Chartists and Fundamentalists in the Currency Market and the Volatility of Exchange Rates*

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February 14, 2003

Abstract

The purpose of this paper is to implement theoretically, the empirical observation that the relative importance of fundamental versus technical analysis in the foreign exchange market depends on the planning horizon. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons. The theoretical framework is the Dornbusch (1976) overshooting model. The perfect foresight path near long-run equilibrium is derived, and it is shown that the magnitude of exchange rate overshooting is larger than in the Dornbusch (1976) model. Specifically, the extent of overshooting depends inversely on the planning horizon.

Keywords: Chartists; Excess Volatility; Foreign Exchange; Fundamentalists; Moving Averages; Overshooting; Technical Analysis.

JEL-Codes: E41; E44; F31; F41.

* This paper was to a large extent written during a longer visit at the University of Wisconsin-Madison, U.S.A., and I am thankful for the hospitality during the stay. A research grant from the Swedish Foundation for International Cooperation in Research and Higher Education (STINT) is also gratefully acknowledged.

1 Introduction

The empirical literature demonstrates that there are often large movements in nominal exchange rates that are apparently unexplained by macroeconomic fundamentals. Frankel and Froot (1990, p. 73), for example, write:

> "[...] the proportion of exchange rate movements that can be explained even after the fact, using contemporaneous macroeconomic variables, is disturbingly low".

It was Meese and Rogoff (1983) who first demonstrated that it is very difficult to explain systematically, much less predict, movements in nominal exchange rates. They found that at horizons up to one year, none of the foreign exchange models could outperform the predictions of a random walk model. Remarkably, this was true even when the predictions of the models were based on realized, and not predicted, values of the explanatory variables. Thus, the out-of-sample fit of the exchange rate models was extremely poor. Today, researchers have continued to find it difficult to firmly demonstrate systematic relationships between movements in nominal exchange rates and macroeconomic fundamentals.[1]

In a seminal paper by Dornbusch (1976), it was shown that large movements in nominal exchange rates could be consistent with perfect foresight. Specifically, the paper develops a theory of exchange rate movements under perfect capital mobility, a slow adjustment of the real sector relative to the monetary sector due to sticky goods prices, and an expectations formation that is consistent with the model. After a monetary disturbance, the exchange rate responds more strongly than necessary to maintain long-run equilibrium. The reason for this overshooting effect is the stickiness of goods prices since it initially restricts prices from making their required contribution to overall adjustment of the economy to long-run equilibrium.

However, which is also the origin of this paper, Dornbusch (1976) disregard from the fact that a very high proportion of chief foreign exchange dealers also use other tools than macroeconomic analysis in their currency trade. Taylor and Allen (1992), for example, conducted a questionnaire survey on the use of technical analysis among chief foreign exchange dealers based in London in 1988.[2] They found that at least 90 percent of the respondents reported placing some weight on technical analysis. As a consequence, a foreign exchange model that tries to mimic observed movements in nominal exchange rates must take this fact into account in order to be successful.

In Frankel and Froot (1986), three kinds of actors were introduced into an exchange rate model: fundamentalists, chartists and portfolio managers. The fundamentalists base their expectations about the future development of the exchange rate according to a model that consists of macroeconomic fundamentals only. The chartists use the history of past exchange rates to detect patterns that they extrapolate into the future, i.e., they use technical analysis. Being restricted to the use of technical analysis, however,

[1] See the recent discussion in the *Economic Journal* by Dixon (1999), Flood and Rose (1999), MacDonald (1999), and Rogoff (1999).

[2] Other related studies are by Lui and Mole (1998), Cheung and Wong (2000), and Cheung and Chinn (2001).

is not a shortcoming for the chartists since a primary assumption behind technical analysis is that all relevant information about the future development of the exchange rate is contained in the history of past exchange rates. Finally, the portfolio managers, the actors who buy and sell currencies, form their expectations about the future development of the exchange rate as a weighted average of the expectations of fundamentalists and chartists.[3]

The purpose of this paper is to integrate these three actors into the Dornbusch (1976) overshooting model in order to address the excess volatility of nominal exchange rates mentioned in the quote by Frankel and Froot (1990). The specific theoretical contribution is found in the way in which portfolio managers weight the expectations of fundamentalists and chartists. This consists of explicitly modelling the empirical observation that the relative importance of fundamental versus technical analysis in the foreign exchange market depends on the planning horizon. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons (e.g., Taylor and Allen, 1992).

In the model developed in this paper, the fundamentalists use the Dornbusch (1976) overshooting model when forming their expectations and the chartists use moving averages. The model used by the fundamentalists is, however, simplified since it renders the complete model a lot more transparent without qualitatively changing its major implications. Specifically, it is assumed that aggregate demand is not affected by changes in the interest rate. The chartists use moving averages since, in practice, it is the most common model used in technical analysis (e.g., Taylor and Allen, 1992). The question in focus in this paper is: how does exchange rate dynamics change relative to the benchmark case of the Dornbusch (1976) model when technical analysis is introduced into the model?

The results turn out to be quite pleasing. The perfect foresight path near long-run equilibrium is derived, and it is shown that the magnitude of exchange rate overshooting is larger than in the Dornbusch (1976) model. It is also shown that the extent of overshooting depends inversely on the planning horizon since for shorter horizons, more weight is placed on technical analysis and, in addition, technical analysis is a destabilizing force in the foreign exchange market. Technical analysis is a destabilizing force since the chartists expect that the exchange rate more and more will diverge from long-run equilibrium. In what way the perfect foresight planning horizon is affected by changes in the structural parameters in the model is also derived. Thus, the introduction of technical analysis into the Dornbusch (1976) model help to better explain the excess volatility of nominal exchange rates.

The remainder of this paper is organized as follows. The benchmark model and the expectations formations are presented in Section 2. The formal analysis of the model is carried out in Section 3, where the focus is on exchange rate dynamics near long-run equilibrium. Section 4 contains a discussion of the main results in the paper. Therein, it will be discussed in what way the model in this paper may shed

[3] Throughout this paper, we assume that it is the portfolio managers who do the currency trade. But without affecting the theoretical results in the paper, we could equally well assume that it is the fundamentalists and chartists who do the trade and, therefore, leave the portfolio managers out of account in the model.

some light on the fact that countries with flexible exchange rates have more volatile rates than countries with target zones, but equally volatile macroeconomic fundamentals (Flood and Rose, 1995 and 1999). The relationship in the model between exchange rate volatility and the interest rate semi-elasticity of money demand will also be emphasized. This connect on to the discussion within monetary economics whether the money demand function is stable or not, i.e., if the parameters in the demand function are constant over the time or if they change. Finally, we will discuss how the so-called Tobin tax on foreign exchange transactions probably will affect the volatility of exchange rates in the model.

2 Theoretical model

2.1 Benchmark model

The formal structure of the model is presented below. All variables, except the interest rates, are in natural logarithms, i.e., the model is linear in the logarithms. Greek letters denote positive structural parameters and $E(\cdot)$ is the expectations operator. The model consists of a real sector and a monetary sector, where the goods market constitutes the real sector, and the money and the international asset markets constitute the monetary sector.

Eqs. (1)-(2) below constitute the goods market, where eq. (1) is a Phillips curve without inflation expectations and eq. (2) describes the determinants of aggregate demand. The Phillips curve is

$$\frac{dp}{dt} = \alpha\left(y^d - y\right), \tag{1}$$

where $\frac{dp}{dt}$, y^d and y denote the domestic inflation rate, aggregate demand for domestic goods and aggregate supply of domestic goods, respectively. Goods prices are assumed to be sticky in the short-run. This means that they respond to market disequilibria, but not fast enough to eliminate the disequilibria instantly. Two extremes are obtained by letting $\alpha \to \infty$, which is the case of perfectly flexible prices, and by setting $\alpha = 0$, which is the case of completely rigid prices. A permanent and fully employed work force is assumed, which implies that fluctuations in demand for goods result only in price movements and not in output movements. The exclusion of inflation expectations in eq. (1) is motivated by the fact that the inflation rate is zero in long-run equilibrium, and that we investigate the dynamics of the exchange rate near long-run equilibrium.

Aggregate demand for domestic goods is

$$y^d = \beta(s - p) + \gamma y, \tag{2}$$

where s and p denote the spot exchange rate and the domestic price level, respectively. The exchange rate is defined as the amount of the domestic currency one has to pay for one unit of the foreign currency. Thus, a rising exchange rate indicates that the domestic currency is losing value. The first term in eq. (2) represents net exports which depend on the real exchange rate, $s - p$. The second term represents income-dependent demand for domestic goods. Contrary to the Dornbusch (1976) model, the dependence of aggregate demand on the domestic interest rate is not considered here. This renders the complete model a lot more transparent without qualitatively changing its major implications.

Eq. (3) constitutes the money market:

$$m = p + \delta y - \zeta i, \tag{3}$$

where m and i denote the domestic money supply and the domestic interest rate, respectively. The real money demand, $m - p$, depends on aggregate income and the domestic interest rate. The money market is assumed to be in permanent equilibrium, i.e., disturbances are immediately intercepted by a perfectly flexible interest rate.

Eq. (4) constitutes the international asset market:

$$i = i^* + E\left(\frac{ds}{dt}\right), \tag{4}$$

where i^* and $E\left(\frac{ds}{dt}\right)$ denote the foreign interest rate and the expected rate of change of the exchange rate, respectively. This asset market equilibrium condition, also known as uncovered interest parity, is based on the assumption that domestic and foreign assets are perfect substitutes, which can only be the case if there is perfect capital mobility. Since the capital mobility is assumed to be perfect, only the slightest difference in expected yields would draw the entire capital into the asset that offers the highest expected yield. Thus, the international asset market can only be in equilibrium if domestic assets offer the same expected yield as foreign assets. The equilibrium condition is maintained by the assumption of a perfectly flexible exchange rate.

2.2 Expectations formations

Empirically, it has been found that the relative importance of fundamental versus technical analysis in the foreign exchange market depends on the planning horizon. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons (e.g., Taylor and Allen, 1992). In this paper, we model this empirical observation in the following way:

$$E\left(\frac{ds}{dt}\right) = E_c\left(\frac{ds}{dt}\right)\exp(-\tau) + E_f\left(\frac{ds}{dt}\right)(1 - \exp(-\tau)), \tag{5}$$

where $E(\cdot)$, $E_c(\cdot)$ and $E_f(\cdot)$ denote expectations of portfolio managers, chartists and fundamentalists, respectively. $E(\cdot)$ is a weighted average of these expectations, where τ, the planning horizon, determines the weights.

The fundamentalists base their expectations about the future development of the exchange rate according to a model that consists of macroeconomic fundamentals only. The chartists use the history of past exchange rates to detect patterns that they extrapolate into the future. Being restricted to the use of technical analysis, however, is not a shortcoming for the chartists since a primary assumption behind technical analysis is that all relevant information about the future development of the exchange rate is contained in the history of past exchange rates. Finally, the portfolio managers, the actors who buy and sell currencies, form their expectations about the future development of the exchange rate as a weighted average of the expectations of fundamentalists and chartists.

The most common model used by chartists is the moving average model (e.g., Taylor and Allen, 1992). In this model, buying and selling signals are generated by two moving averages; a short-period and a long-period moving average, where a buy (sell) signal is generated when the short-period moving average rises above (falls below) the long-period moving average. In its simplest form, the short-period moving average is the current exchange rate and the long-period moving average is an exponential moving average of past exchange rates (e.g., Bishop and Dixon, 1992). Thus, the chartists expect an increase (a decrease) in the exchange rate when the current exchange rate is above (below) an exponential moving average of past exchange rates:

$$E_c\left(\frac{ds}{dt}\right) = \eta\left(s - MA\right), \tag{6}$$

where η and MA denote the expected adjustment speed of the exchange rate and an exponential moving average of past exchange rates, respectively. Moreover, the long-period moving average can be written as

$$MA\left(t\right) = \int_{-\infty}^{t} \upsilon\left(\mu\right) s\left(\mu\right) d\mu, \tag{7}$$

where

$$\int_{-\infty}^{t} \upsilon\left(\mu\right) d\mu = \int_{-\infty}^{t} \omega \exp\left(\omega\left(\mu - t\right)\right) d\mu = 1. \tag{8}$$

Finally, following Dornbusch (1976), the expectations of fundamentalists are

$$E_f\left(\frac{ds}{dt}\right) = \theta\left(\bar{s} - s\right), \tag{9}$$

where θ and $\bar{s}$ denote the expected adjustment speed of the exchange rate and the long-run exchange rate, respectively.

3 Formal analysis of the model

Since the long-period moving average in eqs. (7)-(8) is a function of the infinite history of past exchange rates, the complete model is rather hard to analyze. This is still true even if eqs. (7)-(8) can be rewritten as the current exchange rate plus an infinite series of time derivatives (of increasing orders) of the current exchange rate (see Proposition 4 in this paper). However, by assuming that the economy has, for a long time, been in long-run equilibrium before a monetary disturbance occurs, the moving average in eqs. (7)-(8) is approximately equal to the long-run exchange rate. This assumption makes the model much more tractable to analyze. However, a saddle-path stability result is also derived for the more general case when it is not assumed that the economy is in long-run equilibrium when a monetary disturbance occurs.

3.1 Long-run equilibrium

If we assume that the economy has been in long-run equilibrium for a long time, the long-period moving average in eqs. (7)-(8) is approximately equal to the long-run exchange rate:

$$MA(t) \approx \int_{-\infty}^{t} \upsilon(\mu)\,\bar{s}\,d\mu = \bar{s}\int_{-\infty}^{t} \upsilon(\mu)\,d\mu = \bar{s}\,. \tag{10}$$

Substitution of eq. (10) into the expectations of chartists, i.e., eq. (6), then yields

$$E_c\left(\frac{ds}{dt}\right) \approx \eta\left(s - \bar{s}\right), \tag{11}$$

i.e., chartists expect an increase (a decrease) in the exchange rate when the current exchange rate is above (below) the long-run exchange rate. Thus, chartists expect that the exchange rate more and more will diverge from long-run equilibrium. As a consequence, technical analysis is a destabilizing force in the foreign exchange market.

Substitution of the expectations of fundamentalists and chartists, i.e., eq. (9) and eq. (11) (assuming equality in the equation), into the expectations formation in eq. (5) then yields

$$E\left(\frac{ds}{dt}\right) = \eta\left(s - \bar{s}\right)\exp\left(-\tau\right) + \theta\left(\bar{s} - s\right)\left(1 - \exp\left(-\tau\right)\right). \tag{12}$$

Thus, the portfolio managers expect a constant exchange rate in long-run equilibrium, i.e., $E\left(\frac{ds}{dt}\right) = 0$ when $s = \bar{s}$, which also means, according to the asset market equilibrium in eq. (4), that the domestic and foreign interest rates are equal in long-run equilibrium.

The equations that describe the money and the international asset markets, i.e., eqs. (3)-(4), can be solved to yield the price level in long-run equilibrium, i.e., $\bar{p}$, if the aforementioned equality of the interest rates in long-run equilibrium is used:

$$\bar{p} = m - \delta y + \zeta i^{*}. \tag{13}$$

Thus, the quantity theory of money holds in the long-run since $\frac{d\bar{p}}{dm} = 1$. Furthermore, if we evaluate the equations that describe the goods market, i.e., eqs. (1)-(2), in long-run equilibrium and note that the price level is constant in the long-run, i.e., $\frac{dp}{dt} = 0$, the exchange rate in long-run equilibrium can be solved to yield

$$\bar{s} = \bar{p} + \frac{1-\gamma}{\beta}y. \tag{14}$$

Thus, purchasing-power parity holds in the long-run since $\frac{d\bar{s}}{d\bar{p}} = 1$.

3.2 Stability condition and adjustment to long-run equilibrium

In order to derive the stability condition for the model near long-run equilibrium, we start by combining the equations that describe the money and the international asset markets, i.e., eqs. (3)-(4), with the expectations formation in eq. (12) and the price level in long-run equilibrium in eq. (13):

$$s = \bar{s} + \frac{p - \bar{p}}{\zeta\left(\eta + \theta\right)\exp\left(-\tau\right) - \zeta\theta}, \quad \tau \neq \log\left(1 + \frac{\eta}{\theta}\right). \tag{15}$$

Thereafter, we rewrite the equations that describe the goods market, i.e., eqs. (1)-(2), by using the exchange rate in long-run equilibrium in eq. (14) and the relationship between the exchange rate and the price level in eq. (15):

$$\frac{dp}{dt} = \underbrace{\left(\frac{\alpha\beta}{\zeta(\eta+\theta)\exp(-\tau)-\zeta\theta} - \alpha\beta\right)}_{\equiv r_0(\tau)} \left(p - \bar{p}\right), \quad \tau \neq \log\left(1+\frac{\eta}{\theta}\right). \tag{16}$$

The price adjustment equation in eq. (16) can be solved to yield

$$p(t) = \bar{p} + \left(p(0) - \bar{p}\right)\exp\left(r_0(\tau)t\right), \quad \tau \neq \log\left(1+\frac{\eta}{\theta}\right). \tag{17}$$

Thus, after a change in the money supply, the price level begins to adjust to the new long-run equilibrium at the rate $r_0(\tau)$. Then, substitution of eq. (17) into eq. (15) gives the time path of the exchange rate:

$$s(t) = \bar{s} + \left(s(0) - \bar{s}\right)\exp\left(r_0(\tau)t\right), \quad \tau \neq \log\left(1+\frac{\eta}{\theta}\right). \tag{18}$$

Of course, the exchange rate adjusts to the new long-run equilibrium at the same rate as the price level.

The model is stable if both the exchange rate and the price level, after a monetary disturbance, do converge to long-run equilibrium. Thus, the stability condition is

$$r_0(\tau) < 0, \tag{19}$$

or, if solving for the planning horizon,

$$\begin{cases} \tau > \log\left(1+\frac{\eta}{\theta}\right) \\ \tau < \log\frac{\zeta\eta+\zeta\theta}{1+\zeta\theta} \end{cases}. \tag{20}$$

The latter inequality in eq. (20) is relevant only when $\zeta\eta > 1$ since the planning horizon must be non-negative. However, as will clear in the next section, by assuming that the portfolio managers have consistent expectations, the latter inequality in eq. (20) will be ruled out.

3.3 Consistent expectations

It is important that the expectations of portfolio managers are not arbitrary and, given the model, do not involve persistent prediction errors. On the other hand, the expectations of chartists do involve persistent prediction errors.[4] This is not surprising since they do not use an economic model to predict exchange rate movements. Instead, they use the history of past exchange rates. Moreover, since it is not possible for both fundamentalists and portfolio managers to have consistent expectations in the same model, we focus on the portfolio managers, the actors who buy and sell currencies, and assume that they have model consistent expectations. However, the fundamentalists still correctly predict the exchange rate and the price level in long-run equilibrium.[5]

[4] It will be clear in the section on saddle-path stability that the expectations of chartists do involve persistent prediction errors since, according to eq. (37), $E\left(\frac{ds}{dt}\right) \neq \frac{ds}{dt}$.

[5] Mark (1995) has presented empirical evidence that there is an economically significant predictable component in long-horizon changes in nominal exchange rates. See also the critique by Berkowitz and Giorgianni (2001) of Mark's (1995) findings.

Clearly, for the expectations formation in eq. (12) to correctly predict the path of the exchange rate, it must be true that

$$E\left(\frac{ds}{dt}\right) = \frac{ds}{dt}, \tag{21}$$

which is to say that the portfolio managers have perfect foresight, the deterministic equivalent of rational expectations.[6] By assuming perfect foresight, the expectations formation in eq. (12) can be rewritten as

$$\frac{ds}{dt} = \underbrace{\left((\eta + \theta)\exp(-\tau) - \theta\right)}_{\equiv r_1(\tau)} \left(s - \bar{s}\right). \tag{22}$$

Thus, the condition for the exchange rate to converge to long-run equilibrium along the perfect foresight path is that

$$r_1(\tau) < 0, \tag{23}$$

or

$$\tau > \log\left(1 + \frac{\eta}{\theta}\right). \tag{24}$$

Accordingly, the planning horizon that corresponds to perfect foresight, i.e., τ_{pf}, or, equivalently, that is consistent with the model is given by the solution to

$$\frac{\alpha\beta}{\zeta(\eta + \theta)\exp(-\tau_{pf}) - \zeta\theta} - \alpha\beta = (\eta + \theta)\exp(-\tau_{pf}) - \theta, \tag{25}$$

where the left-hand and right-hand sides of eq. (25) are the convergence rates of the exchange rate to long-run equilibrium in eq. (16) and eq. (22), i.e., $r_0(\tau_{pf}) = r_1(\tau_{pf}) = r(\tau_{pf})$.[7]

It is clear that the fundamentalists and portfolio managers cannot simultaneously have consistent expectations since, according to eq. (9) and eqs. (21)-(22), $-\theta \neq (\eta + \theta)\exp(-\tau) - \theta$. Instead, the fundamentalists consistently predict a faster adjustment speed of the exchange rate to long-run equilibrium than the actual adjustment speed.[8] The exception is when the planning horizon is infinitely long, i.e., $\tau \to \infty$, and the portfolio managers place no weight on technical analysis when predicting the future development of the exchange rate. However, the fundamentalists always correctly predict the exchange rate in long-run equilibrium.

The general solution of eq. (25) is

$$\tau_{pf} = f(\alpha, \beta, \gamma, \delta, \zeta, \eta, \theta). \tag{26}$$

Thus, the perfect foresight planning horizon is a function of the structural parameters in the model and, therefore, endogenously determined within the model.

Proposition 1 The perfect foresight planning horizon depends on the structural parameters in the model in the following way:

$$\begin{cases} \frac{d\tau_{pf}}{d\alpha} > 0, & \frac{d\tau_{pf}}{d\beta} > 0, & \frac{d\tau_{pf}}{d\gamma} = 0, & \frac{d\tau_{pf}}{d\delta} = 0, \\ \frac{d\tau_{pf}}{d\zeta} < 0, & \frac{d\tau_{pf}}{d\eta} > 0, & \frac{d\tau_{pf}}{d\theta} < 0. \end{cases} \tag{27}$$

[6] Eq. (21) also means that market expectations are characterized by perfect foresight.
[7] Recall that the price level adjusts to long-run equilibrium at the same rate as the exchange rate.
[8] Note that a lower $r(\tau)$ corresponds to a faster adjustment speed.

Proof. See Appendix B for a proof. ■

According to Proposition 1, the perfect foresight planning horizon is longer, the faster goods prices respond to market disequilibria (α), the stronger the demand for goods responds to changes in the real exchange rate (β), and the larger the expected adjustment speed of the exchange rate is according to the chartists (η). Thus, in these cases, the portfolio managers place more and more weight on fundamental analysis when forecasting exchange rate movements. The opposite is true when the demand for money responds stronger to changes in the interest rate (ζ), and the larger the expected adjustment speed of the exchange rate is according to the fundamentalists (θ). In these cases, the portfolio managers place more and more weight on technical analysis when forecasting exchange rate movements. The perfect foresight planning horizon is not affected by changes in the response of goods demand and money demand to changes in income (γ and δ).

3.4 Overshooting

Substitution of the price level and the exchange rates in long-run equilibrium, i.e., eqs. (13)-(14), into the relationship between the exchange rate and the price level in eq. (15), and differentiating the resulting equation with respect to the money supply, keeping the price level constant, gives

$$\left.\frac{ds}{dm}\right|_{long-run\,equilibrium} = 1 + \underbrace{\frac{1}{\zeta\theta - \zeta\left(\eta+\theta\right)\exp\left(-\tau_{pf}\right)}}_{\equiv o(\tau_{pf})}, \quad \tau_{pf} > \log\left(1+\frac{\eta}{\theta}\right). \tag{28}$$

The price level is hold constant when deriving eq. (28) since it is assumed to be sticky in the short-run. Thus, eq. (28) describes the short-run impact on the exchange rate near long-run equilibrium, given perfect foresight, of a change in the money supply.[9]

By letting $\tau \to \infty$, the expectations of portfolio managers coincide with the expectations of fundamentalists. Therefore, the equation describing exchange rate overshooting in Dornbusch (1976) is obtained:

$$\left.\frac{ds}{dm}\right|_{Dornbusch\,(1976)} = 1 + \frac{1}{\zeta\theta} > 1. \tag{29}$$

In this case, the magnitude of exchange rate overshooting depends on the interest rate response of money demand (ζ), and the expected adjustment speed of the exchange rate according to the fundamentalists (θ).

According to the stability condition and the condition for perfect foresight, i.e., eq. (20) and eq. (24), the planning horizon must be greater than $\log\left(1+\frac{\eta}{\theta}\right)$. This implies that the extent of exchange rate overshooting is even larger in this model than in the Dornbusch (1976) model:

$$\left.\frac{ds}{dm}\right|_{\substack{long-run\,equilibrium,\\ \tau_{pf}\in\left(\log\left(1+\frac{\eta}{\theta}\right),\infty\right)}} \geq \left.\frac{ds}{dm}\right|_{Dornbusch\,(1976)}. \tag{30}$$

[9] The overshooting phenomenon is investigated in Appendix A for the more general case when it is not assumed that the economy is in long-run equilibrium when a monetary disturbance occurs. However, model consistent expectations are not assumed in the derivations.

Moreover, the magnitude of overshooting depends inversely on the planning horizon:

$$\left.\frac{ds}{dm}\right|_{\substack{long-run\,equilibrium,\\ \tau_{pf}=\tau_0}} > \left.\frac{ds}{dm}\right|_{\substack{long-run\,equilibrium,\\ \tau_{pf}=\tau_1}}, \quad \tau_1 > \tau_0 > \log\left(1+\frac{\eta}{\theta}\right). \tag{31}$$

The extent of overshooting depends inversely on the planning horizon since for shorter horizons, more weight is placed on technical analysis and, in addition, technical analysis is a destabilizing force in the foreign exchange market. Technical analysis is a destabilizing force since the chartists expect that the exchange rate more and more will diverge from long-run equilibrium.

In the short-run, before goods prices have time to react, the exchange rate will rise more than the money supply and, thus, more than is necessary to bring the exchange rate to long-run equilibrium. This means that even though purchasing-power parity holds in the long-run, it does not hold in the short-run. However, after the monetary disturbance and the initial overshooting of the exchange rate, the price level and the exchange rate begin to adjust to the new long-run equilibrium according to eqs. (17)-(18).

Proposition 2 The magnitude of exchange rate overshooting, given perfect foresight, depends on the structural parameters in the model in the following way:

$$\begin{cases} \frac{do(\tau_{pf})}{d\alpha} < 0, & \frac{do(\tau_{pf})}{d\beta} < 0, & \frac{do(\tau_{pf})}{d\gamma} = 0, & \frac{do(\tau_{pf})}{d\delta} = 0, \\ \frac{do(\tau_{pf})}{d\zeta} < 0, & \frac{do(\tau_{pf})}{d\eta} = 0, & \frac{do(\tau_{pf})}{d\theta} = 0. \end{cases} \tag{32}$$

Proof. See Appendix B for a proof. ■

Proposition 3 The adjustment speed of the exchange rate to long-run equilibrium, given perfect foresight, depends on the structural parameters in the model in the following way:

$$\begin{cases} \frac{dr(\tau_{pf})}{d\alpha} < 0, & \frac{dr(\tau_{pf})}{d\beta} < 0, & \frac{dr(\tau_{pf})}{d\gamma} = 0, & \frac{dr(\tau_{pf})}{d\delta} = 0, \\ \frac{dr(\tau_{pf})}{d\zeta} > 0, & \frac{dr(\tau_{pf})}{d\eta} = 0, & \frac{dr(\tau_{pf})}{d\theta} = 0. \end{cases} \tag{33}$$

Proof. See Appendix B for a proof. ■

According to Propositions 2 and 3, the expected adjustment speeds of the exchange rate (η and θ) do not affect the magnitude of exchange rate overshooting nor the adjustment speed of the exchange rate to long-run equilibrium. If, for example, the fundamentalists believe that the adjustment speed (θ) will increase, the perfect foresight planning horizon will, according to Proposition 1, decrease. As a consequence, the extent of exchange rate overshooting will, according to eq. (31), increase. This is the indirect effect of an increased expected adjustment speed. However, the direct effect of an increased expected adjustment speed is that the magnitude of overshooting will decrease:

$$\frac{do\,(\tau = \text{constant})}{d\theta} = -\frac{\zeta\,(1-\exp(-\tau))}{(\zeta\theta - \zeta\,(\eta+\theta)\exp(-\tau))^2} < 0. \tag{34}$$

Hence, there are two effects that cancel out; a direct effect and an indirect effect via a change in the perfect foresight planning horizon. A similar argument applies to a changed expected adjustment speed of the exchange rate according to the chartists (η).

There is also a direct and an indirect effect of a change in the interest rate response of money demand (ζ). The indirect effect of a stronger interest rate response is that the perfect foresight planning horizon will, according to Proposition 1, decrease. As a consequence, the extent of exchange rate overshooting will, according to eq. (31), increase. Turning to the direct effect, however, the magnitude of overshooting will decrease:

$$\frac{do\,(\tau = \text{constant})}{d\zeta} = -\frac{\overbrace{\theta - (\eta + \theta)\exp(-\tau)}^{=-r_1(\tau)>0}}{(\zeta\theta - \zeta(\eta + \theta)\exp(-\tau))^2} < 0. \tag{35}$$

Taken all in all, a stronger interest rate response of money demand will, according to Proposition 2, decrease the extent of overshooting. Also, the adjustment speed of the exchange rate towards long-run equilibrium will, according to Proposition 3, decrease.

Furthermore, the magnitude of exchange rate overshooting will decrease and the adjustment speed of the exchange rate to long-run equilibrium will increase, the faster goods prices respond to market disequilibria (α), and the stronger the demand for goods responds to changes in the real exchange rate (β). Since the extent of overshooting does not directly depend on these parameters, the effects are only indirect via a change in the perfect foresight planning horizon. The magnitude of overshooting and the adjustment speed are, however, not affected by changes of the response of goods demand and money demand to changes in income (γ and δ).

3.5 Saddle-path stability

In general, we cannot assume that the long-period moving average is equal to the long-run exchange rate as is done in eq. (10). Instead, the moving average in eqs. (7)-(8) is a function of the infinite history of past exchange rates. Unfortunately, this makes the model more intractable to analyze than when the economy is near long-run equilibrium. However, according to Proposition 4, eqs. (7)-(8) can be rewritten as the current exchange rate plus an infinite series of time derivatives (of increasing orders) of the current exchange rate. The advantage of this rewriting of the long-period moving average is that the derivation of the saddle-path stability result is simplified.

Proposition 4 The exponential moving average in eqs. (7)-(8) can be written as

$$MA(t) = s(t) + \sum_{i=1}^{\infty} (-1)^i \frac{1}{\omega^i} \frac{d^i s(t)}{dt^i}. \tag{36}$$

Proof. See Appendix B for a proof. ■

Then, substitution of eq. (36) into eq. (6) yields

$$E_c\left(\frac{ds}{dt}\right) = -\eta \sum_{i=1}^{\infty} (-1)^i \frac{1}{\omega^i} \frac{d^i s}{dt^i}, \tag{37}$$

which can be approximated by

$$E_c\left(\frac{ds}{dt}\right) \approx \frac{\eta}{\omega}\frac{ds}{dt} - \frac{\eta}{\omega^2}\frac{d^2 s}{dt^2} + \frac{\eta}{\omega^3}\frac{d^3 s}{dt^3}, \tag{38}$$

if we assume that $\omega > 1$, i.e., more weight is placed on derivatives of lower orders than on derivatives of higher orders, which means that the long-period moving average more strongly depends on recent exchange rates than when $0 < \omega < 1$ holds. The approximation in eq. (38) is used when deriving the saddle-path stability result since eq. (37) still makes the complete model rather hard to analyze.

Proposition 5 When the expectations of chartists is described by eq. (38), the model is characterized by saddle-path stability.

Proof. See Appendix B for a proof. ■

According to eq. (37), but also eq. (38), the expectations of chartists do involve persistent prediction errors since $E_c\left(\frac{ds}{dt}\right) \neq \frac{ds}{dt}$. This is not surprising since they do not use an economic model to predict exchange rate movements. Instead, they use the history of past exchange rates.

4 Discussion

Exchange rates are excessively volatile, i.e., movements in nominal exchange rates are larger than movements in macroeconomic fundamentals. Moreover, a very high proportion of chief foreign exchange dealers view fundamental and technical analysis as complementary forms of analysis. Specifically, it has been found that the relative importance of fundamental versus technical analysis depends on the planning horizon. For shorter planning horizons, more weight is placed on technical analysis, while more weight is placed on fundamental analysis for longer planning horizons (e.g., Taylor and Allen, 1992).

The point of departure in this paper was to implement this empirical observation theoretically into the Dornbusch (1976) model. The results turned out to be quite pleasing. The perfect foresight path near long-run equilibrium was derived, and it was shown that the magnitude of exchange rate overshooting is larger than in the Dornbusch (1976) model. Moreover, the relative importance of fundamental versus technical analysis implied that the extent of overshooting depends inversely on the planning horizon, where the latter is endogenously determined within the model. Thus, the introduction of chartists helped to better explain the excess volatility of nominal exchange rates.

The model developed in this paper may also shed some light on the fact that, as a first approximation, countries with flexible exchange rates have more volatile rates than countries with target zones, but equally volatile fundamentals (Flood and Rose, 1999). In fact, the volatility of macroeconomic fundamentals such as money and output do not change much across currency regimes (Flood and Rose, 1995). Therefore, Flood and Rose (1995, p. 5) suggest that macroeconomic fundamentals alone are unable to explain exchange rate volatility:[10]

> "Intuitively, if exchange rate stability varies across regimes without corresponding variation in macroeconomic volatility, then macroeconomic variables will be unable to explain much exchange rate volatility".

However, by making a clear distinction between the volatility of the exchange rate and the stability of the dynamic system generating the exchange rate, one may resolve the apparent paradox that nominal

[10] "Exchange rate volatility" and "exchange rate stability" are used synonymously by Flood and Rose (1995).

exchange rates have become more volatile while macroeconomic fundamentals have not.[11] Thus, a volatile exchange rate may also be the manifestation of a less stable dynamic system, and not only of volatile fundamentals. In the context of this paper, the dynamic system is more (less) stable when the magnitude of exchange rate overshooting, given the size of the monetary disturbance, is smaller (larger).

Since it is reasonable to think that prices (and wages) are more rigid in countries with flexible exchange rates than in countries with target zones, at least in countries with extensive foreign trade, there is more pressure on the exchange rate to adjust to retain the competitiveness of domestic goods. Therefore, the extent of exchange rate overshooting, according to Proposition 2, is larger in countries with flexible exchange rates than in countries with target zones.[12] Thus, even if the volatility of money supply due to shocks is unchanged, the model predicts a more volatile exchange rate in countries with flexible exchange rates than in countries with target zones. It should be noted, however, that it is necessary to investigate the relationship between exchange rate mechanisms and the volatility of exchange rates in a model that explicitly address the problem.

The model in this paper may also be related to the discussion within monetary economics whether the money demand function is stable or not, i.e., if the parameters in the demand function are constant over the time or if they change. Specifically, the magnitude of exchange rate overshooting, according to Proposition 2, is smaller when the demand for money responds stronger to changes in the interest rate (ζ), i.e., when the interest rate semi-elasticity of money demand ($-\zeta$) is larger in absolute value. Thus, even if the volatility of money supply due to shocks is unchanged, the model predicts a less volatile exchange rate during periods of time when the interest rate semi-elasticity of money demand ($-\zeta$) is larger in absolute value. Ball (2001), who investigates the long-run money demand in the U.S., asserts that the interest rate semi-elasticity of money demand ($-\zeta$) is, in absolute value, smaller in the post-war period than what it was in the pre-war period. Thus, in the context of the model in this paper, the exchange rate should be more volatile in the post-war period than what it was in the pre-war period. It is a matter for empirical research to investigate the relationship between exchange rate volatility and the interest rate semi-elasticity of money demand.

As a final example, the model in this paper may also support the idea that the so-called Tobin tax on foreign exchange transactions will reduce the volatility of exchange rates. The argument is that a tax will make currency trading more expensive and, thus, probably reduce the most shortsighted transactions. As a consequence, the planning horizon will be longer and, according to eq. (31), the volatility smaller. But also in this case, it is necessary to develope a model that explicitly address the problem.

[11] In Bask and de Luna (2002), it is shown how the stability properties of non-linear dynamic systems may be characterized and studied, where the degree of stability is defined by the effects of exogenous shocks on the evolution of the observed stochastic system. It is emphasized that the stability of a dynamic system should be considered when the volatility of a variable is studied. Bask and de Luna (2002) also illustrate how the presented framework can be used to study the degree of stability and volatility of an exchange rate.

[12] Recall that by letting $\alpha \rightarrow 0$, prices become more rigid.

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Appendix A

The overshooting phenomenon is investigated in this Appendix for the more general case when it is not assumed that the economy is in long-run equilibrium when a monetary disturbance occurs. Thus, it is not assumed that the long-period moving average in eqs. (7)-(8) is equal to the long-run exchange rate as is done in eq. (10). However, model consistent expectations are not assumed in the derivations.

Now, substitute the expectations of chartists and fundamentalists, i.e., eq. (6) and eq. (9), into the expectations formation in eq. (5):

$$E\left(\frac{ds}{dt}\right) = \eta\,(s - MA)\exp(-\tau) + \theta\left(\bar{s} - s\right)(1 - \exp(-\tau))\,. \tag{A.1}$$

Then, substitute the equations that describe the money and the international asset markets, i.e., eqs. (3)-(4), and the price level in long-run equilibrium in eq. (13) into eq. (A.1), and differentiate the resulting equation with respect to the exchange rate, the money supply and the long-period moving average, keeping the price level constant, and noting that $d\,\bar{s} = d\,\bar{p} = dm$:

$$\frac{ds}{dm} = \frac{1 + \zeta\theta - \zeta\left(\eta\frac{dMA}{dm} + \theta\right)\exp(-\tau)}{\zeta\theta - \zeta\,(\eta + \theta)\exp(-\tau)}, \quad \tau \neq \log\left(1 + \frac{\eta}{\theta}\right). \tag{A.2}$$

The price level is hold constant when deriving eq. (A.2) since it is assumed to be sticky in the short-run. Then, since

$$\begin{aligned} \frac{dMA}{dm} &= \frac{dMA}{ds}\frac{ds}{dm} = \frac{d}{ds}\int_{-\infty}^{t} \upsilon(\mu)\, s(\mu)\, d\mu \cdot \frac{ds}{dm} \\ &= \int_{-\infty}^{t} \upsilon(\mu)\frac{ds(\mu)}{ds} d\mu \cdot \frac{ds}{dm} = \int_{-\infty}^{t} \upsilon(\mu)\, d\mu \cdot \frac{ds}{dm} = \frac{ds}{dm}, \end{aligned} \tag{A.3}$$

eq. (A.2) can be rewritten as

$$\frac{ds}{dm} = \frac{1 + \zeta\theta - \zeta\left(\eta\frac{ds}{dm} + \theta\right)\exp(-\tau)}{\zeta\theta - \zeta\,(\eta + \theta)\exp(-\tau)}, \quad \tau \neq \log\left(1 + \frac{\eta}{\theta}\right), \tag{A.4}$$

which, if solved for $\frac{ds}{dm}$, gives

$$\frac{ds}{dm} = 1 + \frac{1}{\zeta\theta\,(1 - \exp(-\tau))}, \quad \tau \neq 0. \tag{A.5}$$

Eq. (A.5) describes the short-run impact on the exchange rate of a change in the money supply.

Recall that we no longer assume that the economy is in long-run equilibrium when a monetary disturbance occurs. This also explains why the expression for exchange rate overshooting is different in eq. (28) and eq. (A.5).

Appendix B

Proof of Proposition 1 Let

$$x_0(\tau) \equiv (\eta + \theta)\exp(-\tau) - \theta. \tag{B.1}$$

Then, according to eq. (25), the perfect foresight planning horizon satisfies

$$\frac{\alpha\beta}{\zeta x_0(\tau_{pf})} - \alpha\beta = x_0(\tau_{pf}), \tag{B.2}$$

which has the solution

$$x_0(\tau_{pf}) = -\frac{\alpha\beta}{2} \pm \sqrt{\frac{\alpha^2\beta^2}{4} + \frac{\alpha\beta}{\zeta}}. \tag{B.3}$$

However, since the perfect foresight planning horizon has to satisfy $\tau_{pf} > \log\left(1 + \frac{\eta}{\theta}\right)$, which implies that $x_0(\tau_{pf}) < 0$, it follows that

$$x_0(\tau_{pf}) = -\frac{\alpha\beta}{2} - \sqrt{\frac{\alpha^2\beta^2}{4} + \frac{\alpha\beta}{\zeta}}. \tag{B.4}$$

The perfect foresight planning horizon can be solved for by combining eq. (B.1) and eq. (B.4):

$$\tau_{pf} = -\log\left(\frac{\theta}{\eta+\theta} - \frac{\alpha\beta}{2(\eta+\theta)} - \sqrt{\frac{\alpha^2\beta^2}{4(\eta+\theta)^2} + \frac{\alpha\beta}{\zeta(\eta+\theta)^2}}\right) = \log \underbrace{\frac{2\sqrt{\zeta}(\eta+\theta)}{\sqrt{\zeta}(2\theta-\alpha\beta) - \sqrt{4+\alpha\beta\zeta}\sqrt{\alpha\beta}}}_{\equiv x_1}, \tag{B.5}$$

where $x_1 > 1$ since $\tau_{pf} > 0$. Then,

$$\begin{aligned} \frac{d\tau_{pf}}{d\alpha} &= \frac{d\tau_{pf}}{dx_0(\tau_{pf})}\frac{dx_0(\tau_{pf})}{d\alpha} = \left(\frac{dx_0(\tau_{pf})}{d\tau_{pf}}\right)^{-1}\frac{dx_0(\tau_{pf})}{d\alpha} \\ &= \underbrace{\left(-(\eta+\theta)\exp(-\tau_{pf})\right)^{-1}}_{<0} \cdot \underbrace{\left(-\frac{\beta}{2} - \frac{\frac{2\alpha\beta^2}{4} + \frac{\beta}{\zeta}}{2\sqrt{\frac{\alpha^2\beta^2}{4} + \frac{\alpha\beta}{\zeta}}}\right)}_{<0} > 0, \end{aligned} \tag{B.6}$$

$$\begin{aligned} \frac{d\tau_{pf}}{d\beta} &= \frac{d\tau_{pf}}{dx_0(\tau_{pf})}\frac{dx_0(\tau_{pf})}{d\beta} = \left(\frac{dx_0(\tau_{pf})}{d\tau_{pf}}\right)^{-1}\frac{dx_0(\tau_{pf})}{d\beta} \\ &= \underbrace{\left(-(\eta+\theta)\exp(-\tau_{pf})\right)^{-1}}_{<0} \cdot \underbrace{\left(-\frac{\alpha}{2} - \frac{\frac{2\alpha^2\beta}{4} + \frac{\alpha}{\zeta}}{2\sqrt{\frac{\alpha^2\beta^2}{4} + \frac{\alpha\beta}{\zeta}}}\right)}_{<0} > 0, \end{aligned} \tag{B.7}$$

and

$$\begin{aligned} \frac{d\tau_{pf}}{d\zeta} &= \frac{d\tau_{pf}}{dx_0(\tau_{pf})}\frac{dx_0(\tau_{pf})}{d\zeta} = \left(\frac{dx_0(\tau_{pf})}{d\tau_{pf}}\right)^{-1}\frac{dx_0(\tau_{pf})}{d\zeta} \\ &= \underbrace{\left(-(\eta+\theta)\exp(-\tau_{pf})\right)^{-1}}_{<0} \cdot \underbrace{\left(-\frac{-\frac{\alpha\beta}{\zeta^2}}{2\sqrt{\frac{\alpha^2\beta^2}{4} + \frac{\alpha\beta}{\zeta}}}\right)}_{>0} < 0. \end{aligned} \tag{B.8}$$

The expression for the perfect foresight planning horizon in eq. (B.5) has the derivatives

$$\frac{d\tau_{pf}}{d\gamma} = 0, \tag{B.9}$$

$$\frac{d\tau_{pf}}{d\delta} = 0, \tag{B.10}$$

$$\frac{d\tau_{pf}}{d\eta} = \frac{1}{x_1}\frac{dx_1}{d\eta} = \frac{1}{x_1}\frac{2\sqrt{\zeta}}{\sqrt{\zeta}\,(2\theta-\alpha\beta)-\sqrt{4+\alpha\beta\zeta}\sqrt{\alpha\beta}} = \frac{1}{x_1}\frac{x_1}{\eta+\theta} = \frac{1}{\eta+\theta} > 0, \tag{B.11}$$

and

$$\begin{aligned}\frac{d\tau_{pf}}{d\theta} &= \frac{1}{x_1}\frac{dx_1}{d\theta} = \frac{1}{x_1}\frac{2\sqrt{\zeta}\cdot\left(\sqrt{\zeta}\,(2\theta-\alpha\beta)-\sqrt{4+\alpha\beta\zeta}\sqrt{\alpha\beta}\right)-2\sqrt{\zeta}\,(\eta+\theta)\cdot 2\sqrt{\zeta}}{\left(\sqrt{\zeta}\,(2\theta-\alpha\beta)-\sqrt{4+\alpha\beta\zeta}\sqrt{\alpha\beta}\right)^2} \\ &= \frac{1}{x_1}\left(\frac{x_1}{\eta+\theta}-x_1\cdot\frac{x_1}{\eta+\theta}\right) = \frac{1-x_1}{\eta+\theta} < 0,\end{aligned} \tag{B.12}$$

and the proof is completed. ■

Proof of Proposition 2 According to eq. (28) and eqs. (B.1)-(B.2),

$$o(\tau_{pf}) \equiv \frac{1}{\zeta\theta-\zeta(\eta+\theta)\exp(-\tau_{pf})} = -\frac{1}{\zeta x_0(\tau_{pf})} = -1-\frac{x_0(\tau_{pf})}{\alpha\beta}, \tag{B.13}$$

which has the derivatives

$$\frac{do(\tau_{pf})}{d\alpha} = \underbrace{\frac{1}{(\zeta x_0(\tau_{pf}))^2}}_{>0}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\alpha}}_{<0} < 0, \tag{B.14}$$

$$\frac{do(\tau_{pf})}{d\beta} = \underbrace{\frac{1}{(\zeta x_0(\tau_{pf}))^2}}_{>0}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\beta}}_{<0} < 0, \tag{B.15}$$

$$\frac{do(\tau_{pf})}{d\gamma} = -\frac{1}{\alpha\beta}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\gamma}}_{=0} = 0, \tag{B.16}$$

$$\frac{do(\tau_{pf})}{d\delta} = -\frac{1}{\alpha\beta}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\delta}}_{=0} = 0, \tag{B.17}$$

$$\frac{do(\tau_{pf})}{d\zeta} = \underbrace{-\frac{1}{\alpha\beta}}_{<0}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\zeta}}_{>0} < 0, \tag{B.18}$$

$$\frac{do(\tau_{pf})}{d\eta} = -\frac{1}{\alpha\beta}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\eta}}_{=0} = 0, \tag{B.19}$$

and

$$\frac{do(\tau_{pf})}{d\theta} = -\frac{1}{\alpha\beta}\cdot\underbrace{\frac{dx_0(\tau_{pf})}{d\theta}}_{=0} = 0, \tag{B.20}$$

where results in the proof of Proposition 1 are utilized in the derivations, and the proof is completed. ■

Proof of Proposition 3 According to eq. (22) and the fact that $r_1(\tau_{pf}) = r(\tau_{pf})$,

$$r(\tau_{pf}) \equiv (\eta+\theta)\exp(-\tau_{pf}) - \theta = x_0(\tau_{pf}), \tag{B.21}$$

which has the derivatives

$$\frac{dr(\tau_{pf})}{d\alpha} = \frac{dx_0(\tau_{pf})}{d\alpha} < 0, \tag{B.22}$$

$$\frac{dr(\tau_{pf})}{d\beta} = \frac{dx_0(\tau_{pf})}{d\beta} < 0, \tag{B.23}$$

$$\frac{dr(\tau_{pf})}{d\gamma} = \frac{dx_0(\tau_{pf})}{d\gamma} = 0, \tag{B.24}$$

$$\frac{dr(\tau_{pf})}{d\delta} = \frac{dx_0(\tau_{pf})}{d\delta} = 0, \tag{B.25}$$

$$\frac{dr(\tau_{pf})}{d\zeta} = \frac{dx_0(\tau_{pf})}{d\zeta} > 0, \tag{B.26}$$

$$\frac{dr(\tau_{pf})}{d\eta} = \frac{dx_0(\tau_{pf})}{d\eta} = 0, \tag{B.27}$$

and

$$\frac{dr(\tau_{pf})}{d\theta} = \frac{dx_0(\tau_{pf})}{d\theta} = 0, \tag{B.28}$$

where results in the proof of Proposition 1 are utilized in the derivations, and the proof is completed. ■

Proof of Proposition 4 By using integration by parts repeatedly, eqs. (7)-(8) can be rewritten as

$$\begin{aligned} MA(t) &= s(t) - \frac{1}{\omega}\int_{-\infty}^{t} \omega \exp(\omega(\mu - t))\frac{ds(\mu)}{d\mu}d\mu \qquad \text{(B.29)} \\ &= s(t) - \frac{1}{\omega}\frac{ds(t)}{dt} + \frac{1}{\omega^2}\int_{-\infty}^{t} \omega \exp(\omega(\mu - t))\frac{d^2 s(\mu)}{d\mu^2}d\mu \\ &= s(t) - \frac{1}{\omega}\frac{ds(t)}{dt} + \frac{1}{\omega^2}\frac{d^2 s(t)}{dt^2} - \frac{1}{\omega^3}\int_{-\infty}^{t} \omega \exp(\omega(\mu - t))\frac{d^3 s(\mu)}{d\mu^3}d\mu \\ &= s(t) - \frac{1}{\omega}\frac{ds(t)}{dt} + \frac{1}{\omega^2}\frac{d^2 s(t)}{dt^2} - \frac{1}{\omega^3}\frac{d^3 s(t)}{dt^3} + \frac{1}{\omega^4}\int_{-\infty}^{t} \omega \exp(\omega(\mu - t))\frac{d^4 s(\mu)}{d\tau^4}d\mu \\ &= \ldots \\ &= s(t) + \sum_{i=1}^{\infty}(-1)^i \frac{1}{\omega^i}\frac{d^i s(t)}{dt^i}, \end{aligned}$$

and the proof is completed. ■

Proof of Proposition 5 The dynamic system consisting of eqs. (1)-(5), eq. (9) and eq. (38) (assuming equality in the equation) can be written as a system of four first-order differential equations:

$$\begin{cases} \frac{dp}{dt} = \alpha\beta(s - p) + \alpha\beta\left(\bar{p} - \bar{s}\right) \\ \frac{ds}{dt} = u \\ \frac{du}{dt} = v \\ \frac{dv}{dt} = \frac{\omega^3 \exp(\tau)}{\zeta\eta}\left(p - \bar{p}\right) + \frac{\theta\omega^3(1-\exp(\tau))}{\zeta\eta}\left(s - \bar{s}\right) - \omega^2 u + \omega v \end{cases}, \tag{B.30}$$

where we have utilized the expressions for the price level and exchange rate in long-run equilibrium, i.e., eqs. (13)-(14). The Jacobian matrix evaluated at long-run equilibrium, i.e., $\frac{dp}{dt} = \frac{ds}{dt} = \frac{du}{dt} = \frac{dv}{dt} = 0$, is then

$$J = \begin{pmatrix} -\alpha\beta & \alpha\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\omega^3 \exp(\tau)}{\zeta\eta} & \frac{\theta\omega^3(1-\exp(\tau))}{\zeta\eta} & -\omega^2 & \omega \end{pmatrix}. \tag{B.31}$$

The dynamic system has four roots, which are denoted by λ_0, λ_1, λ_2 and λ_3. Then,

$$\lambda_0\lambda_1\lambda_2\lambda_3 = \det(J) = \frac{\alpha\beta\omega^3\exp(\tau)}{\zeta\eta} > 0, \tag{B.32}$$

which means that zero, two or four roots has a negative real part. However, the case of four roots with a negative real part can be ruled out since the Routh-Hurwitz conditions are not fulfilled (Coppel, 1965, p. 158). Specifically, the Routh-Hurwitz conditions state that the necessary and sufficient conditions for a real polynomial of degree four,

$$a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0, \tag{B.33}$$

to have four roots with a negative real part are that

$$\begin{cases} a_0 > 0, & a_1 > 0, \quad a_2 > 0, \\ a_4 > 0, & a_3(a_1a_2 - a_0a_3) > a_1^2a_4. \end{cases} \tag{B.34}$$

In this particular case, the characteristic equation, $\det(J - \lambda I) = 0$, is

$$\lambda^4 + (\alpha\beta - \omega)\lambda^3 - \alpha\beta\omega\lambda^2 + \frac{\alpha\beta\omega^3\exp(\tau)}{\zeta\eta} = 0, \tag{B.35}$$

where, for example, $a_2 = -\alpha\beta\omega \ngtr 0$. Now, let $\chi_{a,b,c}$ be the 3×3 principal minor of J associated with the rows and columns a, b and c. Then, according to Theorem 1.2.12 in Horn and Johnson (1985, p. 42),

$$\begin{aligned} \lambda_0\lambda_1\lambda_2 + \lambda_0\lambda_1\lambda_3 + \lambda_0\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_3 &= \chi_{1,2,3} + \chi_{1,2,4} + \chi_{1,3,4} + \chi_{2,3,4} \\ &= \frac{\theta\omega^3(1 - \exp(\tau))}{\zeta\eta} - \alpha\beta\omega^2 < 0, \end{aligned} \tag{B.36}$$

which rules out the case of zero roots with a negative real part. Therefore, two of the four roots have a negative real part, which means that the model is characterized by saddle-path stability, and the proof is completed. ■