Towards a Theory of Capacity Utilization: Shiftwork and the Workweek of Capital

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Among the large number of economic indicators that provide information on the current or future state of the economy, the index of capacity utilization (CU) published by the Federal Reserve Board is one of the more prominent. Low levels of the CU index number tend to be associated with below-average aggregate activity, and high levels are supposed to indicate inflationary pressures (Corrado and Mattey 1997).[1] For the most part, CU is an empirical concept that is only loosely related to economic theory, and until recently it has not played an important role in models of the business cycle. There are various interpretations of what CU means, but in this article I address one particular aspect of CU from the point of view of standard production theory. I first review some empirical evidence on the workweek of capital, a measure that makes the concept of CU operational. I then extend standard production theory to incorporate the workweek of capital into the neoclassical growth model. Finally, I argue that recent attempts to use variations in the workweek of labor in order to get CU-adjusted measures of short-term productivity growth are potentially misguided, since the workweek of labor is not an unbiased measure of the workweek of capital.

The Board's CU index is defined as the ratio of actual output to potential output. Potential output reflects "sustainable practical capacity, defined as the

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[1] Finn (1995) argues that the CU index is not particularly useful in forecasting future inflation rates.

greatest level of output each plant in a given industry can maintain within the framework of a realistic work schedule, taking account of normal downtime and assuming sufficient availability of inputs to operate machinery and equipment in place" (Corrado and Mattey 1997, 152). The capacity measures are limited to the manufacturing industries, mining, and electric and gas utilities, and they are based on the *Survey on Plant Capacity,* which is produced by the U.S. Census Bureau in the fourth quarter of each year. Capacity measures for quarters are obtained by smooth interpolation of the fourth-quarter numbers. Given the smooth interpolation of capacity and the volatility of output, movements in the CU index are mainly due to movements in actual output.

The standard theory of production views the quantity of output produced as a function of the quantities of inputs used. How does the concept of CU fit into this theory? I focus on the "utilization" aspect of CU and disregard the "capacity" aspect.[2] When we measure inputs to production we usually consider the capital stock, total hours worked by production workers/employees, and the quantities of intermediate inputs (materials and energy) used. We implicitly assume that the input services provided by capital are proportional to its accumulated stock. Looking at the production of a plant, we can see that its output obviously depends on the extent to which it uses its existing capital stock: how many machines are running and for how long? That is, the plant can vary the service flow per unit of capital, and this input variation is not covered in the usual input measures. This opportunity to vary the flow of services from capital creates a problem for productivity measurement when we want to attribute movements in output to movements in inputs and changes in productivity.

The workweek of capital is supposed to capture the service flow of the capital stock, which is proportional to the average duration of time for which a unit of capital is operated. The workweek of capital is different from the workweek of labor, which is the average duration of time a unit of labor (worker) is employed. To the extent that labor and capital are complementary inputs (for example, a certain number of workers are needed to operate a machine), the workweek of capital and the workweek of labor are related, but they need not be the same. For instance, if a plant is operating multiple shifts, then the workweek of capital will be a multiple of the workweek of labor. Furthermore, if the extent to which a plant uses shiftwork changes over the cycle, the cyclical behavior of the workweeks of capital and labor will be different.

[2] The capacity concept implies that there is a maximal output level in the short run, the "capacity constraint," and that this level cannot be exceeded no matter how many variable inputs are hired. Economic variables will respond differently to changes in the environment, depending on whether the capacity constraint is binding or not. This behavior can introduce a nonlinearity into observed economic relations. For a recent model with occasionally binding capacity constraints, see Hansen and Prescott (2000).

In the remainder of the article I first review evidence on the workweek of capital from micro- and macrostudies. I then describe a simple model with variable employment in late shifts. Finally, I discuss the implications of variations in shiftwork for the measurement of productivity changes.

1. OBSERVATIONS ON THE WORKWEEK OF CAPITAL

The workweek of capital varies widely across industries and across plants. The average length of the workweek of capital in an industry depends on common elements of the production processes used by different plants in an industry. Within an industry, plants deviate from the industry average in response to variations in demand across plants because production cannot be reallocated between plants. Depending on the structure of the production process, there are limits on the extent to which firms can vary the workweek of capital. Microevidence indicates that plants can adjust the workweek of capital along a number of margins. Aggregating plant level data to get industry data shows that the workweek of capital varies substantially over time and indeed is more volatile than the workweek of labor. It also appears that at the aggregate level a substantial fraction of the capital workweek's volatility is due to movements in the share of the labor force that works on late shifts.

How long do plants operate in a quarter and how do they change the duration for which they operate? Mattey and Strongin (1997) answer this question based on plant level data on actual and capacity hours worked per quarter from the *Survey on Plant Capacity*. They break down total operating hours in a quarter as follows:

$$\frac{Weeks}{Quarter} \cdot \frac{Days}{Week} \cdot \frac{Shifts}{Day} \cdot \frac{Hours}{Shift}.$$

Note that as long as plants do not operate 24 hours a day for every day of the quarter, there is scope for variation in the workweek of capital. Mattey and Strongin (1997) find that in their sample 35 percent of all plants do not operate every week, 62 percent do not operate every day of the week, 20 percent have only one shift, and 13 percent do not work overtime. Plants design their operating margins in order to vary the workweek of capital. For example, in some industries almost all plants seem to operate essentially every hour of the quarter. Within other industries the workweek of capital varies substantially across plants. Mattey and Strongin (1997) classify industries according to operating margins, and they distinguish between "continuous process" industries and "variable workweek" industries. The two classifications do not exhaust their sample.

An industry is classified as continuous process if its plants report that at capacity they essentially operate every hour of the quarter. Continuous process industries contain one-fourth of all plants in the sample. Even within this class, not all plants actually operate at full capacity: 20 percent shut down for five weeks in a quarter, 11 percent work only five days a week, and 9 percent work only two shifts. For continuous process plants, output variations take place mainly through the variation in material inputs use and not through variations in the workweek of capital. Petroleum is an example of a continuous process industry.

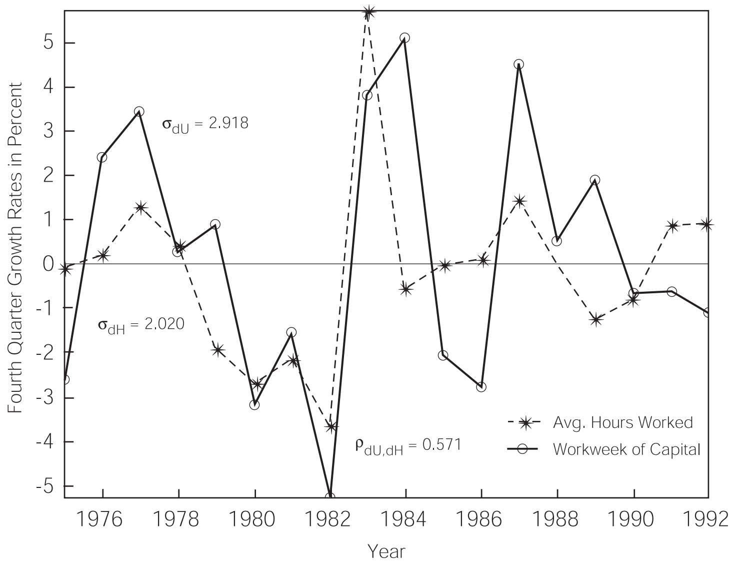
An industry is classified as variable workweek if individual plants in that industry show substantial variation in their actual workweek over time. Most plants in these industries (85 percent) operate at least 12 weeks per quarter and five to six days a week (93 percent), so that differences in the workweek of capital across plants are evident mainly in the number of shifts operated: 27 percent operate one shift, 40 percent operate two shifts, and 32 percent operate three shifts. A substantial share (16 percent) of these plants also use overtime as an option.

Obviously the boundaries between continuous process and variable workweek industries are not clear cut. For example, the steel industry appears to be close to the continuous process ideal: an individual blast furnace is a continuous process technology. Yet Bertin, Bresnahan, and Raff (1996) argue that for iron production, the relevant unit of observation is a plant that may operate several blast furnaces, and therefore the average time its furnaces are operated (workweek) is the more relevant measure. Another example is an apparel workshop that can vary the average time its sewing machines are used along two margins: (1) the length of time of time an individual machine is used, and (2) the number of machines actually operated.

The prototypical example of a variable workweek industry is automobile production, particularly its assembly plants. Bresnahan and Ramey (1994) study 50 U.S. automobile plants from 1972–1983. They find that variations in the workweek of capital are mainly due to weekly shutdowns related to model changeovers, inventory adjustment, and holidays. Individual plants only infrequently change the number of shifts they operate. Nevertheless, since shift changes have a huge impact on output, variations in the number of shifts operated make a substantial contribution to output volatility (about 25 percent). Hall (2000) in a study of 14 Chrysler plants from 1990–1994 also finds that most of the variation in an individual plant's workweek is due to short-term shutdowns rather than to variations in the number of shifts operated. Even though at the individual plant level the shift margin does not appear to be important, it can still be important at the industry level if the relative shares of plants operating at different shifts change systematically over the cycle.

What are the implications of plant-level patterns for industrywide movements in the workweek of capital? Beaulieu and Mattey (1998) construct capital workweek series for two-digit Standard Industrial Classification (SIC) industries in the manufacturing sector for the period 1974–1992. They find that the capital workweek is both longer and more volatile than the workweek of labor. For the overall manufacturing sector, the mean workweek of capital

Figure 1 Workweek of Capital and Workweek of Labor in Manufacturing



Notes: The average workweek of capital for manufacturing is from Beaulieu and Mattey (1998). The average hours worked are total hours worked divided by the number of employees for manufacturing only. For both series, the growth rate is calculated from the fourth quarter in the previous year to the fourth quarter of the current year.

is 97 hours. In Figure 1, I plot the fourth-quarter to fourth-quarter percentage growth rates of the workweek of capital and average hours worked in manufacturing.[3] We can see that the workweek of capital is substantially more volatile than average hours worked, and although the two series tend to move together, the fit is not very tight: the correlation coefficient is about 0.6. Beaulieu and Mattey (1998) also find significant differences in the statistical properties of the workweek of capital across industries. The mean workweek of capital ranges from 44 hours in apparel to as high as 156 hours for the continuous-process-type petroleum refining industry. The volatility of the workweek of capital is also quite different across industries; for example, the standard deviation of percentage changes in the workweek of capital ranges from a high of 10.0 in primary metals to a low of 3.0 for chemicals and petroleum.

[3] The SPC is undertaken in the fourth quarter of each year, that is, the workweek of capital refers only to that quarter. For consistency I have used the same procedure for average hours worked.

Unfortunately, the work by Beaulieu and Mattey (1998) is limited to the manufacturing sector. Shapiro (1996) constructs an alternative measure of the workweek of capital based on the employment pattern of shiftwork in the *Current Population Survey*, and his work covers manufacturing and nonmanufacturing industries of the economy. With respect to the manufacturing sector, Shapiro (1996) suggests that the capital workweek is shorter (52 hours) and only half as volatile as Beaulieu and Mattey (1998) argue. Like Beaulieu and Mattey (1998), Shapiro (1996) also observes substantial variation of the mean and volatility of the capital workweek across industries in manufacturing. With respect to nonmanufacturing industries, Shapiro (1996) finds that the capital workweek is only 44 hours, which is close to the workweek of labor, and that the capital workweek is substantially less volatile than in manufacturing.

Why does the capital workweek tend to be more volatile than the workweek of labor, especially in the manufacturing sector? Shapiro (1996) points to variations in the extent to which shiftwork is used in production. He finds that in overall manufacturing about 25 percent of all production workers are working late shifts.[4] Furthermore, in each industry the late-shift share of employment is quite volatile and tends to increase with overall employment. In particular, Shapiro estimates that a 1 percent increase of employment increases late-shift employment by 1.5 percent. For nonmanufacturing industries, where the capital workweek is less volatile, late-shift work is not as prevalent, and for most service industries the late-shift employment share tends to decline with overall employment.

2. A MODEL OF SHIFTWORK AND THE WORKWEEK OF CAPITAL

I now construct a simple model where capacity utilization is reflected in the workweek of capital and the workweek of capital is closely related to the share of late-shift work in total employment. This model builds on work by Kydland and Prescott (1991), Bils and Cho (1994), and Hall (1996). I argue that there are systematic differences between the workweek of capital and the workweek of labor. The model serves as an illustration only, and I do not provide a complete analysis of all of its properties in this article.

Shiftwork in Production

In the standard model of production, we view output y as a function of the capital stock k and total hours worked nh:

$$y = zk^{\alpha} (hn)^{1-\alpha}, 0 < \alpha < 1, \tag{1}$$

[4] The employment share of late-shift work ranges from 4 percent in apparel to 40 percent in tobacco.

where n is employment and h is average hours worked per worker (that is, the workweek of labor), and z denotes productivity.[5] We usually assume that output increases as inputs increase, the marginal product of each input is positive, and the marginal product of each input is declining. Furthermore, production is constant-returns-to-scale (CRS) in the capital stock and total hours worked: if we double the capital stock and total hours worked, then output doubles. This structure assumes that the contribution of the input capital is proportional to the stock of capital.

We can allow for variations in the utilization of capital through changes in the workweek of capital, assuming that labor works a single shift:

$$y = zk^{\alpha}n^{1-\alpha}h = z\,(hk)^{\alpha}\,(hn)^{1-\alpha}\,. \tag{2}$$

We now assume that production per unit of time is CRS in the capital stock and workers employed. Total output is then proportional to the hours capital and workers are employed. The relevant inputs for this production structure are the services capital and labor provide, and these services are proportional to the hours worked. Furthermore, the production structure continues to be CRS with respect to the capital and labor services employed. This production structure has been used by Kydland and Prescott (1991), Bils and Cho (1994), and Hall (1996). Note that for this production structure, the workweeks of capital and labor are the same.

What happens if we allow for more than one shift and if we can vary the relative employment levels on the two shifts? I consider the case where the economy can operate the capital stock with two employment shifts. That is, in any given period the existing capital stock can be used twice in production. For this case I assume that production takes place with machines and that machines and workers are complementary, meaning that if a worker is matched with a machine containing $\tilde{k}$ units of capital, then output per worker per unit of time is $z\tilde{k}^{\alpha}$. Assuming that all machines are the same, the number of machines m is limited by the available total capital stock, $m\tilde{k} \leq k$. Given the available machines, the economy can employ $n_1 \leq m$ workers, each working for h_1 hours on the first shift, and output from the first shift is $\tilde{k}^{\alpha}h_1n_1$. The same machines can be used on the second shift with employment $n_2 \leq m$ and shift length h_2, with corresponding output $\tilde{k}^{\alpha}h_2n_2$. The sum of shift lengths is bounded by the duration of a period, $h_1 + h_2 \leq \bar{h}$. Total production in a period is then

$$y = z\tilde{k}^{\alpha}\,(n_1h_1 + n_2h_2)\,.$$

For an efficient allocation of capital, all capital is used in machines and for at least one shift all machines are employed. An efficient allocation means that

[5] For concreteness I have assumed that the production function is Cobb-Douglas. All the arguments apply for a general concave constant-returns-to-scale production function.

for a given employment decision, the vector $(\tilde{k}, m)$ maximizes output. Since the marginal product of capital is positive, and there is no additional cost of building a machine besides the amount of capital it contains, it is always efficient to use all available capital, that is, $\tilde{k}m = k$. For the same reason, the efficient number of machines is equal to the maximum of the two employment levels. Without loss of generality let $n_1 \geq n_2$, in which case we need at least $m = n_1$ machines. It is not efficient to create more machines, because that would only reduce the capital-labor ratio and therefore reduce output for a given employment decision. This argument simplifies the representation of the production function to

$$y = z\,(k/n_1)^{\alpha}\,(n_1h_1 + n_2h_2)\,, \text{ with } h_1 + h_2 \leq \bar{h} \tag{3}$$

$$= z\,(uk)^{\alpha}\,(hn)^{1-\alpha}\,. \tag{4}$$

In this economy, the workweek of capital (that is, the average duration a unit of capital is operated) is $u = (n_1h_1 + n_2h_2)\,/n_1$. The workweek of labor (that is, the average duration a worker is employed) is $h = (n_1h_1 + n_2h_2)/n$, where n is total employment $n = n_1 + n_2$. Variations in the employment share of the first shift $\omega = n_1/n$ drive a wedge between the workweek of capital and the workweek of labor:

$$u = h/\omega. \tag{5}$$

I make one more assumption regarding the dynamic structure of production. Specifically, I assume that it is costly to change the capital-labor ratio in machines. For simplicity, I choose an extreme case where the capital-labor ratio is fixed at the beginning of the period. This means that the capital-labor ratio cannot be adjusted in response to new information on the state of the economy. The capital-labor ratio, however, can be adjusted at no cost at the end of a period. This assumption essentially makes employment on the first shift a predetermined variable. On the other hand, variations of employment on the second shift allow the economy to respond more flexibly to contemporaneous shocks. This feature of the economy will give rise to variations in the employment ratio of the second shift, and it will increase the volatility of the workweek of capital relative to the workweek of labor.

The remainder of the production structure is standard:

$$\begin{aligned} y &= c + x + g \\ k' &= (1-\delta)\,k + x \\ \ln\left(g'/\bar{g}\right) &= \rho_g \ln\left(g/\bar{g}\right) + \varepsilon_g \\ \ln z' &= \rho_z \ln z + \varepsilon_z. \end{aligned}$$

Output can be used for private consumption c, government spending g, and investment x. Investment augments the capital stock, which depreciates at a constant rate δ. Primes denote the next period's values. Government spending is financed by lump-sum taxation, and log-deviations from its mean $\bar{g}$ follow an

AR(1) process with autocorrelation coefficient ρ_g. Productivity also follows an AR(1) process with autocorrelation coefficient ρ_z. The disturbance terms ε_g and ε_z in the government spending and productivity equations are i.i.d. and uncorrelated with each other.[6]

Capital is not always utilized to the fullest extent. In our setup full capital utilization means that both shifts use all available machines, $n_1 = n_2$, and machines are continuously operated through the period, $h_1 + h_2 = \bar{h}$. Capital will not be fully utilized if there are increasing marginal costs to the utilization of capital. One way to model these costs of capital utilization is to assume that the rate at which capital depreciates increases as capital utilization increases (see Greenwood, Hercowitz, and Huffman [1988], Burnside and Eichenbaum [1996], and Basu and Kimball [1997]). In my setup there is another reason why capital would not always be operated at full capacity. Since capital utilization is tied to the use of labor, higher capital utilization requires more employment at less desirable times and longer work hours. If there is a wage premium for work at extended hours and that wage premium is sufficiently high, then capital may never be used at full capacity. I now describe preferences that give rise to a wage premium for shiftwork and overtime work.

Preferences and the Wage Premium

There is an infinitely-lived representative household with a large number of members. In any time period the household can send n_1 of its members to the first shift, where they will work h_1 hours, and it can send n_2 of its members to the second shift, where they will work h_2 hours. The number of employed household members cannot exceed the total number of household members, $n_1 + n_2 \leq \bar{n}$. The household's expected utility from a random consumption and labor supply process is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t - \sum_{i=1,2} \sigma_i \left[\frac{n_{it}^{1+\gamma}}{1+\gamma} + \psi_i n_{it} \frac{h_{it}^{1+\phi}}{1+\phi} \right] \right\}, \tag{6}$$

with discount rate $\beta \in (0, 1)$, and $\sigma_i, \psi_i, \gamma, \phi \geq 0$.[7]

The household is assumed to maximize the expected present value of utility subject to budget constraints. The household purchases consumption goods, saves, and supplies its labor. The market wage rates for employment on the two shifts are given by the wage functions $w_{1t}(h_{1t})$ and $w_{2t}(h_{2t})$. The

6 My treatment of government spending follows Hall (1996). In the last section I will discuss some issues in the measurement of productivity within the context of the capacity utilization model. The methods I discuss there use Instrumental Variable (IV) techniques, that is, they require the use of a variable which is exogenous to productivity but affects production decisions in the economy. In my model economy, government spending is such an instrumental variable.

7 These preferences are based on those described by Bils and Cho (1994).

household's period budget constraint is then

$$c_t + a_{t+1} \leq R_t a_t + w_{1t}(h_{1t})\, n_{1t} + w_{2t}(h_{2t})\, n_{2t},$$

where R_t is the return on asset holdings a_t. Note that the household can choose only which shifts to work, but not how long to work on each shift. On the other hand, when firms make their employment decisions, I assume that they choose employment in each shift and the length of each shift, given the wage functions they see in the labor market. The cost minimization problem of a firm is

$$\begin{aligned} &\min_{h_i, n_i} w_1(h_1)\, n_1 + w_2(h_2)\, n_2 \\ &\text{s.t.} \quad y = z\,(k/n_1)^{\alpha}\,(n_1 h_1 + n_2 h_2)\,, \\ &\qquad\; 0 \leq n_2 \leq n_1. \end{aligned}$$

We can define a competitive equilibrium for this economy, and it turns out to be the solution to the planning problem where we choose an allocation that maximizes the household's utility subject to the constraint that the allocation is feasible.[8]

The optimal employment decision by a household implies that the marginal disutility of employment at a given shift length is equal to the wage for a shift of that length:

$$w_{it}(h_i) = \frac{\sigma_i}{\lambda_t}\left\{n_{it}^{\gamma} + \psi_i h_{it}^{1+\phi}/(1+\phi)\right\}, \tag{7}$$

where λ_t is the Lagrange multiplier on the period budget constraint. In an equilibrium we can take this condition as the definition of the wage function. We can see that for the same employment levels and shift lengths, work on the second shift will require a higher wage than work on the first shift if $\sigma_1 \leq \sigma_2$ or $\psi_1 \leq \psi_2$, which means that work on the first shift creates less disutility than work on the second shift.

A Quantitative Evaluation of the Model

What does this model say about the behavior of the workweeks of capital and labor? In particular, does it predict that the workweek of capital is more volatile than the workweek of labor and that the late-shift employment share is procyclical? The model is sufficiently complicated that analytical characterizations are not feasible. I therefore parameterize the model, obtain a numerical solution, and calculate the response of the workweeks of capital and labor to a productivity shock.

8 In a more general formulation we have shifts of different lengths with wages depending on the length of the shift. Nevertheless, in an equilibrium, the household and firms would choose to operate each shift at one particular length. See Hornstein and Prescott (1993).

The main difference between the model described above and the standard growth model relates to the description of employment, in particular how labor market variables enter preferences and production. With respect to preferences, hours worked and employment are separate arguments in the utility function; furthermore, there are two types of employment (shifts). For the specification of the employment and hours elasticities, I follow Bils and Cho (1994) and select $\phi = 2$ and $\gamma = 1.6$.[9] I calibrate the scale parameters on the disutility of work based on assumptions on the relative steady state values of employment and hours worked for the two shifts.

Total employment is normalized at one, and I assume that 20 percent of total employment is in the second shift, $n_1 = 0.8$ and $n_2 = 0.2$. As stated above, Shapiro (1996) reports a mean of 25 percent for late-shift employment in manufacturing, but manufacturing represents only a subset of the economy, and late-shift work is less prevalent outside manufacturing. Since I do not have any information on the relative length of shifts, I simply assume that in the steady state both shifts are of equal length, which I normalize to one, $h_1 = h_2 = 1$. The two assumptions on relative employment and shift length imply that the workweek of capital is 25 percent longer than the workweek of labor. The calibrated capital workweek is substantially shorter than what Shapiro (1996) reports for the capital workweek in manufacturing based on *Survey on Plant Capacity* data, but it is comparable to his capital workweek estimates based on the *Consumer Population Survey*. The assumptions on steady state employment and hours worked and the assumptions on the elasticity parameters together determine the scale coefficients σ_i and ψ_i in the utility function.

We can evaluate the parameterization of labor supply based on the implied shift premium and labor supply elasticities. First, the implied steady state shift premium of the second shift is quite high, about 70 percent. This premium is substantially higher than the 10 percent shift premium Bils (1995) argues for or the 20 percent night-shift premium Shapiro (1996) suggests. Second, from the equilibrium wage function the implied elasticity of shift employment and hours worked to changes in the wage rate are[10]

$$1/\eta_n \equiv \frac{\partial w(h)}{\partial n} \frac{n}{w(h)} = \gamma \frac{n^\gamma}{n^\gamma + \psi h^{1+\phi}/(1+\phi)},$$
$$1/\eta_h \equiv \frac{\partial w(h)}{\partial h} \frac{h}{n} = (1+\phi) \frac{\psi h^{1+\phi}/(1+\phi)}{n^\gamma + \psi h^{1+\phi}/(1+\phi)}.$$

[9] I should note that the argument which Bils and Cho (1994) make for these particular parameter values is not strictly applicable to my model since their interpretation of the employment and hours worked variables is different from mine.

[10] These are wealth compensated supply elasticities since the Lagrange multiplier (marginal utility of consumption) is taken as constant.

For the given parameterization of preferences these labor supply elasticities are[11]

$$\eta_{n1} = 1.46;\ \eta_{n2} = 0.94;\ \eta_{h1} = 0.58;\ \text{and}\ \eta_{h2} = 1.00.$$

The labor supply elasticities are relatively low compared to other specifications used in dynamic general equilibrium models, where supply elasticities around 2 are more common. The labor supply elasticities in this model and in a standard growth model are, however, not directly comparable for two reasons. First, standard dynamic general equilibrium models usually do not distinguish between the labor supply elasticity of employment and the labor supply elasticity of hours worked per worker as I have done. Second, supply elasticities are usually stated in terms of percentage response of hours to a percentage change in the hourly wage rate.[12]

The remaining parameter values are comparable to those used in other studies. I choose the time discount factor $\beta = 0.99$ such that the annual steady state interest rate is 4 percent, the depreciation $\delta = 0.02$ such that the annual depreciation rate is 8 percent, and the capital coefficient $\alpha = 1/3$ such that the capital income share is one-third. The parameterization of productivity and government spending is based on Hall (1996). For the productivity process I assume that $\rho_z = 0.97$ and the standard deviation of innovations to the productivity process is 0.006. For the government spending process I assume that $\rho_g = 0.97$ and the standard deviation of innovations to the productivity process is 0.009. The steady state share of government spending in output is $\bar{g}/y = 0.18$.

Figure 2 displays the responses of the workweeks of capital and labor to a percentage point deviation of productivity from the steady state value. We can see that on impact, the workweek of capital increases more than does the workweek of labor. Since employment on the first shift is predetermined, the economy cannot use this margin when it responds to the contemporaneous productivity shock. The economy is, however, free to increase employment

[11] Some algebra shows that the steady state values of these labor supply elasticities are

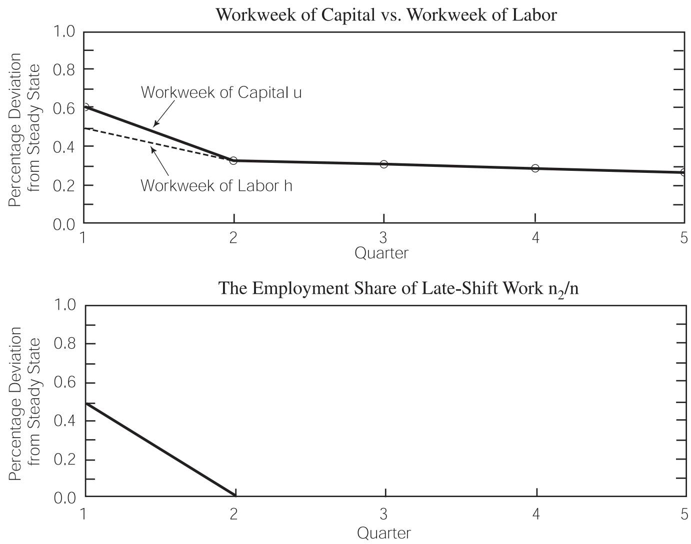
$$\begin{aligned}
1/\eta_{n_1} &= \gamma\left[\rho/(1-\rho) - \alpha u/h_1\right]/\left[1-\alpha u/h_1\right],\\
1/\eta_{n_2} &= \gamma\rho/(1-\rho),\\
1/\eta_{h_1} &= 1/(1-\alpha u/h_1),\\
1/\eta_{h_2} &= 1.
\end{aligned}$$

[12] A more appropriate procedure would define the labor supply elasticity with respect to average wage changes. This does not affect the measure of employment supply elasticity, but it does change the measure of hours elasticity:

$$1/\hat{\eta}_h \equiv \frac{\partial\left[w(h)/h\right]}{\partial h}\frac{h}{w(h)/h} = 1/\eta_h - 1.$$

According to this alternative measure, the hours supply elasticity is actually quite high.

Figure 2 Workweek of Capital and Workweek of Labor in Model



on the second shift, which increases the employment share of late-shift work. This increase in the employment share of the second shift in turn amplifies the response of the workweek of capital.

The model does not generate persistent differences between the workweek of capital and the workweek of labor. That is to say, only unanticipated shocks create a divergence between the two workweek definitions. After the first period, when employment on the first shift can be adjusted again, the economy attains its target late-shift employment ratio. This feature of the model is due to the particular assumption on adjustment costs for the capital-labor ratio, namely infinite adjustment costs at the beginning of the period and zero adjustment costs at the end of the period. Alternatively, we could assume that any changes in the production structure, specifically the capital content of machines, involve some resource costs (see, for example, Bils and Cho [1994]). With this alternative assumption, there will be persistent deviations between the workweek of capital and the workweek of labor in response to a productivity shock.

Another way to evaluate the role of shiftwork in the model is to compare its business cycle properties to those of the U.S. economy and other models without shiftwork. The business cycle properties are defined with respect to Hodrick-Prescott filtered time series of output, consumption, investment, capital stock, total hours worked, employment, and the workweek of labor (see columns 1 and 5 of Table 1). For our purposes it is of interest to note

Table 1 Business Cycle Properties of U.S. Data and Model Economies

	Percentage Standard Deviations				Correlation with Output			
	U.S.	**I**	**II**	**III**	**U.S.**	**I**	**II**	**III**
Output	1.74	1.08	1.00	1.02	1.00	1.00	1.00	1.00
Consumption	1.29	0.50	0.48	0.48	0.85	0.89	0.89	0.89
Investment	8.45	3.58	3.30	3.37	0.91	0.99	0.99	0.99
Capital Stock	0.63	0.25	0.24	0.24	0.05	0.03	0.02	0.02
Total Hours Worked	1.74	0.40	0.34	0.35	0.77	0.94	0.90	0.92
Employment	1.50	0.26	0.23	0.22	0.81	0.94	0.74	0.83
Workweek of Labor	0.46	0.14	0.15	0.15	0.76	0.94	0.88	0.90
Workweek of Capital		0.14	0.15	0.19		0.94	0.88	0.83

Notes: U.S. Data are from Bils and Cho (1994) and they cover the time period 1955:III–1984:I. Model I is the one-shift model with employment not predetermined; Model II is the one-shift model with employment predetermined; Model III is the two-shift model with employment of the first shift predetermined. All time series are detrended with the Hodrick-Prescott filter. The model statistics are based on 100 simulations where each simulation consists of 30 years of quarterly observations.

that employment is more volatile than the workweek of labor (average hours worked), and employment is slightly more correlated with output than is the workweek of labor. I do not have quarterly observations on the workweek of capital, but in the previous section I note that for annual growth rates, the workweek of capital is more volatile than the workweek of labor and that the workweeks of capital and labor are only weakly correlated.

To evaluate the contribution of shiftwork as modeled in this article, I consider three models. The first and second models assume that there is a distinction between employment and the workweek of labor, but that all employment is in one shift only. Preferences and production are as previously described, with the restriction that $n_2 = h_2 = 0$. For Model I, employment and the workweek of labor are determined at the beginning of the period, after the productivity and government spending shock have been observed. For Model II, employment is determined before observations on the current productivity and government spending shock are available. Model I is similar to Kydland and Prescott (1991) and Bils and Cho (1994), whereas Model II is similar to Burnside, Eichenbaum, and Rebelo (1993) and Hall (1996) in that part of the employment decision is predetermined. Finally, Model III is the economy with shiftwork as described above.

Table 2 The Workweeks of Capital and Labor, Annual Growth Rates

	Std. Dev. Δu	**Std. Dev.** Δh	**Corr.** $(\Delta u, \Delta h)$
U.S.	2.92	2.02	0.57
Model III	1.14	0.99	0.98

Notes: For U.S. data, see Figure 1.

For each model I generate 100 random samples, each with 30 years of quarterly observations. The artificial time series are detrended with the Hodrick-Prescott filter, and I calculate the average standard deviations and correlations with output of the detrended series. The results are listed in Table 1. We can see that given the exogenous disturbances, the model economies are not as volatile as the U.S. economy. The model economies capture the fact that investment is more volatile than consumption, but consumption tends to be too smooth. The model economies also capture the fact that employment is more volatile than the workweek of labor, but overall labor is too smooth relative to the U.S. economy. Concerning the comovement with output, we see that in the models employment is more closely correlated with output than it is in the U.S. economy.

In Model III, the workweek of capital is indeed more volatile than the workweek of labor for detrended quarterly data. Since quarterly U.S. data on the workweek of capital are not available, I have calculated the standard deviations and correlations for the annual percentage growth rates of the workweeks of capital and labor (see Table 2). We can see that in the U.S. manufacturing sector, the workweek of capital is relatively more volatile than the workweek of labor. Furthermore, the relationship between the workweek of capital and the workweek of labor is much tighter in the model than in the data. While the model captures the qualitative features of the workweek of capital, it does not come close yet to replicating its quantitative properties.

3. IMPLICATIONS FOR PRODUCTIVITY MEASUREMENT

I now study the implications of variable shiftwork for the measurement of productivity changes. Since productivity change is defined as output changes that cannot be attributed to input changes, unobserved input movements obscure our measures of productivity change. Changes in capital services, such as changes in the workweek of capital, represent important input movements that are not reflected in our standard measures of inputs. Recently, Basu and Kimball (1997) have argued that unobserved variation in the utilization of inputs is related to the observed variation in the workweek of labor.[13] They use this relationship to obtain a utilization-corrected measure of productivity change. There are two potential problems with the approach of Basu and Kimball (1997). First, their procedure requires that the workweek of capital be strictly proportional to the workweek of labor. If this is not true, as suggested by the available evidence and theory on the workweek of capital, then their procedure does not necessarily generate unbiased estimates of the volatility of productivity change. Second, even if the estimates of the volatility of productivity are unbiased, they may not be precise because the estimates rely on instrumental variables that may be quite poor.

Consider the standard production function (1), which takes as inputs the capital stock and total hours worked. We can measure productivity growth through the Solow residual, which defines productivity growth as output growth less input growth weighted by the output elasticities of inputs:

$$\hat{z}^m \equiv \hat{y} - \alpha\hat{k} - (1-\alpha)\left(\hat{h} + \hat{n}\right) = z.$$

The Solow residual is an operational concept because in a competitive equilibrium with CRS production, we can identify the elasticities with the factor income shares of inputs. Consider now the production function (2) with a variable workweek of capital but only one shift. If we continue to assume that the relevant inputs are the stock of capital and total hours worked, then measured productivity growth no longer reflects true productivity growth:

$$\hat{z}^m = \hat{z} + \alpha\hat{h}.$$

However, a simple correction of the Solow residual, made by subtracting the growth rate of average hours worked weighted by the factor income share of capital, retrieves the true productivity change:

$$\hat{z}^m - \alpha\hat{h} = \hat{z}.$$

Empirically, this simple correction does not deliver measures of true productivity change. Suppose you have variables—call them instrumental variables—that on a priori grounds are considered to be independent of true productivity change. On these grounds, the instrumental variables should be uncorrelated with the Solow residual corrected for average hours worked. In empirical applications, however, it turns out that the Solow residual corrected for average hours worked remains correlated with these instrumental variables. However, in an instrumental variables regression of the measured Solow residual on average hours growth, Basu and Kimball (1997) find that the coefficient on average hours growth is around one, larger than the factor income share of capital, which is substantially less than one. They argue that the relatively large coefficient on average hours worked reflects other unobserved input utilization, which is strictly proportional to average hours worked. Furthermore,

[13] See also Basu, Fernald, and Kimball (2000).

they argue that once they correct the Solow residual for movements related to movements in average hours worked, they can recover exogenous movements in productivity. By contrast, I argue here that this contention is not true when the workweek of capital and the workweek of labor are not perfectly correlated.

Consider the production function (3) of the two-shift model described above. With this production structure, measured productivity growth based on changes in the capital stock and total hours worked is

$$\hat{z}^m = \hat{z} + \alpha \hat{u}. \tag{8}$$

Again, the true productivity disturbance can be recovered by correcting for the workweek of capital. Shapiro (1996) argues that industry Solow residuals that are corrected for the workweek of capital are essentially uncorrelated with instrumental variables. A problem with this approach is that only a limited number of observations on the workweek of capital are available. Shapiro (1996) uses the workweek of capital numbers constructed by Beaulieu and Mattey (1998), and this sample is limited to the years 1974–1992. Given the limited availability of direct observations on the workweek of capital, the argument of Basu and Kimball (1997) for the use of average hours worked as a proxy for different forms of capacity variation is attractive. Especially so since, as they argue, average hours worked not only covers variations in the workweek of capital, but also variations in capital utilization that are not related to corresponding changes in the worktime of labor.

Basu and Kimball (1997) suggest estimating the regression equation

$$\hat{z}^m = b\hat{h} + e \tag{9}$$

using instrumental variable techniques.[14] Let q denote an instrumental variable that is uncorrelated with the true productivity shock, then the two-stage instrumental variable estimator of b is

$$\bar{b} = \frac{E\left[\hat{z}^m \hat{q}\right]}{E\left[\hat{h}\hat{q}\right]} = \frac{E\left[\hat{q}\left(\alpha\hat{u} + \hat{z}\right)\right]}{E\left[\hat{q}\hat{h}\right]}$$

given the true relationship (8). If we suppose for the moment that changes in the workweek of capital are proportional to changes in the workweek of labor, $\hat{u} = \mu\hat{h}$, then the estimator simplifies to

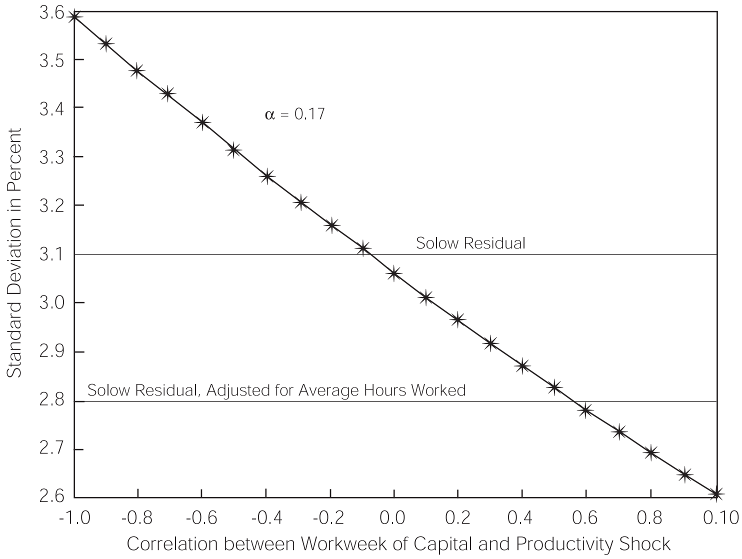
$$\bar{b} = \alpha\mu.$$

[14] Basu and Kimball's (1997) approach is actually somewhat more complicated in that they consider the possibility of noncompetitive behavior. They estimate the equation

$$\hat{y} = \gamma\hat{m} + b\hat{h} + e,$$

where γ is the average markup of price over marginal cost. I assume competitive behavior, that is, $\gamma \equiv 1$, and only the coefficient b must be estimated.

Figure 3 Productivity Volatility Implied by Workweek Volatility



Notice that the estimator does not have a structural interpretation, since μ reflects a relation between two endogenous variables that depends on elements of a fully specified equilibrium model. Nevertheless, correcting the Solow residual recovers the true changes in productivity

$$\hat{z}^m - \bar{b}\hat{h} = \left(\alpha\mu\hat{h} + \hat{z}\right) - \bar{b}\hat{h} = \hat{z}.$$

The only problem with this approach is that the above model of the workweek of capital predicts that for a reasonable specification of the production structure, the changes in the workweek of capital are not strictly proportional to changes in the workweek of labor. Furthermore, in my review of the empirical evidence on the workweek of capital, I have shown that the relation between the workweek of capital and the workweek of labor is not very tight; the correlation coefficient between their respective percentage changes is only 0.6.

If the workweek of capital is not tightly related to the workweek of labor, then the average hours-corrected estimates of the volatility of productivity disturbances obtained by Basu and Kimball (1997) are not unbiased. Suppose that the relation between the workweek of capital and the workweek of labor is $\hat{u} = \mu\hat{h} + \hat{v}$, where $\hat{v}$ is an endogenous movement in the workweek of capital that is orthogonal to the workweek of labor. Since $\hat{v}$ is endogenous, we would expect it to be correlated with the instrumental variable $\hat{q}$, $E\left[\hat{v}\hat{q}\right] \neq 0$. The estimated parameter and the hours-corrected Solow residual are then

$$\bar{b} = \alpha\mu + \frac{E\left[\hat{v}\hat{q}\right]}{E\left[\hat{h}\hat{q}\right]} \text{ and } \hat{z}^m - \bar{b}\hat{h} = \hat{z} + \left(\hat{v} - \bar{b}\hat{h}\right).$$

Notice that the hours-corrected Solow residual no longer provides an estimate of true productivity movements.

Can we say anything about the volatility of the true productivity shocks conditional on what we know about the workweek of capital? From equation (8) we can write the variance of the Solow residual $\sigma^2_{z^m}$ as a function of the variance of the true productivity shock σ^2_z, the variance of the workweek of capital σ^2_u, and the correlation of true productivity shocks and the workweek of capital ρ_{zu}. This expression defines an implicit equation in the volatility of the true productivity shock conditional on the volatility of the measured Solow residual, the volatility of the workweek of capital, and an assumption on the correlation coefficient between the true productivity shock and the workweek of capital:

$$0 = \sigma^2_z + \left(2\alpha\rho_{zu}\sigma_u\right)\sigma_z + \left(\alpha^2\sigma^2_u - \sigma^2_{z^m}\right).$$

Consider now the manufacturing sector. From Beaulieu and Mattey (1998), the volatility of the workweek of capital for the time period 1974–1992 is 2.9 percent, and from Basu, Fernald, and Kimball (1999), the volatility of the Solow residual is $\sigma_{z^m} = 3.1$ percent. Because Basu, Fernald, and Kimball (1999) estimate the Solow residual from gross output data, and intermediate inputs make up a substantial share of total payments to inputs, I choose a capital coefficient $\alpha = (1/3)\,(1/2)$. In Figure 3, I plot the implied volatility of the true productivity shock for values of the correlation coefficient between negative one and positive one. The two horizontal lines indicate the volatility of the unadjusted Solow residual and the hours-worked-adjusted Solow residual from Basu, Fernald, and Kimball (1999) for the manufacturing sector. We can see that the workweek-of-labor-corrected Solow residual underestimates (overestimates) the true volatility of the productivity shocks when the workweek of capital and the true productivity disturbance are weakly (strongly) correlated. The critical value for the correlation coefficient is 0.6. Since we expect a relatively strong positive correlation between the capital workweek and productivity disturbances, the actual bias might not be very large.

Suppose that Basu and Kimball's (1997) estimates of the volatility of production are unbiased. Can we say anything about how precise these estimates are? This question is relevant since the estimate of the coefficient b in equation (9) and the estimated productivity change $\hat{z}^m - \hat{b}\hat{h}$ are based on instrumental variables that are quite poor and the sample size is quite small (only 40 years). We can evaluate the uncertainty surrounding the estimates using the workweek of capital model described above. This model captures the qualitative features that the workweek of capital is more volatile than the workweek of

Table 3 Ratio of Estimated to True Volatility of Productivity Growth

Sample Size	Mean	Std. Dev.
40 years	0.88	0.35
200 years	1.02	0.18
400 years	1.03	0.12

labor and that the two workweeks are not perfectly correlated. In order to evaluate the uncertainty about the estimates, I generate 1,000 samples of 40 years of quarterly observations for the model. For each sample I construct annual data from the quarterly data and then estimate equation (9) with the annual data. In the model, government spending is exogenous and affects other endogenous variables, that is, it can be used as an instrumental variable. For the estimation of equation (9), I use contemporaneous and lagged growth of annual government spending as instruments. I then use equation (9) to calculate the volatility of estimated productivity growth rates. The model tells me the volatility of the two productivity growth rates. In Table 3, I display the means and standard deviations of the ratio of estimated to true productivity volatility across the samples. We can see that for a small sample of 40 years, on average we underestimate the true productivity volatility. More important, the standard deviation on the estimates is very large: the two-standard deviation error band for the ratio reaches from 0.18 to 1.58.

Given the small sample of available data, the estimate of true productivity volatility is very imprecise. Basu and Kimball (1997) increase the sample size by using industry data rather than aggregate data. They assume that in equation (9), the coefficient b is the same across industries, and then they pool industry data. For manufacturing they pool durable-goods-producing industries (ten industries) and nondurable-goods-producing industries (seven industries). I replicate the industry pooling approach by assuming that each industry represents another 40 years of observations. With more observations the estimate of productivity volatility appears to be unbiased (see Table 3). This result should not be too surprising since the differences between the workweek of capital and the workweek of labor are not quantitatively important in the model, as opposed to the data. We might therefore expect that the use of the workweek of labor rather than the workweek of capital in equation (9) would not generate a large bias in the estimate of the volatility of productivity. Even with a larger data set, however, the estimate of productivity volatility is not very precise: the two-standard deviation error band for the ratio still ranges from 0.79 to 1.27.

4. CONCLUSION

Variations in capital utilization as measured by the workweek of capital are large; indeed, the workweek of capital is substantially more volatile than the workweek of labor. This observation suggests that for output fluctuations, short-term variations in the utilization of the capital input are at least as important as short-term variations in the utilization of the labor input. Yet, official statistics are collected only for variations in the workweek of labor, not for the workweek of capital. Improved measurement of the workweek of capital is clearly called for (Shapiro 1996). Improved data would allow for a better assessment of the role of productivity disturbances.

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