

How Well Do Diffusion Indexes Capture Business Cycles? A Spectral Analysis

Raymond E. Owens and Pierre-Daniel G. Sarte

Regional Federal Reserve banks expend considerable effort preparing for FOMC meetings, culminating in a statement presented to the committee. Statements typically begin with an assessment of regional economic conditions, followed by an update on national economic conditions and other developments pertinent to monetary policy.

This article examines whether the regional economic information produced by the Federal Reserve Bank of Richmond (FRBR), in the form of diffusion indexes, can be tied to the business cycle. Such a link is of direct interest because of its applicability to policy decisions. Very short cycles (such as a month in length) are potentially just noise and of little policy interest. Very long cycles (such as a long-term trend) are typically thought to be driven by technological considerations over which policy has little bearing. In contrast, one generally thinks of monetary policy decisions as affecting primarily medium-length cycles or business cycles. The objective of the research herein, therefore, is to identify which of the FRBR's indexes tend to reflect primarily business cycle considerations. Indeed, indexes for which such considerations are small or nonexistent have little hope of providing any information about the state of aggregate production measures over the business cycle, and their calculation would be of limited value.

At the regional level, economic data are less comprehensive and less timely than at the national level. For example, no timely data are published on state-level manufacturing output or orders. In addition, published data on Gross State Product (GSP) are available with lags of 18 months or more. Also, these

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published data are available to FOMC members as soon as they are available to the Reserve Banks so that their analysis by the latter adds little to the broader monetary policy process. These shortcomings have led a number of organizations—including several regional Federal Reserve banks—to produce their own regional economic data. These efforts mostly have taken the form of high-frequency surveys. Surveys provide speed and versatility, overcoming the obstacles inherent in the traditional data. But surveys are often relatively expensive per respondent, leading organizations to maintain relatively small sample sizes. Further, to limit the burden on respondents, survey instruments often ask very simple questions, limiting the information set and level of analysis.

The Richmond Fed conducts monthly surveys of both manufacturing and services sector activity. The number of survey respondents is usually around 100 and respondents report mostly whether a set of measures increased, decreased, or was unchanged. However, there are several measures—primarily changes in prices—reported as an annual percentage change. Results from these surveys, along with Beige Book information, comprise the foundation of regional economic input into monetary policy discussions.

That said, there are several reasons why one may be skeptical of diffusion indexes' ability to capture useful variations in the business cycle. Specifically, the usefulness of diffusion indexes hinges critically on the following aspects of survey data:

- Diffusion indexes are produced from data collected at relatively high frequency—with new indexes being typically released every month—and therefore potentially quite noisy.
- The types of questions being asked allow for very little leeway in respondents' answers. For example, the regional diffusion indexes produced by the FRBR are calculated from survey answers that only distinguish between three states from one month to the next. Thus, we ask only whether shipments, say, are up, down, or unchanged relative to last month. In particular, let I , D , and N denote the number of respondents reporting increases, decreases, and no change respectively, in the series of interest. The diffusion index is then simply calculated as

$$I = \left(\frac{I - D}{I + N + D} \right) \times 100. \quad (1)$$

Observe that I is bounded between -100 and 100 , and takes on a value of zero when an equal number of respondents report increases and decreases.

- The surveys must contain a large enough sample in order that a diffusion index capture potentially meaningful variations at business cycle

frequencies. As a stark example, note that if only two firms were surveyed, the index I above would only ever take on five values, $\{-100, -50, 0, 50, 100\}$. If three firms were sampled, I in (1) would only ever take on the values $\{-100, -66, -33, 0, 33, 66, 100\}$. Evidently, I will take on more and more values the more firms are sampled. This may not be a problem for identifying whether the resulting index is driven mainly by business cycle considerations *per se*, but will affect the degree to which such indexes commove with more continuous aggregate measures of production over the cycle.

- Composition effects will also affect this last observation. To see this, suppose that periods of recessions and expansions are characterized by all firms decreasing and increasing their shipments respectively as changes in demand occur. Then, even with a large sample, the diffusion index in (1) could never take on any other value than -100 and 100 and would, therefore, offer no information on the relative strength of economic conditions. This will not be the case, however, when the number of firms reporting decreases or increases in shipments, say, varies in a systematic fashion with the extent of recessions and expansions.
- Finally, respondents possess much discretion in the way they answer survey questions. Thus if a given manufacturer's new orders, say, increased or decreased this month by only a "small" amount relative to last month, she may decide to report no change in her orders. But the key point here is that the definition of "small" is left entirely to the respondent's discretion.

1. SOME KEY CONCEPTS IN FREQUENCY DOMAIN ANALYSIS

Before tackling the issue of whether regional diffusion indexes have anything to do with business cycles, let us briefly review some important concepts that we shall use in our analysis. In particular, the material in this section summarizes central notions of frequency domain analysis that can be found in Hamilton (1994), Chapter 6; Harvey (1993), Chapter 3; as well as King and Watson (1996).

The *spectral representation theorem* states that any covariance-stationary process $\{Y_t\}_{t=-\infty}^{\infty}$ can be expressed as a weighted sum of periodic functions

of the form $\cos(\lambda t)$ and $\sin(\lambda t)$:¹

$$Y_t = \mu + \int_0^\pi \alpha(\lambda) \cos(\lambda t) d\lambda + \int_0^\pi \delta(\lambda) \sin(\lambda t) d\lambda, \quad (2)$$

where λ denotes a particular frequency and the weights $\alpha(\lambda)$ and $\delta(\lambda)$ are random variables with zero means.

Generally speaking, given that any covariance-stationary process can be interpreted as the weighted sum of periodic functions of different frequencies, a series' *power spectrum* gives the variance contributed by each of these frequencies. Thus, summing those variances over all relevant frequencies yields the total variance of the original process. Moreover, should certain frequencies, say $[\lambda_1, \lambda_2]$, mainly drive a given series, then the variance of cycles associated with these frequencies will account for the majority of the total variance of that series.

A Simple Example

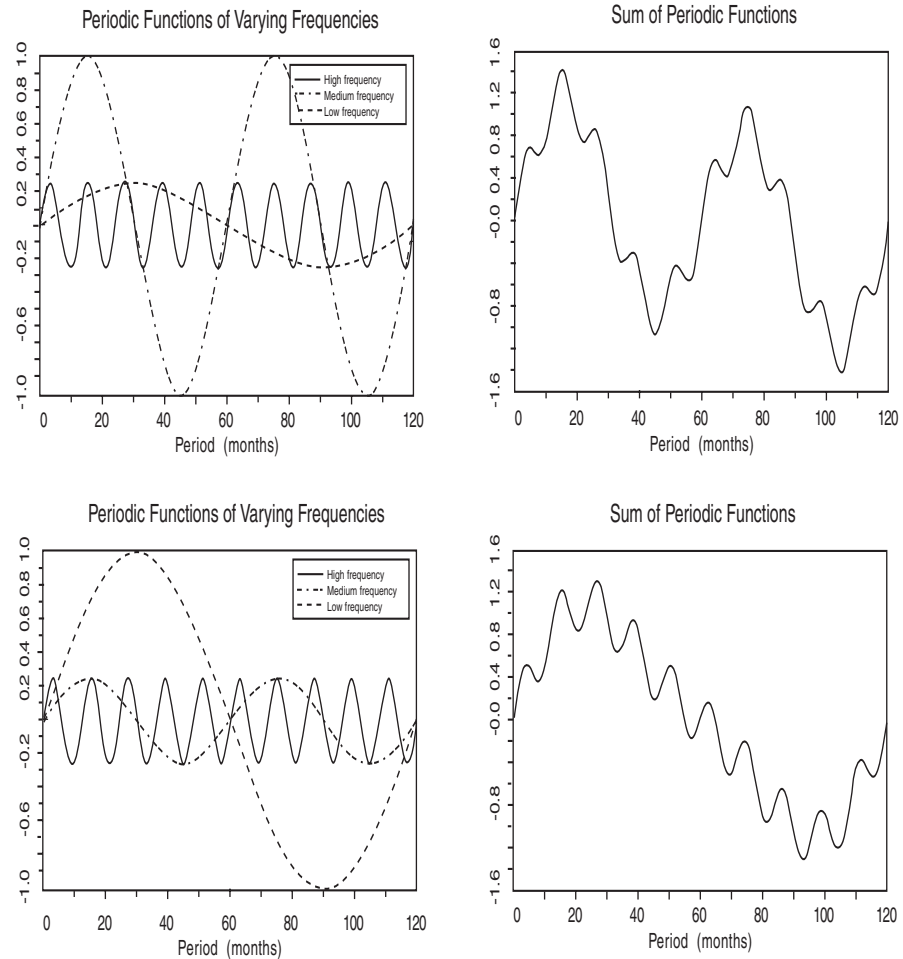
In order to make matters more concrete, consider the following example. Define the following process for a hypothetical economic time series, Y_t ,

$$Y_t = \alpha_1 \sin(\lambda_1 t) + \alpha_2 \sin(\lambda_2 t) + \alpha_3 \sin(\lambda_3 t), \quad (3)$$

where the α_i 's and λ_i 's are strictly positive real numbers. A sine function is bounded between -1 and 1 , so that the first term on the right-hand side of equation (3) will oscillate between $-\alpha_1$ and α_1 , the second term between $-\alpha_2$ and α_2 , etc. We refer to α_i as the amplitude of the component of Y_t associated with $\alpha_i \sin(\lambda_i t)$. A function is periodic with period T when the function repeats itself every T periods. The period of a sine function is defined as 2π divided by its frequency. Thus, the first term on the right-hand side of (3) will repeat itself every $2\pi/\lambda_1$ periods, the second term every $2\pi/\lambda_2$ periods, etc. Furthermore, observe that the higher the frequency, the faster a periodic function repeats itself.

For additional concreteness, assume now that one unit of time is a month, and that in the above example, $\{\alpha_1, \lambda_1\} = \{0.25, \frac{\pi}{6}\}$, $\{\alpha_2, \lambda_2\} = \{1, \frac{\pi}{30}\}$, and $\{\alpha_3, \lambda_3\} = \{0.25, \frac{\pi}{60}\}$. Then, the components of Y_t given by $\alpha_1 \sin(\lambda_1 t)$ and $\alpha_3 \sin(\lambda_3 t)$ have the shortest and longest periods, one year (i.e., a seasonal cycle) and 10 years, respectively, as well as the smallest amplitude, 0.25. We refer to these components as the high- and low-frequency components of Y_t , respectively. In contrast, the component of Y_t given by $\alpha_2 \sin(\lambda_2 t)$ repeats itself every $2\pi/(\pi/30) = 60$ months, or five years. Thus, we refer to this component as the medium-frequency or business cycle component of Y_t . Note

¹ A stochastic process, Y_t , is covariance stationary if $E(Y_t) = \mu$ and $E(Y_t Y_{t-s}) = \sigma_s \forall t$ and s .

Figure 1 Examples of Aggregation of Periodic Functions

also that $\alpha_2 \sin(\lambda_2 t)$ has the largest amplitude of all three components since $\alpha_2 = 1$. The upper left-hand panel of Figure 1 illustrates these periodic functions separately over a period of 10 years. We can clearly see that the slowest moving periodic function (i.e., the low-frequency component) repeats itself exactly once over that time span. In contrast, the business cycle component repeats itself twice and dominates in terms of its amplitude.

The upper right-hand panel of Figure 1 illustrates the sum of these periodic components. It is clear that Y_t repeats itself twice over the 10-year time span. Put another way, Y_t in this case is primarily driven by its business cycle or medium-frequency component. This is because this component has

the largest amplitude and matters most, while the high- and low-frequency components have relatively small amplitude. In particular, the amplitude of Y_t is $\alpha_1 + \alpha_2 + \alpha_3 = 1.5$, with two-thirds of that amplitude being contributed by the medium-frequency component. Since, strictly speaking, the power spectrum relates to variances, the fraction of total variance of Y_t explained by the component $\alpha_2 \sin(\lambda_2 t)$ in this case is $1/(0.25^2 + 0.25^2 + 1)$, or 89 percent.²

As an alternative example, suppose that $\alpha_2 = 0.25$ while $\alpha_3 = 1$, with all other parameters unchanged. This case is depicted in the lower left-hand panel of Figure 1, where it is the component that repeats itself just once over 10 years that now evidently dominates in terms of amplitude. The sum of low-, medium-, and high-frequency components, Y_t , is given in the lower right-hand side panel of Figure 1, and notice that it reflects mainly its slowest moving element, $\alpha_3 \sin(\lambda_3 t)$. And indeed, contrary to our earlier example, it is now this low-frequency component that accounts for the bulk of the total variance of Y_t , or two-thirds of its amplitude.

Formally, one defines the population spectrum of Y as

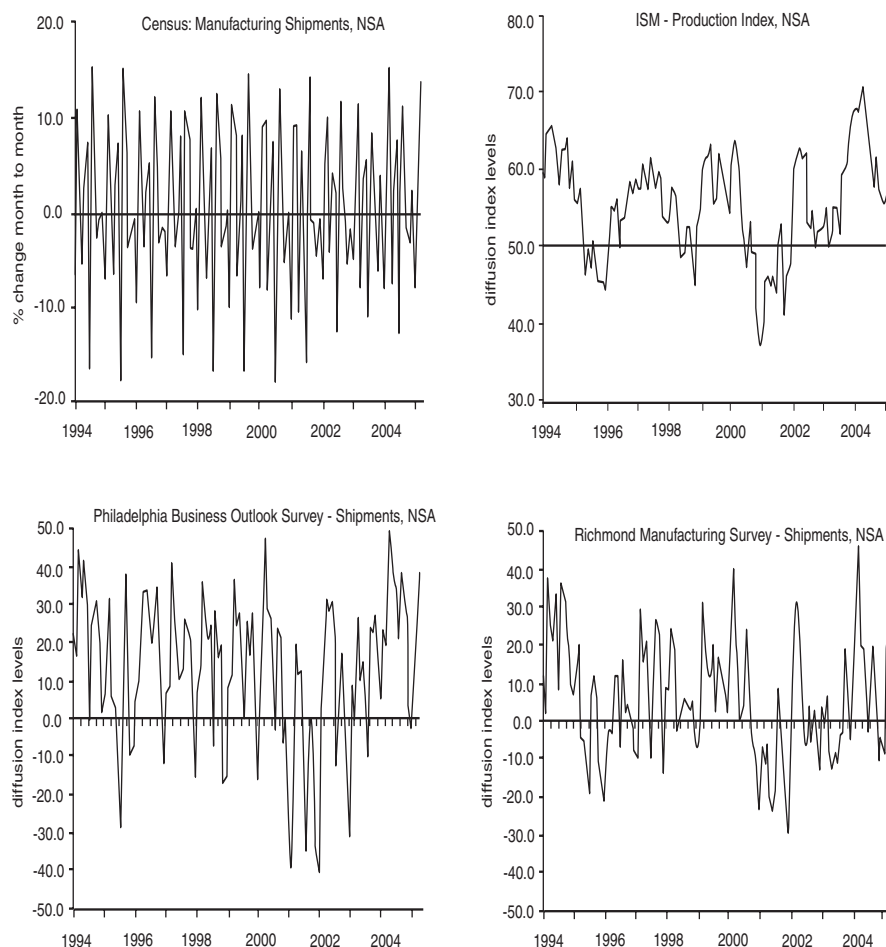
$$\begin{aligned} f(\lambda) &= \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j e^{-i\lambda j}, \quad -\pi \leq \lambda \leq \pi \\ &= \frac{1}{2\pi} \left[\gamma_0 + 2 \sum_{j=1}^{\infty} \gamma_j \cos(\lambda j) \right], \end{aligned} \quad (4)$$

where $i^2 = -1$ and γ_j is the j^{th} auto-covariance of Y , $\text{cov}(Y_t, Y_{t \pm j})$. In a manner similar to our example above, economic time series that are driven principally by business cycle forces will have most of their variance (or amplitude) associated with cycles lasting between one and a half to eight years. We can think of $f(\lambda)$ in equation (4) as the variance of the periodic component with frequency λ . Similarly, in the above example, the components $\alpha_i \sin(\lambda_i t)$ have different amplitude or variance. More specific attributes of the power spectrum are given in Appendix A. Details of estimation and calculations for the results that follow are given in Appendix B.

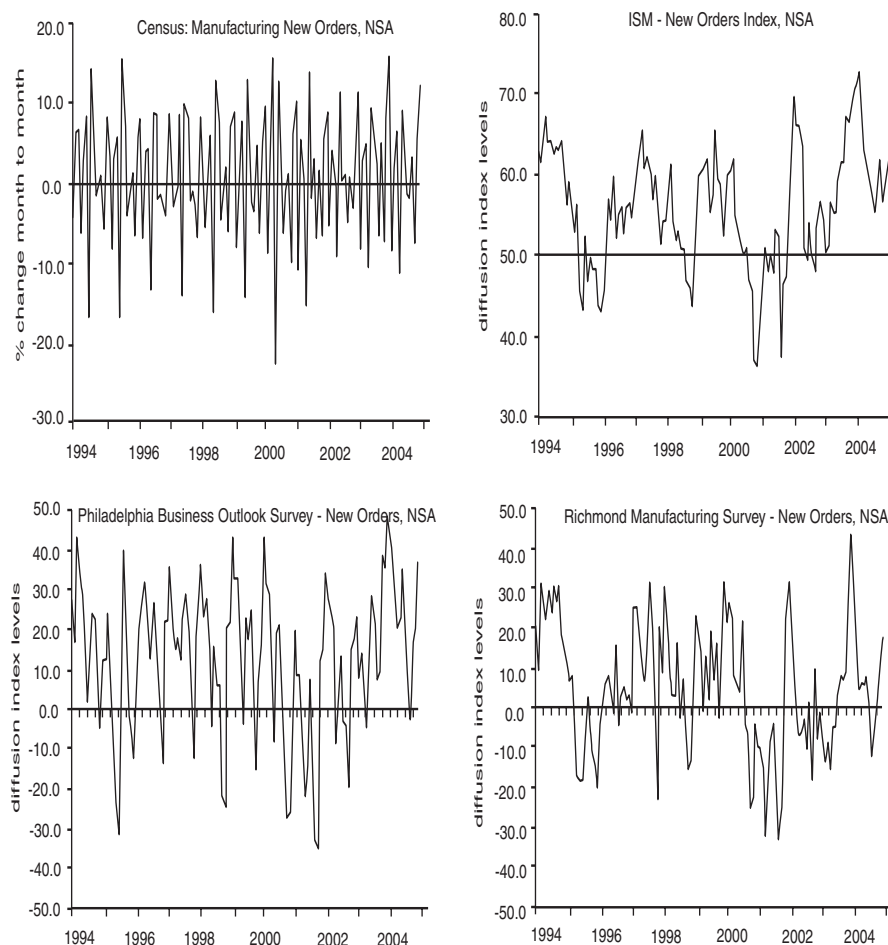
2. EXAMPLES WITH MANUFACTURING DATA

Figure 2 plots the behavior of manufacturing shipments as actually recorded by the Census at the national level, and as captured by different indexes including the Institute of Supply Management (ISM) index, the Federal Reserve Bank of Philadelphia (FRBP) Business Outlook survey, and the FRBR regional survey.

²In particular, amplitude and variance are closely related here since $\text{var}(\alpha_i \sin(\lambda_i t)) = \alpha_i^2 \text{var}(\sin(\lambda_i t))$ and $\text{var}(\sin(\lambda_i t)) = \text{var}(\sin(\lambda_j t))$ for $i \neq j$. Therefore, the fraction of total variance explained by the component $\alpha_i \sin(\lambda_i t)$ is $\alpha_i^2 / \sum_i \alpha_i^2$.

Figure 2 Measures of Manufacturing Shipments

Because the FRBR only began to produce its diffusion indexes in November 1993, we chose to homogenize our samples in Figure 2 and show the behavior of the series over the same period. Although the actual monthly manufacturing shipments and the ISM index are meant to reflect similar information, there are clear differences between the two series. The ISM does not make public the formula it uses for translating its respondents' answers into an actual diffusion index, but it is apparent that it produces a much smoother series. At the same time, observe that we can clearly see a common cyclical pattern between the FRBR's manufacturing shipments survey and the corresponding ISM index. The regional diffusion indexes are also smoother than the actual national data,

Figure 3 Measures of Manufacturing New Orders

but this could be indicative of the specific regional industrial makeup of the Third and Fifth Federal Reserve Districts. These observations all apply to the behavior of new orders in Figure 3.

A presumption of our analysis is that manufacturing data fluctuates over time to reflect evolving business cycle conditions. However, this is certainly not obvious from the upper left-hand panel in Figures 2 and 3, where the series seem primarily driven by very fast-moving random components. Economic analysts implicitly recognize this fact when commenting on the behavior of manufacturing data and, indeed, informal discussions of the current data are often framed relative to other episodes. In other words, analysis of the data

Table 1 Aggregate National Data
Percent of variance attributable to cycles with different periods

	periods > 8 years	1.5 years < periods < 8 years	periods > 6 mo.
Shipments	19.00	71.30	97.90
New Orders	17.29	67.89	93.75
Employment	33.80	62.76	99.64

often involves the use filters, whether implicitly or explicitly, in the hope to gain some insight from the series about evolving economic conditions.³ In principle, one can apply any filter one wishes to the data (that leaves the resulting series covariance stationary) and estimate the corresponding power spectrum to determine to what degree business cycle components are actually being emphasized.

To illustrate this last point, Figure 4 shows estimated power spectra for manufacturing shipments, new orders, and employment data based on both the series' month-to-month and year-to-year changes. The solid vertical lines in the figures cover the frequencies associated with the conventional definition of business cycles, $[\pi/9, \pi/48]$, which correspond to cycles with periods ranging from one and a half to eight years. The dashed vertical line corresponds to cycles with a period of six months, $\lambda = \pi/3$. Observe that cycles have longer and longer periods as we move toward zero on the horizontal axis.

Figure 4 shows that month to month, both national manufacturing shipments and new orders power spectra exhibit multiple peaks at high frequencies. Thus, the monthly observations are driven mainly by short-lived random periodic cycles that are not necessarily informative for the purposes of policymaking. In contrast, the power spectra for the 12-month difference of the manufacturing data series all contain a high notable peak in the business cycle interval, as well as a lower peak at roughly frequency $\lambda = 0.3$ (i.e., cycles of length close to two years). King and Watson (1996) refer to the shape of the power spectra in the right-hand panels of Figure 4 as the typical spectral shape for differences in macroeconomic time series. Cycles that repeat themselves on a yearly basis, and are thus associated with seasonal changes, have frequency $\lambda = \pi/6 = 0.53$, and we can see that the spectra in the right-hand panels of Figure 4 also display a small peak just to the right of that frequency.

Table 1 gives the fraction of total variance attributable to cycles of different lengths for the manufacturing series depicting year-to-year changes.

As in the analysis of King and Watson (1996), the business cycle interval contains the bulk of the variance of the yearly change in these macroeconomic

³ By filters, we mean a transformation of the original time series such as a moving average or an $n > 1$ order difference.

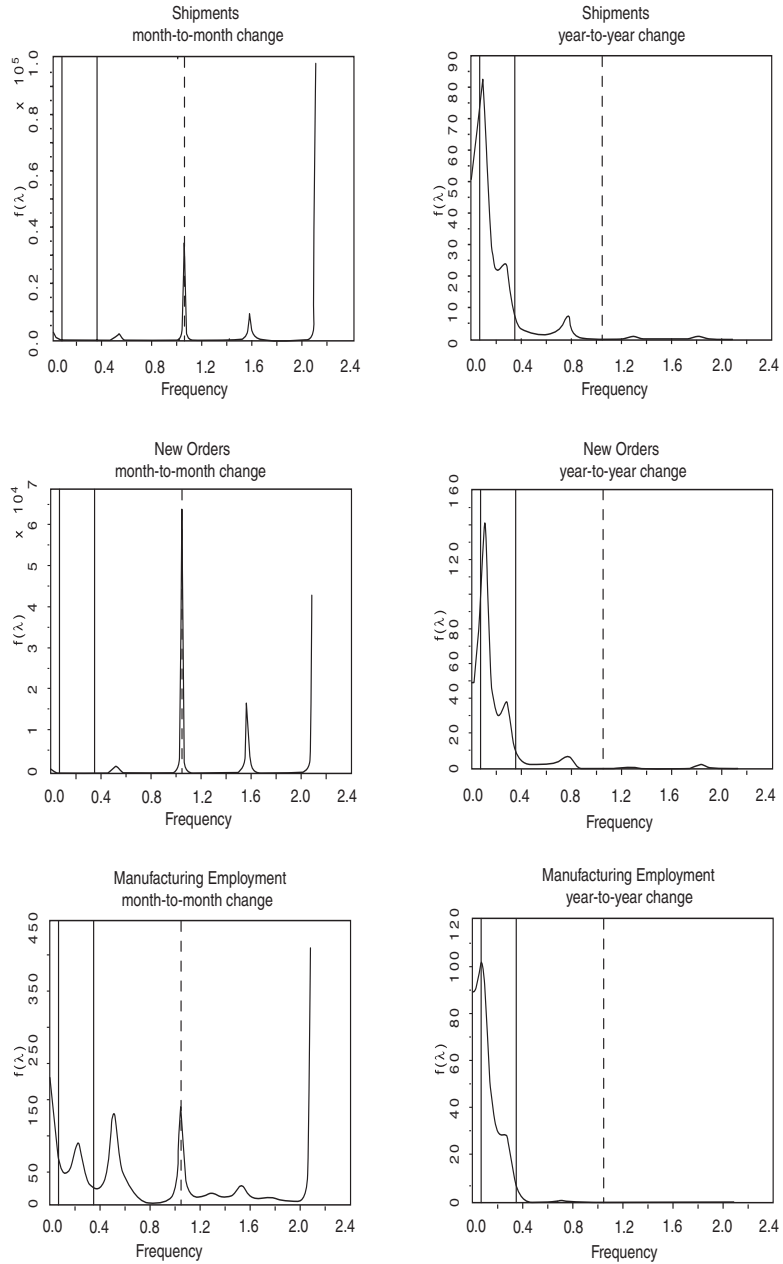
Figure 4 Power Spectra for Actual Manufacturing Data

Table 2 ISM Indexes
Percent of variance attributable to cycles with different periods: ISM indexes

	periods > 8 years	1.5 years < periods < 8 years	periods > 6 mo.
Composite Index	18.31	59.64	94.86
Shipments	11.40	57.12	87.78
New Orders	11.69	56.40	89.12
Employment	17.83	61.62	95.80

time series. Some nontrivial contribution to total variance does stem from longer-lived cycles (i.e., those with periods greater than eight years). At the other extreme, virtually no contribution to variance is attributable to cycles with periods less than six months. Observe also in Figure 4 that, outside of the business and seasonal cycles, the power spectra are close to zero.⁴

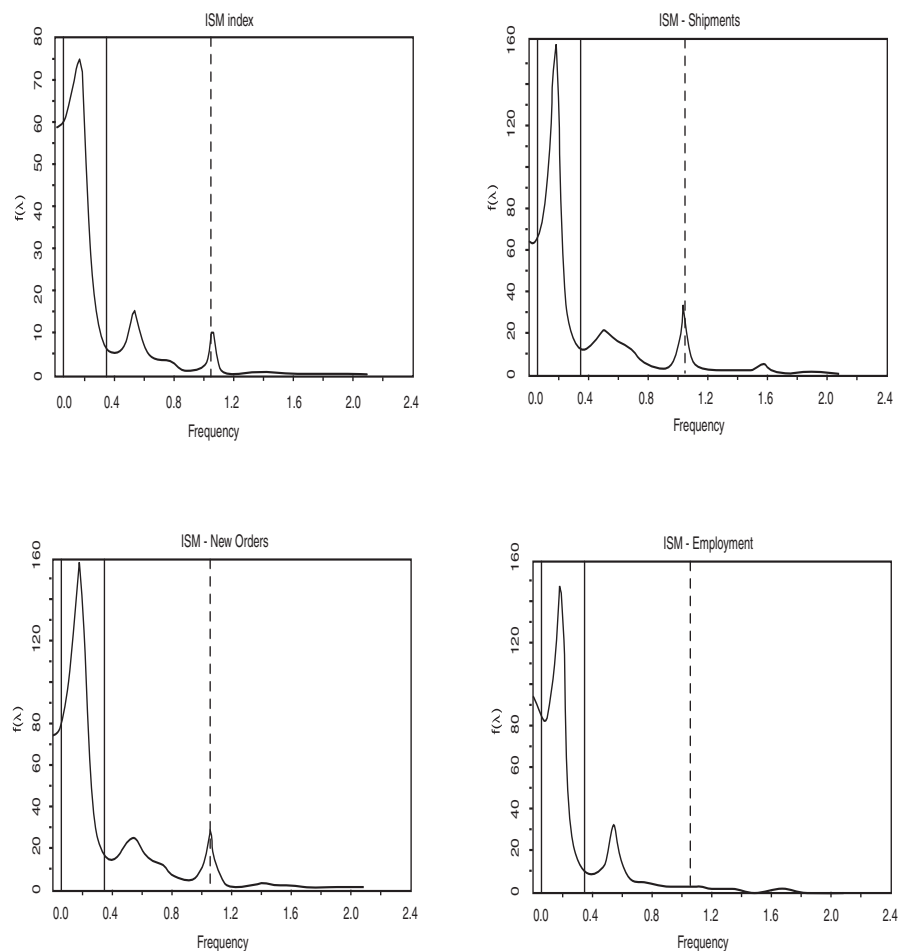
3. POWER SPECTRA FOR DIFFUSION INDEXES

Figure 5 displays estimated power spectra for the ISM diffusion indexes corresponding to the manufacturing series in Figure 4. Interestingly, even though the indexes not filtered in any way, all possess the typical spectral shape associated with differences in macroeconomic time series. In particular, a principle and notable peak in each case occurs well within the business cycle interval. The spectra for the diffusion indexes associated with shipments and new orders suggest an important six-month cycle, and all indexes further emphasize a yearly cycle with a peak occurring almost exactly at frequency $\lambda = \pi/6$. The estimated spectra associated with the ISM indexes suggest virtually no contribution from cycles with periods less than six months.

Thus, although many caveats are associated with survey-generated indexes, it appears that these indexes nonetheless capture systematic aspects of changes in economic time series that virtually mimic those of actual data. This observation is particularly important in that survey data can be much less costly, and always much faster, to produce than measuring changes in actual economic data. In the case of federal regional districts, for instance, state manufacturing data is not even collected; but corresponding diffusion indexes can be produced by the various Federal Reserve Banks in a relatively inexpensive and timely manner.

Finally, the power spectra shown in Figure 5 are indicative of two important aspects of changes in economic conditions. First, it is noteworthy that the untransformed survey data and the year-over-year changes in the national

⁴ Results in this case do not depend only on the natural properties of the data, but also on the specific form of the filter. For instance, a 12-month difference filter will by construction eliminate all variations in cycles shorter than one year.

Figure 5 Power Spectra for ISM Diffusion Indexes

aggregate display similar spectral shapes. Second, and related to this last observation, while surveys allow for much discretion in the way respondents answer questions, this discretion does not obscure the informational content of the responses in such a way as to simply produce statistical noise, or even emphasize high-frequency changes.

Table 2 gives a decomposition of variance for the different diffusion indexes in Figure 5 according to cycles of different frequencies.

As with actual manufacturing data in Table 1, the bulk of the overall variance in diffusion indexes is contained within the business cycle frequencies, albeit to a somewhat lesser extent. This reinforces the notion that diffusion

indexes capture specific aspects of changes in economic conditions. In this case in particular, and unlike the 12-month difference of actual manufacturing data, the power spectra suggest that some nontrivial portion of the overall variance in the indexes stem from shorter seasonal cycles, those associated with six-month and one-year periods. Shorter cycles, however, appear to play no role in respondents' answers.

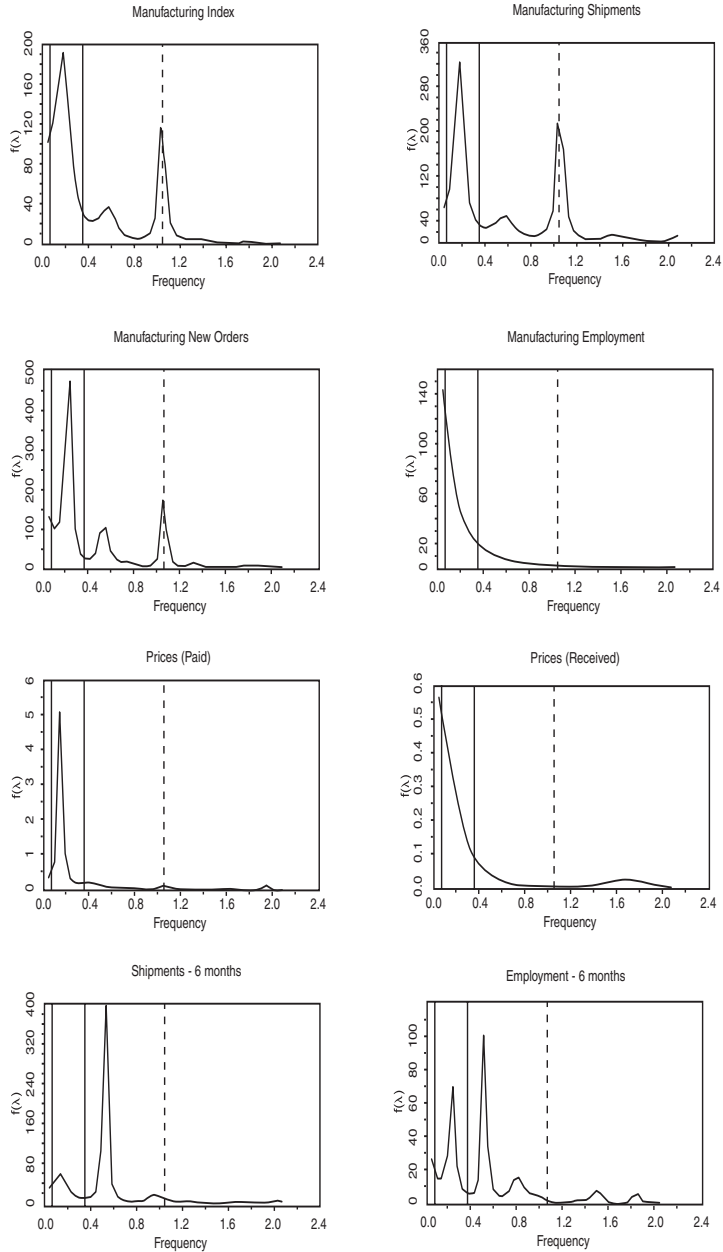
Power Spectra for FRBR Regional Diffusion Indexes

The FRBR's manufacturing survey produces diffusion indexes according to the formula described in the introduction for shipments, new orders, employment, and an overall index. Fifth District businesses are also surveyed regarding prices, as well as expected shipments and employment six months ahead.

Cyclical Properties of Manufacturing Indexes in the Fifth Federal Reserve District

Figure 6 shows estimated power spectra for the various raw (i.e., unfiltered) diffusion indexes produced by the FRBR in manufacturing. Perhaps most surprisingly, it is not the case that the power spectra are indicative of mostly short-lived cyclical noise, even at the relatively narrow regional level. On the contrary, the diffusion indexes display distinctive patterns. More specifically, it appears that the survey respondents do not strictly answer the questions they are asked—(relating simply to changes relative to the previous month)—but instead carry out some implicit deseasonalization. In particular, as with the ISM, the spectrum for the untransformed survey display distinct similarities with the year-over-year changes in the national aggregates. The overall manufacturing index, as well as shipments and new orders, display three distinctive peaks: one in the business cycle interval, a smaller one that captures approximately a 12-month seasonal cycle at $\lambda = 0.53$, as well as distinct evidence of a six-month cycle. Prices paid and received reported by survey respondents also emphasize business cycle frequencies, rather than shorter-lived cycles where the power spectrum is essentially zero. Therefore, it appears that despite the simplicity of the questions asked, which essentially restrict respondents to three states, the questions are asked of enough agents that the corresponding diffusion index captures time variations that move strongly either with business or seasonal cycles.

The figures for expected shipments and employment six months ahead are somewhat less informative. Indeed, the power spectra capture variations that are principally driven by a 12-month seasonal cycle, possibly suggesting that respondents are basing their answers mainly on what they expect during the course of a given year. Thus, key dates that occur on a yearly basis,

Figure 6 Power Spectra for FRBR Manufacturing Diffusion Indexes

such as Christmas or even, say, yearly shut-down periods driven by retooling considerations, seem to play a key role in shaping their expectations.

Table 3 FRBR Manufacturing Diffusion Indexes: (Unadjusted)
Percent of variance attributable to cycles with different periods

	periods>8 years	1.5 years<periods<8 years	periods>6 mo.
Overall Index	8.26	47.76	77.52
Shipments	3.48	38.27	65.68
New Orders	7.34	43.83	72.64
Employment	26.29	41.05	82.88
Prices (paid)	5.43	75.90	94.83
Prices (received)	20.45	52.89	83.06
Shipments-6M	2.36	14.32	52.92
Employment-6M	6.43	20.88	61.13

Table 3 gives the fraction of variance attributable to cycles of different periods for the various manufacturing regional indexes. On the whole, these indexes capture more movement stemming from short-lived cycles relative to the actual manufacturing data in Table 1. Cycles with periods greater than six months can leave up to 47 percent of the total series' variance unaccounted for (e.g., expected shipments six months ahead). However, except for expected future employment and shipments, the business cycle interval does contain a nontrivial fraction of the total variance for the various series, ranging from 38.27 to 75.90 percent. Prices paid, as simply reported in the monthly survey, appear to move most strongly with business cycle frequencies. As suggested above, expected employment and shipments six months ahead have the least to do with business cycles.

Because the unfiltered manufacturing diffusion indexes are driven to a non-negligible extent by relatively short-lived cycles that are presumably less relevant to policymaking decisions, we also consider a six-month difference of all the regional diffusion indexes. The idea is to eliminate variations in the indexes that are quickly reversed in order to acquire a sharper picture of the business cycle. In particular, it should be clear by now that spectral analysis represents a natural tool in searching for a filter that helps isolate changes associated with these specific frequencies.

Figure 7 displays power spectra associated with the six-month difference of the diffusion indexes produced by the FRBR. Except for expected shipments and employment six months ahead, all power spectra now have the typical spectral shape for differences, and their main peaks lie squarely in the business cycle interval. Evidence of a small seasonal cycle lasting one year is also clearly distinguishable. Furthermore, as indicated in Table 4, the business cycle interval now contains a very large fraction of the total variation

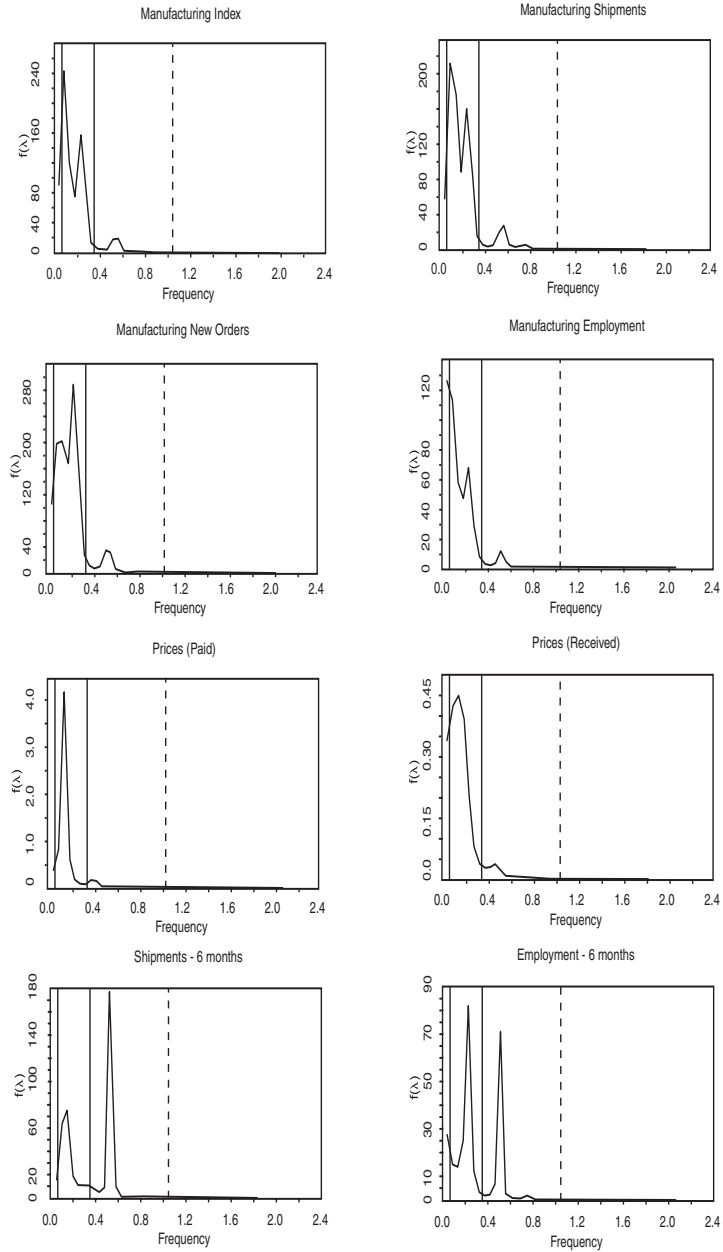
**Figure 7 Power Spectra for FRBR Manufacturing Diffusion Indexes
6-Month Difference**

Table 4 FRBR Manufacturing Diffusion Indexes: (6-Month Difference)
Percent of variance attributable to cycles with different periods

	periods>8 years	1.5 years<periods<8 years	periods>6 mo.
Overall Index	12.63	82.87	1.00
Shipments	7.62	84.67	1.00
New Orders	10.62	82.45	1.00
Employment	30.07	63.67	1.00
Prices (paid)	8.15	79.30	96.25
Prices (received)	20.37	72.59	99.43
Shipments-6M	4.60	49.18	79.53
Employment-6M	12.07	36.33	69.52

in the series. Interestingly, the six-month difference filter leaves the spectra associated with prices relatively unchanged.

4. FINAL REMARKS

Information on economic activity gathered from high-frequency surveys offers a timely gauge of conditions in the sector surveyed. The value of this timely information to monetary policymakers depends not only on whether the information accurately reflects conditions within the sector, but also on whether the information infers something about conditions that monetary policy can address, such as movements in the business cycle. That is, if survey results typically deviate from trend very often or very seldom, the information gained from the results may suggest changes in economic conditions at frequencies largely immune to monetary policy capabilities and may be of little value to policymakers, even if the results are an accurate reading of sector conditions. In contrast, if the deviations occur with a frequency similar to that of the business cycle, monetary policymakers can use the information to better shape policy.

In this article, we estimate power spectra for the results from two high-frequency surveys and show that deviations from trend generally occur at business-cycle-length frequencies in manufacturing indexes. The proportion of variation captured in business-cycle-length frequencies is strongest for a six-month moving average of the Richmond results.

APPENDIX A

Some important features of the power spectrum are as follows:

- $\gamma_0 = \int_{-\pi}^{\pi} f(\lambda)d\lambda$. In other words, the area under the population spectrum between $-\pi$ and π integrates to the overall variance of Y .
- Since $f(\lambda)$ is symmetric around 0, $\gamma_0 = 2 \int_0^{\pi} f(\lambda)d\lambda$. More generally, $2 \int_0^{\lambda_1} f(\lambda)d\lambda$ represents the portion of the variance in Y that can be attributed to periodic random components with frequencies less than or equal to λ_1 .
- Recall that if the frequency of a cycle is λ , the period of the corresponding cycle is $2\pi/\lambda$. Thus, a conventional frequency domain definition of business cycles, deriving from the duration of business cycles isolated by NBER researchers using the methods of Burns and Mitchell (1946), is that these are cycles with periods ranging between 18 and 96 months. Therefore, in the frequency domain, business cycles are characterized by frequencies $\lambda \in [\pi/48, \pi/9] \approx [0.065, 0.35]$.
- The power spectrum is not well defined for frequencies larger than π radians. The frequency $\lambda = \pi$ is known as the *Nyquist* frequency and corresponds to a period of $2\pi/\pi = 2$ time units. To see the relevance of this concept, note that with quarterly data, no meaningful information can be obtained regarding cycles shorter than two quarters since, by definition, the shortest observable changes in the data are measured from one quarter to the next. Hence, changes within the quarter are not observable. In contrast, with monthly data, one can refine the calculation of the power spectrum up to a two-month cycle.
- When Y is a white noise process, $Y_t \sim^{iid} N(0, \sigma^2)$, $f(\lambda)$ is simply constant and equal to $\sigma^2/2\pi$ on the interval $[-\pi, \pi]$. If survey-generated data were mainly noise, therefore, one might expect a relatively flat power spectrum with no specific frequencies being emphasized.

APPENDIX B

Estimation of the power spectrum:

Given data $\{Y_t\}_{t=1}^T$, the power spectrum can be estimated using one of two approaches: a non-parametric or a parametric approach. Evidently, the

simplest (non-parametric) way to estimate the power spectrum is by replacing (4) by its sample analog,

$$\widehat{f}(\lambda) = \frac{1}{2\pi} \left[\widehat{\gamma}_0 + 2 \sum_{j=1}^{T-1} \widehat{\gamma}_j \cos(\lambda j) \right], \quad (5)$$

where the “hat” notation denotes the sample analog of the population autocovariances. Since our hypothetical sample contains only T observations, autocovariances for j close to T will be estimated very imprecisely and, although unbiased asymptotically, $\widehat{f}(\lambda)$ will generally have large variance. One way to resolve this problem is simply to reduce the weight of the autocovariances in (5) as j approaches T . The Bartlett kernel, for example, assigns the following weights:

$$\omega_j = \begin{cases} 1 - \frac{j}{k+1} & \text{for } j = 1, 2, \dots, k \\ 0 & \text{for } j > k \end{cases},$$

where k denotes the size of the Bartlett bandwidth or window. When k is small, $\widehat{f}(\lambda)$ has relatively small variance since the autocovariances that are estimated imprecisely (i.e., those for which j is close to T) are assigned small or zero weight. However, given that the true power spectrum is based on all the autocovariances of Y , $\widehat{f}(\lambda)$ also becomes asymptotically biased. The reverse is true when k is large; the periodogram becomes asymptotically unbiased but acquires large variance. How does one then choose k in practice? Hamilton (1994) suggests that one “practical guide is to plot an estimate of the spectrum using several different bandwidths and rely on subjective judgment to choose the bandwidth that produces the most plausible estimate.”

Another popular way to go about estimating the spectrum of a series is to adopt a parametric approach. Specifically, one can show that for any AR(P) process, $Y_t = \mu + \phi_1 Y_{t-1} + \dots + \phi_p Y_{t-p} + \varepsilon_t$ such that $\text{var}(\varepsilon_t) = \sigma^2$, the power spectrum (4) reduces to

$$f(\lambda) = \frac{\sigma^2}{2\pi} \cdot \left\{ \left| 1 - \sum_{j=1}^p \phi_j e^{-i\lambda j} \right| \right\}^{-1}, \quad \text{where } i^2 = -1. \quad (6)$$

Therefore, since any linear process has an AR representation, one can estimate an AR(P) by OLS and substitute the coefficient estimates, $\widehat{\phi}_1, \dots, \widehat{\phi}_p$, for the parameters ϕ_1, \dots, ϕ_p in (6). Put another way, one can fit an AR(P) model to the data, and the estimator of the power spectrum is then taken as the *theoretical* spectrum of the *fitted* process. Note that the spectrum estimated in this way will converge to the true spectrum (as the sample size becomes large) under standard assumptions that guarantee that the coefficient estimates, $\widehat{\phi}_1, \dots, \widehat{\phi}_p$, converge to the true parameters, ϕ_1, \dots, ϕ_p . Of course, the difficulty lies in deciding on the order of the AR process. When P is small, the estimated spectrum may be badly biased but a large P increases its variance. The

trade-off, therefore, is similar to that encountered in using the non-parametric approach described above. Harvey (1993) suggests that one solution that works well in practice is to actively determine the order of the model on a goodness-of-fit criterion, such as maximizing the adjusted R^2 statistic or minimizing Akaike's information criterion.

For the purpose of this article, power spectra will be estimated using the parametric method we have just described. Since we shall be analyzing time series with monthly data, we fit an $AR(P)$ to each series with P being at most 24. The actual value of P is then chosen by maximizing the adjusted R^2 in each series' estimation.

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