Input-Output Analysis, Linear Programming and Modified Multipliers

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Abstract: The input-output (IO) analysis explores changes in final demand through the regional economy using multipliers. However, it isn't flexible to investigate the regional impact from the capacity limitations which are directly imposed on *production*, not final demand. This is because the multipliers are changing with exogenous restrictions on production. Conventionally, the IO analysis is performed assuming exogenous production restrictions being the changes in final demands or assuming the sector being exogenous sector like the final demand. If researchers or policy makers are interested in only economic impacts from production restrictions, there is no need to look into the modified multipliers. The modified multipliers should be considered when researchers and policy makers attempt to analyze the compensation of impact, especially recovery of loss using government expenditure. We suggest that the linear programming is a useful and efficient tool to derive modified multipliers and estimate correct regional impact from the policy changes.

Key Words: Input-Output Analysis, Multipliers, Regional Impact Analysis

JEL Classifications: C67, R15, R5

Introduction

The input-output (IO) analysis is well-known in regional economics and has been applied to numerous economic issues for a long time. The IO method is based on the

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interrelationship between sectors in the economy and how each is affected by a change in the final demand for a sector's output. The IO analysis can be summarized as the multiplier analysis, which outlines individual changes in final demand through the regional economy over short periods of time (Schaffer, 1999, p33). As elaborated in the following section, various types of multiplier exist. The output multiplier refers to an increase in the final demand can lead to an even greater increase in output. The employment and income multipliers refer to the concepts that the increase in numbers of employees or household income will lead to an increase in total value of output, employment, and income as well.

However, the IO analysis or the multiplier analysis is not flexible to investigate the regional impact from the policies or capacity limitations which are *directly* imposed on *production*, for example, limiting production in power generation sector to reduce greenhouse gas emissions for meeting the national or international requirement, or government's ban on meat production due to food safety issues for instance BSE, or reduction in cattle production due to limitation of public land grazing in Western US in Fadali, Harris and Alevy (2007). This is because it is expected that the multipliers are changing when the exogenous production restrictions exist. We call these as the modified multipliers.

Conventionally, the IO analysis is performed assuming that exogenous production restrictions are the changes in final demands or the sector being restricted is treated as exogenous sector like the final demand. If researchers or policy makers are interested in only economic impacts from these restrictions, there is no need to look into the modified multipliers. The modified multipliers should be considered when researchers and policy makers attempt to analyze the compensation or recovery of impact (mostly economic loss) from production restrictions using promoting other sectors' final demand or increasing government spending. This is important because the conventional IO analysis with additional restrictions is apt to overestimate multipliers and lead insufficient investment to recover the loss from production change.

In order to obtain the modified multipliers responding to direct restrictions on production the IO transaction matrix should be rebuilt, which is not possible before implementing policies. In this sense, it is required to figure out how to derive the modified multipliers without rebuilding the transaction matrix and explore the regional impact analysis. We suggest that the linear programming (LP) approach is one of the candidates. In the LP, the shadow price has the same meaning as multipliers in the IO analysis (Brink and McCarl, 1977). Previous works using the LP in place of the IO analysis are Wilfred and Boehlje (1971) who analyzes the capital budgeting with multiple goals, and Penn et al. (1976) for modeling and simulating the U.S economy with alternative energy availabilities. These papers use the LP approach mainly because of computational problem rather than inflexibility of the IO analysis. As argued in Brink and McCarl (1977) the LP algorithms are simpler, easier and more accurate than matrix inversion algorithms. During 1970's and early 1980's, the computer system doesn't allow invert the huge Leontief matrix, which is essential in the IO analysis. The advent of the fast and stable computer removes advantages to use the LP approach in the regional impact analysis.

In this paper, the LP approach is recalled. The multipliers in the conventional IO analysis are fixed and constant regardless of restrictions such as reduction of production in a specific sector, but the multipliers in LP formulation are updated accordingly when additional restrictions are added on the sector's production directly. As mentioned earlier, if researchers and policy makers want to recover economic loss from exogenous production restrictions, the modified multiplier should be used. Otherwise economic boosting policy tends to be overestimated.

This paper consists of the following five parts. Section 2 discusses the IO analysis and multipliers, and section 3 shows how to derive multipliers from the LP formulation analytically. Section 4 contains extension of the LP formulation with the additional constraints and how to derive the modified multipliers responding to this change. Section 5 includes a numerical example and empirical application, and section 6 concludes the findings.

Input-Output Analysis and Multipliers

For an economy of *n* sectors (industries) the standard IO model is represented by $\mathbf{X} = \mathbf{Y} + \mathbf{AX}$, where **X** is the output vector, **Y** is the final demand vector, and **A** is the direct requirement matrix, which elements, a_{ij}, are calculated as $a_{ij} = \frac{x_{ij}}{x_j}$, where x_{ij} is the transaction between sector *i* and *j*, and x_j is the sectoral output which is $x_j = \sum_i x_{ij}$. This relation indicates that the sum of output **X** equals to the direct uses in final demand **Y** and its indirect uses in intermediate production **AX**. The solution can be obtained by rewriting as:

(1) $\mathbf{X} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{Y}$,

where **I** is the $n \times n$ identity matrix. The $(\mathbf{I}-\mathbf{A})$ matrix is called the Leontief matrix and $(\mathbf{I}-\mathbf{A})^{-1}$ is called the Leontief inverse matrix which shows the total-requirements matrix for the economy. Equation (1) can be interpreted as $\Delta\mathbf{X}=(\mathbf{I}-\mathbf{A})^{-1}\Delta\mathbf{Y}$, which means changes in total industry output are predicted using the Leontief inverse matrix. Thus the column sum of $(\mathbf{I}-\mathbf{A})^{-1}$ is interpreted as the total changes in output from the changes in final demand, which is called output multiplier

(2) $\boldsymbol{\alpha}'=\mathbf{i}'(\mathbf{I}-\mathbf{A})^{-1}$,

where $\boldsymbol{\alpha}$ is the output multiplier column vector and **i** is an $n \times 1$ column vector of ones. Thus *k*th element in $\boldsymbol{\alpha}$ implies there is exogenous change in final demand for *k*th sector total industry output change by α_k. Likewise, the employment multiplier can be defined as follows

(3) $\mathbf{e}'=\mathbf{i}'\mathbf{N}(\mathbf{I}-\mathbf{A})^{-1}$,

where **N** is the matrix with diagonal of $n_1, n_2, \ldots, n_n$ and off diagonal all zeros, where $n_i=\dfrac{\text{Employment}_\text{i}}{\text{Output}_\text{i}}$ (i = 1, 2, … n). Hence, the *k*th element in e implies there is an exogenous change in employment for *k*th sector, total industry output change by e_k. Similarly, the income multiplier can be defined as

(4) $\mathbf{h}'=\mathbf{i}'\mathbf{H}(\mathbf{I}-\mathbf{A})^{-1}$,

where **H** is the symmetric matrix with diagonal of $h_1, h_2, \ldots, h_n$ and off diagonal all zeros, where $h_i=\dfrac{\text{household income}_\text{i}}{\text{output}_\text{i}}$ (i = 1, 2, … n). Again, the *k*th element in h implies there is an exogenous change in household income for kth sector, total industry output change by $h_{k.}$

Input-Output Analysis and Linear Programming

The linear programming (LP) is applied to input-output analysis by Brink and McCarl (1977) and they demonstrate how the output multiplier can be obtained from LP by setting as

$$
(5)\qquad \begin{array}{ll} \max & \mathbf{i}'\mathbf{X} \\ \text{s.t.} & (\mathbf{I}-\mathbf{A})\mathbf{X} \le \mathbf{Y} \\ & \mathbf{X} \ge 0 \end{array} \qquad \text{or} \qquad \begin{array}{ll} \max & \mathbf{i}'\mathbf{X} \\ \text{s.t.} & (\mathbf{I}-\mathbf{A})\mathbf{X} + \mathbf{IS} = \mathbf{Y}, \\ & \mathbf{X},\ \ \mathbf{S} \ge 0 \end{array}
$$

where **S** is slack variables matrix. The problem is to maximize the value of the sum of outputs from all industries under the constraint that the output from each industry does not exceed the use of that output in final demand and as input to other industries. As argued in Brink and McCarl (1977), the matrix (**I** – **A**) is the *basis* in LP formulation[1]. It is easily understood because the optimal solution should be identical to the level of production from input-output table and thus all elements in **X** are positive, which implies elements in X are basic variables and thus (**I** – **A**) is basis.

Shadow price in LP formulation is defined as the expected rate of change in the objective function (here, $\mathbf{i}'\mathbf{X}$) when the right hand sides (here **Y**) are changed. In other words, $\frac{\partial z}{\partial \mathbf{b}} = \mathbf{C}_B\mathbf{B}^{-1}$, where z is the objective function, **b** is the right hand sides, $\mathbf{C}_B$ is the

1 LP theory (Bazaraa, Jarvis and Sherali, 1990, p53; McCarl and Spreen, 2006, Chapter 3, pp3 reveals that a solution to the LP problem will have a set of nonzero variables equal in number to the number of constraints. Such a solution is called a basic (feasible) solution and the associated variables are commonly called basic variables. The matrix containing the coefficients of the basic variables as they appear in the constraints is called basic matrix or basis, which is $n \times n$ square matrix.

objective function coefficients for basic variables and **B** is the basis (McCarl and Spreen, 2006, Chapter 3, p12). Shadow price for the LP formulation in equation (5) is given by

(6) $$\mathbf{C}_B\mathbf{B}^{-1} = \mathbf{i}'(\mathbf{I}-\mathbf{A})^{-1}.$$

Obviously, shadow price in equation (6) is identical to output multipliers in equation (2) as shown in Brink and McCarl (1977). Using the similar logic the employment and income multipliers are derived from the following models,

(7) $$\begin{array}{ll} \max & \mathbf{n}'\mathbf{X} \\ \text{s.t.} & (\mathbf{I}-\mathbf{A})\mathbf{X} \le \mathbf{Y} \\ & \mathbf{X} \ge 0 \end{array} \quad \text{and} \quad \begin{array}{ll} \max & \mathbf{h}'\mathbf{X} \\ \text{s.t.} & (\mathbf{I}-\mathbf{A})\mathbf{X} \le \mathbf{Y}, \\ & \mathbf{X} \ge 0 \end{array}$$

where $\mathbf{n}' = [n_1, n_2, ..., n_n]$ and $\mathbf{h}' = [h_1, h_2, ..., h_n]$. Shadow prices from these models are given by $\mathbf{n}'(\mathbf{I}-\mathbf{A})^{-1} \equiv \mathbf{i}'\mathbf{N}(\mathbf{I}-\mathbf{A})^{-1}$ and $\mathbf{h}'(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{i}'\mathbf{H}(\mathbf{I}-\mathbf{A})^{-1}$, which are identical to employment and income multipliers in equations (3) and (4), respectively.

Modified Multipliers using LP

As alluded in introduction, the LP approach is attractive because it allows us to study the effects of exogenous capacity limitations in some industries, for example limiting production to reduce greenhouse gas emissions from power generation sector, or government's ban on the cattle production due to the food safety issues. We suggest that shadow prices from the restricted LP model with the additional exogenous capacity limitations provide the *modified* output, employment and income multipliers. It can be argued that these modified multipliers are crucial for the further policy or regional impact analysis.

The additional exogenous capacity limitations can be represented as $\mathbf{DX} \le \mathbf{Z}$, where **D** is the $m \times n$ design matrix to impose restrictions on industries. Note that m is the number of industries restricted and n is the number of industries in the economy. The elements of matrix **D** are zero or one (or it could be other values) and one indicates restriction is imposed. **Z** is the capacity limitations vector and its dimension is $m \times 1$. The equation (5) is now

$$
\begin{array}{llr}
\max & \mathbf{i}'\mathbf{X} & \\
\text{s.t.} & (\mathbf{I}_n - \mathbf{A})\mathbf{X} + \mathbf{I}_n\mathbf{S}_1 & = \mathbf{Y} \\
 & \mathbf{DX} \qquad\qquad + \mathbf{I}_k\mathbf{S}_2 & = \mathbf{Z} \\
 & \mathbf{X}, \quad \mathbf{S}_1, \quad \mathbf{S}_2 \ge 0 &
\end{array}
\quad \text{or} \quad
\begin{array}{ll}
\max & \mathbf{i}'\mathbf{X} \\
\text{s.t.} & \begin{bmatrix} (\mathbf{I}_n - \mathbf{A}) & \mathbf{I}_n & \mathbf{0} \\ \mathbf{D} & \mathbf{0} & \mathbf{I}_m \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{S}_1 \\ \mathbf{S}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{Y} \\ \mathbf{Z} \end{bmatrix} \\
 & \mathbf{X}, \mathbf{S}_1, \mathbf{S}_2 \ge 0
\end{array}
\tag{8}
$$

Note that $\mathbf{I}_n$ is $n \times n$ and $\mathbf{I}_m$ is $m \times m$ identity matrices, and $\mathbf{S}_1$ and $\mathbf{S}_2$ are slack variables correspondingly. In this formulation the matrix $(\mathbf{I}_n - \mathbf{A})$ is not the basis anymore because of additional constraints and in turn, the shadow prices are different from those of LP formulation in equation (5). This fact implies that the output multiplies with additional constraints cannot be the same as multipliers from the input-output analysis. Because $\mathbf{0} \le \mathbf{Z} \le \mathbf{X}$ by construction, the slack variables for restricted industries should be nonzero and they come into the basic variables. The $(n + m) \times (n + m)$ basis of the problem in equation (8) is given by

$$
\mathbf{B} = \begin{bmatrix} \mathbf{I} - \mathbf{A} & \mathbf{D}' \\ \mathbf{D} & \mathbf{0} \end{bmatrix} \text{ and } \mathbf{B}^{-1} = \begin{bmatrix} (\mathbf{I}_n - \mathbf{A})^{-1}\{\mathbf{I}_n + \mathbf{D}'\mathbf{FD}(\mathbf{I}_n - \mathbf{A})^{-1}\} & -(\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{D}'\mathbf{F} \\ -\mathbf{FD}(\mathbf{I}_n - \mathbf{A})^{-1} & \mathbf{F} \end{bmatrix}
\tag{9}
$$

where $\mathbf{F} = \{-\mathbf{D}(\mathbf{I}_n - \mathbf{A})^{-1}\mathbf{D}'\}^{-1}$. Thus, the modified output multipliers (for sectors) are obtained by

(10) $$\boldsymbol{\alpha} = [\mathbf{i}',\mathbf{0}]\mathbf{B}^{-1} = \mathbf{i}'(\mathbf{I}_n - \mathbf{A})^{-1}\{\mathbf{I}_n + \mathbf{D}'\mathbf{F}\mathbf{D}(\mathbf{I}_n - \mathbf{A})^{-1}\}$$

Similarly the modified employment and income multipliers can be derived. It is noteworthy that some elements in matrix in equation (10) are zero due to **D** matrix of which elements are zero and ones, and the modified multipliers are always smaller than the original multipliers. This indicates that economic impact would be overestimated when the original multiplier is used with additional capacity limitations on production.

Numerical Example

An example application of equation (8) through (10) is shown in this section. The hypothetical data from table 4.2 in Schaffer (1999) is used (See Table 1). In this hypothetical economy, there exist five sectors; Extraction, Construction, Manufacturing, Trade and Service. Suppose that the central government imposes production limit on manufacturing sector for some reasons, for example to reduce air pollution, by 10%. From equation (8) we set up the LP problem as follows:

$$\max \ [1 \quad 1 \quad 1 \quad 1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

$$\text{s.t} \quad \begin{bmatrix} 0.891 & -0.012 & -0.042 & -0.001 & -0.001 \\ -0.008 & 0.999 & -0.003 & -0.003 & -0.026 \\ -0.085 & -0.164 & 0.902 & -0.023 & -0.031 \\ -0.031 & -0.089 & -0.037 & 0.985 & -0.023 \\ -0.061 & -0.088 & -0.061 & -0.116 & 0.825 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \le \begin{bmatrix} 783 \\ 2{,}156 \\ 11{,}749 \\ 3{,}694 \\ 7{,}613 \\ 12{,}745 \end{bmatrix},$$

where Z = [12,745] and D = [0 0 1 0 0]. Using equations (9) and (10)

$$\mathbf{B}^{-1} = \begin{bmatrix} 1.123 & 0.015 & 0 & 0.003 & 0.009 & 0.048 \\ 0.012 & 1.004 & 0 & 0.007 & 0.032 & 0.006 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.039 & 0.094 & 0 & 1.019 & 0.031 & 0.041 \\ 0.090 & 0.121 & 0 & 0.144 & 1.221 & 0.084 \\ 0.101 & 0.172 & 1 & 0.029 & 0.045 & -0.893 \end{bmatrix}.$$

In turn, the (output) multipliers for restricted model are calculated as follows

$$\boldsymbol{\alpha} = [\mathbf{i}',\mathbf{0}]\mathbf{B}^{-1} = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0]\begin{bmatrix} 1.123 & 0.015 & 0 & 0.003 & 0.009 & 0.048 \\ 0.012 & 1.004 & 0 & 0.007 & 0.032 & 0.006 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.039 & 0.094 & 0 & 1.019 & 0.031 & 0.041 \\ 0.090 & 0.121 & 0 & 0.144 & 1.221 & 0.084 \\ 0.101 & 0.172 & 1 & 0.029 & 0.045 & -0.893 \end{bmatrix}.$$

$$= [1.264 \quad 1.234 \quad 0 \quad 1.172 \quad 1.293 \quad 1.179]$$

Note that the original multipliers are $\boldsymbol{\alpha} = \left[1.397 \quad 1.461 \quad 1.320 \quad 1.211 \quad 1.353\right]$.

There are two things should be addressed here. First, the modified multiplier for the restricted sector is zero (in the short-run). This is because the final demand should decrease proportionally to reductions in production. Until then increases in final demand doesn't have any effect. Second, the last element in $\boldsymbol{\alpha}$ vector, $\alpha_6 = 1.179$, is the marginal value of restriction. If the exogenous restriction on the production decreases by \$1, which means production increases by \$1, overall economic impact would be \$1.179. In other words, if manufacturing sector has \$1 more restriction, overall economy will lose \$1.179.

If there are 10% reduction in production from manufacturing sector, the whole economy will lose \$1,869 (= \$1,416×1.32). Suppose that the central government try to recover this loss by increasing government expenditure or investing service sector. The output multiplier for service sector is given by 1.353 from the unrestricted IO model (original multiplier) and 1.293 from the restricted IO model (modified multiplier). The central government may calculate the amount of investment in service sector as \$1,381 (= \$1,869/1.353) using the original multiplier, which in fact is not enough to recover the loss. Government's investment increases only \$1,786 (= \$1,381×1.293) in economy and the economy is still losing \$83. Actually, the final demand in service sector should rise by \$1,445 (= \$1,869/1.293) to recover all of economic loss, which is \$64 more investment comparing to amount of expenditure based on the original multiplier. Net benefit to use the modified multiplier is \$19 (= \$83 – \$64). If the economy is relatively large, say millions of dollars, the difference might be substantial. Clearly the conventional way underestimates the economic impact after imposing exogenous production restriction.

Empirical Analysis

As shown in above sections, the IO analysis deals with final demand changes and rippling effects on the regional economy. However, when exogenous capacity limitation on production is imposed, the multipliers are changing as in equation (10) and the difference might be substantial as illustrated above numerical example. For the real example, the US input-output table is formulated using IMPLAN 2006 data and linear programming model accordingly. IMPLAN sectors are aggregated into 21 sectors which is 2 digit NAICS with power generation and supply sector (MIG, Inc, 2004). See Table 2 for sectoral aggregation. As in equations (8), the LP model is run and the output multipliers are obtained, which are reported in the second column in Table 3.

Suppose that the US government imposes production limit on power generation and supply sector in order to reduce the greenhouse gas emissions. For more discussions about reduction in greenhouse gas emissions, see McKinsey & Company (2007). For illustration purpose we assume that power generation sector should reduce its production by 20%*2* to meet an international requirement. This requirement is evidently burden to the US economy. Economic loss to U.S. is calculated using the output multiplier for power generation sector, which is 1.26 (Table 2). 20% reduction in power generation causes direct loss which is $51.7 billion, and additional indirect loss which is $13.5 billion. In total US economy would be suffering from the loss of $65.2 billion.

Suppose that US government has a plan to recover this loss by increasing government expenditure. Under the conventional IO approach, we may use the original multiplier, 1.86 (Table 2), and thus government expenditure would be expanded by $35.1 billion. However, the modified multiplier for government sector with production restriction is given by 1.84 (Table 2) and thus expenditure should be expanded by $35.4 billion to recover the loss not $35.1 billion. Even if government succeeds to promote the economy using the government expenditure by $35.1 billion with original multiplier, the U.S. still loses $616 million because the output multiplier is overestimated. This implies that US economy may not be recovered fully. The modified multiplier tells us that US government invests $300 million more to recover the economic loss from the production restriction on power generation sector. The net gain to use the modified multiplier might be $316 million (= $616 million – $300 million).

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2 US should reduce its greenhouse gas emission to 7% below 1990 emission level under the Kyoto Protocol, which is equivalent to 2.5 gigatons per year or approximately 30% of current emissions (McKinsey & Company, 2007; Kim and McCarl, 2008).

Conclusion and Implications

This paper analyzes the multipliers from the conventional IO analysis and reinforces the LP method to calculate modified multipliers, from both a theoretical aspect and numerical examples. In short, if exogenous capacity limitations are imposed on production directly, the modified multipliers should be used for regional economic analysis. This is because the conventional approach tends to overestimate the output multipliers. This is important especially when researchers and/or policy makers design the policies for recovering or boosting economy which might be suffering from the capacity limitations on production. Otherwise, economic loss would not be fully recovered. Net gain to use the modified multipliers can be huge in a relatively large scales economy such as national or state levels.

One caveat is that this analysis is short-run analysis. In the long run, the final demand in restricted sector would be adjusted, most likely decreases, which means the final sector is not exogenous any more, and in turn all the coefficients in the direct requirement matrix and multipliers are readjusted. This is not possible here. However, one possibility is that we might update IO table using another LP set up as discussed in Ghanem (2004), RAS method (Schneider and Zenios, 1990), or Minimum Cross Entropy (CE) method (Robinson et al., 2001). This would be the further study.

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Table 1: Hypothetical IO Data in Schaffer (1999)

	Extract	Construct	Manufact	Trade	Service	HH	Gov't	X	Total
Extract	183	31	599	6	73	99	88	596	1,675
Construct	14	1	43	14	293	0	1,803	353	2,521
Manufact	142	414	1,390	110	356	1,275	1,130	9,344	14,161
Trade	52	224	520	72	257	2,563	161	970	4,819
Service	102	221	862	558	1,990	4,262	523	2,828	11,346
Labor	595	665	3,696	2,385	4,603				
Oth Pymt	261	191	1,624	1,365	2,402				
Import	326	774	5,427	309	1,372				
Col. Total	1,675	2,521	14,161	4,819	11,346				

Table 2: Sector Aggregation

Industries	IMPLAN code	NAICS code
Agriculture, Forestry, Fishing & Hunting	1	11
Mining	19	21
Power generation and supply	30	
Utilities	31	22
Construction	33	23
Manufacturing	46	31-33
Wholesale Trade	390	42
Retail Trade	391	48-49
Transportation &Warehousing	401	44-45
Information	413	51
Finance & Insurance	425	52
Real Estate & rental	431	53
Professional- scientific & tech services	437	54
Management of companies	451	55
Administrative & waste services	452	56
Educational services	461	61
Health & social services	464	62
Arts- entertainment & recreation	475	71
Accommodation & food services	479	72
Other services	482	81
Government & non NAICs	495	92

Table 3: Output Multipliers (US)

Industries	Multipliers w/o restriction	Modified multipliers
Agriculture, Forestry, Fishing & Hunting	2.26	2.24
Mining	1.65	1.63
Power generation and supply	1.26	**0.00**
Utilities	1.81	1.80
Construction	2.04	2.04
Manufacturing	2.48	2.46
Wholesale Trade	1.57	1.56
Retail Trade	1.85	1.84
Transportation &Warehousing	1.58	1.57
Information	1.93	1.92
Finance & Insurance	1.69	1.68
Real Estate & rental	1.58	1.56
Professional- scientific & tech services	1.74	1.73
Administrative & waste services	1.69	1.67
Educational services	1.67	1.66
Health & social services	1.68	1.67
Arts- entertainment & recreation	1.69	1.68
Accommodation & food services	1.67	1.65
Other services	1.89	1.87
Government & non NAICs	1.86	1.84
Production limit on power generation sector		1.16