Optimal Paternalism: Sin Taxes and Health Subsidies*

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Abstract

The starting point for this paper is the potential self-control problem underlying the consumption of unhealthy food. The purpose is to analyze public policies, which are designed to correct for the welfare loss associated with such behavior. Contrary to previous studies, our analysis suggests that subsidies on wealth and health capital are part of the policy package, which can be used to implement a socially optimal resource allocation.

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1 Introduction

The classical approach to studying the corrective role of government is based on the notion of market failure; for instance, external effects (as well as other forms of imperfect competition) may provide rationales for a government

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to intervene in a specific market. More recently, a literature on optimal paternalism has evolved, which focuses on another motive for government intervention; individuals may not be fully rational. This is a consequence of a self-control problem resulting from 'present biased' preferences. The idea is that some agents may, at any time, apply a higher utility discount rate for the tradeoff between present and future utility than the utility discount rate applied to similar tradeoffs in the future. As a consequence, the optimal consumption plan decided upon at time t may no longer be optimal for the individual at time $t+1$. This is interpretable such that the individual's current self imposes external effects on future selves, which provides a corrective role for the government in addition to the motives associated with market failures.

This paper addresses the potential self-control problem underlying the consumption of unhealthy food (for instance, high-calorie food that is low in nutritional value), and our purpose is to analyze how policy intervention may be used to improve the resource allocation[1]. In a study by O'Donoghue and Rabin (2003), paternalism is exemplified in the context of optimal commodity taxation, and the self-control problem is dealt with by a modification of the commodity tax structure. In particular, the results show that the optimal public policy may imply a higher tax on unhealthy commodities (exemplified by potato chips) and lower taxes on other commodities (exemplified by carrots) than in the absence of the underlying self-control problem. Their analysis is based on a model, where the instantaneous utility increases with the current consumption of three commodities (among which potato chips is one) as well as depends negatively on the consumption of potato chips in the previous period. The latter is motivated by the future health consequences of the current consumption, and the authors argue that it is

[1] Despite the purpose of our study, we realize that the consumption of unhealthy food does not necessarily imply that individuals fail to reach an optimal resource allocation in a longer time perspective. It may, instead, be the outcome of a fully rational choice. See also the related literature on rational addiction; for instance, Becker and Murphy (1988).

not essential that this relationship only goes one period forward.

While sympathetic to the analysis carried out by O'Donoghue and Rabin, we believe that the relationship between the current instantaneous utility and the past consumption of unhealthy goods is critical for our understanding of policy intervention. O'Donoghue and Rabin do not address the capital aspects of health. If the instantaneous utility depends on the current consumption and on the current stock of health capital, while the current stock of health capital, in turn, depends on (among other things) all previous consumption of the unhealthy good - which appears to us to be at least as realistic as the corresponding assumption in the paper by O'Donoghue and Rabin - then the optimal corrective policy is likely to involve a subsidy directed explicitly to the health capital stock instead of a sin tax. Furthermore, we show that a wealth subsidy is also part of the policy package, which can be used to implement the socially optimal resource allocation.

This paper is based on a discrete Ramsey type model with two capital stocks; physical capital and health capital. There are two explicit objectives behind the analysis carried out below. The first is to show that introducing a sin tax in an otherwise uncontrolled market economy is likely to improve the resource allocation. Here, our analysis does not disagree with O'Donoghue and Rabin. The second is to analyze the public policy that implements a social optimum, where individuals behave as if the self-control problem is absent. We show how the socially optimal resource allocation can be implemented by using subsidies on wealth and health capital.

2 The Model and the Main Results

To begin with, we assume that all consumers are identical and normalize the number of consumers to one. The instantaneous utility function facing the consumer is written

$$u_t = u(c_t, x_t, z_t, h_t) \tag{1}$$

where c is the consumption of an ordinary (not unhealthy) good, x the consumption of the unhealthy good, z leisure and h the stock of health capital. Leisure is, in turn, defined as a time endowment, H, less the time in market work, l. We assume that the function $u(\cdot)$ is increasing in each argument and strictly concave. We operationalize the concept of present biased preferences by using an approach developed by Phelps and Pollak (1968) and later used by e.g. Laibson (1997) and O'Donoghue and Rabin (1999, 2003). The intertemporal objective at time t is given by

$$U_t = u_t + \beta \sum_{s=t+1}^{\infty} u_s \Theta^{s-t} \tag{2}$$

where $\Theta^t = 1/(1+\theta)^t$ is a conventional utility discount factor with utility discount rate θ, whereas β is a time-inconsistent preference for immediate gratification, meaning that $\beta < 1$.

The consumer holds an asset in the form of physical capital. This asset accumulates according to

$$k_{t+t} - k_t = r_t k_t + w_t l_t - e_t - c_t - x_t \tag{3}$$

in which k is the physical capital stock, w the wage rate, r the interest rate and e the private resources spent on health. The prices of the two consumption goods are set equal to one (and we have not yet introduced taxation or other government interventions). We assume that the consumer treats the paths for w and r as exogenous during optimization. The health capital stock accumulates according to the equation

$$h_{t+t} - h_t = g(x_t, e_t) \tag{4}$$

where $g(\cdot)$ is a health production function with the properties $\partial g(x_t, e_t)/\partial x_t <$

0 and $\partial g(x_t, e_t)/\partial e_t > 0$. The initial capital stocks, k_0 and h_0, are exogenously given.

In this section, we assume that the consumer is naive in the sense of not recognizing that the preference for immediate gratification is present also when the future arrives. This assumption, which was also used by O'Donoghue and Rabin (2003), will be relaxed in the next section. In each time period, the consumer behaves as if he/she is maximizing equation (2) subject to equations (3) and (4). The consumer decides upon the levels of his/her control variables together with capital investments (in physical capital and health capital), while treating the initial stocks as given. In period t, this means that the consumer behaves as if he/she is choosing c_t, x_t, e_t, l_t, k_{t+1} and h_{t+1} conditional on k_t and h_t.

The goods market is competitive and consists of identical firms, and we assume that the production technology is characterized by constant returns to scale. The number of firms is normalized to one for notational convenience. The production function is written $f(l, k)$, and the objective of the firm is to choose labor and capital to maximize profits, implying that $\partial f(l_t, k_t)/\partial l_t - w_t = 0$ and $\partial f(l_t, k_t)/\partial k_t - r_t = 0$ for all t. In addition, constant returns to scale means $f(l_t, k_t) - w_t l_t - r_t k_t = 0$, which is the zero profit condition.

By combining the first order conditions for the consumer and the firm, while eliminating the Lagrange multipliers associated with equations (3) and (4), we can derive

$$\frac{\partial u_t}{\partial x_t} - \frac{\partial u_t}{\partial c_t} + \frac{\partial u_t}{\partial c_t}\frac{\partial g_t/\partial x_t}{\partial g_t/\partial e_t} = 0 \tag{5}$$

$$-\frac{\partial u_t}{\partial z_t} + \frac{\partial u_t}{\partial c_t}\frac{\partial f_t}{\partial l_t} = 0 \tag{6}$$

$$-\frac{\partial u_t}{\partial c_t} + \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\Theta[1 + \frac{\partial f_{t+1}}{\partial k_{t+1}}] = 0 \tag{7}$$

$$-\frac{\partial u_t/\partial c_t}{\partial g_t/\partial e_t} + \beta[\frac{\partial u_{t+1}/\partial c_{t+1}}{\partial g_{t+1}/\partial e_{t+1}} + \frac{\partial u_{t+1}}{\partial h_{t+1}}]\Theta = 0 \tag{8}$$

where $u_t = u(c_t, x_t, z_t, h_t)$ and $f_t = f(l_t, k_t)$. Finally, by using equation (3) and the zero profit condition, the resource constraint for period t can be written as

$$k_{t+1} - k_t = f(l_t, k_t) - e_t - c_t - x_t, \tag{9}$$

Taken together, equations (4)-(9) characterize the equilibrium in the uncontrolled market economy. Equation (5) is interpretable as the first order condition for x_t, in which we have recognized that the shadow price associated with health capital is equal to $(\partial u_t/\partial c_t)/(\partial g_t/\partial e_t)$ at the equilibrium[2], whereas equation (6) is the standard first order condition for the hours of work. Similarly, equations (7) and (8) refer to the optimal choices of k_{t+1} and h_{t+1}, respectively. Equations (4)-(9) can be used to solve for the equilibrium values of c_t, x_t, l_t, e_t, k_{t+1} and h_{t+1} conditional on k_t and h_t. Repeating this decision procedure for all t gives the equilibrium path associated with the uncontrolled market economy. Let $\{c_t^0, x_t^0, l_t^0, e_t^0, k_t^0, h_t^0\}$ for all t represent this equilibrium path, where the superindex "0" is used to denote the resource allocation in the absence of government intervention.

To be able to carry out the policy analysis, it is necessary to define the social objective function. Following O'Donoghue and Rabin (2003), we assume that $\beta = 1$ from the point of view of a social planner, meaning that the social objective function can be written as

$$\tilde{U}_0 = \sum_{t=0}^{\infty} u_t \Theta^t \tag{10}$$

Let us begin by considering a marginal intervention, where the government permanently imposes a small tax, τ, on the unhealthy good and returns the

[2]This is seen by noting that the first order condition for e_t (which we suppressed above) can be written as

$-\frac{\partial u_t}{\partial c_t} + \mu_t \frac{\partial g_t}{\partial e_t} = 0$

where μ_t is the Lagrange multiplier associated with equation (4). As such, μ_t measures the shadow price (in utility terms) of health capital.

revenues lump-sum to the consumer. This cost benefit rule can be derived by differentiating equations (4), (9) and (10) with respect to τ and then evaluating the resulting derivative at the initial equilibrium, in which $\tau = 0$. In the initial equilibrium, therefore, behavior is governed by equations (5)-(8). Consider Proposition 1;

Proposition 1*If the consumer obeys equations (4)-(9) for all t, and the social welfare function is represented by equation (10), then the cost benefit rule for τ can be written as*

$$\frac{\partial \tilde{U}_0^0}{\partial \tau} = (1-\beta)\sum_{t=1}^{\infty}[\frac{\partial u_t^0}{\partial c_t}(1+r_t^0)\frac{\partial k_t^0}{\partial \tau} + \{\frac{\partial u_t^0/\partial c_t}{\partial g_t^0/\partial e_t} + \frac{\partial u_t^0}{\partial h_t}\}\frac{\partial h_t^0}{\partial \tau}]\Theta^t$$

where $r_t^0 = \partial f(l_t^0, k_t^0)/\partial k_t$.

Proof: See the Appendix.

Note first that, if we were to relax the assumption that the consumers are time-inconsistent, implying that $\beta = 1$, then $\partial \tilde{U}_0^0/\partial \tau = 0$. In this case, therefore, there would be no first order welfare effects of the reform. On the other hand, if the consumers are time-inconsistent in the sense that $\beta < 1$, as we assume here, a sufficient condition for the reform to be welfare improving is that $\partial k_t/\partial \tau > 0$ and $\partial h_t/\partial \tau > 0$ for all t.

To be able to relate the welfare effects of policy to the resulting changes in k and h along the general equilibrium path, it is convenient to decompose the cost benefit rule in two parts, as we have done in Proposition 1. The first part measures the sum of welfare gains over time of marginal increases in the stock of physical capital, where each instantaneous welfare gain is multiplied by the corresponding tax induced change in the stock of physical capital. Similarly, the second part measures the sum of welfare gains over time of marginal increases in the stock of health capital, where each instantaneous welfare gain is weighted by the corresponding tax induced change in the stock of health capital. The instantaneous welfare gain of a marginal increase in the stock of health capital is, in turn, decomposable in two separate effects. First, an increase in the stock of health capital increases the future choice set of the consumer, and the associated welfare gain is summarized by the shadow price of health capital, which is equal to $(\partial u_t/\partial c_t)/(\partial g_t/\partial e_t) > 0$ at the equilibrium. An interpretation is that the increase in the stock of health capital leads the consumer to reduce his/her private health expenditures, ceteris paribus, which increases the resources available for private consumption. Second, an increase in the stock of health capital also implies a direct welfare gain, since health capital is an argument in the instantaneous utility function.

The intuition behind Proposition 1 is, of course, that present biased preferences imply weaker incentives for capital formation (both with respect to physical capital and health capital) than the preferences represented by the social welfare function. A similar result could have been derived if we, instead, introduced a small subsidy towards private investments in health, e, financed by a lump-sum tax. As long as the policy contributes to increase the capital stocks, it will also increase the social welfare.

Note that Proposition 1 would also apply, if we were to assume that part of the consumers is time-consistent, while the other part is time-inconsistent in the way described above. For time-consistent consumers, $\beta = 1$, implying that the policy reform has no first order welfare effect. Instead, only those that impose external effects on their future selves are directly affected.

On the other hand, although we may be able to increase welfare by taxing the unhealthy good, such a tax is not necessarily part of the policy package that implements the socially optimal resource allocation. The reason is that the external effect the individual imposes on his/her future selves does not reflect an incorrect choice of x conditional on the relevant shadow prices; it reflects an underestimation of the shadow prices of physical capital and health capital. To see this more clearly, let us derive the socially optimal resource allocation by maximizing equation (10) subject to equations (4) and (9). The first order conditions for x_t and l_t remain as in equations (5) and (6), whereas equations (7) and (8) change to read

$$-\frac{\partial u_t}{\partial c_t}+\frac{\partial u_{t+1}}{\partial c_{t+1}}\Theta[1+\frac{\partial f_{t+1}}{\partial k_{t+1}}] = 0 \tag{11}$$

$$-\frac{\partial u_t/\partial c_t}{\partial g_t/\partial e_t}+[\frac{\partial u_{t+1}/\partial c_{t+1}}{\partial g_{t+1}/\partial e_{t+1}}+\frac{\partial u_{t+1}}{\partial h_{t+1}}]\Theta = 0 \tag{12}$$

since $\beta = 1$ from the point of view of the social planner. Let $\{c_t^*, x_t^*, l_t^*, e_t^*, k_t^*, h_t^*\}$ for all t represent the equilibrium implicit in equations (4)-(6), (9) and (11)-(12), where the superindex "$*$" is used to denote the socially optimal resource allocation.

To implement the social optimum in the decentralized economy, suppose that we were to announce, in each period, that the consumer will receive two subsidies in the next period, which are proportional to the value of private wealth and the stock of health capital, respectively, and that the subsidies are financed by a lump-sum tax[3]. Note also that these subsidies must be part of a 'surprise policy' introduced in each period, since the consumer does not expect to be time-inconsistent in the future. To illustrate, consider once again the decisions made by the consumer in period t, i.e. when the consumer chooses c_t, x_t, l_t, e_t, k_{t+1} and h_{t+1} conditional on k_t and h_t. Introducing the two subsidies at the rates s_{t+1}^* and p_{t+1}^*, respectively, means that the capital accumulation equation for period $t+1$ changes to read

$$\begin{aligned} k_{t+2} &= (1+r_{t+1})k_{t+1}(1+s_{t+1}^*)+w_{t+1}l_{t+1}+p_{t+1}^*h_{t+1}-T_{t+1} \\ &\quad -e_{t+1}-c_{t+1}-x_{t+1} \end{aligned} \tag{13}$$

in which case s_{t+1}^* and p_{t+1}^* directly affect the choice set in period t. The variable T is a lump-sum tax such that the government's budget constraint is

[3]The use of interest rate subsidies to implement different savings policies in an economy where the (sophisticated) agents have a self-control problems due to quasi-hyperbolic discounting has been addressed by Laibson (1996). However, since we are considering health aspects of consumption as well as focus (in this section) on naive consumers, our framework differs in a fundamental way from that of Laibson.

satisfied; $s_t^*(1+r_t)k_t + p_t^* h_t = T_t$ for all $t > 0$. We assume that the consumer treats the lump-sum tax as exogenous. Define $u_t^* = u(c_t^*, x_t^*, z_t^*, h_t^*)$ and $g_t^* = g(x_t^*, e_t^*)$ and consider Proposition 2;

Proposition 2 *Suppose that the consumer at any time, t, expects to receive the subsidies $s_{t+1}^*(1+r_{t+1})k_{t+1}$ and $p_{t+1}^* h_{t+1}$ in period $t+1$, while he/she expects to receive no subsidies beyond period $t+1$. If*

$$\begin{aligned} s_{t+1}^* &= \frac{(1-\beta)}{\beta} \text{ and} \\ p_{t+1}^* &= \frac{(1-\beta)}{\beta} \frac{1}{\partial u_{t+1}^*/\partial c_{t+1}} \left[\frac{\partial u_{t+1}^*/\partial c_{t+1}}{\partial g_{t+1}^*/\partial e_{t+1}} + \frac{\partial u_{t+1}^*}{\partial h_{t+1}} \right], \end{aligned}$$

then the equilibrium in the decentralized economy is equivalent to the social optimum.

The proof of Proposition 2 is straight forward; given the subsidies described in the proposition, and if the first order conditions associated with the market economy controlled by these subsidies are evaluated in the social optimum, they coincide with the first order conditions that can be derived directly from the social optimization problem. Each formula serves the purpose of eliminating a divergence between an Euler equation associated with the private optimization problem and the corresponding Euler equation resulting from the social optimization problem. The terms within the square bracket of the expression for p_{t+1}^* are equivalent to, and have the same interpretations as, the corresponding terms in Proposition 1. The intuition behind Proposition 2 is that the difference between the uncontrolled market economy and the social optimum arises as time-inconsistent individuals underestimate the shadow prices of physical capital and health capital. As a consequence, the policy required to internalize the external effect, which the individual imposes on his/her future selves, must be designed to make the individual value physical capital and health capital in the same way as the social planner. The subsidies towards the stocks of physical capital and health capital described above will have precisely this effect.

3 Briefly on Sophisticated Consumers

So far, we have assumed that the consumer is naive in the sense of not recognizing that the preference for immediate gratification is present also when the future arrives. This assumption may, or may not, be correct. Another possibility discussed in previous studies on self-control problems due to quasi-hyperbolic discounting is that the consumer is sophisticated; in this case, the consumer recognizes that his/her future selves will also apply quasi-hyperbolic discounting. As such, the consumer understands that his/her future selves also face a self-control problem and uses this information when solving his/her optimization problem. Therefore, the best the current self can do is to decide upon a plan, which the future selves will follow. With sophisticated consumers, the resource allocation is commonly described as a subgame perfect equilibrium resulting from a game played by the different intertemporal selves.

Since the step from naive to sophisticated consumers only affects the Euler equations, while the static first order conditions for the control variables remain as in equations (5) and (6), the qualitative implications for policy intervention will, in technical terms, resemble those derived in the previous section. As a consequence, our treatment of the case with sophisticated consumers is brief[4]. Following Fischer (1999), we describe the optimization problem for self t in terms of a modified Bellman equation

$$V_t = \underset{c_t, x_t, e_t, l_t}{Max} \; u(c_t, x_t, z_t, h_t) + [V_{t+1} - (1-\beta)u(c_{t+1}, x_{t+1}, z_{t+1}, h_{t+1})]\Theta \quad (14)$$

subject to equations (3) and (4). The term V_t is interpretable as the value function for self t, while V_{t+1} is the corresponding value function for self

[4]For a more thorough treatment of sophisticated consumers under quasi-hyperbolic discounting, although in a different context than ours, see e.g. Laibson (1996, 1997), Fischer (1999) and Gruber and Köszegi (2001, 2002). For an excellent discussion about the distinction between naive and sophisticated consumers, see O'Donoghue and Rabin (1999).

$t+1$. The relevant difference between selves t and $t+1$ is that self t believes that self $t+1$ overvalues his/her instantaneous utility, which explains why self t subtracts the second terms within the square bracket from the value function of self $t+1$. Note also that the strategy chosen by each self depends on the initial endowment, so c_{t+1}, x_{t+1}, e_{t+1} and l_{t+1} are treated as functions of k_{t+1} and h_{t+1} by self t. The Euler equations can be written as

$$-\frac{\partial u_t}{\partial c_t} + [\frac{\partial u_{t+1}}{\partial c_{t+1}}(1+\frac{\partial f_{t+1}}{\partial k_{t+1}}) - (1-\beta)\frac{du_{t+1}}{dk_{t+1}}]\Theta = 0 \quad (15)$$

$$-\frac{\partial u_t/\partial c_t}{\partial g_t/\partial e_t} + [\frac{\partial u_{t+1}/\partial c_{t+1}}{\partial g_{t+1}/\partial e_{t+1}} + \frac{\partial u_{t+1}}{\partial h_{t+1}} - (1-\beta)\frac{du_{t+1}}{dh_{t+1}}]\Theta = 0 \quad (16)$$

where

$$\frac{du_{t+1}}{dk_{t+1}} = \frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial k_{t+1}} + \frac{\partial u_{t+1}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial k_{t+1}} - \frac{\partial u_{t+1}}{\partial z_{t+1}}\frac{\partial l_{t+1}}{\partial k_{t+1}}$$

$$\frac{du_{t+1}}{dh_{t+1}} = \frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial h_{t+1}} + \frac{\partial u_{t+1}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial h_{t+1}} - \frac{\partial u_{t+1}}{\partial z_{t+1}}\frac{\partial l_{t+1}}{\partial h_{t+1}} + \frac{\partial u_{t+1}}{\partial h_{t+1}}$$

A combination of subsidies to wealth and health capital can also in this case be used to implement the resource allocation that would be chosen by the social planner with exponential discounting (the outcome of which was described in the previous section); let be that the exact policy rules differ from those described in the context of Proposition 2. This can be seen by comparing equations (15) and (16) with equations (11) and (12).

The main qualitative difference between the policies required here and the policies discussed in the context of Proposition 2 refer to the nature of the underlying self-control problem. In the previous section, where the consumer was treated as naive, it was necessary to implement a 'surprise policy', the purpose of which is to address that the consumer, in each period, does not fully understand his/her future decision problem. With a sophisticated consumer, on the other hand, the same policy instruments will be used for a different purpose; to correct for a self-control problem, which the consumer is fully aware of.

4 Discussion

Despite the simplicity of the model, our policy analysis contains a message of importance for policy intervention. If we treat health as a capital concept - which is arguably realistic - then the optimal policy required to implement the resource allocation, which would be chosen by a social planner with an exponential discount factor, is likely to include a subsidy directed to the stock of health capital instead of a tax on the consumption of the good that gives rise to bad health. At the same time, subsidies to health capital might be more difficult to implement in practice than a tax on unhealthy food, because the stock of health capital is largely unobserved at the individual level. Therefore, much more research is needed before - if ever - we can use these ideas as a basis for practical policy intervention.

It is important to emphasize that the study of policy intervention due to health aspects of self-control problems is still in its infancy. By focusing on other policy instruments than most previous studies dealing with self-control problems, our paper contributes to the understanding of how these instruments can be used for purposes of policy intervension as well as constitutes a starting point for future research. One such natural extension would be to address policy intervention in a situation, where part of the information needed to implement the first best is unobservable to the government. For instance, health status at the individual level is, at least to some extent, likely to be private information. As such, analyzing this policy problem in a model with asymmetric information - in which the informational asymmetries are thoroughly specified - may provide additional insights. We leave this and other questions for future research.

Appendix

Proof of Proposition 1:

To begin with, note that the control, state and costate variables characterizing the general equilibrium path can be written as functions of the parameters of the problem. Therefore, by differentiating equation (10) with respect to τ, we obtain

$$\frac{\partial \tilde{U}_0^0}{\partial \tau} = \sum_{t=0}^{\infty} [\frac{\partial u_t^0}{\partial c_t}\frac{\partial c_t^0}{\partial \tau} + \frac{\partial u_t^0}{\partial x_t}\frac{\partial x_t^0}{\partial \tau} - \frac{\partial u_t^0}{\partial z_t}\frac{\partial l_t^0}{\partial \tau} + \frac{\partial u_t^0}{\partial h_t}\frac{\partial h_t}{\partial \tau}]\Theta^t \tag{A1}$$

where $u_t^0 = u(c_t^0, x_t^0, z_t^0, h_t^0)$. Equations (5) and (9) imply for any $t > 0$

$$\frac{\partial f_t^0}{\partial l_t}\frac{\partial l_t}{\partial \tau} + [1 + \frac{\partial f_t^0}{\partial k_t}]\frac{\partial k_t^0}{\partial \tau} - \frac{\partial e_t^0}{\partial \tau} - \frac{\partial c_t^0}{\tau} - \frac{\partial x_t^0}{\tau} - \frac{\partial k_{t+1}^0}{\partial \tau} = 0 \tag{A2}$$

$$\frac{\partial g_t^0}{\partial x_t}\frac{\partial x_t^0}{\partial \tau} + \frac{\partial g_t^0}{\partial e_t}\frac{\partial e_t^0}{\partial \tau} - \frac{\partial h_{t+1}^0}{\partial \tau} + \frac{\partial h_t^0}{\partial \tau} = 0 \tag{A3}$$

where $f_t^0 = f(l_t^0, k_t^0)$ and $g_t^0 = g(x_t^0, e_t^0)$. The next step is to use equations (A2) and (A3) to substitute for $\partial c_t^0/\partial \tau$ and $\partial e_t^0/\partial \tau$, respectively, in equation (A1). Finally use the necessary conditions given by equations (5)-(8) together with the initial conditions that k_0 and h_0 are exogenous. Rearranging gives the cost benefit rule in Proposition 1. ■

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