the computation of productivity measures either by Tornqvist-Theil index and/or programming approach to provide similar outcome irrespectively of the approach. However, for the unpriced non-market goods like environmental pollution, the unavailability of price information would motivate researchers to apply the programming approach to estimate the productivity measures as well as to recover the shadow prices.

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Table 1. State-wise Annual Output and Input Growth Rates[1], 1960-1996

State	Aggregate Output	Livestock	Crops	Aggregate Input	Capital	Land	Labor	Intermediate	TFP
AL	1.979	2.665	0.924	0.168	0.406	-1.588	-3.225	1.630	1.808
AR	3.608	4.784	2.641	1.085	0.878	-0.429	-2.547	3.028	2.496
AZ	1.356	1.609	1.216	-0.053	0.576	-0.669	-1.081	0.411	1.410
CA	2.164	1.760	2.384	0.443	0.299	-0.701	-0.735	1.408	1.713
CO	2.015	2.336	1.661	0.630	0.021	-0.523	-1.262	1.674	1.377
CT	0.680	0.594	0.729	-2.070	-1.240	-2.697	-2.636	-1.666	2.808
DE	2.853	3.231	2.077	0.951	0.290	-0.786	-2.888	2.264	1.884
FL	2.656	2.199	2.804	0.556	1.269	-1.439	-0.434	1.791	2.088
GA	2.759	3.164	2.269	0.291	0.144	-1.823	-2.808	1.917	2.461
IA	1.047	-0.273	2.270	-0.583	0.172	-0.255	-2.386	-0.134	1.640
ID	2.631	2.218	2.828	0.321	0.276	-0.171	-1.849	1.813	2.302
IL	1.027	-1.563	2.100	-0.608	-0.035	-0.304	-2.682	-0.026	1.645
IN	1.089	-0.240	1.839	-0.839	0.042	-0.589	-3.092	0.134	1.945
KS	1.886	2.331	1.522	0.559	-0.147	-0.303	-1.894	2.006	1.320
KY	2.017	2.021	1.976	-0.393	1.116	-0.697	-2.579	0.956	2.419
LA	2.439	1.253	2.933	-0.374	0.139	-0.636	-3.454	1.368	2.824
MA	-0.695	-2.145	0.321	-2.734	-1.158	-1.856	-4.327	-2.026	2.096
MD	1.716	1.661	1.741	-0.212	0.071	-1.467	-2.559	0.935	1.932
ME	0.122	0.807	-0.701	-1.883	-0.984	-2.628	-3.210	-1.396	2.043
MI	1.350	0.442	1.951	-1.193	-0.442	-1.010	-3.228	0.598	2.573
MN	1.519	0.372	2.499	-0.252	0.049	-0.342	-2.009	0.773	1.776
MO	1.164	0.305	1.795	-0.533	0.303	-0.525	-2.215	0.377	1.707
MS	2.181	2.676	1.700	-0.405	-0.039	-1.229	-3.948	1.737	2.597
MT	1.461	0.725	1.748	0.065	0.130	-0.358	-0.952	0.883	1.395
NC	2.254	4.748	0.430	-0.400	0.218	-1.768	-3.537	1.973	2.665
ND	2.406	0.013	2.935	0.099	0.021	-0.324	-1.181	1.196	2.305
NE	2.530	2.220	2.706	0.563	0.073	-0.159	-1.518	1.867	1.955
NH	-0.428	-1.105	0.556	-2.394	-1.184	-2.926	-3.134	-2.238	2.014
NJ	-0.244	-1.166	0.450	-1.946	-1.363	-1.417	-2.628	-1.855	1.736
NM	2.563	3.159	1.433	0.450	0.337	-0.558	-1.834	2.225	2.103
NV	2.090	1.057	3.538	0.931	0.685	-0.185	-0.209	1.769	1.148
NY	0.209	0.458	-0.202	-1.277	-0.474	-1.713	-2.251	-0.839	1.505
OH	0.879	0.068	1.381	-0.888	-0.149	-0.609	-2.224	0.054	1.783
OK	1.409	2.239	0.149	0.430	0.058	-0.360	-1.347	2.117	0.975
OR	2.190	0.679	2.966	0.196	0.402	-0.536	-0.477	1.198	1.990
PA	1.485	1.444	1.502	-0.689	0.002	-1.237	-1.952	0.232	2.189
RI	-0.736	-2.451	0.572	-2.414	-1.693	0.000	-4.550	-1.313	1.719
SC	1.048	2.587	0.191	-1.258	-0.414	-1.969	-4.579	0.726	2.335
SD	1.717	0.341	2.891	-0.148	-0.305	-0.255	-1.524	0.697	1.867
TN	1.118	0.433	1.658	-0.754	0.550	-0.997	-2.756	0.531	1.885
TX	1.733	2.264	1.020	0.396	0.417	-0.627	-1.525	1.848	1.332
UT	1.656	1.595	1.705	-0.118	0.367	-0.464	-1.409	0.766	1.777
VA	1.443	2.087	0.721	-0.692	0.175	-1.217	-2.912	0.695	2.150
VT	0.398	0.741	-0.832	-1.427	-0.257	-2.210	-2.885	-0.509	1.852
WA	2.990	2.676	3.131	0.695	0.385	-0.263	-0.801	2.256	2.279
WI	0.792	0.393	1.428	-0.587	0.095	-0.806	-2.437	0.376	1.387
WV	0.581	0.807	0.098	-1.208	-0.415	-1.516	-2.479	0.050	1.811
WY	1.180	0.956	1.589	0.229	0.166	-0.134	-0.614	1.013	0.949
Average[2]	1.506	1.233	1.568	-0.402	-0.003	-0.943	-2.266	0.735	1.916

[1] Annual growth rate is $((X_{t+1}/X_t)^{1/T}-1)*100$ where X is input or output variable and T is the time period.

[2] A simple average across states.

Table 2. State-wise Annual Productivity Growth Rates, 1960-1996.

		Output based Malmquist -Time series				Output based Malmquist -Panel Model			
State	Tornqvist-Theil Index	$OMP^T(1,1)$	$OMP^T(4,1)$	$OMP^T(1,2)$	$OMP^T(4,2)$	$OMP_{t-1}^t(1,1)$	$OMP_{t-1}^t(4,1)$	$OMP_{t-1}^t(1,2)$	$OMP_{t-1}^t(4,2)$
AL	1.808	1.808	0.592	1.084	0.222	1.808	1.563	1.199	1.183
AR	2.496	2.496	0.649	1.539	0.140	2.496	1.722	1.886	1.591
AZ	1.410	1.410	0.775	1.269	0.635	1.410	1.778	1.406	1.729
CA	1.713	1.714	1.151	1.311	0.587	1.714	1.657	1.538	1.552
CO	1.377	1.377	0.417	1.190	0.235	1.377	1.719	1.343	1.628
CT	2.808	2.808	1.944	2.720	1.859	2.808	2.568	2.787	2.565
DE	1.884	1.884	1.084	1.233	0.824	1.884	2.503	1.300	1.994
FL	2.088	2.088	0.973	1.634	0.493	2.088	1.497	1.932	1.567
GA	2.461	2.461	1.022	1.973	0.633	2.460	2.003	2.193	2.073
IA	1.640	1.640	0.922	0.321	0.035	1.640	1.830	1.323	1.534
ID	2.302	2.302	1.137	1.891	0.497	2.302	2.132	2.103	1.934
IL	1.645	1.645	1.126	-0.066	-0.058	1.645	1.436	0.399	0.555
IN	1.945	1.945	0.954	0.604	-0.029	1.945	1.780	1.399	1.375
KS	1.320	1.319	0.031	1.128	0.000	1.319	1.026	1.274	1.006
KY	2.419	2.419	0.891	2.384	0.850	2.419	1.626	2.399	1.819
LA	2.824	2.824	1.057	1.633	0.225	2.824	2.108	2.304	1.768
MA	2.096	2.096	0.480	0.854	0.071	2.096	1.449	1.516	1.110
MD	1.932	1.932	0.774	1.877	0.740	1.932	1.689	1.939	2.048
ME	2.043	2.043	1.117	1.205	0.444	2.043	1.923	1.770	1.678
MI	2.573	2.573	0.967	1.654	0.384	2.573	1.761	2.265	1.481
MN	1.776	1.776	0.907	0.627	0.106	1.776	2.053	1.402	1.515
MO	1.707	1.707	0.792	0.844	0.162	1.707	1.392	1.401	1.247
MS	2.597	2.597	0.786	2.596	0.782	2.597	1.848	2.649	2.110
MT	1.395	1.395	0.961	0.660	0.330	1.395	1.220	0.980	0.814
NC	2.665	2.665	0.546	1.171	0.146	2.665	1.530	1.542	0.969
ND	2.305	2.305	1.500	0.474	0.275	2.305	1.929	1.287	1.015
NE	1.955	1.956	0.747	1.705	0.513	1.956	1.692	1.969	1.767
NH	2.014	2.014	0.765	1.320	0.341	2.014	1.870	1.991	2.162
NJ	1.736	1.736	1.135	0.795	0.199	1.736	1.954	1.344	1.643
NM	2.103	2.103	0.331	0.995	0.000	2.103	1.964	1.712	1.541
NV	1.148	1.148	0.382	0.370	0.085	1.148	1.018	0.951	0.848
NY	1.505	1.505	0.687	1.304	0.428	1.505	1.406	1.450	1.436
OH	1.783	1.783	0.859	0.964	0.293	1.783	1.166	1.411	0.895
OK	0.975	0.975	-0.184	0.667	-0.071	0.975	0.116	0.305	-0.299
OR	1.990	1.990	1.036	0.774	0.139	1.990	1.743	1.497	1.421
PA	2.189	2.189	1.256	2.148	1.209	2.189	1.868	2.179	2.075
RI	1.719	1.719	-0.079	0.227	0.000	1.719	0.765	0.543	-0.006
SC	2.335	2.335	0.695	1.951	0.308	2.335	1.230	2.030	1.302
SD	1.867	1.867	1.096	1.147	0.485	1.867	1.811	1.959	1.870
TN	1.885	1.885	0.564	1.615	0.159	1.885	0.991	2.106	1.664
TX	1.332	1.332	0.048	0.726	-0.077	1.332	0.804	0.955	0.689
UT	1.777	1.776	1.013	1.715	0.823	1.776	1.455	1.749	1.501
VA	2.150	2.150	0.931	1.502	0.380	2.150	1.459	1.619	1.257
VT	1.852	1.852	0.657	1.276	0.070	1.852	1.557	1.034	0.947
WA	2.279	2.280	1.070	1.978	0.485	2.280	1.946	2.141	1.849
WI	1.387	1.387	0.612	0.986	0.338	1.387	1.383	1.478	1.461
WV	1.811	1.811	0.730	1.621	0.587	1.811	1.163	1.687	1.165
WY	0.949	0.949	0.190	0.725	-0.015	0.949	0.734	1.015	0.712
Average	1.916	1.916	0.794	1.257	0.360	1.916	1.580	1.597	1.412

Where $OMP(i, j)$ represents the output based Malmquist productivity measures with i and j indicating number of inputs and outputs. Annual growth rate and average is defined in Table 1.

Table 3. U.S. Agricultural Sector Arithmetic Mean of Input and Output Share-Weights for various Levels of Commodity Aggregation, 1948-94

Approach	Input Share-weights			
	Capital	**Farm Real Estate**	**Farm Labor**	**Intermediate Inputs**
Tornqvist-Theil Index	0.111	0.162	0.265	0.463
Input based Malmquist				
$IMP^T(1,4)$	0.003	0.023	0.019	0.956
$IMP^T(2,4)$	0.004	0.028	0.020	0.948
Malmquist Input Index of MTFP $MI^T(0,4)$	0.017	0.318	0.354	0.312
	Output Share-weights			
			Crops	**Livestock**
Tornqvist-Theil Index			0.554	0.446
Output based Malmquist				
$OMP^T(1,2)$			0.064	0.936
$OMP^T(4,2)$			0.085	0.915
Malmquist Output Index of MTFP $MO^T(0,2)$			0.064	0.936

Where $OMP(i,j)$ represents the output based Malmquist productivity measures, $IMP(j,i)$ represents the input based Malmquist productivity measures, $MI(i)$ represents Malmquist input index of the Malmquist total factor productivity, and $MO(i)$ represents Malmquist output index of the Malmquist total factor productivity with i and j indicating number of inputs and outputs.

Table 4. U.S. Agricultural Sector Annual Productivity Growth Rates for various Levels of Commodity Aggregation, 1948-94

Approach	**Annual Productivity Growth Rate**
Tornqvist-Theil Productivity Index	1.963
Output/Input based Malmquist Productivity	
$OMP^{T}(1,1)$ or $IMP^{T}(1,1)$	1.963
$OMP^{T}(4,1)$ or $IMP^{T}(1,4)$	0.486
$OMP^{T}(1,2)$ or $IMP^{T}(2,1)$	1.729
$OMP^{T}(4,2)$ or $IMP^{T}(2,4)$	0.267
Malmquist Total Factor Productivity	
$MTFP^{T}(1,1)$	1.963
$MTFP^{T}(4,1)$	1.893
$MTFP^{T}(1,2)$	1.728
$MTFP^{T}(4,2)$	1.659

Where $OMP(i,j)$ represents the output based Malmquist productivity measures, $IMP(j,i)$ represents the input based Malmquist productivity measures, $MTFP(i,j)$ represents Malmquist total factor productivity with i and j indicating number of inputs and outputs.