Externalities, Border Trade and Illegal Production: An Optimal Tax Approach to Alcohol Policy

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Abstract

This paper deals with optimal income and commodity taxation in an economy, where alcohol is an externality-generating consumption good. In our model, alcohol can be bought domestically, imported (via border trade) or produced illegally. Border trade implies an incentive to set the domestic alcohol tax below the marginal social damage of alcohol, and to tax (subsidize) commodities which are complementary with (substitutable for) alcohol. In addition, since leisure and alcohol consumption are generally nonseparable, the income tax will also be used as a corrective instrument. On the other hand, the desire to reduce the illegal production may generally affect the optimal income and commodity taxes in either direction. One possible (and arguably realistic) outcome is, nevertheless, that the desire to avoid the illegal production works to reduce both the alcohol tax and the marginal income tax rate.

Keywords: Optimal taxation, external effects, alcohol, border trade.

JEL Classification: D61, D62, H21, H23.

1 Introduction

Between 1996 and 2003, alcohol consumption increased by approximately 29 per cent in Sweden[1]. This is, at least in part, attributable to higher per capita import quotas in combination with a 45 per cent decrease in the Danish alcohol tax. A higher level of alcohol consumption is typically expected to have several negative effects for society. First, it may increase the frequency of alcohol-related diseases in the population. In case health care in publicly provided, this implies welfare costs due to fiscal external effects. Second, a higher level of alcohol consumption may also affect the production side of the economy via increases in sick-absenteeism and/or reduced productivity.

Alarmed by this development, the Swedish government appointed a commission during 2003, the aim of which was to put forward proposals regarding our future alcohol policy; in particular alcohol taxation. In an interim report, presented in August 2004, one of the key suggestions was to reduce the Swedish alcohol tax by 40 per cent. Although this may lead to an increase in the total consumption of alcohol, the main motivation was that a lower tax is likely to reduce the amount of imported alcohol; an indication that the commission believed that the current alcohol policy does not fulfill its main objective of reducing the alcohol consumption[2]. The Swedish situation

[1] See SOU 2004:86.

[2] A related issue addressed in the economics literature is that border trade makes the domestic alcohol taxes less efficient as means of collecting revenues; see Crawford and

is not unique. The reduction of the alcohol tax in Denmark mentioned above was a response to the lower level of taxation in Germany. Similarly, Finland reduced its alcohol tax by 44 per cent due to the lower level of taxation in Estonia. Also in Denmark and Finland, the main motivation for the tax reductions seems to have been to avoid private import of alcohol.

The political environment in which these policy reforms are suggested features at least two important characteristics. First, the countries do not seem to cooperate with respect to alcohol policy. Instead, each country chooses its policy in isolation while treating the policies of other countries as exogenous. Second, in today's world, free trade agreements may effectively rule out the use of tariffs and similar trade barriers. The latter implies that individual countries do not have access to instruments, which would make it possible to control the import of alcohol. Therefore, these characteristics suggest that 'alcohol policy' should be thought upon as a decision problem in a second best framework. An interesting and important question then emerges; what does 'the optimal alcohol policy' look like at the national level, if neither international cooperation nor trade policy is implementable?

The purpose of this paper is to address alcohol taxation in the context of an optimal tax problem. Our analysis is based on a representative agent model with two consumption goods, one of which is alcohol. There are three ways for the consumers of acquiring alcohol; (i) buying alcohol on the domestic market, (ii) importing alcohol via border trade and (iii) illegal

Tanner (1995) and Crawford et al. (1999). Both studies estimate the demand for different types of alcohol using data from the U.K. The results show that, while the duties on beer and wine appear to be set below their revenue maximizing levels, the authors were not able to the reject that the duty on spirits is revenue maximizing. The main difference between the two studies is that Crawford et al. incorporate cross-price effects between different types of alcohol.

production of alcohol (moonshining). Excessive alcohol consumption causes health damage which may, in turn, necessitate medical treatment. We assume that health care is publicly provided and financed via tax revenues, meaning that alcohol consumption gives rise to a fiscal external effect[3]. In addition, although our study focuses on alcohol taxation, the qualitative results can be generalized to any externality-generating good, which is subject to border trade.

The government in our paper faces a mixed tax problem, where the set of tax instruments contains a nonlinear income tax and linear commodity taxes. Such a framework provides a reasonably realistic description of the tax instruments which a government has at its disposal. It also implies that the use of distortionary taxation is a consequence of optimization; it is not a consequence of restrictions on the tax instruments. To be able to concentrate on the corrective role of taxation, we follow some of the earlier literature on optimal taxation under imperfect competition, e.g. Fuest and Huber (1997) and Aronsson and Sjögren (2004a, 2004b), by disregarding motives for distortionary taxes that also apply under perfect competition (e.g. asymmetric information). Therefore, the motives for using distortionary taxes discussed here are solely related to the external effects of alcohol consumption. This simplification does not reflect a belief that other motives for using distortionary taxes, such as the desire to redistribute among consumers in an economy with asymmetric information, are unimportant; only that they are well understood from earlier studies.

[3] One can also think of other external effects of alcohol (e.g. external effects in the production due to influences on the stock of human capital). Although we limit our analysis to a fiscal health cost externality, the results are easily generalized to any negative externality associated with alcohol.

The optimal tax policy derived in the paper will be compared with that of a 'standard' model, which neither contains the possibility of importing alcohol via border trade nor illegal production[4]. Since private imports erodes the tax base for alcohol, we cannot solely rely on the domestic alcohol tax as a means of internalizing the external effect of alcohol. The reason is that the number of variables one would like to control via tax policy exceeds the number of policy instruments. As a consequence, the commodity taxes on other (nonalcoholic) goods as well as the income tax should also be used for corrective purposes. The results show that the external effect associated with alcohol consumption provides an incentive to tax (subsidize) goods, which are complementary with (substitutable for) alcohol. This result can be related to the 'additivity property', which was first derived by Sandmo (1975) in the environmental economics literature and further discussed by Pirttilä and Tuomala (1997) in the context of mixed taxation[5]. The additivity property means that the marginal social damage should enter the tax formula for the externality-generating good, although it should have no direct effect on the tax formulas for the other goods. Therefore, in our framework, the additivity property does not apply, since the marginal social damage of alcohol directly

[4]For previous research on alcohol taxation in the absence of border trade and illegal production, see e.g. Pogue and Sgontz (1989) and Sgontz (1993). The basic issue in the first paper is how to set the alcohol tax in an economy, which distinguishes between abusers and nonabusers of alcohol. This analysis relates, in turn, to Diamond (1973), who addresses corrective pricing in case the external effects vary with the individuals causing them (although prices are uniform). The second paper extends the analysis by considering the mix of alcohol and other taxes under a tax revenue requirement. See also Parry (2003) for a welfare analysis of alcohol and other taxes in the context of a numerical model applied to the U.K.

[5]See also the related literature on environmental policy in the context of mixed taxation, i.e. Cremer and Gahvari (2001), Cremer et al. (2001) and Aronsson and Blomquist (2004).

affects the tax formula for the nonalcoholic good. The results also imply that, if alcohol and labor are complements (substitutes), there will an incentive to increase (decrease) the hours of work via the income tax system. Adding illegal production of alcohol to the analysis makes the model much more complex. The joint influence of boarder trade and illegal production does not provide a clear incentive to choose a lower commodity tax on alcohol than in the standard model, where the government has full control of the externality-generating tax base. However, a possible (and arguably realistic) outcome is, nevertheless, that the desire to avoid illegal production works to reduce both the alcohol tax and the marginal income tax rate.

The outline of the paper is as follows. In Section 2, we describe the basic model. The optimal tax policy is characterized in Section 3, and we divide the analysis in two parts. First, we characterize the optimal tax policy in the absence of illegal production of alcohol, implying that we are focusing on the consequences of border trade for domestic policy. This approach seems reasonable, considering that the implications the for optimal tax policy of allowing consumers to buy alcohol abroad differ substantially from those associated with illegal production. It is also motivated by evidence suggesting that the domestic illegal production may be small relative to the size of private import[6]. Second, we discuss the consequences of allowing for illegal production of alcohol, meaning that the implications of border trade and illegal production are addressed simultaneously. The results are summarized and discussed in Section 4.

[6]For Sweden, it has been estimated that out of the total alcohol consumption during 2003, 26 per cent came from private imports (where the numbers for legal and illegal imports are 22 per cent and 4 per cent, respectively), while 6 per cent came from domestic illegal production (SOU 2004:86).

2 The Model

Consider an economy with identical consumers[7], the number of which is normalized to one. Consumer preferences are defined by the utility function $u(c, x, z)$, where c is a commodity with no alcoholic content, to be called 'nonalcoholic good' in what follows, x is alcohol and z is leisure. We assume that the function $u(\cdot)$ is increasing in c and z as well as strictly quasiconcave. We also assume that alcohol causes health damage[8]; a property which for simplicity is embedded in the relationship between $u(\cdot)$ and x[9]. The health consequences of alcohol imply that the individual may need medical treatment. Since health care is publicly provided by assumption, the cost of medical treatment is financed via tax revenues.

[7]Recall that our paper focuses on the consequences of border trade and illegal production for the optimal tax mix; issues which can be addressed in the context of a representative agent model.

[8]Since alcohol is a commodity to which consumers may become addicted, another possible approach would be to analyze consumer behavior within the framework of a rational addiction model; see Becker and Murphy (1988). However, this approach may necessitate a dynamic model, which is analytically more complicated. Since the aspects to be addressed in this paper are captured by a static model, analytical convenience motivates our choice of using such a model.

[9]This means that, although the marginal utility of alcohol must be positive at the optimum, the relationship between $u(\cdot)$ and x needs not necessarily be monotonous. A more formal (yet equivalent) approach would be to incorporate a health indicator, $h(x)$, in the utility function, such that the utility function changes to read $\breve{u}(c, x, z, h(x))$, where $\partial\breve{u}/\partial x > 0$, $\partial\breve{u}/\partial h > 0$ and $\partial h/\partial x < 0$. The 'net' marginal utility of alcohol then becomes

$$\frac{\partial\breve{u}}{\partial x} + \frac{\partial\breve{u}}{\partial h}\frac{\partial h}{\partial x}$$

For 'normal' levels of x, the first positive terms dominates, whereas the second negative term may take over for sufficiently large quantities.

In our model, there are three ways of acquiring alcohol for the consumer; (i) buying on the domestic market, (ii) imports and (iii) illegal production. The amount of alcohol bought on the domestic market is denoted by x^d, whereas x^f denotes the amount of alcohol imported via border trade. The consumer prices on domestic and imported alcohol are denoted by q_x and q_x^f, respectively, where q_x^f is treated as fixed by the domestic government. Buying alcohol abroad is also assumed to be associated with a transportation cost, $r\left(x^f\right)$, which is increasing and strictly convex in its argument. The private, illegal, production of alcohol is defined by the production function $x^u = f\left(l^u\right)$, where l^u is the amount of labor used to produce alcohol and avoid detection. We assume that $f\left(l^u\right)$ is increasing and strictly concave in its argument. The concavity of $f(\cdot)$ reflects the idea that, as an individual increases his/her illegal production of alcohol, relatively more time must be spent avoiding detection and relatively less time can be spent in the actual production.

The optimal tax problem below will be defined in terms of a conditional indirect utility function and conditional demand functions. Therefore, following Christiansen (1984), it is convenient to solve the consumer's optimization problem in two stages. In the first stage, we maximize utility conditional on the hours of work in the official labor market, l. This problem is written

$$\max_{c,x,x^d,x^f,l^u} \quad u\left(c, x, H - l - l^u\right) \tag{1}$$

subject to

$$b = q_x x^d + q_x^f x^f + r\left(x^f\right) + q_c c \tag{2}$$

$$x = x^d + x^f + f\left(l^u\right) \tag{3}$$

where q_c is the consumer price of the nonalcoholic good, and b is the after-tax income which is treated as fixed in the first stage. Note also that we have substituted the time constraint, $z = H - l - l^u$, into the utility function, where H is the time endowment. To simplify the analysis, we assume a linear technology, where the wage rate and the producer prices are fixed[10]. The consumer prices are given by $q_c = p_c + t_c$ and $q_x = p_x + t_x$, respectively, where p_c and p_x are producer prices, while t_c and t_x are commodity taxes. By assuming interior solutions, i.e. $x, x^d, x^f, c, l^u > 0$, the first stage optimization implicitly defines the conditional demand and 'supply' functions

$$\begin{aligned} x &= x\left(b, l, q_c, q_x\right), \; x^d = x^d\left(b, l, q_c, q_x\right), \; x^f = x^f\left(q_x\right), \\ c &= c\left(b, l, q_c, q_x\right) \text{ and } l^u = l^u\left(b, l, q_c, q_x\right) \end{aligned} \tag{4}$$

where the fixed parameter q_x^f has been suppressed for notational convenience. Note that x^f is written as a function only of q_x, since the first order condition for x^f can be written as $r'\left(x^f\right) = q_x - q_x^f$. Strict convexity of the cost function implies that x^f is increasing in q_x. Substituting the conditional demand and supply functions into the direct utility function and using the time constraint gives the conditional indirect utility function

$$v = v\left(b, l, q_c, q_x\right) \tag{5}$$

In the second stage, l is chosen to maximize the conditional indirect utility function subject to the budget constraint $b = wl - T\left(wl\right)$, where w is the wage rate earned in the official labor market and $T\left(\cdot\right)$ the income tax function. The first order condition is given by

[10] This assumption is not important for the qualitative results derived below.

$$v_b w \left(1 - T^{'}\right) + v_l = 0 \tag{6}$$

in which $v_b = \partial v(\cdot)/\partial b$ and $v_l = \partial v(\cdot)/\partial l = -\partial u(\cdot)/\partial z$ denote the marginal utilities of private income and labor, respectively, whereas $T^{'} = \partial T(wl)/\partial(wl)$ is the marginal income tax rate.

3 The Optimal Tax Problem

The objective of the government is to maximize the welfare of the representative individual, $v = v\,(b, l, q_c, q_x)$, subject to its budget constraint. The tax instruments are income and commodity taxes, and the tax revenues are used to finance the expenditure on health care. Since we are focusing on tax policy, we disregard public policies aimed at detecting illegal production. The budget constraint can be written

$$T\,(wl) + t_c c + t_x x^d - \rho\,(x) = 0 \tag{7}$$

where $\rho\,(x)$ is the cost of health care, which is increasing in the total consumption of alcohol, x. Note also that the amount of alcohol bought in the domestic market, x^d, constitutes the tax base for the domestic alcohol tax. The consumer price on imported alcohol, q_x^f, is treated as fixed by the government.

Recall that $T(\cdot)$ is a general income tax, meaning that it can be used to implement any desired combination of (l, b). It is convenient to use l and b directly, instead of the parameters of $T\,(\cdot)$, as decision variables in the optimal tax problem. Therefore, l, b, t_c and t_x, together, constitute the set of decision variables. The Lagrangean becomes

$$\mathfrak{L} = v(\cdot) + \gamma \left[wl - b + t_c c(\cdot) + t_x x^d(\cdot) - \rho(x(\cdot)) \right] \tag{8}$$

where $x^d(\cdot) = x(\cdot) - x^f(\cdot) - f(l^u(\cdot))$, while γ is the Lagrange multiplier associated with the budget constraint. The expression within square brackets is computed by using $T = wl - b$.

The first order conditions are presented in the Appendix. As we mentioned above, the analysis will be divided in two parts. First, we characterize the optimal public policy in the absence of illegal production of alcohol, implying that we are focusing on the consequences for the optimal tax policy of allowing for alcohol imports. Second, we discuss the consequences of allowing for illegal production of alcohol, meaning that the implications of imports and illegal production are addressed simultaneously.

3.1 Case 1: No Illegal Production of Alcohol

In terms of the original model, this case means that $x^u = f(l^u) = 0$. Let us begin by discussing the commodity tax structure. Denote the compensated demand functions by $\tilde{x}$, $\tilde{x}^f$ and $\tilde{c}$, respectively. By substituting equation (A2) into equations (A3) and (A4), the first order conditions for t_c and t_x can be written as

$$\begin{bmatrix} \frac{\partial \tilde{c}}{\partial q_c} & \frac{\partial \tilde{x}^d}{\partial q_c} \\ \frac{\partial \tilde{c}}{\partial q_x} & \frac{\partial \tilde{x}^d}{\partial q_x} \end{bmatrix} \times \begin{bmatrix} t_c \\ t_x \end{bmatrix} = \begin{bmatrix} \rho' \frac{\partial \tilde{x}}{\partial q_c} \\ \rho' \left(\frac{\partial \tilde{x}}{\partial q_x} - \frac{\partial x}{\partial b} x^f \right) \end{bmatrix} \tag{9}$$

To be able to interpret the optimal tax policy implicit in equation system (9), consider first the special case with no private import of alcohol, i.e. when $x^f = 0$. Solving equation system (9) for $x = x^d$, and then using the first order condition for l in the Appendix, gives

$$t_c = 0,\ t_x = \rho^{'} \text{ and } T^{'} = 0 \tag{10}$$

This situation, which will be referred to as the 'Standard Case', summarizes the conventional results of optimal taxation in a representative agent model with an external effect. The intuition is that the government is, in this case, able to reach the first best by fully internalizing the external effect. This is accomplished by choosing t_x equal to the marginal cost of health care, $\rho^{'}$. On the other hand, and regardless of the pattern of complementarity or substitutability between c and x, the optimal tax on the nonalcoholic good, t_c, is zero. These results confirm the additivity property of Sandmo (1975). Note also that the marginal income tax rate is equal to zero in the Standard Case, implying that the income tax becomes a pure lump-sum tax.

Having briefly addressed the Standard Case, let us return to the general case with border trade summarized by equation system (9), in which we have $x = x^d + x^f$. In addition, define

$$|\mathbf{H}| = \frac{\partial \tilde{c}}{\partial q_c}\frac{\partial \tilde{x}^d}{\partial q_x} - \frac{\partial \tilde{c}}{\partial q_x}\frac{\partial \tilde{x}^d}{\partial q_c} > 0$$

to be the determinant of the matrix on the left hand side of equation system (9). By applying Cramer's rule, while using $x^d = x - x^f$, we can derive[11];

[11]Note that the tax rules in Proposition 1 are written in terms of derivatives of the compensated (not the Mashallian) conditional demand functions. The reason is that the tax formulas are derived by combining the first order conditions for t_c, t_x and b, since the efficient tax structure presupposes an optimal income tax. Therefore, changes in t_c and t_x will be accompanied by a simultaneous change in b in order to balance the government's budget. Since the latter is also interpretable as a change in b such that the utility is unchanged, the derivatives of the conditional compensated demand functions appear in the tax rules.

Proposition 1 *In the presence of border trade, efficient taxation means that the commodity taxes can be written as*

$$
\begin{aligned}
t_c &= -\frac{\rho'}{|\mathbf{H}|}\frac{\partial \tilde{x}^f}{\partial q_x}\frac{\partial \tilde{x}}{\partial q_c} \\
t_x &= \rho' + \frac{\rho'}{|\mathbf{H}|}\frac{\partial \tilde{x}^f}{\partial q_x}\frac{\partial \tilde{c}}{\partial q_c}
\end{aligned}
$$

Consider first the interpretation of the optimal alcohol tax, t_x. In comparison with the Standard Case, where $t_x = \rho'$, there is an additional term on the right hand side of the tax formula, which arises as a direct consequence of border trade. Since $\partial\tilde{x}^f/\partial q_x > 0$ and $\partial\tilde{c}/\partial q_c < 0$, this additional term is negative and constitutes an incentive to set the alcohol tax below the marginal social damage of alcohol[12], ρ'. The intuition is that border trade erodes the tax base for the alcohol tax, meaning that t_x is no longer a perfect instrument to control the total consumption of alcohol, $x = x^d + x^f$. Instead, since t_x is now a less efficient policy instrument than in the Standard Case, there is an incentive to set the alcohol tax such that $t_x < \rho'$. In addition, note that the more sensitive border trade is to alcohol taxation, i.e. the greater $\partial\tilde{x}^f/\partial q_x$, the lower will be the optimal alcohol tax relative to the marginal social damage of alcohol.

Another difference, in comparison with the Standard Case, is that the tax on the nonalcoholic good, t_c, will now be used as an additional instrument to correct for the external effect. The intuition is that, since t_x does not fully

[12] In Christiansen (2003), a similar qualitative result is established; although in a different context. He finds that the optimal commodity tax on an externality-generating consumption good falls short of the marginal social damage, if the externality-generating consumption good is subject to border trade. At the same time, his paper has a different focus than ours, and he does not address how other policy instruments (other than taxes the externality-generating consumption goods) can be used for corrective purposes.

internalize the external effect, the government has an incentive to use other instruments in order to influence the alcohol consumption. Proposition 1 is interpretable to mean that, if the nonalcoholic good is complementary with (substitutable for) alcohol in the sense that $\partial\tilde{x}^d/\partial q_c < 0$ (> 0), then the optimal tax on the nonalcoholic good is positive (negative). In other words, there is an incentive to tax (subsidize) goods which are complementary with (substitutable for) alcohol. As a consequence, the additivity property no longer applies. This is basically due to the fact that the government wants to influence four goods (c, x^d, x^f and l), although it has only three instruments at its disposal (t_c, t_x and T).

Let us continue by characterizing the marginal income tax rate. Consider Proposition 2;

Proposition 2 *In the presence of border trade, efficient taxation means that the marginal income tax rate can be written as*

$$T^{'} = -\frac{\rho^{'}}{w\left|\mathbf{H}\right|}\frac{\partial\tilde{x}^f}{\partial q_x}\frac{\partial\tilde{c}}{\partial q_c}\left[\frac{\partial\tilde{x}}{\partial l} - \frac{\partial\tilde{c}/\partial l}{\partial\tilde{c}/\partial q_c}\frac{\partial\tilde{x}}{\partial q_c}\right]$$

Proof: See the Appendix.

It is once again useful to compare the results with those that would emerge in the Standard Case, where the marginal income tax rate is equal to zero. In an economy with border trade, on the other hand, Proposition 2 suggests that the government has an incentive to use the income tax as an additional instrument to reduce the alcohol consumption. The intuition behind the first part of the expression within the square bracket relates to whether the hours of work are complementary with, or substitutable for, for alcohol. As such, if the hours of work are complementary with (substitutable for) alcohol in the sense that $\partial\tilde{x}/\partial l > 0$ (< 0), there is an incentive to decrease (increase) the hours of work from the point of view of the government. This can, in turn, be accomplished by choosing a higher (lower) marginal income tax rate than otherwise. For instance, if alcohol and leisure are complements (which appears to be a reasonable assumption), then this argument provides a rationale for lowering the marginal income tax rate.

To interpret the second part of the expression within square brackets, it is necessary to bear in mind that the corrective role of income taxation in this economy is to reduce the consumption of alcohol; there is no reason to directly distort the consumption of the other commodity (the nonalcoholic good does not give rise to external effects). Therefore, if a change in l (induced by a change in the income tax), nevertheless, distorts the choice underlying the consumption of the nonalcoholic good, there will be an incentive for the government to adjust the tax structure accordingly. The second part of the expression within the square bracket can be understood in terms of the answer to the following question: if a change in l (via an adjustment of the income tax) is used to reduce the consumption of alcohol, and if $\partial\tilde{c}/\partial l \neq 0$, how should q_c be changed in order to keep $\tilde{c}$ fixed at, say, $\bar{c}$? Differentiating $\bar{c} = \tilde{c}(u, l, q_c, q_x)$ with respect to l and q_c, we have

$$\frac{\partial q_c}{\partial l} \mid_{\tilde{c}=\bar{c}} = -\frac{\partial\tilde{c}/\partial l}{\partial\tilde{c}/\partial q_c} \tag{11}$$

Substituting into the tax formula in Proposition 2 gives

$$T' = -\frac{\rho'}{w\,|\mathbf{H}|}\frac{\partial\tilde{x}^f}{\partial q_x}\frac{\partial\tilde{c}}{\partial q_c}\left[\frac{\partial\tilde{x}}{\partial l} + \frac{\partial\tilde{x}}{\partial q_c}\left(\frac{\partial q_c}{\partial l} \mid_{\tilde{c}=\bar{c}}\right)\right] \tag{12}$$

The interpretation now becomes straight forward. Suppose that $\partial q_c/\partial l > 0$, implying that an increase in l must be accompanied by a higher q_c for $\tilde{c}$ to be constant. Then, if the nonalcoholic good is complementary with (substitutable for) alcohol in the sense that $\partial\tilde{x}/\partial q_c < 0$ (> 0), the second part of the expression within the square bracket constitutes an incentive to increase (decrease) the hours of work, which is accomplished by choosing a lower (higher) marginal income tax rate than otherwise. If $\partial q_c/\partial l < 0$, on the other hand, the opposite argument applies.

As a final concern in this subsection, let us briefly turn to the marginal cost of public funds, $MCPF$, which is defined as γ/v_b. In the Standard Case, it is easy to show that $MCPF = 1$, since the Standard Case means that we are able to implement the first best resource allocation. For the more general model with border trade, $MCPF$ will generally differ from one. By using equation (A2) together with the tax formulas in Proposition 1, we obtain

$$MCPF = \frac{|\mathbf{H}|}{|\mathbf{H}| - \phi} \tag{13}$$

where

$$\phi = \rho' \frac{\partial \tilde{x}^f}{\partial q_x} \left[\frac{\partial \tilde{c}}{\partial q_c} \frac{\partial x}{\partial b} - \frac{\partial \tilde{x}}{\partial q_c} \frac{\partial c}{\partial b} \right]$$

Although $MCPF \lesseqgtr 1$ in general, one would be inclined to argue that $MCPF < 1$ is the most likely outcome for the model set out here. To see this, suppose (as one would normally expect) that $\partial x/\partial b > 0$ and $\partial c/\partial b > 0$ at the optimum. Then, since $\partial \tilde{x}^f/\partial q_x > 0$ and $\partial \tilde{c}/\partial q_c < 0$, the first part of the formula for ϕ works to reduce $MCPF$ below one. Then, a sufficient condition for $MCPF < 1$ is that the two consumption goods are weak substitutes in the sense that $\partial \tilde{x}/\partial q_c \geq 0$. Note also that this condition is not necessary; if the two goods are complementary, implying that $\partial \tilde{x}/\partial q_c < 0$, $MCPF$ will still fall short of one, provided that the compensated cross price effect is not too large in absolute value. The intuition is straight forward; since $t_x < \rho'$, possibly in combination with $t_c < 0$ (recall that the nonalcoholic good will be subsidized if $\partial \tilde{x}/\partial q_c > 0$), a larger part of the tax revenues will here be collected via the income tax than in the Standard Case. As such, this is likely to imply that the government relies on lump-sum taxation to a greater extent than in the Standard Case, since the general income tax contains a lump-sum element.

3.2 Case 2: Illegal Production of Alcohol

Adding illegal production of alcohol to the analysis, the domestic demand for alcohol changes to read $x^d = x - x^f - x^u$, where $x^u = f(l^u)$ is the amount illegally produced with labor input l^u. To simplify the presentation, let us introduce the short notations

$$\begin{aligned}
f' &= \frac{\partial f(\cdot)}{\partial l^u} \\
\frac{\partial \tilde{l}^u}{\partial q_c} &= \frac{\partial l^u}{\partial q_c} + \frac{\partial l^u}{\partial b} c \\
\frac{\partial \tilde{l}^u}{\partial q_x} &= \frac{\partial l^u}{\partial q_x} + \frac{\partial l^u}{\partial b} x \\
\frac{\partial \tilde{x}_s}{\partial q_x} &= \frac{\partial \tilde{x}^d}{\partial q_x} + \frac{\partial \tilde{x}^f}{\partial q_x} + \frac{\partial \tilde{x}^u}{\partial q_x}
\end{aligned}$$

By applying the same technique as in Proposition 1, we obtain the following result regarding commodity taxation;

Proposition 3 *In the presence of border trade and illegal production, efficient taxation means that the commodity taxes can be written as*

$$\begin{aligned}
t_c &= \frac{\rho' f'}{|\mathbf{H}|} \frac{\partial \tilde{x}_s}{\partial q_x} \left[\frac{\partial \tilde{l}^u}{\partial q_c} - \frac{\partial \tilde{x}/\partial q_c}{\partial \tilde{x}_s/\partial q_x} \frac{\partial \tilde{l}^u}{\partial q_x} \right] \\
&\quad - \frac{\rho'}{|\mathbf{H}|} \frac{\partial \tilde{x}}{\partial q_c} \frac{\partial \tilde{x}^f}{\partial q_x} \\
t_x &= \frac{\rho' f'}{|\mathbf{H}|} \frac{\partial \tilde{c}}{\partial q_c} \left[\frac{\partial \tilde{l}^u}{\partial q_x} - \frac{\partial \tilde{c}/\partial q_x}{\partial \tilde{c}/\partial q_c} \frac{\partial \tilde{l}^u}{\partial q_c} \right] \\
&\quad \rho' + \frac{\rho'}{|\mathbf{H}|} \frac{\partial \tilde{c}}{\partial q_c} \frac{\partial \tilde{x}^f}{\partial q_x}
\end{aligned}$$

The second row of each tax formula in Proposition 3 corresponds to the special case with no illegal production discussed in the previous subsection. As such, these terms are equivalent to, and have the same interpretations as, the tax formulas presented in Proposition 1. On the other hand, the terms in the first row of each formula did not appear in the previous subsection; they are due to the assumption that part of the time endowment is spent on illegal production of alcohol.

According to the tax formulas in Proposition 3, t_x does no longer necessarily fall short of the marginal social damage of alcohol (as it did in the absence of border trade). In addition, we can no longer determine whether the nonalcoholic good should be taxed or subsidized simply by analyzing whether the two consumption goods are complements or substitutes in terms of compensated cross price effects. The intuition is that the government now wants to influence five variables (c, x^d, x^f, x^u and l), although it still has only three instruments at its disposal (t_c, t_x and T). As a consequence, the number of incentive effects to be included in each tax formula is greater here than in the previous subsection. Note also that each such additional incentive effect is related to the influence of policy on the time spent in illegal production, l^u.

Since the second row of each tax formula was thoroughly discussed in the previous subsection, we concentrate on the incentive effects associated with illegal production, which are summarized by the terms within square brackets in the first row. Consider first the formula for t_x. Since $\partial\tilde{c}/\partial q_c < 0$, it follows that $\partial\tilde{l}^u/\partial q_x > 0$ ($<$) works to decrease (increase) the optimal alcohol tax, ceteris paribus. Therefore, if an increase in the domestic alcohol tax increases the illegal production, which appears to be a reasonable assumption, there is an additional cost associated with alcohol taxation. As such, this effect works to reduce the alcohol tax further below the marginal social damage of alcohol, implying that it strengthens the results of the previous subsection. On the other hand, if higher alcohol taxation leads to reduced illegal production of alcohol (which cannot be excluded on theoretical grounds), this effect works the other way around.

To interpret the second term within the square bracket, define $dq_c = -[(\partial\tilde{c}/\partial q_x)/(\partial\tilde{c}/\partial q_c)]dq_x$ to measure the change in q_c necessary to hold $\tilde{c}$ constant, if q_x increases marginally. Therefore, this part of the formula explores the relationship between the illegal production of alcohol and the two commodity taxes without adding any additional distortion to the nonalcoholic good. If $dq_c < 0$, implying that the two goods are complements in terms of the utility function, then $\partial\tilde{l}^u/\partial q_c > 0$ (< 0) provides an incentive to increase (decrease) the alcohol tax. The intuition is that the induced effect on q_c works to decrease the illegal production of alcohol, ceteris paribus. If, on the other hand, $dq_c > 0$, meaning that the two goods are substitutes, we have the opposite incentive effect, although the intuition in terms of the desire to reduce the illegal production of alcohol remains the same. The interpretation of the first row in the formula for t_c is analogous.

As a final concern, let us briefly analyze the marginal income tax rate. Introducing the short notations

$$\begin{aligned}
\Delta^c &= \frac{\partial\tilde{c}}{\partial q_c}\left[\frac{\partial\tilde{l}^u}{\partial q_x} - \frac{\partial\tilde{c}/\partial q_x}{\partial\tilde{c}/\partial q_c}\frac{\partial\tilde{l}^u}{\partial q_c}\right] \\
\Delta^x &= \frac{\partial\tilde{x}_s}{\partial q_x}\left[\frac{\partial\tilde{l}^u}{\partial q_c} - \frac{\partial\tilde{x}/\partial q_c}{\partial\tilde{x}_s/\partial q_x}\frac{\partial\tilde{l}^u}{\partial q_x}\right]
\end{aligned}$$

our result is summarized as follows;

Proposition 4 *In the presence of border trade and illegal production, efficient taxation means that the marginal income tax rate can be written as*

$$\begin{aligned} T^{'} &= -\frac{\rho^{'}}{w\left|\mathbf{H}\right|}\frac{\partial \tilde{x}^{f}}{\partial q_{x}}\frac{\partial \tilde{c}}{\partial q_{c}}\left[\frac{\partial \tilde{x}^{d}}{\partial l}-\frac{\partial \tilde{x}}{\partial q_{c}}\frac{\partial \tilde{c}/\partial l}{\partial \tilde{c}/\partial q_{c}}\right] \\ &\quad +\frac{\rho' f'}{w}\left[\frac{\partial \tilde{l}^{u}}{\partial l}-\frac{1}{\left|\mathbf{H}\right|}\left(\Delta^{c}\frac{\partial \tilde{x}^{d}}{\partial l}-\Delta^{x}\frac{\partial \tilde{c}}{\partial l}\right)\right] \end{aligned}$$

The proof of Proposition 4 is analogous to the proof of Proposition 2 and is, therefore, omitted. With the exception that $\partial \tilde{x}^d/\partial l = \partial \tilde{x}/\partial l - f'(\partial \tilde{l}^u/\partial l)$ in Proposition 4, the first row takes the same general form, and has the same interpretation, as the corresponding formula in the absence of illegal production. As such, it was thoroughly discussed in the context of Proposition 2. Let us, therefore, concentrate on the terms associated with illegal production of alcohol, which are given in the second row. The first part of the second row is likely to reduce the optimal marginal income tax rate; if $\partial \tilde{l}^u/\partial l < 0$, so that an increase in the hours of work spent in the official labor market reduces the illegal production, then there is an incentive to simulate increased hours of work by a lower marginal income tax rate.

The second part of the second row is, in a technical sense, analogous to the first row; both parts of the tax formula reflect how the time spent in market work affects compensated demand functions. Therefore, the appearance of the derivatives $\partial \tilde{x}^d/\partial l$ and $\partial \tilde{c}/\partial l$ in the tax formula is no longer attributable only to border trade (as in the previous subsection), since their presence in the second part of the second row is a consequence of illegal production of alcohol. The expressions Δ^c and Δ^x summarize how the relationships between the consumer prices and the illegal production of alcohol affect the commodity tax structure; information that was also part of (and interpreted in the context of) Proposition 3. Here, the roles of Δ^c and Δ^x are to interact with the derivatives of the compensated demand functions with respect to l. The intuition is that efficiency necessitates a broader spectrum of interaction effects between the policy instruments than in the absence of illegal production, indicating a greater need to use the income tax for corrective purposes.

4 Discussion

In this paper, we characterize the optimal income and commodity tax structure in a representative agent model with border trade and illegal production of alcohol. Alcohol is assumed to generate a negative fiscal external effect, which the government wants to internalize. The analysis first concentrates on the implications of border trade alone and then continues by simultaneously addressing the consequences of border trade and illegal production for optimal taxation.

Introducing border trade in an otherwise standard model with mixed taxation implies that we can no longer solely rely on the domestic alcohol tax as a means of internalizing the external effect. Contrary to what appears to be the conventional wisdom underlying practical policy, this does not only provide an incentive to reduce the alcohol tax below the marginal social damage of alcohol; it means, more generally, that the number of variables we would like to control via tax policy exceeds the number of policy instruments at our disposal. In other words, it is no longer possible to implement the first best resource allocation (or any other resource allocation that would solve the social planner problem in the absence of border trade). As a consequence, the commodity taxes on other (nonalcoholic) goods as well as the income tax should also be used for corrective purposes. The results show that the external effect associated with alcohol consumption provides an incentive to tax (subsidize) goods, which are complementary with (substitutable for) alcohol. This means, in turn, that the additivity property does not apply. The results also explain the corrective role of income taxation; if alcohol and leisure are complements (substitutes), there will an incentive to increase (decrease) the hours of work via the income tax system.

Adding illegal production of alcohol to the analysis strengthens one of the main messages; the commodity tax on the nonalcoholic good and the income tax should supplement the alcohol tax for corrective purposes. At the same time, illegal production complicates the analysis, indicating that the basic results described above - which are due to the effects of border trade in the absence of illegal production of alcohol - need no longer apply. Let be that some of the additional mechanisms work in the direction of decreasing the optimal alcohol tax further below the marginal social damage of alcohol, as well as constitute incentives for lowering the optimal marginal income tax rate in order to reduce the time spent on illegal production.

Appendix

The first order conditions of the optimal tax problem are

$$\frac{\partial \mathfrak{L}}{\partial l} = v_l + \gamma \left(t_x \frac{\partial x^d}{\partial l} - \rho^{'} \frac{\partial x}{\partial l} + t_c \frac{\partial c}{\partial l} + w \right) = 0 \tag{A1}$$

$$\frac{\partial \mathfrak{L}}{\partial b} = v_b + \gamma \left(t_x \frac{\partial x^d}{\partial b} - \rho^{'} \frac{\partial x}{\partial b} + t_c \frac{\partial c}{\partial b} - 1 \right) = 0 \tag{A2}$$

$$\frac{\partial \mathfrak{L}}{\partial t_c} = -c v_b + \gamma \left(t_x \frac{\partial x^d}{\partial q_c} - \rho^{'} \frac{\partial x}{\partial q_c} + t_c \frac{\partial c}{\partial q_c} + c \right) = 0 \tag{A3}$$

$$\frac{\partial \mathfrak{L}}{\partial t_x} = -x^d v_b + \gamma \left(t_x \frac{\partial x^d}{\partial q_x} - \rho^{'} \frac{\partial x}{\partial q_x} + t_c \frac{\partial c}{\partial q_x} + x^d \right) = 0 \tag{A4}$$

where we have used $-cv_b = v_{q_c}$ and $-x^d v_b = v_{q_x}$.

Proof of Proposition 2:

Let us first use equation (A2) solve for γ and then substitute the resulting expression into equation (A1). After some manipulations, we obtain

$$v_b w + v_l = v_l \left(t_x \frac{\partial x^d}{\partial b} - \rho^{'} \frac{\partial x}{\partial b} + t_c \frac{\partial c}{\partial b} \right) - v_b \left(t_x \frac{\partial x^d}{\partial l} - \rho^{'} \frac{\partial x}{\partial l} + t_c \frac{\partial c}{\partial l} \right) \quad \text{(A5)}$$

Note that, by using equation (6), the left hand side of equation (A5) can be written $v_b w T'$. Then, since $\partial x/\partial b = \partial x^d/\partial b$ and $\partial x/\partial l = \partial x^d/\partial l$, and by substituting the commodity tax formulas in Proposition 1 into equation (A5), we have

$$T^{'} = \frac{\rho^{'}}{w\,|\mathbf{H}|} \frac{\partial \tilde{x}^f}{\partial q_x} \left[\frac{\partial \tilde{c}}{\partial q_c} \left(\frac{\partial x^d}{\partial b} \frac{v_l}{v_b} - \frac{\partial x^d}{\partial l} \right) - \frac{\partial \tilde{x}^d}{\partial q_c} \left(\frac{\partial c}{\partial b} \frac{v_l}{v_b} - \frac{\partial c}{\partial l} \right) \right] \quad \text{(A6)}$$

Define

$$u = v\left(e\left(u, l, q_c, q_x\right), l, q_c, q_x\right) \quad \text{(A7)}$$

$$\tilde{c}\left(u, l, q_c, q_x\right) = c\left(e\left(u, l, q_c, q_x\right), l, q_c, q_x\right) \quad \text{(A8)}$$

$$\tilde{x}^d\left(u, l, q_c, q_x\right) = x^d\left(e\left(u, l, q_c, q_x\right), l, q_c, q_x\right) \quad \text{(A9)}$$

where $e\left(u, l, q_c, q_x\right) = b$ is the expenditure function. Differentiating equations (A7) and (A9) with respect to l and combining the resulting expressions give

$$\frac{\partial \tilde{x}^d}{\partial l} = \frac{\partial x^d}{\partial l} - \frac{v_l}{v_b} \frac{\partial x^d}{\partial b} \quad \text{(A10)}$$

By a similar procedure, one can use equations (A7) and (A8) to derive

$$\frac{\partial \tilde{c}}{\partial l} = \frac{\partial c}{\partial l} - \frac{v_l}{v_b} \frac{\partial c}{\partial b} \quad \text{(A11)}$$

Substituting equations (A10) and (A11) into equation (A6) gives the tax formula in Proposition 2. ■

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