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Optimal Taxation in a Two Sector Economy with Heterogeneous Agents

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Abstract

In this paper we examine the optimal taxation problem in a two sector economy with heterogeneous agents. We show that in a steady state of this economy the optimal capital income tax rate can be different from zero. In this economy since capital and labour margins are interdependent, any difference in investment goods and consumption goods prices allows the government to tax capital income in one sector and undo the tax distortion by differential labour income taxation. This policy serves efficiency purpose as it restores the production efficiency condition.

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1 Introduction.

In this paper we show that in a two sector economy with heterogeneous agents, in a steady state the optimal capital income tax rate is in general different from zero. This result is due to the interdependence of capital and labour margins in equilibrium. This interdependence allows the government to choose an optimal policy that taxes/subsidizes capital income from one sector. This policy is optimal since it restores the production efficiency condition.

Our model is a two sector neoclassical growth model with a government that finances an exogenous stream of purchases. We consider two sectors that produce consumption goods (consumption sector, hereafter) and investment goods (investment sector, hereafter), using raw labour and capital on which government levies distorting flat-rate income taxes. We construct the Ramsey (1927) problem, i.e. the planner's problem of determining the optimal settings over time for two labour income tax rates and two capital income tax rates. Our study extends the important works of Judd (1985), Chamley (1986), and Atkeson, Chari and Kehoe (1999), all of which discuss the optimality of zero capital income tax. Chamley (1986) shows that in a steady state of a one sector economy, the optimal policy is to set the tax rate on capital income equal to zero. Judd (1985) extends this result by showing that in a one sector economy with heterogeneous agents, unanticipated redistributive capital taxation has severely limited effectiveness since it depresses wages. We consider a model which is one of the simplest extensions of Chamley (1986) and Judd's (1985) models. We show that in a steady state of our model, the optimal capital income tax rate in the investment sector is zero but the optimal capital income tax rate in the consumption sector is in general different from zero.

We show that in a two sector economy where investment and consumption are produced as two final goods, capital and labour margins are interdependent, and so is the long run optimal policy of taxing income from these factors. Due to this interdependence, capital income taxation in our model can serve the efficiency purpose. We argue that in a steady state since any difference in the relative price of investment and consumption is associated with a difference in the social marginal values of investment and consumption, a tax/subsidy on capital income in one sector, leaving the other capital income tax at a zero rate can undo this difference, which in turns restores the production efficiency condition. The optimal capital income tax distortions can be undone by differential labour income taxation.

The optimality of nonzero capital income tax was primarily hinted in Atkeson et al. (1999), who use a one sector model with heterogeneous agents. Atkeson et al. (1999) impose additional restrictions on the optimal taxation problem in order to restrict capital income tax rates and labour income tax rates to be same across all types of agents. These restrictions are (1) in the Ramsey equilibrium, the intertemporal marginal rate of substitution of consumption across all agent types must be equal, and (2) the intratemporal marginal rate of substitution between consumption and labour across all types of agents is equal to the ratio of marginal products of labour. Their paper argues that zero capital income taxation in the steady state is optimal if these extra constraints (in particular, 2) do not depend on the capital stock, i.e. if the production function is separable between capital and labour. In this paper we extend their analysis; we show that in a two sector economy the interdependence of labour and capital margins in equilibrium explains the long run optimal policy of taxing/subsidizing capital income, and thus it is not necessary to impose additional restrictions on the production function in order to derive a more general result.

2 A Decentralized Economy with Heterogeneous Agents.

Time is discrete and runs forever. The two production sectors, the consumption sector and the investment sector, are indexed by $j \in \{C, X\}$. There is a finite (integer) number of different classes of agents and each class is of measure one. The consumption, labour supply and capital stock of the representative agent in class i are denoted by c_t^i, n_{jt}^i and k_{jt}^i, respectively. Class i's utility function is $u^i\left(c_t^i, n_{ct}^i, n_{xt}^i\right)$ but the discount factor $\beta \in (0,1)$ is same for all agents[1]. The agents purchase consumption and investment goods and supply capital and working time to the firms. Firms return the rented capital stock next period (net of depreciation), pay unit cost of capital, r_j, and wages, w_j. Class i agents are each endowed with one unit of time each period and $k_0^i > 0$ units of capital at period 0. The resource constraints are:

$$\begin{aligned} c_t + g_t &\leq f^c(k_{ct}, n_{ct}) && \text{(1a)} \\ x_{ct} + x_{xt} &\leq f^x(k_{xt}, n_{xt}) && \text{(1b)} \\ x_{jt} &= k_{jt+1} - (1-\delta) k_{jt};\ j \in \{C, X\},\ \delta \in (0,1) && \text{(1c)} \end{aligned}$$

where $g_t = \overline{g} > 0$ is government consumption, c_t is aggregate private consumption, and x_j is new investment goods[2].

The government taxes labour income and capital income at rates τ_t^j per unit and θ_t^j per unit, respectively. The government has access to a *commitment technology* that allows it to commit itself once and for all to the sequence of tax rates announced at period 0. It makes non-negative class-specific lump sum transfer $TR_t^i \geq 0$. The government's social welfare function is a non-negatively weighted average of individual utilities, with the weight $\alpha^i \geq 0$ on class i, $\sum_{i=1}^{N} \alpha^i = 1$. The government's budget constraints are:

$$g_t + TR_t = \tau_t^c w_{ct} n_{ct} + \tau_t^x w_{xt} n_{xt} + \theta_t^c r_{ct} k_{ct} + \theta_t^x r_{xt} k_{xt} \tag{2}$$

In (1) and (2), for $z = c, n_c, n_x, k_c, k_x, x_c, x_x, TR$, let $z_t \equiv \sum_{i=1}^{N} z_t^i$.

With competitive pricing, factors are paid their marginal revenue product. Denoting the relative price of new investment goods as p_t, optimality in the production sectors implies $r_{ct} = f_k^c(t), w_{ct} = f_n^c(t), r_{xt} = p_t f_k^x(t)$, and $w_{xt} = p_t f_n^x(t)$.

The representative agent in class i chooses allocations $\left\{c_t^i, n_{ct}^i, n_{xt}^i, k_{ct+1}^i, k_{xt+1}^i\right\}_{t=0}^{\infty}$ in order to maximize discounted lifetime utility subject to the following budget constraints:

$$c_t^i + p_t\left(k_{ct+1}^i + k_{xt+1}^i\right) \leq (1-\tau_t^c) w_{ct} n_{ct}^i + (1-\tau_t^x) w_{xt} n_{xt}^i + p_t\left(k_{ct}^i R_{ct} + k_{xt}^i R_{xt}\right) + TR_t^i \tag{3}$$

where $R_{jt} \equiv \left[\left(1-\theta_t^j\right)\frac{r_{jt}}{p_t} + 1 - \delta\right]$. Optimality conditions for agent i include transversality conditions, (3), and:

$$\begin{aligned} u_{nc}^i(t) &= -u_c^i(t)(1-\tau_t^c) w_{ct} && \text{(4a)} \\ u_{nx}^i(t) &= -u_c^i(t)(1-\tau_t^x) w_{xt} && \text{(4b)} \\ p_t u_c^i(t) &= \beta u_c^i(t+1)\left\{\left(1-\theta_{t+1}^c\right) r_{ct+1} + p_{t+1}(1-\delta)\right\} && \text{(4c)} \\ p_t u_c^i(t) &= \beta u_c^i(t+1)\left\{\left(1-\theta_{t+1}^x\right) r_{xt+1} + p_{t+1}(1-\delta)\right\} && \text{(4d)} \end{aligned}$$

[1] The utility function is strictly increasing in consumption, decreasing in labour supply, separable in consumption and labour, linear in labour and satisfies standard regularity conditions.

[2] The technology $f^j(.)$ satisfies standard regularity conditions (including linear homogeneity).

- *A feasible allocation is a sequence* $\{c_t, n_{ct}, n_{xt}, g_t, k_{ct}, k_{xt}, x_{ct}, x_{xt}\}_{t=0}^{\infty}$ *that satisfies equation (1);*

- *A price system is a 5-tuple of non-negative bounded sequences* $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty}$*;*

- *A government policy is a 5-tuple of sequences* $\{\tau_t^c, \tau_t^x, \theta_t^c, \theta_t^x, TR_t\}_{t=0}^{\infty}$.

Definition 1 (Competitive equilibrium) *A competitive equilibrium is a feasible allocation, a price system, and a government policy, such that (a) given the price system and the government policy, the allocation solves both sets of the firms' problems and the agents' problems, and (b) given the allocation and the price system, the government policy satisfies the sequence of government budget constraints (2).*

The competitive equilibrium dynamics can be characterized by a system of equations that include the transversality conditions, optimality conditions in the production sectors, (1), (2) and for each class of agent i, (3) and (4).

Notice the interdependence of the capital and labour margins in this two sector economy. From (4a) and (4b), it is straightforward to show that if the competitive equilibrium has a steady state, the relative price of new investment goods is determined by $p\left(1-\tau^x\right) f_n^x u_{nx}^i = \left(1-\tau^c\right) f_n^c u_{nc}^i$. Furthermore, (4c) and (4d) together imply that in a steady state, $p\left(1-\theta^x\right) f_k^x = \left(1-\theta^c\right) f_k^c$. These conditions imply that in a steady state of the Ramsey equilibrium the government can only choose optimal policies that generate allocations which together with the optimal taxes satisfy $\left(1-\tau^c\right)\left(1-\theta^x\right) f_k^x f_n^c u_{nc}^i = \left(1-\tau^x\right)\left(1-\theta^c\right) f_k^c f_n^x u_{nx}^i$. Thus the capital and labour income taxes that can implement the competitive equilibrium allocation will depend on each other.

3 The Ramsey Problem.

We use Chamley's (1986) approach to the Ramsey problem and derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement these second-best wedges. We assume that the government chooses after tax returns to maximize welfare, such that the chosen after tax returns generate an allocation that is implementable in a competitive equilibrium. Using the linear homogeneity property of the production functions, we rewrite (2) as:

$$g_t + TR_t = f^c\left(k_{ct}, n_{ct}\right) + p_t f^x\left(k_{xt}, n_{xt}\right) - \widetilde{r}_{ct} k_{ct} - \widetilde{r}_{xt} k_{xt} - \widetilde{w}_{ct} n_{ct} - \widetilde{w}_{xt} n_{xt} \tag{5}$$

where $\widetilde{r}_{jt} \equiv \left(1-\theta_t^j\right) r_{jt}$ and $\widetilde{w}_{jt} \equiv \left(1-\tau_t^j\right) w_{jt}$.

In a model with only one class of agents, given the preset revenue target the Ramsey problem is the government's problem of choosing the after tax returns that maximize welfare and generate allocations and prices that satisfy the i invariant equations (5), (4), and (1). Since there are many classes of agents, the optimal taxes must generate allocations and prices that satisfy equilibrium conditions for each class of agents. So here the government's problem is one in which the government chooses allocations to maximize welfare subject to (5), (4), (3), and (1) for all class i. The

Lagrangian is:

$$\widehat{L}=\sum_{t=0}^{\infty}\beta^{t}\left\{\begin{array}{l}\sum_{i=1}^{N}\alpha^{i}u^{i}\left(c_{t}^{i},n_{ct}^{i},n_{xt}^{i}\right)\\+\psi_{t}\left[f^{c}\left(k_{ct},n_{ct}\right)+p_{t}f^{x}\left(k_{xt},n_{xt}\right)-\widetilde{r}_{ct}k_{ct}-\widetilde{r}_{xt}k_{xt}-\widetilde{w}_{ct}n_{ct}-\widetilde{w}_{xt}n_{xt}-g_{t}-TR_{t}\right]\\+\phi_{1t}\left[f^{c}\left(k_{ct},n_{ct}\right)-c_{t}-g_{t}\right]\\+\phi_{2t}\left[f^{x}\left(k_{xt},n_{xt}\right)+\left(1-\delta\right)\left(k_{ct}+k_{xt}\right)-k_{ct+1}-k_{xt+1}\right]\\+\sum_{i=1}^{N}\mu_{1t}^{i}\left[u_{nc}^{i}\left(t\right)+u_{c}^{i}\left(t\right)\widetilde{w}_{ct}\right]+\sum_{i=1}^{N}\mu_{2t}^{i}\left[u_{nx}^{i}\left(t\right)+u_{c}^{i}\left(t\right)\widetilde{w}_{xt}\right]\\+\sum_{i=1}^{N}\mu_{3t}^{i}\left[p_{t}u_{c}^{i}\left(t\right)-\beta u_{c}^{i}\left(t+1\right)\left\{\widetilde{r}_{ct+1}+p_{t+1}\left(1-\delta\right)\right\}\right]\\+\sum_{i=1}^{N}\mu_{4t}^{i}\left[p_{t}u_{c}^{i}\left(t\right)-\beta u_{c}^{i}\left(t+1\right)\left\{\widetilde{r}_{xt+1}+p_{t+1}\left(1-\delta\right)\right\}\right]\\+\sum_{i=1}^{N}\varepsilon_{t}^{i}\left[p_{t}R_{ct}k_{ct}^{i}+p_{t}R_{xt}k_{xt}^{i}+\widetilde{w}_{ct}n_{ct}^{i}+\widetilde{w}_{xt}n_{xt}^{i}-c_{t}^{i}-p_{t}k_{ct+1}^{i}-p_{t}k_{xt+1}^{i}+TR_{t}^{i}\right]\end{array}\right\} \tag{6}$$

where $y_t \equiv \sum_{i=1}^{N} y_t^i$, for $y = c, n_c, n_x, k_c, k_x, TR$, and $\psi_t, \phi_{1t}, \phi_{2t}, \mu_{1t}^i, \mu_{2t}^i, \mu_{3t}^i, \mu_{4t}^i$ and ε_t^i are Lagrange multipliers for (5), (1a), (1b&c), (4a), (4b), (4c), (4d) and (3), respectively.

In the Ramsey problem the budget constraint and the first order conditions of each class of agents are included; thus the social marginal value of an increment in the capital stock depends now on whose capital stock is augmented. If in equilibrium all classes behave in the same manner, their unilateral actions determine the social marginal value of capital[3].

We examine the Ramsey equilibrium in the general case for all class i. The Ramsey problem's first order conditions with respect to k_{ct+1}^i, k_{xt+1}^i, n_{ct}^i and n_{xt}^i are:

$$\phi_{2t}+\varepsilon_t^i p_t = \beta\left\{\begin{array}{c}\psi_{t+1}\left[f_k^c(t+1)-\widetilde{r}_{ct+1}\right]+\phi_{1t+1}f_k^c(t+1)+\phi_{2t+1}(1-\delta)\\+\varepsilon_{t+1}^i\left[\widetilde{r}_{ct+1}+p_{t+1}(1-\delta)\right]\end{array}\right\} \tag{7a}$$

$$\phi_{2t}+\varepsilon_t^i p_t = \beta\left\{\begin{array}{c}\psi_{t+1}\left[p_{t+1}f_k^x(t+1)-\widetilde{r}_{xt+1}\right]+\phi_{2t+1}\left[f_k^x(t+1)+1-\delta\right]\\+\varepsilon_{t+1}^i\left[\widetilde{r}_{xt+1}+p_{t+1}(1-\delta)\right]\end{array}\right\} \tag{7b}$$

$$\alpha^i u_{nc}^i(t) = \widetilde{w}_{ct}\left(\psi_t-\varepsilon_t^i\right)-\left(\psi_t+\phi_{1t}\right)f_n^c(t) \tag{7c}$$

$$\alpha^i u_{nx}^i(t) = \widetilde{w}_{xt}\left(\psi_t-\varepsilon_t^i\right)-\left(\psi_t p_t+\phi_{2t}\right)f_n^x(t) \tag{7d}$$

Proposition 1 *In a steady state the optimal tax rates are given by:*

$$(1-\theta^x)=1; \qquad (1-\theta^c)=1+\frac{1}{\psi}\left(\phi_1-\phi_2\frac{f_n^x}{f_n^c}\right)$$

$$(1-\tau^x)=\frac{\psi}{\alpha^i u_c^i+\frac{(\psi-\varepsilon^i)}{f_n^c(\psi+\phi_1)}\left[f_n^c(\psi+\phi_1)-\phi_2 f_n^x\right]}; \qquad (1-\tau^c)=\frac{\psi+\phi_1}{\alpha^i u_c^i+(\psi-\varepsilon^i)}$$

[3]Here we assume that tax rates on capital income and tax rates on labour income do not differ across classes of agents. Atkeson et al. (1999) solve a similar problem for a one sector economy using the primal approach. If one solves the current problem using the primal approach and invokes such a restriction, in addition to the resource and implementability constraints, the Ramsey problem must include additional constraints: *(1)* $u_c^i(t)\,u_c^\iota(t+1)=u_c^\iota(t)\,u_c^i(t+1)$; *(2)* $u_{nc}^i(t)\,u_c^\iota(t)\,f_{n\iota}^c(t)=u_{nc}^\iota(t)\,u_c^i(t)\,f_{ni}^c(t)$; and *(3)* $u_{nx}^i(t)\,u_c^\iota(t)\,p_t f_{n\iota}^x(t)=u_{nx}^\iota(t)\,u_c^i(t)\,p_t f_{ni}^x(t)$, $i\neq\iota$. We use Chamley's (1986) approach where these constraints are incorporated with the detailed equilibrium conditions for all classes of agents.

Proof. In a steady state, the Ramsey equilibrium allocation is implementable as a competitive equilibrium allocation. The time invariant versions of (7b) and (4d) are:

$$\phi_2 + p\varepsilon^i \left[1 - \beta\left(\frac{\widetilde{r}_x}{p} + 1 - \delta\right)\right] = \beta\left[\psi\left(r_x - \widetilde{r}_x\right) + \phi_2\left(\frac{r_x}{p} + 1 - \delta\right)\right] \tag{8a}$$

$$1 = \beta\left(\frac{\widetilde{r}_x}{p} + 1 - \delta\right) \tag{8b}$$

Since the optimal taxes generate the allocation that satisfies both (8a) and (8b), the optimal taxes and the allocation must satisfy $\phi_2\left[1 - \beta\left(\frac{r_x}{p} + 1 - \delta\right)\right] = \beta\psi\left(r_x - \widetilde{r}_x\right)$, which together with (8b) implies:

$$\beta\left(\widetilde{r}_x - r_x\right)\left(\frac{\phi_2}{p} + \psi\right) = 0 \tag{8c}$$

Since $\beta\left(\frac{\phi_2}{p} + \psi\right) \neq 0$, it must be that in a steady state the optimal taxes satisfy $\left(\widetilde{r}_x - r_x\right) = 0$, i.e. $\theta^x = 0$. Similar steps, starting with the time invariant versions of (7a) and (4c) give:

$$\phi_2\left[1 - \beta\left(1 - \delta\right)\right] = \beta\left[\psi\left(r_c - \widetilde{r}_c\right) + \phi_1 r_c\right] \tag{8d}$$

which, together with $1 = \beta\left[\frac{\widetilde{r}_c}{p} + 1 - \delta\right]$, imply

$$\left(1 - \theta^c\right)\left(\frac{\phi_2}{p} + \psi\right) = \psi + \phi_1 \tag{8e}$$

In a steady state, the optimal tax policy must be consistent with the equilibrium price of investment goods, which is given by $p\left(1 - \tau^x\right) f_n^x u_{nx}^i = \left(1 - \tau^c\right) f_n^c u_{nc}^i$. Substituting for the equilibrium price in (8e) we derive

$$\left(1 - \theta^c\right) = \frac{\left(\psi + \phi_1\right)\left(1 - \tau^c\right) f_n^c u_{nx}^i}{\phi_2\left(1 - \tau^x\right) f_n^x u_{nc}^i + \psi\left(1 - \tau^c\right) f_n^c u_{nx}^i} \tag{8f}$$

The time invariant versions of (7c) and (4a) together, and the time invariant versions of (7d) and (4b) together imply that the optimal labour income tax rates are given by $\left(1 - \tau^c\right) = \frac{\psi + \phi_1}{\alpha^i u_c^i + \left(\psi - \varepsilon^i\right)}$ and $\left(1 - \tau^x\right) = \frac{\psi}{\alpha^i u_c^i + \frac{\left(\psi - \varepsilon^i\right)}{f_n^c\left(\psi + \phi_1\right)}\left[f_n^c\left(\psi + \phi_1\right) - \phi_2 f_n^x\right]}$.

Substituting these in (8f) we derive $\left(1 - \theta^c\right) = 1 + \frac{1}{\psi}\left(\phi_1 - \phi_2\frac{f_n^x}{f_n^c}\right)$. ∎

Proposition 2 *In a steady state, the optimal capital income tax rate in the consumption sector is zero if and only if the optimal labour income tax rates are equal across sectors. Otherwise the optimal capital income tax rate in the consumption sector is not zero.*

Proof. From proposition 1, in a steady state optimal labour income taxes satisfy:

$$\frac{1 - \tau^x}{1 - \tau^c} = \frac{\psi}{\psi + \phi_1 - \frac{f_n^x}{f_n^c}\phi_2} \tag{9}$$

and $\tau^x = \tau^c$ if and only if $\frac{\phi_1}{\phi_2} = \frac{f_n^x}{f_n^c}$. The optimal capital income tax rate in the consumption sector is given by $\left(1 - \theta^c\right) = 1 + \frac{1}{\psi}\left(\phi_1 - \phi_2\frac{f_n^x}{f_n^c}\right)$, and $\theta^c = 0$ if and only if $\frac{\phi_1}{\phi_2} = \frac{f_n^x}{f_n^c}$. ∎

From proposition 2, in a steady state the optimal capital income tax rate in the consumption sector is nonzero in general, and zero only conditionally. The intuition behind this result can be drawn from the interdependence of capital and labour margins and the social marginal values of consumption and investment in this economy. Unlike a one sector model where the final good is either consumed or invested in capital, here capital is a good produced in a different sector. This is why equilibrium capital and labour margins are interdependent. It is therefore the initial allocation of capital across the two sectors that determines the social marginal values of investment and consumption in a steady state. Due to this interdependence, the equilibrium price of investment goods depend on the optimal policy of taxing labour income and the equilibrium labour margins. In addition, from (8f) it is clear that in a steady state the optimal policy of taxing income from capital and income from labour are also interdependent. Due to this, there exists a unique equilibrium price of investment goods, or more simply a unique condition explaining the social marginal values of consumption and investment (i.e. $\phi_1 f_n^c = \phi_2 f_n^x$), for which zero capital income tax rate in the consumption sector is optimal. The zero capital income tax policy is therefore one of many implementable optimal policies, supported by the optimal policy that involves equal labour income tax rates across sectors. For any other set of allocations, the government can set a tax/subsidy on capital income from the consumption sector and can use differential labour income taxation to undo the tax distortions.

In order to explain to intuition, we simplify the model by assuming $u_{nc}^i = u_{nx}^i$, i.e. the marginal disutility from working in the two sectors is same. We also rewrite the Ramsey optimality condition $\phi_2 \left[1 - \beta \left(\frac{r_x}{p} + 1 - \delta\right)\right] = \beta\psi \left(r_x - \widetilde{r}_x\right)$ as

$$\phi_2 = \beta \left[\psi \left(r_x - \widetilde{r}_x\right) + \phi_2 \left(f_k^x + 1 - \delta\right)\right] \tag{10a}$$

which assists us in identifying the social marginal value of capital. Equation (10a) states that a marginal increment of capital in the investment sector increases the quantity of capital by the amount $(f_k^x + 1 - \delta)$, which has social marginal value equal to ϕ_2. In addition, there is an increase in tax revenues equal to $(r_x - \widetilde{r}_x)$, which enables the government to reduce other taxes by the same amount. Since ψ is the shadow price of the government's resources, the reduction of this excess burden equals $\psi \left(r_x - \widetilde{r}_x\right)$. The sum of the two effects is discounted by β, and is equal to the social marginal value of capital in the investment sector, given by ϕ_2. Since the optimal policy is to set $\theta^x = 0$, investment in the investment sector is consistent with the condition $1 = \beta \left(f_k^x + 1 - \delta\right)$, which characterizes the socially optimal allocation of capital in the investment sector.

Now notice that in a steady state of the Ramsey equilibrium, in (8e), $\phi_1 = \frac{\phi_2}{p} \Leftrightarrow \theta^c = 0$. This implies that a zero capital income tax rate in the consumption sector is optimal if and only if $p = \frac{\phi_2}{\phi_1}$, i.e. $\theta^c = 0$ is optimal if and only if the relative price of investment goods is equal to the ratio of the social marginal value of investment to the social marginal value of consumption. We rewrite (8d) as:

$$\phi_2 = \beta \left[\psi \left(r_c - \widetilde{r}_c\right) + \phi_1 f_k^c + \phi_2 \left(1 - \delta\right)\right] \tag{10b}$$

If in a steady state of the Ramsey equilibrium, $p = \frac{\phi_2}{\phi_1}$ and the government taxes capital income from the consumption sector at a zero rate, (10b) together with $p = \frac{\phi_2}{\phi_1}$ imply:

$$1 = \beta \left[\frac{\phi_1}{\phi_2} f_k^c + 1 - \delta\right] \tag{10c}$$

The zero capital income tax policy (for the consumption sector) is optimal only if the resulting allocations replicate the socially optimal allocation of capital in the consumption sector, for which

$1 = \beta\left(f_k^c + 1 - \delta\right)$ must hold. Together with (10c) this implies that in a steady state if the optimal policy involves $\theta^c = 0$, it should generate an allocation that is consistent with $\frac{\phi_1}{\phi_2} = 1$, i.e. an allocation consistent with $p = 1$.

We now explain the converse, i.e. if in a steady state the price of investment goods and the price of consumption goods are equal, the optimal policy is to set $\theta^c = 0$. Say in a steady state $p = 1$. From (8e),

$$\left(1 - \theta^c\right)\left(\phi_2 + \psi\right) = \left(\psi + \phi_1\right) \tag{10d}$$

This now defines the steady state optimal capital income tax policy for the consumption sector. This policy must satisfy:

$$1 = \beta\left[\left(\frac{\psi + \phi_1}{\psi + \phi_2}\right) f_k^c + 1 - \delta\right] \tag{10e}$$

$$\phi_2 = \beta\left[\psi f_k^c\left\{1 - \left(\frac{\psi + \phi_1}{\psi + \phi_2}\right)\right\} + \phi_1 f_k^c + \phi_2\left(1 - \delta\right)\right] \tag{10f}$$

Equations (10e) and (10f) together imply that the steady state optimal policy implements the socially optimal level of capital if it is consistent with the condition $(\psi + \phi_1) = (\psi + \phi_2)$. The only optimal policy that satisfies this condition is to set $\theta^c = 0$.

If the price of investment goods and the price of consumption goods are not equal, the government can implement the optimal policy that taxes/subsidizes capital income in the consumption sector and taxes labour income from the two sectors at different rates. With $u_{nc}^i = u_{nx}^i$, the competitive equilibrium condition $\left(1 - \tau^c\right)\left(1 - \theta^x\right) f_k^x f_n^c = \left(1 - \tau^x\right)\left(1 - \theta^c\right) f_k^c f_n^x$ is consistent with the production efficiency condition if $\left(1 - \tau^c\right)\left(1 - \theta^x\right) = \left(1 - \tau^x\right)\left(1 - \theta^c\right)$. If there is no difference in the relative price of the two goods, the policy that satisfies the production efficiency condition must involve $\theta^c = 0$, $\tau^c = \tau^x$. This policy is one of many implementable Ramsey policies, and it is the optimal policy only if $p = 1$. This recovers the Chamley-Judd result in our setting. For all other cases, the optimal policy involves $\left(1 - \theta^c\right)\left(1 - \tau^x\right) = \left(1 - \tau^c\right)$ with $\tau^c \neq \tau^x$ and $\theta^x = 0$.

Let us consider an example. Say the economy is in a steady state with an inefficiently large production of consumption goods and low production of investment goods, such that investment goods are more expensive than consumption goods. The long run optimal policy (starting from that particular steady state) should encourage production of investment goods by setting a tax on capital income and a higher tax on labour income from the consumption sector. This policy, supported by a zero capital income tax rate and a lower labour income tax rate in the investment sector, encourages agents to shift more capital and working hours to the investment sector, which in turns increases the production of investment goods and minimizes the relative price difference. This also restores production efficiency.

The Ramsey equilibrium conditions also explain how the distortions of a capital income tax can be undone. Consider (10b), which states that a marginal increment of capital in the consumption sector increases the quantity of consumption goods by the amount f_k^c, which has social marginal value ϕ_1. This increment is adjusted by capital depreciation in the investment sector, which has social marginal value ϕ_2. The aggregate increment in the quantity of available consumption goods in social marginal value terms is equal to $\left[\phi_1 f_k^c + \phi_2\left(1 - \delta\right)\right]$. The first term is due to an increase in capital in the consumption sector, while the second terms stands for an indirect increase in production of consumption goods through an increase in depreciated capital in the investment sector. This is obvious since in a steady state, with zero capital income tax in the investment sector it is best to keep depreciated capital in the investment sector. The increased tax revenue, equal to $(r_c - \widetilde{r}_c)$, enables the government to reduce other taxes by the same amount, and the reduction of this excess burden equals $\psi (r_c - \widetilde{r}_c)$. The sum of these effects is discounted, and is equal to the social marginal value of the available capital.

It is therefore optimal to set zero tax rate on capital income from the consumption sector when the social marginal value of investment and the social marginal value of consumption are same, implying in turns that their relative prices are same. Any difference in the social marginal value of these two is reflected in a relative price difference, which can be undone by the optimal policy that involves a tax/subsidy to capital income in the consumption sector and differential labour income tax rates.

4 Conclusion.

We examine optimal income taxation in a two sector economy with heterogeneous agents. We contribute by showing that in a steady state of this economy, the optimal capital income tax rate in the consumption sector is in general different from zero. Our analysis shows that any difference in the social marginal value of investment and the social marginal value of consumption is reflected in the relative price difference between the same, and such a difference creates an opportunity for the government to choose the optimal policy that has zero tax on capital income from the investment sector, a tax/subsidy on capital income from the consumption sector, and different rates of labour income taxes across sectors.

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