Documentos CEDE

ISSN 1657-7191 edición electrónica

Conflict and Uncertainty: A Dynamic Approach

Miguel A. Espinosa
Juan D. Prada

6

ENERO DE 2010

Universidad de los Andes
Facultad de Economía

CEDE
Centro de Estudios sobre Desarrollo Económico

Serie Documentos Cede, 2010-6
ISSN 1657-7191

Enero de 2010

Calle 19A No. 1 – 37, Bloque W.
Bogotá, D. C., Colombia
Teléfonos: 3394949- 3394999, extensiones 2400, 2049, 3233
infocede@uniandes.edu.co
http://economia.uniandes.edu.co

Ediciones Uniandes
Carrera 1ª Este No. 19 – 27, edificio Aulas 6, A. A. 4976
Bogotá, D. C., Colombia
Teléfonos: 3394949- 3394999, extensión 2133, Fax: extensión 2158
infeduni@uniandes.edu.co

Edición, diseño de cubierta, preprensa y prensa digital:
Proceditor ltda.
Calle 1ª C No. 27 A – 01
Bogotá, D. C., Colombia
Teléfonos: 2204275, 220 4276, Fax: extensión 102
proceditor@etb.net.co

Impreso en Colombia – *Printed in Colombia*

Universidad de los Andes
Facultad de Economía

CEDE
Centro de Estudios sobre Desarrollo Económico

Conflict and uncertainty: A Dynamic Approach*

Miguel A.Espinosa F.[†] Juan D. Prada S.[‡]

Most of the conflict theory papers have used a one-shot game set-up. This does not correspond to reality and is certainly incapable of modeling real conflict situations. We propose a dynamic model with N-agents in an infinite time frame which allow us to adequately analyze conflicts. The dynamic aspects of the conflict come at least from two sources: first, the preferences on the good in dispute are not static; second, agents in conflict can influence the future of the conflict by making investment in conflict's technology. We use a simple deterministic rule that defines the evolution of the subjective valuation for the good in dispute according to the results obtained by the agents in the recent past. During each period the realization of stochastic variables of the nature's states induces uncertainty in the game. The model is a theoretical approach that can be applied to evaluate the role of uncertainty and valuations' evolution on the optimal choices of forward-looking economic agents that seek to appropriate a share of a divisible resource.

Keywords: Conflict Theory, Dynamic Economic Model, Uncertainty.

JEL Classification: C70, D70, D81, D84.

*This paper was presented in the National Symposium of Microeconomics in Bogotá, Colombia in August 2007. The remaining errors are ours. The views expressed in the paper are those of the authors and do not represent those of the Banco de la República or its Board of Directors.
†Universidad de los Andes. e-mail: migu-esp@uniandes.edu.co
‡Banco de la República. e-mail jpradasa@banrep.gov.co

Conflicto e incertidumbre: Una aproximación dinámica*

Miguel A. Espinosa F.† Juan D. Prada S.‡

Resumen

La mayoría de papers de la teoría de conflictos han usado un contexto teórico tipo one-shot. Esto no corresponde a la realidad pues es incapaz de modelar situaciones reales donde emergen los conflictos. Nosotros proponemos un modelo dinámico con N agentes en un horizonte infinito que permite modelar adecuadamente conflictos. Los principales aspectos dinámicos del conflicto proviene por lo menos de dos fuentes: Primero, las preferencias por el bien en disputa no son estáticas; segundo, los agentes en el conflicto pueden influenciar el futuro del conflicto realizando inversiones en tecnología de conflicto. Nosotros usamos una simple regla determinística que define la evolución de la valuación subjetiva por el bien en disputa, en función de los resultados obtenidos por los agentes en el pasado reciente. Adicionalmente, la realización de variables estocásticas de estados de la naturaleza provee al modelo de incertidumbre. El modelo es una aproximación teórica que puede ser aplicada para evaluar el rol de la incertidumbre y la evolución de las valoraciones en las elecciones óptimas de agentes económicos que buscan apropiarse de una porción de un bien divisible.

Palabras clave: teoría de conflictos, modelo económico dinámico, incertidumbre.

Clasificación JEL: C70, D70, D81, D84.

---

* Este paper ha sido presentado en el Simposio Nacional de Microeconomía en Bogotá, Colombia en agosto de 2007. Los errores son enteramente nuestra responsabilidad. La visión expresada en este artículo es enteramente del autor y no representa la del Banco de la República.

† Universidad de los Andes. e-mail: migu-esp@uniandes.edu.co

‡ Banco de la República. e-mail jpradasa@banrep.gov.co

1 Introduction

Most of the conflict theory papers have used a one-shot game set-up[1]. We believe this does not correspond to reality and is certainly incapable of modeling real conflict situations. The dynamic aspects of the conflict come at least from two sources: first, the preferences on the good in dispute are not static; second, agents in conflict can influence the future of the conflict by making investment in conflict's technology, and by adapting to different environments. To illustrate the first aspect, let's think about sports. We consider that the first time Roger Federer won a Grand Slam he felt happier that when he won his n-th championship. If we assume that each title is a different kind of good, this fact cannot be explained by decreasing marginal utility. However, it seems appealing to assume that Federer's valuation for his titles have changed with past success. The second source of dynamic concern in conflict is the obvious one. Agents try to do the best they can when fighting for the control of a valuable resource. They will try to improve their technology and to adapt to the conflict environment, making explicit choices to achieve their goals. These aspects cannot be accounted for in a static conflict model.

This paper works on a dynamic conflict model that incorporate both aspects at some extent. The aim is to give one more step in the study of dynamic conflicts. Many authors have explored dynamic aspects in conflict modeling. Hirshleifer (1995)[2], Grossman and Kim (1995) and Skaperdas (1992), among other authors, have called attention to the importance of modeling the events developed in a dynamic conflict. However, perhaps the only ones interested in giving formal answers have been Maxwell and Reuveny (2001, 2005) and Eggert, ichi Itaya and Mino (2008). They induce dynamics in a one-period-conflict model introducing differential equations to account for the exogenous evolution of some state variables. Our paper generalizes their work in some dimensions. We explicitly model the endogenous state variables that generate the inter-temporal links needed for a proper dynamic conflict. This is done through investment in the conflict technology, resembling standard macroeconomics models.

Furthermore, although Maxwell and Reuveny (2001, 2005) have constructed a dynamic model for two players, they have recognized that their model's agents are myopic. We try to improve this limitation proposing a model with N forward-looking agents. We assume the existence of a private set of information for each agent and a public information set, and this allow the agents of our model

[1] For example Hirshleifer (1995), Skaperdas (1992), Grossman and Kim (1995), Neary (1997) and Anderton, Anderton and Carter (1999).

[2] This author was the first to propose an extended methodology in the analysis of conflict, allowing for a dynamic set-up. Nevertheless, like is mentioned in Maxwell and Reuveny (2001), Hirshleifer's model does not use equations describing the paths of variables over time.

to plan the future with some knowledge about the underlying distributions associated with random states of nature. This implies that our agents maximize the discounted sum of expected future utility, not only the current revenue. This differentiates a dynamic conflict from a repeated-game conflict.

Finally we include dynamics in the valuations of agents. It is usual to find conflict theory models that assume that the preferences on the good in dispute are static. To incorporate this dynamic aspect, we use a simple deterministic rule that defines the evolution of the subjective valuation for the good in dispute according to the results obtained by the agents in the recent past.

Although through the paper we will give examples about military conflicts, this is not the only application of our model. Using the same set up, the model could be easily extended to analyze political party disputes, R&D competitions, business races, lobbyists in legislatures or any similar dispute. Besides that, we believe that dynamic models of conflict can have applications to dynamic auction processes. Auctions are non-violent conflicts with a specific allocation rule, that could fit the dynamic set-up proposed in this paper.

The paper is organized as follows: the next section introduces the model. The third section goes through some details of the model that are useful for a better understanding of the conflict context. Then we characterize the solution of our game under a specific allocation rule, followed by a simple example with the aim of understanding how the model works. We conclude with a summary of our main findings.

2 The Model

This section presents the set up model concentrating on the main assumptions that we use. We also explain with some detail the timing of the whole game.

2.1 Axiomatization

Suppose there exists a divisible resource R_t and there are $I \in \mathbb{N}^*$ agents (*individuals or groups*) that are competing to obtain a share of valued divisible resource (like the government of some country, the control over a key population, the monopoly on a natural resource, a victory in a military conflict etc.).

Furthermore as mentioned by Hirshleifer (1989) R_t lacks of future well-defined property rights. That is, R_t or any fraction of it, at the beginning of the period t could be property of someone, but if the resource enters into a conflict situation, then there exists a positive probability that at the end of the period it will be the property of someone else.

We define the set of indexes $\mathcal{I} = \{i \in \mathbb{N}^* : 1 \leq i \leq I\}$ and identify each agent by her corresponding index $i \in \mathcal{I}$. We assume that the time is discrete, and the conflict has a time duration of $T \in \mathbb{N}^*$ periods. Let $\mathcal{T} = \{t \in \mathbb{N}^* : 1 \leq t \leq T\}$ be the set of discrete temporal indexes.

We model each conflict event as a one-time-played game with dependence on past events. That is, each period a new game is played, so each time there will be a new assignation of the divisible resource, but the state of the conflict will depend on past choices. We have that the number of games played $T \to \infty$. Then we do not rule out, by assumption, all non-competitive strategies[3], but we focus in non-cooperative games.

Each agent $i \in \mathcal{I}$ has an initial valuation scale for the good that she is able to obtain. We represent an I-dimensional valuation vector $v_0^{\mathcal{I}} = \left(v_0^1, ..., v_0^I\right) > \mathbf{0}$[4]. Agents' initial value scale are drawn from $\Psi = \left\{v_0^i \in [\underline{v}, \overline{v}] : \underline{v} \geq 0, (\forall i \in \mathcal{I})\right\}$.

We assume that each agent has the incentive to exert some effort to the conflict in order to obtain a proportion of the divisible resource: each agent receives utility from her subjective valuation of the good and from the quantity of the valuable resource that is left to use freely. Additionally we allow agents to invest in conflict technology.

The quantity c_t^i is the amount of resource available for free use and is the quantity that the agent $i \in \mathcal{I}$ has in the period $t \in \mathcal{T}$, after all other uses have realized (that is, after the cost incurred to obtain the valuable resource and the investment are realized). That is determined by

$$c_t^i = \mu_t^i R_t - g^i\left(e_t^i; \theta_t^i\right) - x_t^i$$

where μ_t^i is the share of the good obtained, R_t is the total amount of the resource available in the period $t \in \mathcal{T}$, e_t^i is the effort used explicitly in the conflict by agent i, $g^i\left(e^i; \theta^i\right)$ is the cost (in units

[3]The finiteness of the repeated game would allow us to solve the game by backward induction. This would allow us to rule out, by assumption, all non-competitive strategies. For if such strategy exists, some player has the incentive to deviate from it in order to obtain a higher discounted utility. If so, that player would deviate in the last period, to avoid punishment. But knowing that, all other agents would also deviate from the strategy. Solving for the period $T-1$ we would find the same situation. Then, the finiteness of the game, that impedes a credible punishment to the players who deviate from a possible cooperative strategy, would allow us to rule out this possibility.

[4]As standard in the economic literature (in the convex analysis literature) we define the following order relation for vectors.

Let $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ be two vectors, and consider the convex cone $\mathbb{R}_{++}^n = \{\mathbf{r} = (r_1, \ldots, r_n) \in \mathbb{R}_{++}^n : r_1 > 0, \ldots, r_n > 0\}$. Then we define the order relation $>$ as:

$$\mathbf{u} > \mathbf{v} \leftrightarrow \mathbf{u} - \mathbf{v} \in \mathbb{R}_{++}^n$$

of the resource) of exerting a total effort of e_t^i, $\theta^i \in \mathbb{R}^m$ is a vector of cost function parameters on which we will turn briefly[5] and x_t^i is the investment level in conflict technology. We denote the effort I-vector in the period $t \in \mathcal{T}$ as $e_t^{\mathcal{I}} = \left(e_t^1, ..., e_t^I\right)$ and $e_t^{\mathcal{I}\setminus\{i\}} = \left(e_t^1, ..., e_t^{i-1}, e_t^{i+1}, ..., e_t^I\right)$ the effort I-vector of everybody but the agent $i \in \mathcal{I}$. We assume that every period the valuable resource left for free use for every agent is nonnegative[6]

$$(\forall i \in \mathcal{I})\,(\forall t \in \mathcal{T})\,,(c_t^\iota \geq 0)$$

The share μ_t^i is defined by the level of effort exerted in the conflict by all the agents, according to a typical conflict effort function (or contest success function, Skaperdas (1996))

$$p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$$

where $\alpha_t^{\mathcal{I}} = \left(\alpha_t^1, \ldots, \alpha_t^I\right)$ represents the technological coefficients associated to all the agents. That is, we assume that the contest success function summarizes all efforts and relative power of the agents in conflict. The allocation rule μ_t takes this information and indicates how to split the resource between agents. We assume that μ_t^i is nondecreasing with $p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$.

We define $\alpha_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}\setminus\{i\}}$ in the same way as for the effort levels, and let these symbols represent the technological coefficients associated to all the agents, and $I-1$ agents, respectively. This technology satisfies that $(\forall i \in \mathcal{I})\,(\forall t \in \mathcal{T})\left(p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right) : \mathbb{R}_+^I \times \mathbb{R}_+^I \rightarrow [0,1]\right)$ and $\sum_{i\in\mathcal{I}} p^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right) = 1$ and will determine the share of the valuable resource that each agent is able to obtain[7].

Then, there is strategic interdependence between agents, because the effort level chosen by all the opponents affects the share of the good obtained in the conflict. This interdependence is a key factor of other kind of models, as auction models. These models seek not just to understand the decisions of each agent, but the implications of complex interactions between them.

We assume that $\forall i \in \mathcal{I}, \forall t \in \mathcal{T}$, $\frac{\partial}{\partial \alpha_t^i} p_t^i\left(e_t^{\mathcal{I}}, \alpha_{n,t}^{\mathcal{I}}\right) \geq 0$, $\frac{\partial}{\partial e_t^i} p_t^i\left(e_t^{\mathcal{I}}, \alpha_{n,t}^{\mathcal{I}}\right) \geq 0$, and for $j \neq i$, $\frac{\partial}{\partial e_t^j} p_t^i\left(e_t^{\mathcal{I}}, \alpha_{n,t}^{\mathcal{I}}\right) \leq 0$: that is, the share of the resource obtained by agent i is nondecreasing in α_t^i and e_t^i, but is nonincreasing in the effort level exerted by the opponents.

[5] We impose that $g^i : \mathbb{R}_+^{m+1} \rightarrow \mathbb{R}_+ \forall i \in \mathcal{I}$, furthermore $g^i\left(\cdot;\theta^i\right)$ is a continuously differentiable convex function. We also impose that $g^i\left(0;\theta^i\right) = 0$. Note that we allow differences among the model's agents.

[6] That is imposing a restriction where the cost of exerting a total effort should always be not greater than the difference between the good obtained and the investment level: $\mu_t^i R_t - x_t^i \geq g^i\left(e_t^i;\theta_t^i\right)$.

[7] See Section 3. If $\mu_t^i = p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$, this can be understood naturally like a share model such as in Skaperdas (1996), Hirshleifer (1989, 1991, 1995) and Maxwell and Reuveny (2001). In a share model every agent wins a fraction of R_t. A second alternative is the win-approach, in which just one agent wins the whole R_t. To obtain the later approach it is only necessary to assign an arbitrary rule. The most simple rule could be one in which the winner has the greatest value of $p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$. In case of ties whatever random rule could solve the problem.

The parameter α_t^i represents a measure of ex-ante relative effectiveness in the conflict: *ceteris paribus*, higher levels of α_t^i increase the share of the good obtained by agent i. This technological parameter can be influenced by investment in conflict technology. In particular, we have a transition function

$$\alpha_{t+1}^i = \alpha^i \left(x_t^i, \alpha_t^i, z_t, s_t^i\right)$$

satisfying $\frac{\partial}{\partial x_t^i}\alpha^i \left(x_t^i, \alpha_t^i, z_t\right) \geq 0$ (rising investment level x_t^i will never decrease the technological parameter function α^i), where s_t^i is a success measure for the agent $i \in \mathcal{I}$ in the period $t \in \mathcal{T}$. The factor z_t is an exogenous shock to investment in conflict technology.

This investment in conflict technology is a key ingredient in dynamic conflicts. For example, since the "enemy adapts to the methods employed by the attacker" (Weeks (2001)) we could think of a change in the relative effectiveness of the agents according to the average success on each time period. In that case, the weaker players could adapt to the harder environment, and increase their relative effectiveness. This would imply a negative relationship between success and next-period effectiveness. This fact could explain why ex-ante weak fighters can resist fights over long periods. This cannot be modeled without an explicit dynamic mechanism.

We represent the preferences on the net share of the resource obtained with a utility function

$$\pi_t^i = \pi^i \left(v_t^i, u^i \left(c_t^i\right)\right) : \mathbb{R}_+^2 \to \mathbb{R}$$

that is increasing (at a decreasing rate) in its arguments, where v_t^i is the subjective valuation for the good in dispute (is the value scale parameter when π^i is multiplicatively separable) and $u^i \left(c_t^i\right)$ is the standard instantaneous utility function. A higher valuation for the good in dispute implies that the utility obtained by consumption of the good in dispute is more valuable to the agent. We assume that a higher quantity of the good left for free use increases the utility received by the agent with decreasing marginal returns. We impose the standard conditions on the utility function: $\forall i \in \mathcal{I}$, $u^i(\cdot)$ and $\pi^i(\cdot)$ are continuously differentiable concave functions.

3 Simple Characterization

3.1 Allocation Rule

Given the effort level $e_t^{\mathcal{I}}$ and the effectiveness parameters $\alpha_t^{\mathcal{I}}$, the allocation rule (the central planner, the government, nature or the justice or whatever in what you believe) assigns the resource in dispute to the agents of conflict.

The allocation rule of our game μ_t is a function

$$\mu_t : [0,1]^I \to [0,1]^I$$

such that $\sum_{i \in \mathcal{I}} \mu_t^i \left(p_t\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)\right) = 1$.

We now give some examples of possible allocation rules.

- Share model: Let $\mu_t^i = p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$. Then each agent receives a share of the resource that is totally determined by the contest success function. This is the simpler way to allocate the resource in dispute.

- Probabilistic model: Let $\mu_t^i = 1$ with probability $p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$ and $\mu_t^i = 0$ with probability $1 - p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$. This implies a random allocation of the resource, but each agent can increase the probability of receiving the good. This approach is followed by Maxwell and Reuveny (2005).

- Auction model: We can interpret the contest success function $p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)$ as the normalized value offered by agent i for the good in dispute. The parameters $\alpha_t^{\mathcal{I}}$ represent the lobbying power of each agent. In this way, not only explicit effort influences the result of the auction, but also the relative power of the bidders. In this case, the allocation rule μ_t is any standard auction rule. Let $M = \arg\max_{i \in \mathcal{I}} \left\{p_t^i\left(e_t^{\mathcal{I}}, \alpha_t^{\mathcal{I}}\right)\right\}$. For a first-price auction we could have

$$\mu_t^i = \begin{cases} \frac{1}{\#M} & \text{if } i \in M \\ 0 & \text{otherwise} \end{cases}$$

These mechanisms are just examples of many ways of allocating the valuable resource to the conflict agents. Standard conflict theory assumes a conflict technology that according to the effort levels assigns a share of the good (or gives a probability of victory). Other mechanisms could be used as well. There is no reason to think, for instance, that an auction mechanism is better/worse than any other mechanisms available in the literature.

Note that the relevant concept of equilibrium and the solution method to this game depends on the allocation rule. For instance, if we follow an auction approach, then auction theory methods should be applied (see for example Krishna (2002)).

For simplicity, we follow the conflict theory literature and work with the share model from now on. The solution to this game, assuming differentiability, is almost straigthforward.

3.2 Dissipation

Following Esteban and Ray (1999) we understand conflict as "a situation in which, in the absence of a collective decision rule, social groups with opposed interests incur losses in order to increase the likelihood of obtaining their preferred outcome". We define the dissipation of the conflict as the sum of those losses, and this measure could be seen as a measure of the intensity of the conflict. Hirshleifer (1991) notices that generally the cost associated to a conflict includes diverse aspects as attrition of the resources, collateral damages and foregone opportunities etc. Unfortunately our approximation to the cost of conflict, like an intensity measurement, leaves aside these considerations. Nevertheless we believe that these subjects are of fundamental importance, reason why they will be taken implicitly into account in some of the later analyses. The way to include this in our model is from the viewpoint of the opportunity cost. Resources devoted to the conflict (effort levels, its cost and the investment on the technology of the conflict) are resources that are not used in a productive way, in this context c_t^i, the resource for free use. In the literature which relates conflict theory and growth analysis, these resources are means to increase the wealth of a group. In our context, the cost of conflict comes in the form of foregone consumption.

We define the dissipation of the conflict as

$$d_t = \sum_{i \in \mathcal{I}} g^i \left(e_t^i ; \theta_t^i \right)$$

and this is our measure of the intensity of conflict.

3.3 Uncertainty

We model uncertainty in each period. The total amount of the divisible resource R_t is unknown during the choice-making process for all the agents. This reflects the fact that the agents are not capable of determining the total amount of the valuable resource available in each event: in a political process the exact amount of resources that some party is able to capture is not known until all the bureaucratic processes are completed; in a struggle for a natural resource, the amount of the resource (a oil well, a mine etc.) is not known until long after the exploitation begins; in a treasury auction, the total supply may be unknown for the bidders; in a war, the "booty" that can be appropriated in an event is known only when the battle is finished. Although the agents involved in a war could know how big the territory they can win is, the do not know how rich the loots could be. There is uncertainty about the investment as well. We assume that the exogenous factor z_t follows a dynamic process affected by random shocks

$$z_t = z\left(z_{t-1}, \epsilon_t^z\right)$$

We also assume that the cost parameters $\theta_t = \left(\theta_t^1, \theta_t^2, \ldots \theta_t^I\right)$ are random variables.

Let

$$F\left(R_t, \epsilon_t^z, \theta_t\right)$$

be the continuous cumulative joint distribution function of resources and technological characteristics of the agents. All the marginal distributions derived from $F(\cdot)$ are also continuous. We assume that each agent $i \in \mathcal{I}$ knows her own cost parameter vector θ^i that is drawn according to $F(\cdot)$ during each event, before the choice-making process takes place. But the agent is unaware of the vectors $\theta_t^{\mathcal{I}\setminus\{i\}} = \left(\theta_t^1, \ldots, \theta_t^{i-1}, \theta_t^{i+1}, \ldots, \theta_t^I\right)$, the cost vectors associated to all the other agents of the conflict. This reflects the fact that the relative strength of the opponents is unknown during each event, and in this context we treat it as a random variable. Another approach could try to endogenize the evolution of the effort-cost factors, with investment in conflict machinery, investment in research and development or adaptation to the conflict situations (as proposed by Weeks (2001)). Nevertheless this could neglect factors such as luck, climate and geographic influences or an exogenous change taking place in the conflict. Then, as a first approach we prefer to model the effort-cost coefficients as an exogenous process and focus on the role of uncertainty on the conflict costs. And later we will focus in investment in conflict technology, not in effort costs.

We assume rational expectations: the agents take all available information into account in forming expectations. Formally we assume that the agents know the associated distributions for the random variables. This is equivalent to assume that the subjective distributions taken to form expectations are the same objective distributions from the conflict environment.

3.4 This is an incomplete information game

For clarification we summarize the information using a version of game theory axiomatization. Our incomplete information game $(\omega, \varphi, \sigma)$ has the following components:

1. A set of private information $\omega = \{\alpha, \theta, \pi, \mathbf{g}\}$ where $\alpha^i = \left\{\alpha_t^i\right\}_{t\in\mathcal{T}}$ and $\alpha = \left\{\alpha^i\right\}_{i\in\mathcal{I}}$, α is a $TI \times 1$ vector, $\theta = \left\{\theta^i\right\}_{i\in\mathcal{I}}$ is a $I \times 1$ vector, $\mathbf{g} = \left\{g^i\right\}_{i\in\mathcal{I}}$ and $\pi = \left\{\pi^i\right\}_{i\in\mathcal{I}}$ are $I \times 1$ vectors.

2. A set of possible messages $\varphi = \left\{\varphi^i\right\}_{i\in\mathcal{I}}$ and $\varphi^i : \omega \to \mathbb{R}_+^2$ that maps from the private information set to the real set, that is $\varphi = \{\mathbf{e}, \mathbf{x}\}$ is a $2\left(TI \times 1\right)$ vector.

3. A set of common information $\sigma = \{\sigma_t\}_{t\in\mathcal{T}} = \{\mu, I, T, R_{t-1}, z_{t-1}\}$. That is, an allocation rule μ that says how the game works, I agents, T periods, the available resource under dispute in $t-1$ and

the past exogenous technological shock. Agents know the functional forms involved in the game, but are unaware of the value of random parameters and shocks.

Therefore our complete game can be represented in an extensive way as: $(\alpha, \theta, \mathbf{g}, \mathbf{u}, \mathbf{e}, \mathbf{x}, \mu, I, T)$.

4 Dynamic mechanism

The key feature of our model is the evolution with time of the valuation of the resource. We do not believe that in a repeated game the valuation for the good should be kept static. Assume that there is some kind of satiation from the conflict good. For example, the public opinion may get tired of their current politicians, and this generates a cost in terms of "happiness" to the incumbent agent; a military victory after many defeats is more valued than the last of many consecutive victories; the novelty is more valued etc. This all lead us to think that the valuations change according to the success in obtaining the good.

However the adjustments are not immediate. It takes time to change the subjective perceptions: if the agent is a political party, the valuation of the good in dispute comes from an agreement process; the subjective beliefs and valuations do not change from one event to another. This is why we assume the valuation as fixed within each time period. Then, the time frame is defined by the moments when the valuations can be changed.

Each time period the valuation of the good is updated according to a fully deterministic rule

$$v_{t+1}^i = v^i\left(v_t^i, s_t^i\right)$$

where s_t^i is a success measure for the agent $i \in \mathcal{I}$ in the period $t \in \mathcal{T}$. In this way we link the valuation evolution to the results obtained in the recent past. The "success index" is given by a function which depends on the share of good obtained

$$s_t^i = s\left(p_t^i\right)$$

4.1 Solution to the share model

The problem faced by each agent $i \in \mathcal{I}$ during the period $t \in \mathcal{T}$ is:

$$
\begin{aligned}
\max_{\left\{e^i_{t+j},\ x^i_{t+j},\ \alpha^i_{t+j+1}\right\}} \quad & \sum_{j=0}^{\infty} E^i_t \beta^j_i \left\{\pi^i \left(v^i_{t+j}, u^i \left(p^i_{t+j} R_{t+j} - g^i \left(e^i_{t+j}; \theta^i_{t+j}\right) - x^i_{t+j}\right)\right)\right\} \\
\text{s.t.} \quad & p^i_{t+j} = p^i \left(e^i_{t+j}, e^{\mathcal{I}\setminus\{i\}}_{t+j}, \alpha^{\mathcal{I}}_{t+j}\right) \\
& \alpha^i_{t+j+1} = \alpha^i \left(x^i_{t+j}, \alpha^i_{t+j}, z^i_{t+j}, s^i_{t+j}\right) \\
& p^i_{t+j} R_{t+j} - g^i \left(e^i_{t+j}; \theta^i_{t+j}\right) - x^i_{t+j} \geq 0 \\
& t + j \in \mathcal{T}
\end{aligned}
$$

where E^i_t is the expectation operator referred to the information set available to the agent i during period t.

This problem simply states that each agent chooses the effort level given the effort of the other agents and the uncertainty about the total amount of the resource and investment in order to maximize the discounted sum of the expected utility.

This is a standard dynamic programming problem and we solve it using the method of Lagrange. The first order conditions for agent i are:

$$
\begin{aligned}
\frac{\partial}{\partial e^i_t} p^i \left(e^i_t, e^{\mathcal{I}\setminus\{i\}}_t, \alpha^{\mathcal{I}}_t\right) R_t &= \frac{\partial}{\partial e^i_t} g^i \left(e^i_t; \theta^i_t\right) && (1) \\
\frac{\partial}{\partial u^i} \pi^i_t \left(v^i_t, u^i_t\right) \frac{\partial}{\partial c^i_t} u^i \left(p^i_t R_t - g^i \left(e^i_t; \theta^i_t\right) - x^i_t\right) &= \gamma^i_t \frac{\partial}{\partial x^i_t} \alpha^i \left(x^i_t, \alpha^i_t, z^i_t, s^i_t\right) && (2) \\
\alpha^i_{t+1} &= \alpha^i \left(x^i_t, \alpha^i_t, z^i_t, s^i_t\right) \\
p^i_t &= p^i \left(e^i_t, e^{\mathcal{I}\setminus\{i\}}_t, \alpha^{\mathcal{I}}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
&\beta_i E^i_t \frac{\partial}{\partial u^i} \pi^i_t \left(v^i_{t+1}, u^i_{t+1}\right) \frac{\partial}{\partial c^i_{t+1}} u^i \left(p^i_{t+1} R_{t+1} - g^i \left(e^i_{t+1}; \theta^i_{t+1}\right) - x^i_{t+1}\right) \frac{\partial}{\partial a^i_{t+1}} p^i \left(e^i_{t+1}, e^{\mathcal{I}\setminus\{i\}}_{t+1}, \alpha^{\mathcal{I}}_{t+1}\right) R_{t+1} \\
&\qquad = \gamma^i_t - \beta_i E^i_t \gamma^i_{t+1} \frac{\partial}{\partial a^i_{t+1}} \alpha^i \left(x^i_{t+1}, \alpha^i_{t+1}, z_{t+1}, s^i_{t+1}\right) \qquad (3)
\end{aligned}
$$

where γ^i_t is the Lagrange multiplier associated with the investment technology.

The condition (1) represents the optimal choice of effort level. This effort depends on the technology parameters (whose values were decided the period before) and the optimal effort of all the other agents in conflict. Therefore this is the reaction curve for effort. It can be easily shown that the optimal effort level of agent i is increasing in the effort level of agent j $(i \neq j)$.

Note that this reaction curve does not depend on $\frac{\partial}{\partial v_t^i}\pi^i$. Then the dissipation in this conflict is independent of the exogenous valuation of the good. Also, the choice of effort is an intra-temporal problem. Then the static model of conflict partially describes this part of the conflict.

The conditions (2) and (3) summarize all the dynamic choices in the conflict. They just say that the expected cost (with information available to agent i) of investing a marginal unit of the valuable resource (given by the current loss of utility) must be equal to the expected profit of this investment, which is the additional resource that can be expected to be obtained the next period thanks to the marginal investment in the conflict technology.

We have developed a dynamic stochastic general equilibrium model of a simple conflict with endowments. The equilibrium of the model is an optimal strategy that takes into account the present and expected future of all variables. Our conflict agents solve a well defined micro-founded problem, fully consistent with the static conflict model, but with a true dynamic behaviour.

4.2 Deterministic steady state

We define the "deterministic steady state" as the equilibrium in which the variables would not change with the time, being all stochastic variables in their unconditional mean. In order for the deterministic steady state to exist, we require some properties for the functions. First, we define the steady state effort levels $\overline{e}^{\mathcal{I}}$ as the solution to the system given by the deterministic version of the optimal rule (1):

$$\frac{\partial}{\partial e^i} p^i \left(e^i, e^{\mathcal{I}\setminus\{i\}}, \alpha^{\mathcal{I}} \right) \overline{R} = \frac{\partial}{\partial e^i} g^i \left(e^i; \theta^i \right)$$

where $\overline{R}$ is the unconditional mean of the resource.

From (2) and (3) we get that in the deterministic steady state

$$\beta_i \frac{\partial}{\partial x^i} \alpha^i \left(x^i, \alpha^i, z^i, s^i \right) \frac{\partial}{\partial a^i} p^i \left(e^i, e^{\mathcal{I}\setminus\{i\}}, \alpha^{\mathcal{I}} \right) \overline{R} = 1 - \beta_i \frac{\partial}{\partial a^i} \alpha^i \left(x^i, \alpha^i, \overline{z}, s^i \right)$$

where $\overline{z}$ is the long-run value of the stochastic shock to investment. In the steady state, the following must also hold

$$\alpha^i = \alpha^i \left(x^i, \alpha^i, z^i, s^i \right)$$

and we are able to solve for α^i and x^i.

If $\overline{v}^i > 0$ is the long-run value scale parameter, the value scale parameter $\overline{v}^i$ must satisfy the following relation:

$$\overline{v}^i = v^i\left(\overline{v}^i, s^i\right)$$

where s^i is the success rate in steady state.

5 Example: The symmetric information case.

In this paper we will carry on with an applied example of the model presented before. First we will explain the symmetric information equilibrium. We assume the following functional forms, which are quite standard in the literature:

$$\begin{aligned}
\pi^i\left(v_t^i, u^i\right) &= v_t^i u^i \\
u^i\left(c_t\right) &= \frac{\left(c_t\right)^{1-\sigma_i}}{1-\sigma_i} \\
g^i\left(e_t^i\right) &= \frac{\left(e_t^i\right)^{1+\eta_i}}{1+\eta_i} \\
p^i &= \frac{\alpha_t^i e_t^i}{\sum_{j\in\mathcal{I}} \alpha_t^j e_t^j} \\
\alpha_{t+1}^i &= \left(1-\delta^i\left(s_t^i\right)\right)\alpha_t^i + z_t^i x_t^i \\
\delta^i\left(s_t^i\right) &= \overline{\delta}^i + a^i\left(\frac{s_t^i}{s_{t-1}^i}-1\right) + b^i\left(\frac{s_t^i}{s_{t-1}^i}-1\right)^2
\end{aligned}$$

where $(\forall i \in \mathcal{I})\left(\sigma_i, \eta_i, \overline{\delta}^i \geq 0\right)$.

Assume that there is no asymetric information in the conflict environment. Then all the shocks are unknown to all the agents, so everybody has the same information set each period. Then the optimal conditions simplify, because now $E_t^i = E_t$, the expected value is taken with the same information set for all the agents.

All functions are differentiable, so the solution for the problem of agent $i \in \mathcal{I}$ is characterized by:

$$\frac{\alpha_t^i\left(\sum_{j\in\mathcal{I}\setminus\{i\}} \alpha_t^j e_t^j\right)}{\left(\sum_{j\in\mathcal{I}} \alpha_t^j e_t^j\right)^2} R_t = \left(e_t^i\right)^{\eta_i}$$

$$v_t^i\left(\frac{\alpha_t^i e_t^i}{\sum_{j\in\mathcal{I}} \alpha_t^j e_t^j} R_t - \frac{\left(e_t^i\right)^{1+\eta_i}}{1+\eta_i} - x_t^i\right)^{-\sigma_i} = \gamma_t^i z_t^i$$

$$\beta_i E_t v_{t+1}^i \left(\frac{\alpha_{t+1}^i e_{t+1}^i}{\sum_{j\in\mathcal{I}} \alpha_{t+1}^j e_{t+1}^j} R_{t+1} - \frac{\left(e_{t+1}^i\right)^{1+\eta_i}}{1+\eta_i} - x_{t+1}^i \right)^{-\sigma_i} \frac{e_{t+1}^i \left(\sum_{j\in\mathcal{I}\setminus\{i\}} \alpha_{t+1}^j e_{t+1}^j\right)}{\left(\sum_{j\in\mathcal{I}} \alpha_{t+1}^j e_{t+1}^j\right)^2} R_{t+1} =$$

$$\gamma_t^i - \beta_i E_t \gamma_{t+1}^i \left(1 - \delta\left(s_{t+1}\right)\right)$$

$$\alpha_{t+1}^i = \left(1 - \delta^i\left(s_t^i\right)\right)\alpha_t^i + z_t^i x_t^i$$

$$\delta\left(s_t^i\right) = \overline{\delta}^i + a^i \left(\frac{s_t^i}{s_{t-1}^i} - 1\right) + b^i \left(\frac{s_t^i}{s_{t-1}^i} - 1\right)^2$$

We assume ad hoc the following rule for adjusting the subjective valuation:

$$v_{t+1}^i = \left(v_t^i\right)^{\rho_i} \left(\overline{v}^i \left(\frac{s_t^i}{s_{t-1}^i} \right)^{-\varphi_i} \right)^{1-\rho_i}$$

where $\rho_i \in [0,1]$ is a smoothing parameter, $\varphi_i \geq 0$ represents the sensitivity of the rule to the results from the recent past and

$$s_t^i \equiv \left(\frac{p_t^i}{\overline{p}^i} \right)$$

is our "success index"[8], being $\overline{v}^i > 0$ the long-run value for the value scale parameter and $\overline{p}^i$ the optimal share of the good appropriated by agent $i \in \mathcal{I}$ in the deterministic steady state.

We are assuming that greater success reduces the value scale parameter for the next period. Note that in the deterministic steady state we have $s_t^i = 1$ and $v_t^i = v_{t+1}^i = \overline{v}^i$.

The first one of the optimality conditions implicitly defines a "reaction function" of the form

$$e_t^i = e^i \left(e_t^{\mathcal{I}\setminus\{i\}}, \alpha_t^{\mathcal{I}}, R_t \right)$$

Because the utility function is concave, the cost function is convex and the conflict technology is concave, then we know that the "reaction functions" are maximizers. By solving the system of "reaction functions" we obtain the effort levels of equilibrium.

For the case in which all the agents are totally equal to each other, formally $\forall i \in \mathcal{I}$, $\sigma_i = \sigma$,

[8] This success index s_t^i simply is the difference between what each player really won and what should have won in a hypothetical steady state.

$\eta_i = \eta$, $\alpha_0^i = \alpha_0$, $v_0^i = v_0$, $\overline{v}^i = \overline{v}$ and $z_t^i = z_t$ we can explicitly solve for effort levels and dissipation:

$$\begin{aligned} e_t &= \left(\frac{I-1}{I^2} R_t\right)^{\frac{1}{1+\eta}} \\ d_t &= I\left(\frac{I-1}{I^2} R_t\right)^{\frac{1}{1+\eta}} \end{aligned}$$

The dissipation, the total cost of conflict, is increasing in R_t, the amount of the valuable resource in dispute, and is increasing in I, the total number of agents in conflict.

In this particular case the dissipation does not depend on the dynamics of the model. This is because everybody make the same choices and the conflict technology does not change with time (even if α_t^i changes, it changes in the same amount for all the agents, so the relative strength of the conflict groups does not change). The dynamic change is observed in the investment in conflict technology and in consumption, but not in effort nor dissipation.

Therefore it is uninteresting for the conflict theory the game where all the agents are the same, because the dynamic choices are not reflected in dissipation.

5.1 Two-group's case

For a conflict with two groups which differ (we can extend this to any number of agents), the first order conditions are

$$\frac{\alpha_t^1 \alpha_t^2 e_t^2}{\left(\alpha_t^1 e_t^1 + \alpha_t^2 e_t^2\right)^2} R_t = \left(e_t^1\right)^{\eta_1}$$

$$\frac{\alpha_t^1 \alpha_t^2 e_t^1}{\left(\alpha_t^1 e_t^1 + \alpha_t^2 e_t^2\right)^2} R_t = \left(e_t^2\right)^{\eta_2}$$

$$v_t^1 \left(\frac{\alpha_t^1 e_t^1}{\alpha_t^1 e_t^1 + \alpha_t^2 e_t^2} R_t - \frac{\left(e_t^1\right)^{1+\eta_1}}{1+\eta_1} - x_t^1\right)^{-\sigma_1} = \gamma_t^1 z_t^1$$

$$v_t^2 \left(\frac{\alpha_t^2 e_t^2}{\alpha_t^1 e_t^1 + \alpha_t^2 e_t^2} R_t - \frac{\left(e_t^2\right)^{1+\eta_2}}{1+\eta_2} - x_t^2\right)^{-\sigma_2} = \gamma_t^2 z_t^2$$

$$\beta_1 E_t \gamma_{t+1}^1 z_{t+1}^1 \frac{e_{t+1}^1 \alpha_{t+1}^2 e_{t+1}^2}{\left(\alpha_{t+1}^1 e_{t+1}^1 + \alpha_{t+1}^2 e_{t+1}^2\right)^2} R_{t+1} = \gamma_t^1 - \beta_1 E_t \gamma_{t+1}^1 \left(1 - \delta\left(s_{t+1}^1\right)\right)$$

$$\beta_2 E_t \gamma_{t+1}^2 z_{t+1}^2 \frac{e_{t+1}^2 \alpha_{t+1}^1 e_{t+1}^1}{\left(\alpha_{t+1}^1 e_{t+1}^1 + \alpha_{t+1}^2 e_{t+1}^2\right)^2} R_{t+1} = \gamma_t^2 - \beta_2 E_t \gamma_{t+1}^2 \left(1 - \delta\left(s_{t+1}^2\right)\right)$$

$$\alpha_{t+1}^1 = \left(1-\delta^1\left(s_t^1\right)\right)\alpha_t^1 + x_t^1 z_t^1$$
$$\alpha_{t+1}^2 = \left(1-\delta^2\left(s_t^2\right)\right)\alpha_t^2 + x_t^2 z_t^2$$
$$\delta\left(s_t^1\right) = \overline{\delta}^1 + a^1\left(\frac{s_t^1}{s_{t-1}^1}-1\right) + b^1\left(\frac{s_t^1}{s_{t-1}^1}-s^1\right)^2$$
$$\delta\left(s_t^2\right) = \overline{\delta}^2 + a^2\left(\frac{s_t^2}{s_{t-1}^2}-1\right) + b^2\left(\frac{s_t^2}{s_{t-1}^2}-1\right)^2$$

This is a standard dynamic model. If we assume rational expectations we can use any solution method proposed by the macroeconomic theory to solve this model.

5.2 Solving a rational expectations dynamic model

The equilibrium in this conflict is in correspondence with a highly non-linear system of dynamic stochastic equations, which cannot easily be solved. A linear approximation around the non-stochastic steady state is used. The method consists in linearizing each first order and/or equilibrium condition around the steady state by a simple first order application of Taylor's Theorem. A linear system is obtained, with transformed variables of the form $\hat{u}_t = u_t - u$, where u is the steady state value of variable u_t.

The linear system of differential equations is solved by the method explained by Schmitt-Grohe and Uribe (2004). A general description of the methodology can be found in Heer and Maussner (2005). The solution is given by the **H**, **M** and **R** matrices that generate the dynamic solution by the iteration of the equations:

$$\begin{aligned} \mathbf{u}_t &= \mathbf{H}\mathbf{s}_t \\ \mathbf{s}_{t+1} &= \mathbf{M}\mathbf{s}_t + \mathbf{R}\varepsilon_{t+1} \end{aligned}$$

where **u** is the vector of forward-looking variables (controls, co-states, flow variables), **s** is the vector of endogenous and exogenous backward-looking state variables, **H** characterizes the optimal policy function, **M** is the state transition matrix, ε is the innovation vector and **R** specifies how the exogenous shocks (innovations) affect the dynamic system.

Note that the uniqueness of the solution can be assured for the stable case. Henceforth a solution to our conflict model a lá macroeconomic theory needs to be, at a first instance, a stable solution.

The stable solution rules out, by definition, the chaotic behaviour of the endogenous variables[9]. But even with this strong constraint we can obtain several conclusions about the dynamic of the

[9] If the model is stable, eventually the endogenous variables return to the steady state. We could then predict the future value of the variables, and therefore they would not be chaotic.

endogenous variables of the model. We can make an economic interpretation of the model, before we focus on the possibility of chaotic behaviour. Nevertheless we can observe some dynamics that could potentially be chaotic if we drop the stability assumption.

This is the approach we follow.

With the dynamic solution of the stable model, we can analyze the economic behaviour behind the dynamic model. We choose the impulse response function for this task.

5.3 Some dynamic properties of the simplified model

We propose several exercises that illustrate the dynamic characteristics of this simplified conflict model. In order to do that, we define the "baseline model" as the simplified conflict model with the following characteristics:

- The valuations follow the simple auto-regressive process

$$v_{t+1}^{i} = \left(v_t^i\right)^{\rho_i} \left(\overline{v}^i\right)^{1-\rho_i} + z_{t+1}^{vi}$$

  that does not depend on the success rate.

- The depreciation of conflict technology is given by

$$\delta^i \left(s_t^i\right) = \overline{\delta}^i$$

  and does not depend on the success rate.

- We make use of the following calibration:

σ_1	σ_2	η_1	η_2	$\overline{\delta}^1 = \overline{\delta}^2$	$\beta_1 = \beta_2$	ρ_R	$\overline{R}$	$\rho_1^z = \rho_2^z$	$\rho_1^v = \rho_2^v$	$\overline{v}_1 = \overline{v}_2$
2	5	2	1	0.03	0.98	0.98	1	0.50	0.50	1

We make this assumptions because we are interested in assessing which mechanism is important for the dynamics of the model: the valuation evolution, the adaptation to harder environment, or both.

5.3.1 Impulse response in the baseline model

First we want to analyze the effects of exogenous changes in the valuation for the conflict good in the conflict with the simplest dynamic response. Therefore we solve the "baseline model" and study a shock in the resource R_t, in the investment z_t^1 and in the valuations of one agent v_t^2.

The graphics show impulse response to exogenous shocks, in linear differences from the steady state.

An exogenous increase in the total amount of the resource in dispute increases the possibilities of the agents. Because of their higher endowment, they are able to devote more resources to the conflict. Both agents increase the effort, and the dissipation increases. This happens because the reaction function of both agents is increasing in the total amount of the valuable resource. The consumption and the investment of both agents increases. The share obtained by each agent depends on the relative strength and the cost of effort.

The shock to investment increases the effectiveness of conflict technology of the agent that receives the positive shock. Then this agent increases her investment because each unit of investment is more effective than the last. In order to keep up, the other agent also invests more in technology. This decreases the consumption and effort of both agents. The increase in both technology coefficients has ambiguous effects on the share obtained by each agent. This depends on the relative strength and the cost of effort.

The shock to the valuation of the agent two increases the utility of present consumption with respect to future consumption, because the shock is transitory. This generates a decrease in the investment made by agent two, and an increase in consumption. To increase present consumption, agent two must exert more effort and dissipation increases at the moment of the shock. However, due to less investment, effort costs are higher and this reduces effort for the next periods. Dissipation is lower after the shock.

5.3.2 Impulse response with adaptation to harder environment

Now we analyze the dynamic response of the stable model with adaptation to harder environments. We capture the fact that the weak agent adapts to the environment and increases the effectiveness of her conflict technology according to the variation of the success rate. In this case

$$\delta^i\left(s_t^i\right) = \overline{\delta}^i + a^i\left(\frac{s_t^i}{s_{t-1}^i} - 1\right) + b^i\left(\frac{s_t^i}{s_{t-1}^i} - 1\right)^2$$

The economic interpretation of the impulse response does not change. However the adaptive behaviour of the agents has important effects in the dynamic response to exogenous shocks.

Note that the endogenous response of the depreciation rate to the success index induces oscillating behaviour in the dissipation. Then we cannot rule out this mechanism as generator of rich dynamics and possibly as a chaos source in the model.

When a positive shock to investment technology of agent one arrives, the share obtained by agent two is higher with this adaption to harder environment than in the other cases considered. This is because agent two can adapt to the new technological challenge.

5.3.3 Impulse response with valuation rule

Finally we analyze the impulse response of the conflict model when the scale factor of the valuations evolves according to the ad-hoc rule

$$v_{t+1}^{i} = \left(v_{t}^{i}\right)^{\rho_i} \left(\overline{v}^{i} \left(\frac{s_t^i}{s_{t-1}^i}\right)^{-\varphi_i}\right)^{1-\rho_i}$$

in which the successful agent decreases her valuation for the conflict resource. We are able to replicate oscillating behaviour on the endogenous variables, specially consumption and investment, but under this specification it seems harder to get a non-smooth dynamic behaviour of the dissipation.

For example, if we set $\varphi_1 = 10$ and $\varphi_2 = 5$ we obtain oscillating consumption:

From the graphics the differences between the optimal paths under different configurations of the model are clear. The valuation rule induces a longer deviation of effort levels and dissipation than the other configurations of the model. This implies a higher long-run cost of conflict, because consumption is away from the long-run path for more time.

5.4 The equilibrium concept

Because of the recursive nature of the problem faced by each agent we can show that there is dynamic consistency: decisions about future choices made in period $t \in \mathcal{T}$ remain optimal for period $t+j \in \mathcal{T}$, $j \geq 0$. In a game theory context, this is equivalent as saying that the equilibrium found is a subgame perfect Nash equilibrium. In this equilibrium we are characterizing the long run optimal strategies of all the players. Note that the optimal effort level of each agent, and therefore the reaction functions, are chosen in the same way that they would be in a static game. The dynamics are incorporated in the determination of the technological parameters, and this is one aspect that other dynamic conflict models have left behind.

Our model is a truly dynamic one, not only a repeated game model.

Figure 1: Shock to R_t

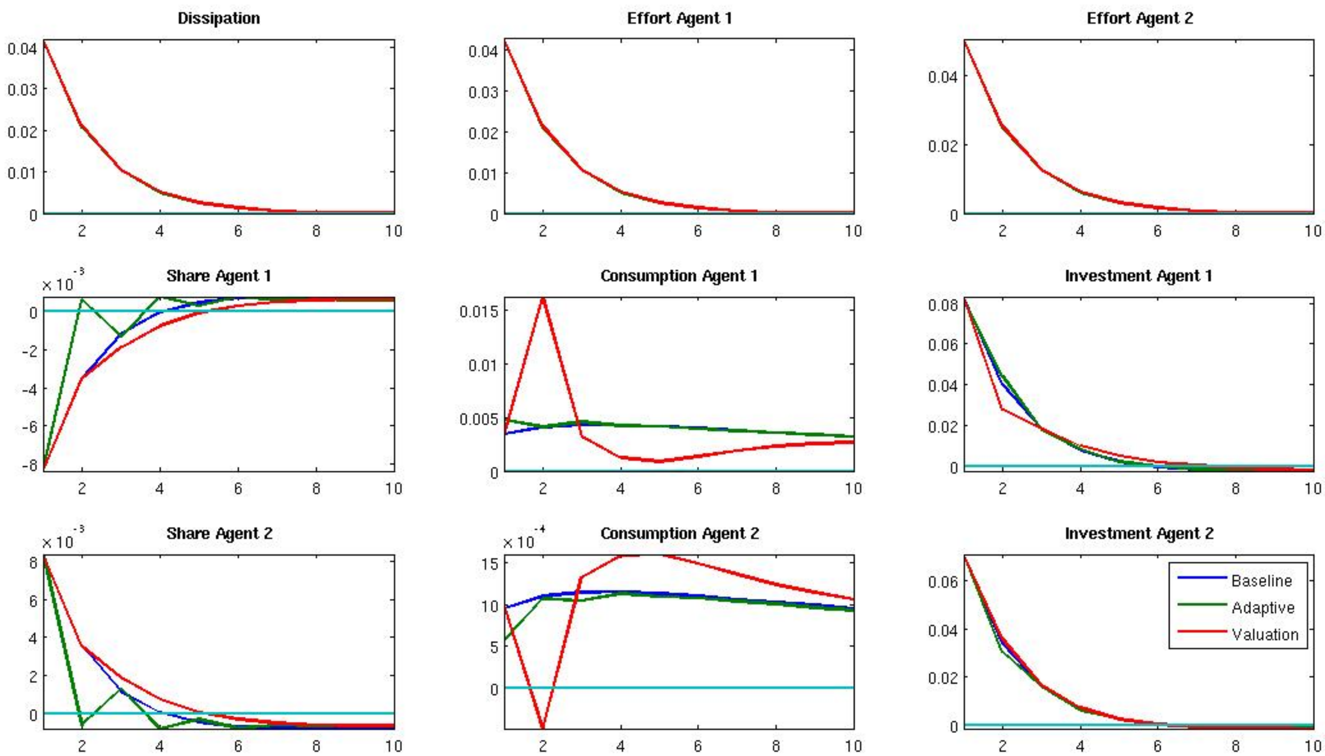


Figure 2: Shock to z_t^1

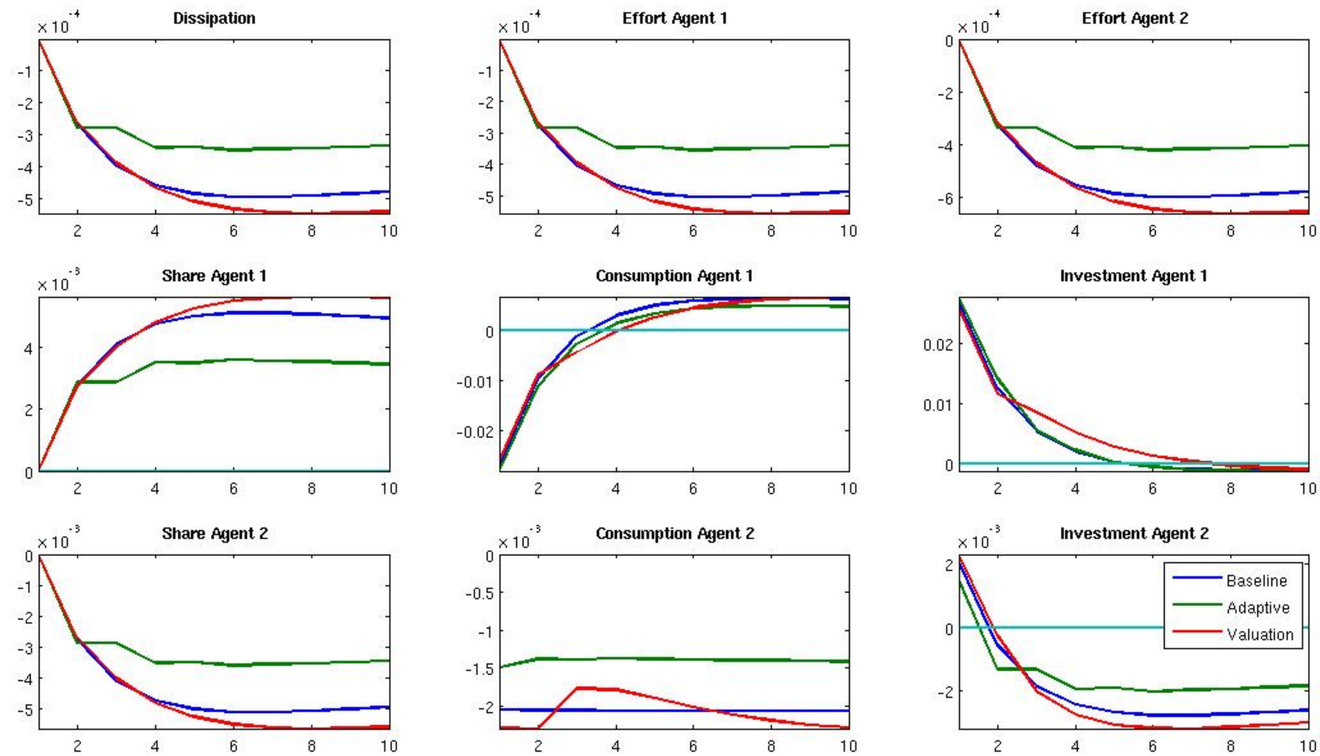
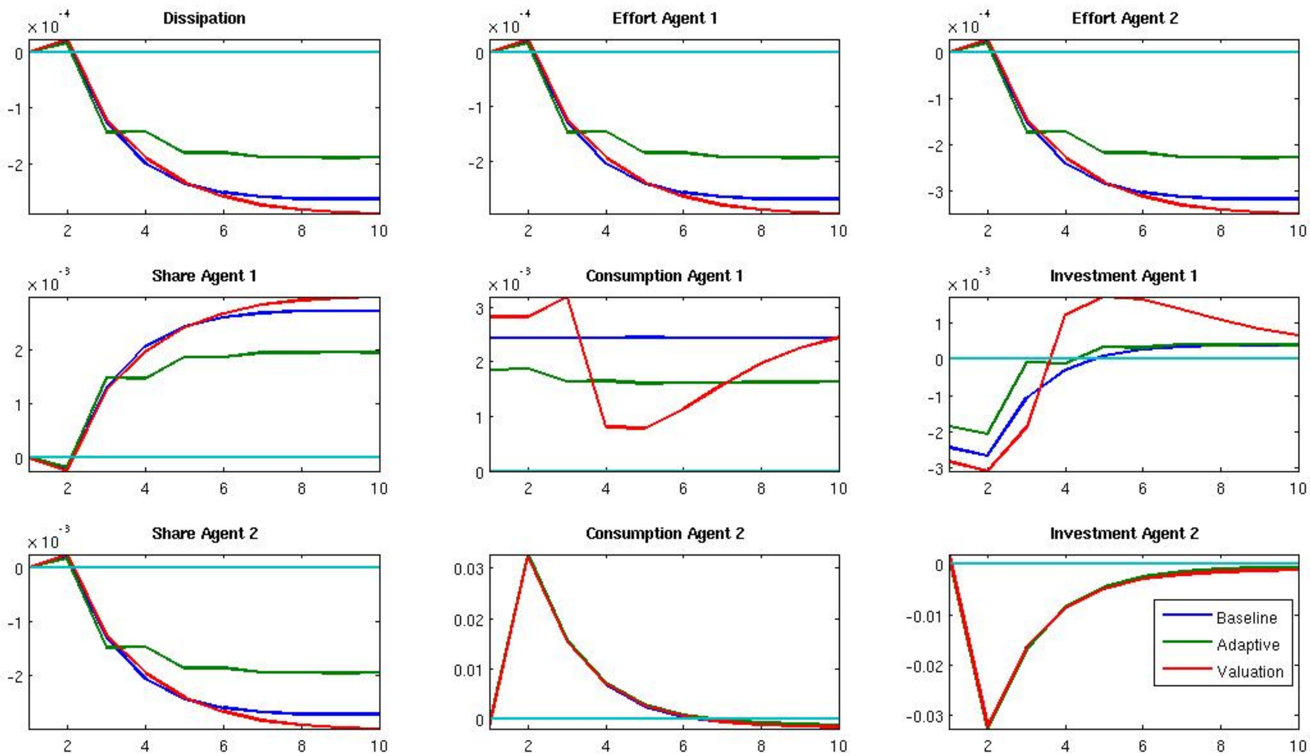


Figure 3: Shock to z_t^{v2}



6 Conclusion

We propose a general framework in which non-myopic agents choose effort levels and investment in conflict technology under uncertainty. The model is a dynamic game with N agents in infinite periods of time.

The allocation rule for the resource in dispute will determine the equilibrium concept and method needed to solve the model. This allows to think of many economic situations involving the distribution of a valuable resource without defined property rights: conflicts, bussiness races, auctions etc.

The model can be easily solved for a stable equilibrium under a differentiable share rule through dynamic programming techniques. We considered different dynamic mechanisms. Dependence of investment technology on past success allows the weaker agents to adapt to harder environments and keep on struggling in the conflict.

We consider that the valuation on the resource on dispute should change over the time. We include this fact by endogenizing the valuation with the recent success on the conflict. This mechanism increases the deviations of the variables with respect to the deterministic steady-state, generating a longer impact of transitory shocks.

References

ANDERTON, CHARLES H; ANDERTON, ROXANE A and CARTER, JOHN R: «Economic Activity in the Shadow of Conflict». *Economic Inquiry*, 1999, **37(1)**, pp. 166–79.

EGGERT, WOLFGANG; ICHI ITAYA, JUN and MINO, KAZUO: «A Dynamic Model of Conflict and Cooperation». *Discussion papers in economics and business*, Osaka University, Graduate School of Economics and Osaka School of International Public Policy (OSIPP), 2008.

ESTEBAN, JOAN and RAY, DEBRAJ: «Conflict and Distribution». *Journal of Economic Theory*, 1999, **(87)**, pp. 379–415.

GROSSMAN, HERSCHEL I and KIM, MINSEONG: «Swords or Plowshares? A Theory of the Security of Claims to Property». *Journal of Political Economy*, 1995, **103(6)**, pp. 1275–88.

HEER, BURKHARD and MAUSSNER, ALFRED: *Dynamic General Equilibrium Modelling*. Springer, Berlin, 2005. ISBN 3-540-22095-X.

HIRSHLEIFER, JACK: «Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success». *Public Choice*, 1989, **63(2)**, pp. 101–112.

——: «The Technology of Conflict as an Economic Activity». *American Economic Review*, 1991, **81(2)**, pp. 130–34.

——: «Anarchy and Its Breakdown». *Journal of Political Economy*, 1995, **103(1)**, pp. 26–52.

KRISHNA, VIJAY: *Auction Theory*. San Diego Academic Press, 1st editionth edition, 2002.

MAXWELL, JOHN W. and REUVENY, RAFAEL: «Conflict and Renewable Resources». *Journal of Conflict Resolution*, 2001, **45**, pp. 719–742.

——: «Continuing conflict». *Journal of Economic Behavior & Organization*, 2005, **58(1)**, pp. 30–52.

NEARY, HUGH: «A comparison of rent-seeking models and economic models of conflict». *Public Choice*, 1997, **93(3)**, pp. 373–388.

SCHMITT-GROHE, STEPHANIE and URIBE, MARTIN: «Solving dynamic general equilibrium models using a second-order approximation to the policy function». *Journal of Economic Dynamics and Control*, 2004, **28(4)**, pp. 755–775.

Skaperdas, Stergios: «Cooperation, Conflict, and Power in the Absence of Property Rights». *American Economic Review*, 1992, **82(4)**, pp. 720–39.

——: «Contest Success Functions». *Economic Theory*, 1996, **7(2)**, pp. 283–90.

Weeks, Michael R.: «Chaos, Conplexity and Conflict». Air and Space Power Journal - Chronicles Online Journal, 2001.
http://www.airpower.maxwell.af.mil/airchronicles/cc/Weeks.html