# Limited Participation and the Neutrality  of Money

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Money is useful in overcoming two types of frictions. First, in barter exchange, money helps to mitigate double-coincidence frictions that arise in developed economies where economic agents are specialized in production and consumption. Second, in trades involving credit, information frictions may imply that one economic agent has difficulty getting another economic agent to accept his or her IOUs in exchange for goods and services. In an economy with monetary exchange, much more can be achieved than in an economy without money. Even so, one of the key lessons of monetary economics is that circumstances exist in which changing the quantity of money will not matter at all for what can be produced and consumed in an economy. For example, governments sometimes engage in currency reforms, particularly in circumstances where there has been a recent history of high inflation.

One such instance occurred in January 2005, when Turkey introduced a new Turkish lira, equivalent in all respects to the old Turkish lira, except that one new lira trades for one million old lira. That is, the central bank of Turkey declared itself willing to exchange old lira for new lira at a rate of one million to one. What would be the effects of this? To help frame the problem, suppose that the U.S. government were to announce a currency reform, where a new U.S. dollar was introduced, defined as being equivalent to 10 old U.S. dollars. Suppose also that the Federal Reserve did not otherwise change its behavior. The result would be that the money stock and all prices and wages in terms of the new U.S. dollar would be one-tenth of what they would have been under the old U.S. dollar, and all real economic activity would be unchanged.

Money would clearly be neutral under these circumstances. That is, changing the quantity of money by simply redefining the units in which we measure the money stock can have no implications for real economic activity.

However, when changing the stock of money through an open market operation, the Federal Reserve System is hardly carrying out a currency reform. For example, when the Fed conducts an open market operation, the economic agents on the receiving end of this transaction typically are large financial institutions that are not directly connected to all other economic agents in the economy through exchange. Initially, an open market operation can affect only the financially interconnected sector of the economy—mainly banks and other financial intermediaries and the economic agents who transact frequently with these institutions. In contrast to what happens in a currency reform, a typical open market operation will, in the short run, have different effects in the financially interconnected sector of the economy from what happens in the decentralized sector of the economy. This difference will be important for short-run movements in interest rates, aggregate output, and the distribution of wealth across the population.

The idea that monetary policy matters in the short run because of financial disconnectedness in the economy is captured in *limited participation models.* The first models of this type were constructed by Grossman and Weiss (1983) and Rotemberg (1984). These early contributions were heterogeneous-agent models that proved to be very difficult to work with due to complications in tracking the distribution of wealth across the population over time. Lucas (1990) finessed these complications by working with a representative-household construct, as did Fuerst (1992). Later important contributions were made by Alvarez and Atkeson (1997) and Alvarez, Atkeson, and Kehoe (2002). Much of this research, which focuses mainly on the asset pricing implications of limited participation, is weak on the microfoundations of money and, as a result, may be misleading. More recently, Williamson (forthcoming) and Shi (2004) constructed monetary search models that treat monetary exchange seriously and permit the study of the role of limited participation in generating short-run nonneutralities of money.

This article reviews the literature on limited participation and points out new directions for research by constructing and building on a baseline limited participation model. The baseline model is a cash-in-advance model similar to the one studied in Lucas (1990).[1] I first show how limited participation provides an explanation for the liquidity effect—the short-run negative response of the nominal interest rate to an open market purchase. The baseline model does not provide a rationale for monetary policy though, as in this setup monetary policy has no implications for real economic variables and economic

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[1] I do not provide a rigorous microfoundation for monetary exchange in this model, but it would be straightforward to do this, for example, along the lines of Williamson (2004a).

welfare. As I discuss in Section 2, an extension of the baseline model along the lines of Fuerst produces a short-run nonneutrality of money in that an open market purchase of interest-bearing government securities by the central bank reduces the nominal interest rate, increases the real wage, and increases real output. In that environment, however, it would be optimal for the monetary authority not to intervene in the economy.

   The conclusion from examining the implications of the first two versions of the model is that, while these variations provide an explanation for the liquidity effect, they do not teach us much about the real effects of monetary policy or how to conduct policy.

   In Section 3, in the third incarnation of the model, I extend the framework in a new direction by permitting a persistent distributional effect of monetary policy. Here, money is nonneutral whether monetary intervention is anticipated or not (a feature not shared with the previous two incarnations of this model). In this version of the model, characterizing the effects of monetary policy is difficult without getting outside the scope of this article, but determining the model's implications for optimal monetary policy is relatively straightforward. Here, optimal monetary policy is in one sense a Friedman rule (the nominal interest rate is always zero at the optimum), but in another sense is much more complicated than a typical Friedman rule. This complication arises because the goal of the monetary authority is to control monetary conditions in the financially disconnected sector of the economy, but monetary control can be achieved only indirectly—through intervention in the financially connected sector of the economy. Finally, Section 4 serves as a conclusion.

## 1.   LIMITED PARTICIPATION AND THE LIQUIDITY EFFECT

This section contains the baseline model—closely related to the model in Lucas (1990)—used throughout this article. Lucas considered a simple asset-pricing model without production, while my model allows for production and endogenous labor supply. As well, there are some minor differences from Lucas's work in how I specify asset markets.

### The Representative Household

In the model, a representative infinitely lived household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - v(n_t)],$$

where $E_0$ is the expectation operator conditional on information in period 0; $\beta$ is the household's discount factor with $0 < \beta < 1$; $c_t$ is the household's consumption; and $n_t$ is household labor supply. Assume that $u(\cdot)$ is strictly

increasing, strictly concave, and twice differentiable with $u'(0) = \infty$, and that $v(\cdot)$ is strictly increasing, strictly convex, and twice differentiable with $v'(0) = 0$ and $v'(h) = \infty$ where $h$ is the household's endowment of time each period. The household has a technology that permits it to produce one unit of the perishable consumption good in period $t$ for each unit of labor supplied.

One of the innovations in Lucas (1990) was to model a household as having many agents, with household members engaged in different activities during each period. This device is used here, and its purpose is to make the model analytically tractable in that monetary policy in this model will only cause changes in the distribution of wealth within the household and within a period, not persistent changes in the distribution of wealth across economic agents. Thus, in the baseline version of the model, the household consists of three agents: a worker, a shopper, and a financial transactor.

As is typical in models with cash-in-advance constraints, the timing of transactions within a period is critical to how the model works. At the beginning of the period, the household has $M_t$ units of money on hand and must then decide how to split these money balances between the shopper, who will go to the goods market to purchase consumption goods from other households, and the financial transactor, who will go to the asset market to purchase assets. Let $X_t$ denote the quantity of money that the household sends to the goods market with the shopper, where $X_t \leq M_t$. For the shopper, the value of goods purchased cannot exceed $X_t$; that is, the shopper faces the cash-in-advance constraint

$$P_t c_t \leq X_t, \tag{1}$$

where $P_t$ is the price level, the price of consumption goods in terms of money. Now, in the asset market I will assume that there is only one asset bought and sold, which is a nominal bond issued by the government. One nominal government bond issued in period $t$ sells at a price $q_t$ in terms of money and is a promise to pay one unit of money at the end of the period. Bonds must be purchased with money, so the financial transactor, like the shopper, faces a cash-in-advance constraint, which in this case is

$$q_t B_t \leq M_t - X_t. \tag{2}$$

The worker stays at the household's location where he produces and sells goods to other households. As is usual in cash-in-advance models, I assume that the household cannot consume its own output and that money acquired by the household from the sale of its output cannot be used within the period to purchase consumption goods or government bonds. The household then faces the budget constraint

$$P_t c_t + M_{t+1} + q_t B_t \leq M_t + B_t + P_t z_t n_t - \Upsilon_t, \tag{3}$$

where $M_{t+1}$ is the quantity of money that the household carries into period $t+1$ and $\Upsilon_t$ is a lump-sum tax that the household pays in money to the

government at the end of the period. The left-hand side of the budget constraint (3) consists of the value of consumption goods purchased by the shopper, plus money balances at the end of the period, plus the value of bonds purchased by the financial transactor. On the right-hand side is the quantity of money possessed by the household at the beginning of the period, plus the total payoff on government bonds held by the household, plus the proceeds from sales of goods by the worker, minus the lump-sum tax paid to the government. It may seem unusual to have government bond purchases $B_t$ appear on the right-hand and left-hand sides of the budget constraint (3). However, these are within-period bonds for which the household gives up $q_t B_t$ units of money on the asset market and receives $B_t$ units of money as a payoff at the end of the period.

**The Government**

Each period, the government must choose the quantity of nominal bonds to issue, which I denote by $\overline{B}_t$ (I will use *overbar* throughout to denote the supplies of assets determined by the government). I will assume that

$$\overline{B}_t = \theta_{t+1}\overline{M}_t, \tag{4}$$

where $\overline{M}_t$ is the quantity of money outstanding at the beginning of period $t$ and $\theta_{t+1}$ is a random variable that is not realized until after the shopper and financial transactor have left the household in period $t$. At this point, perceptive readers might quarrel with the assumption that the government behaves randomly. This assumption proves useful in making my argument, and I will comment later on what happens if $\theta_{t+1}$ is a choice variable for the central bank.

To obtain a clean policy experiment in the model, I will assume that the government sets the lump-sum tax $\Upsilon_t$ so that the money stock at the end of the period is identical to the beginning-of-period money stock. That is,

$$\Upsilon_t = (1 - q_t)\overline{B}_t. \tag{5}$$

Given equation (5), the money stock will remain fixed for all time, and $\theta_t$ will not affect the money growth rate. The following is one interpretation of how policy is conducted in this model, consistent with the notion that it is desirable here to set up the policy experiment so that it captures monetary policy and is not some mix of fiscal and monetary policies. Each period, the fiscal authority issues $\hat{\theta}\overline{M}_t$ nominal bonds, where $\hat{\theta}$ denotes the maximum possible realization of $\theta_{t+1}$. Then, the central bank determines (randomly) how much of the bond issue to acquire from the fiscal authority, purchasing $\left(\hat{\theta} - \theta_{t+1}\right)\overline{M}_t$ bonds, thus leaving $\theta_{t+1}\overline{M}_t$ bonds to be purchased by the public at price $q_t$. Then, at the end of the period, the fiscal authority has $q_t\theta_{t+1}\overline{M}_t$ units of money acquired from bond sales and must pay the bondholders $\theta_{t+1}\overline{M}_t$ units of money, as promised. It then makes up the difference $(1-q_t)\theta_{t+1}\overline{M}_t$ through a lump-sum tax on the representative household, so that the tax is given by (5). Transactions

between the fiscal authority and the central bank merely yield accounting entries, and the central bank's account balance with the fiscal authority is reset to zero at the end of each period.

Assume that information is not transmitted during the period between the asset market and the goods market, so workers and shoppers do not learn $\theta_{t+1}$ until the end of the period after all decisions have been made.

### Optimization and Equilibrium

To specify the household's optimization problem in a tractable way, it proves useful to divide the left-hand and right-hand sides of equations (1) through (3) by $\overline{M}_t$ and let lower-case variables denote the corresponding upper-case variable scaled by the money supply, $p_t \equiv \frac{P_t}{\overline{M}_t}$, for example. For convenience, assume for now that $\theta_t$ is an i.i.d. random variable. Further, drop $t$ subscripts and let primes denote variables dated $t+1$. Then, I can specify the representative household's optimization problem as a dynamic program, where $V(m, \theta)$ is the household's value function. The household solves

$$V(m, \theta) = \max_{x,c,n} \left[ u(c) - v(n) + \beta E_\theta \max_{b,m'} E_{\theta'} V(m', \theta') \right] \qquad (6)$$

subject to

$$pc \leq x, \qquad (7)$$

$$qb \leq m - x, \text{ and} \qquad (8)$$

$$pc + m' + qb \leq m + b + pn - \tau. \qquad (9)$$

In the objective function (6), $E_\theta$ is the expectation operator conditional on information before $\theta'$ is known, while $E_{\theta'}$ conditions on $\theta'$.

To solve the household's problem, first note that the optimal choice of $b$ in the inner maximization problem in (6) gives

$$b = \frac{m - x}{q}, \text{ if } q \leq 1, \text{ and} \qquad (10)$$

$$b = 0, \text{ if } q > 1.$$

Given that the government always issues a strictly positive quantity of nominal bonds, we must have $q \leq 1$ in equilibrium, so confining attention to this case and substituting for $b$ in (9) using (10) gives

$$pc + m' \leq m + \frac{(m - x)(1 - q)}{q} + pn - \tau. \qquad (11)$$

I can then specify the household's problem as solving (6) subject to (7) and (11). Let $\lambda_1$ and $\lambda_2$ denote the multipliers associated with constraints (7) and (11), respectively. From the choice of $m'$ in the inner maximization in (6), I

obtain the first order condition (assuming the value function is strictly concave and differentiable, and using the relevant envelope condition),

$$-\lambda_2 + \beta E_{\theta'}\left(\frac{\lambda_2'}{q'}\right) = 0, \tag{12}$$

and the choices of $x$, $c$, and $n$ in the outer maximization problem in (6) give the following first order conditions, respectively:

$$\lambda_1 - E_\theta\left[\lambda_2\left(\frac{1}{q} - 1\right)\right] = 0, \tag{13}$$

$$u'(c) - p(\lambda_1 + E_\theta\lambda_2) = 0, \text{ and} \tag{14}$$

$$-v'(n) + pE_\theta\lambda_2 = 0. \tag{15}$$

In equilibrium, the bond market clears, or

$$b = \theta'; \tag{16}$$

the representative household willingly holds the existing stock of money, or

$$m = 1; \tag{17}$$

and the market for consumption goods clears, or

$$c = n. \tag{18}$$

Given the assumption that $\theta$ is an i.i.d. random variable, I can solve for an equilibrium in which $x$, $c$, $n$, and $p$ are constant. First assume that the cash-in-advance constraint (7) binds. Then, (12) through (18) give

$$x = 1 - \beta E(\theta), \tag{19}$$

$$v'(c) - \beta u'(c) = 0, \tag{20}$$

$$n = c, \tag{21}$$

$$p = \frac{1 - \beta E(\theta)}{c}, \text{ and} \tag{22}$$

$$q = \frac{\beta E(\theta)}{\theta}. \tag{23}$$

In (19) through (23), $E(\theta)$ is the expected value of $\theta$. Note, from (19) that the fraction of money balances allocated to the shopper for the purchase of consumption goods is decreasing in the expected size of the government's open market operation. Monetary policy has no effect on any variables of consequence, as equation (20) determines consumption (equal to labor supply) and, thus, $c$ is independent of $\theta$ and the distribution of $\theta$. The only effect of $\theta$ is on the price of the nominal bond $q$ in equation (23). Clearly $q$ is decreasing

in $\theta$, so that the nominal interest rate increases as $\theta$ increases. This is the liquidity effect—if the government withdraws more outside money through an open market sale, the nominal interest rate will be higher.

An interesting feature of the setup here is that I have designed the policy experiment to imply no Fisher effect—the positive effect of money growth and inflation on the nominal interest rate. Because monetary policy leaves the money supply constant over time, the only effect on the nominal interest rate is the liquidity effect. Note that the presence of $E(\theta)$ in equations (19), (22), and (23) has nothing to do with the Fisher effect. Instead, if $E(\theta)$ is high, then it is expected that a higher quantity of bonds will be sold to private agents, and the household therefore also predicts that the expected payoff from holding government bonds will be higher. Thus, the household will tend to allocate more cash to the asset market as opposed to the goods market ($x$ declines in equation (19)). Both the price level and the price of the nominal bond are in turn determined in part by $x$, as (22) and (23) indicate.

It is straightforward to show that, given the solutions (19) through (23), we will have $\lambda_1 > 0$, so that the cash-in-advance constraint binds. As well, my solution requires that $q \leq 1$ in equilibrium, so from (23) we require that $\theta \geq \beta E(\theta)$ for all realizations of the random variable $\theta$.

The implication of this model is that, while monetary policy can produce variability in the nominal interest rate, policy is irrelevant for economic welfare as it does not affect consumption and employment. The fact that the asset market and goods market are segmented implies that nominal interest rate movements will have no real effects. Note as well that the price level is in some sense "sticky," as in equation (22) $p$ does not depend on $\theta$—monetary policy can change the supply of liquidity in the asset market but has no effect on the quantity of money in the goods market. However, $p$ depends on $E(\theta)$, so anticipated monetary policy matters for the determination of the price level, though not for any real variables of consequence.

If the model as it stands were a good description of reality, we would conclude that central bank intervention in asset markets was a useless exercise. The central bank might just as well do nothing rather than cause the nominal interest rate to fluctuate, but doing so certainly does not cause any harm. The liquidity effect on nominal interest rates is an important element in the religion of central bankers [2], and Lucas (1990) provides an explanation for the liquidity effect. However, the model does nothing to show why monetary policy matters

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[2] In my opinion, belief in the liquidity effect by central bankers is much like religious belief: It seems impossible to disprove its existence because of the endogeneity of the money stock brought about by endogenous policy and the endogenous response of the banking system to exogenous shocks to the economy. If monetary policy were exogenous, then it would be straightforward to measure the effect of a money shock on nominal interest rates. The problem is that the right natural experiment does not appear to have occurred in practice.

or how it should be conducted. To address these matters, I need to modify the baseline model in ways that make money nonneutral.

## 2. LIMITED PARTICIPATION WITH REAL EFFECTS ON OUTPUT

In this section, I alter the baseline model constructed in the previous section to produce nonneutralities of short-run monetary policy. The basic idea comes from Fuerst (1992).

In the baseline model, I assumed that a household must buy consumption goods from other households with cash acquired in advance of the current period. The extension here is to assume that the household can produce only with labor supplied by other households and that labor, like consumption goods, is subject to a cash-in-advance constraint. To be more explicit, the timing works as follows: At the beginning of the period, the representative household has $M_t$ units of money and sends $X_t$ units of money with the shopper to the goods market. The worker also leaves the household at the same time as the shopper to sell labor to other households. The financial transactor is left behind to trade on the asset market and to purchase $y_t$ units of labor at the real wage rate of $w_t$ from other households, using the remaining quantity of money balances, $M_t - X_t$. Using the same notation as in the previous section, replace the constraint (8) with

$$qb + pwy \leq m - x, \tag{24}$$

and rewrite the household's budget constraint as

$$pc + m' + qb + pwy \leq m + b + pwn + py - \tau. \tag{25}$$

The representative household's dynamic programming problem is now

$$V(m, \theta) = \max_x E_\theta \left\{ \max_{c,n,y,b,m'} \left[ u(c) - v(n) + \beta V(m', \theta') \right] \right\}, \tag{26}$$

subject to (7), (24), and (25). Note that on the right-hand side of the Bellman equation (26) the household must choose $x$, the fraction of cash sent with the shopper, before the central bank intervenes in the asset market, but all other household choices are made with knowledge of the central bank's action $\theta'$, albeit with $x$ already locked in place. That is, $\theta'$ is revealed to the shopper through the price $p$ and to the worker through the real wage rate $w$.

Now, given that $\theta$ is an i.i.d. random variable and that the central bank sets the lump sum tax at the end of the period according to (5) so as to keep the aggregate money stock constant from period to period, it is straightforward to characterize the effects of monetary policy on prices, the nominal interest rate, and output. Let $\lambda_1$, $\lambda_2$, and $\lambda_3$ denote the multipliers associated with the constraints (7), (24), and (25), respectively. An equilibrium is the solution to:

$$E_\theta(\lambda_1) = E_\theta(\lambda_2), \tag{27}$$

$$u'(c) - p(\lambda_1 + \lambda_3) = 0, \tag{28}$$

$$-v'(n) + pw\lambda_3 = 0, \tag{29}$$

$$-w(\lambda_2 + \lambda_3) + \lambda_3 = 0, \tag{30}$$

$$-q\lambda_2 + (1 - q)\lambda_3 = 0, \text{ and} \tag{31}$$

$$-\lambda_3 + \beta E_{\theta'}(\lambda_2' + \lambda_3') = 0, \tag{32}$$

in addition to (7), (24), and (25). This solution is derived from the first order conditions characterizing a solution to the constrained optimization problem on the right-hand side of (26), the relevant envelope conditions, arbitrage conditions, (16) through (18), and the equilibrium condition $y = n$.

From (30) and (31), an interesting feature of the equilibrium is that $w = q$, so that the real wage and the price of the nominal bond are identical, and this equality arises for the following reason. On the one hand, if the financial transactor purchases labor, he gives up money mid-period and receives money in exchange for output at the end of the period. On the other hand, the financial transactor could give up money to purchase a bond with the return on the bond received at the end of the period. In equilibrium, the financial transactor must be indifferent between purchasing labor and acquiring a bond, which requires that $w = q$. Then, if the second cash-in-advance constraint (30) binds so that $\lambda_2 > 0$, we will have $q = w < 1$, or the nominal interest rate is positive and the real wage is less than labor's marginal product. That is, cash received for output cannot be spent until the next period, so no missed profit opportunity necessarily results if the market wage is less than labor's marginal product ($w < 1$).

Now, for convenience, consider an equilibrium where both of the cash-in-advance constraints (7) and (24) bind for all $\theta$. First, given that $\theta$ is an i.i.d. random variable, $x$ will be independent of $\theta$ in equilibrium. Then, given $x$, from (7), (24), (25), and (27) through (32), $w$, $q$, and $c$ are the solutions to

$$w = q = \frac{1 - x}{x + \theta} \text{ and} \tag{33}$$

$$cv'(c) = \left(\frac{1 - x}{x + \theta}\right)\psi, \tag{34}$$

where $\psi$ is a constant, and, given $x$ and $c$, the price level is determined by

$$p = \frac{x}{c}. \tag{35}$$

From (33), the real wage and the price of the nominal bond are decreasing in $\theta$. Therefore, a larger open market sale implies a higher nominal interest rate and a lower real wage because of the tightening of the second cash-in-advance

constraint (24). Thus, as in the baseline model in the previous section, a liquidity effect exists, but here this effect extends to a change in the wage rate. In addition, a real effect of monetary policy now exists. A smaller open market purchase (smaller $\theta$) relaxes the second cash-in-advance constraint (24) from equation (34), implying that the demand for labor rises, and in equilibrium, the increased demand leads not only to an increase in the wage rate but also to an increase in employment, output, and consumption. That is, in equation (34), the left-hand side is increasing in $c$, and the right-hand side is decreasing in $\theta$, so that $c$ is decreasing in $\theta$.

The nonneutrality of money here works in an essentially identical manner to the mechanism in Fuerst (1992), which was later adapted in work by Christiano and Eichenbaum (1995). In these models an additional embellishment makes the model seem more plausible. Rather than having the representative household purchase labor directly subject to a cash-in-advance constraint, as is the case here, Fuerst, for example, supposes that a financial intermediary takes cash deposits from the household and makes cash loans to firms and that the firm then pays workers in cash. This construct amounts to the same thing as specified here—labor is purchased subject to a cash-in-advance constraint, highlighting a key defect in this attempt to understand the short-run role for monetary policy. In the United States, few workers are paid in cash, and even if they are, it seems difficult to argue that firms subject to cash-in-advance constraints account for a significant fraction of U.S. employment. Most firms have sufficient access to banking services and financial markets so that they will not face serious cash constraints in paying their workers. To see why this fact is important in the model, suppose that the representative household can issue IOUs in order to pay workers and purchase government bonds on the asset market, with the IOUs being repaid at the end of the period (equivalent in the Fuerst [1992] model to allowing the "bank" to issue within-period IOUs). Then the nominal interest rate is zero in equilibrium, the liquidity effect goes away, and output is constant for all $\theta$ in my model.

Another problem with this extension of the baseline model is that it does not provide a rationale for short-run central bank intervention. In spite of the fact that the central bank can cause the nominal interest rate, employment, output, and consumption to fluctuate, these fluctuations are inefficient. Randomness in $\theta$ implies randomness in consumption, only making the risk-averse representative household worse off. One way to resurrect a role for monetary policy in this model might be to add a shock to productivity that is not learned until after the representative household has chosen $x$. In this case, conducting open market operations to vary the quantity of liquidity in the asset (labor) market in response to the technology shock would be efficient for the central bank. The conjecture is that an optimal policy would involve "leaning against the wind" by injecting more liquidity when the technology shock is high, which would relax the household's second cash-in-advance constraint

when the demand for labor is high. This rationale for monetary policy relies on the central bank being capable of acting faster than private agents to increase liquidity in the asset market. As well, this rationale relies on cash-in-advance producers, which is problematic, as discussed above.

Perhaps a more plausible approach to the nonneutrality of money and liquidity effects in this vein is taken in Williamson (2004a), in a model where cash-in-advance constraints are derived endogenously from first principles, and these cash-in-advance constraints apply to purchases of retail and whole-sale goods. Credit is permitted so that the results do not depend on all purchases being made with outside money. A key result in Williamson (2004a) is that permitting private intermediaries to issue close substitutes for government-provided outside money alters the nature of cash-in-advance constraints, takes away the liquidity effect, and substantially changes optimal monetary policy rules.

## 3.    LIMITED PARTICIPATION AND THE DISTRIBUTIONAL EFFECTS OF MONETARY POLICY

Much of the literature on limited participation and monetary policy has focused on liquidity effects in asset markets (e.g., Lucas [1990], Alvarez and Atkeson [1997], Alvarez, Atkeson and Kehoe [2002]) while neglecting the implications of limited participation for the distributional effects of monetary policy on output, consumption, and wealth. The heterogenous agent models studied by Grossman and Weiss (1983) and Rotemberg (1984) captured some of these distributional effects but not in a tractable way. Some economic agents receive the first-round impacts of monetary policy actions while others do not, making a difference for the distribution of wealth and for production and consumption across economic agents. Indeed, these distributional effects may be very important for how monetary policy works, if not *the* reason we should care about monetary policy.

In focusing on asset pricing implications, those working in the limited participation literature have also paid scant attention to normative issues. As I have shown in the previous two sections, my baseline model (which captures the key results in the literature) does not have much to say about how to conduct monetary policy. Focusing almost exclusively on optimal monetary policy, as I do here, will be helpful in showing how interesting policy conclusions arise when we are serious about modeling the distributional effects of monetary policy. The model here can also be used to explore the dynamic effects of monetary injections on output, prices, consumption, and interest rates, but that would turn this into a much longer article than the editor would allow.

In this section, I will modify the baseline model to incorporate distributional effects of monetary policy. To cleanly focus on these effects, I will leave out discussion of asset pricing implications. Most of the ideas in this

section come from Williamson (forthcoming), but the model studied there is a monetary search model that builds on Lagos and Wright (forthcoming). The Lagos-Wright model is an approach to handling the distribution of wealth in search models with monetary exchange through the use of quasi-linear preferences rather than a representative household. I can capture the same ideas here as in Williamson (forthcoming) by extending my baseline cash-in-advance model. Though typical cash-in-advance models lack the microfoundations that make monetary search models such as Lagos and Wright (forthcoming) and Williamson (forthcoming) attractive, it is possible to generate cash-in-advance constraints while remaining true to monetary fundamentals, as, for example, in Williamson (2004a).

Suppose now that the representative household consists of two shoppers, two workers, and a continuum of financial transactors with mass 2, and that two locations, denoted location 1 and location 2, exist. One shopper and one worker live at each location, and a unit mass of financial transactors is always at each location. At the beginning of the period, a unit mass of financial transactors arrives at each location to deliver beginning-of-period money balances. In location 1, the shopper buys goods from other households on credit. That is, the shopper exchanges IOUs for goods, and the IOUs are redeemed by the household at the end of the period. At location 2, shoppers buy goods with money. The worker at each location sells goods in exchange for IOUs at location 1 and for money at location 2. At the end of the period, after IOUs clear, the financial transactors take possession of the household's money balances. Financial transactors are then randomly allocated (by nature) to each location. A financial transactor who is at a given location at the end of the period will be at the other location with probability $\pi$ and in the same location with probability $1 - \pi$, where $0 < \pi < 1$. Given the random relocation of financial transactors, it is optimal for the household to allocate its money balances equally among financial transactors within a location.

The reason the financial transactors play the role they do in this version of the model is to have a convenient device for allowing money to diffuse through the economy. I could have accomplished a similar goal by having economic agents randomly allocated to the two locations to buy or sell goods. The key feature of the model I need to achieve my results is some friction associated with moving money and goods across locations. The exact form this friction takes is not so important, and for my purposes it is convenient that producers and consumers cannot move and that the household can move money across locations, though in a random fashion.

For convenience I have included only one asset—money—in this version of the model, so I cannot model central bank intervention as open market operations. Here, the government injects money into the economy through lump-sum transfers, which, for my purposes, is harmless in that the policy implications should not be qualitatively different from what I would get with

a pure monetary policy experiment. The household receives the transfer at location 1 before financial transactors are randomly relocated. It is a key feature of the model that only some agents (those at location 1) receive the money transfer. Note also that the transfer is received by agents who have access to the more sophisticated transactions technology that involves within-period credit, capturing the fact that central bank intervention occurs in markets where financial transactions are relatively more complex than in other sectors of the economy.

In this version of the model, it will be interesting to explore optimal monetary policy in the context of aggregate shocks to the economy, so I will add an aggregate technology shock. Assume that one unit of labor produces $\phi_t$ units of the consumption good in period $t$, where $\phi_t$ follows a first order Markov process. I did not consider technology shocks in the previous versions of the model because the implications of doing so would be no different from those obtained in standard cash-in-advance models. Studying the behavior of the economy under technology shocks will yield important new results in this instance.

Since consumption goods cannot be moved between locations, consumption will in general differ between the two locations. It will be useful to suppose that the shoppers in the household do the consuming, with $c_{it}$ denoting consumption by the shopper at location $i$. Similarly, $n_{it}$ denotes labor supply by the worker at location $i$. Then, the household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_{1t}) + u(c_{2t}) - v(n_{1t}) - v(n_{2t}) \right]. \qquad (36)$$

The household must abide by its budget constraint at location 1,

$$P_{1t} c_{1t} + M_{1,t+1} \leq P_{1t} \phi_t n_{1t} + (1 - \pi) M_{1t} + \pi M_{2t} + \Upsilon_t, \qquad (37)$$

where $P_{it}$ denotes the price of goods in terms of money at location $i$, $M_{it}$ is the quantity of money held by the household at location $i$ at the end of period $t - 1$, and $\Upsilon_t$ is the lump-sum money transfer from the government. As well, the household must satisfy the cash-in-advance constraint at location 2,

$$P_{2t} c_{2t} \leq \pi M_{1t} + (1 - \pi) M_{2t}. \qquad (38)$$

Finally, the household faces its budget constraint at location 2,

$$P_{2t} c_{2t} + M_{2,t+1} \leq P_{2t} \phi_t n_{2t} + \pi M_{1t} + (1 - \pi) M_{2t}. \qquad (39)$$

Let $\overline{M}_{it+1}$ denote the money supply in location $i$ in period $t$ after the government executes the transfer and before financial transactors are relocated. Then

$$\overline{M}_{1,t+1} = (1 - \pi) \overline{M}_{1t} + \pi \overline{M}_{2t} + \Upsilon_{t+1}, \text{ and} \qquad (40)$$

$$\overline{M}_{2,t+1} = \pi \overline{M}_{1t} + (1 - \pi) \overline{M}_{2t}. \qquad (41)$$

As in the previous sections, I can write the household's optimization problem as a dynamic program with analogous notation, except that the scaling variable I use is $\overline{M}_{2t}$, the quantity of money in location 2. Let $z_{it}$ denote the gross growth rate in the money stock in location $i$, and let $V(m_1, m_2, \phi, z_2')$ denote the household's value function. Note here that it is sufficient to include only the money growth factor in location 2 in the state vector. Then, from (36) through (41), the household's dynamic programming problem is

$$
V(m_1, m_2, \phi, z_2') = \max_{c_1, c_2, n_1, n_2, m_1', m_2'} u(c_1) + u(c_2) - v(n_1) - v(n_2)
$$
$$
+ \beta E_t V(m_1', m_2', \phi', z_2'')], \tag{42}
$$

subject to

$$
p_1 c_1 + z_2' m_1' \leq p_1 \phi n_1 + (1 - \pi) m_1 + \pi m_2 + \tau, \tag{43}
$$

$$
p_2 c_2 \leq \pi m_1 + (1 - \pi) m_2, \text{ and} \tag{44}
$$

$$
p_2 c_2 + z_2' m_2' \leq p_2 \phi n_2 + \pi m_1 + (1 - \pi) m_2. \tag{45}
$$

Then, assuming that the cash-in-advance constraint (44) binds, and given the equilibrium conditions $\pi m_1 + (1 - \pi) m_2 = 1$ (money demand equals money supply at location 2) and $c_i = \phi n_i$ for $i = 1, 2$ (the demand for consumption goods equals the supply at each location), the first order conditions from the optimization problem on the right-hand side of the Bellman equation (42) yield

$$
u'(\phi n_1)\phi - v'(n_1) = 0, \tag{45a}
$$

which solves for $n_1$. The first order conditions and the appropriate envelope conditions yield the two Euler equations

$$
z_2' v'(n_2) n_2 = \beta E_t \left[ \pi \psi' + (1 - \pi) u'(\phi' n_2') \phi' n_2' \right] \text{ and} \tag{46}
$$

$$
z_2' \psi = \beta E_t \left[ (1 - \pi) \psi' + \pi u'(\phi' n_2') \phi' n_2' \right], \tag{47}
$$

which then solve for $(\psi, n_2)$ as a function of the state $(\phi, z_2')$, where

$$
\psi \equiv \frac{v'(n_1)}{\phi p_1}. \tag{47a}
$$

## No Aggregate Uncertainty

First, consider the case where there is no uncertainty about productivity; that is, $\phi_t$ is known at date 0 for all $t$. This setup is useful for studying how the central bank should behave in response to predictable events. While some of these predictable events—having to do with the day of the week, the month of the year, or the time until the end of the reserve-averaging period—are not necessarily appropriately modeled as related to aggregate productivity,

the case of predictable productivity fluctuations will nevertheless be quite instructive.

When no aggregate uncertainty exists, then from (46) and (47) an equilibrium consists of sequences $\{\psi_t\}_{t=0}^{\infty}$, $\{n_{2t}\}_{t=0}^{\infty}$ that solve the difference equations

$$z_{2,t+1}v'(n_{2t})n_{2t} = \beta\left[\pi\psi_{t+1} + (1-\pi)u'(\phi_{t+1}n_{2,t+1})\phi_{t+1}n_{2,t+1}\right] \text{ and } (48)$$

$$z_{2,t+1}\psi_t = \beta\left[(1-\pi)\psi_{t+1} + \pi u'(\phi_{t+1}n_{2,t+1})\phi_{t+1}n_{2,t+1}\right], \quad (49)$$

for $t = 0, 1, 2, \ldots$. The first important implication is that money is not neutral here because of a distribution effect. Suppose, for example, that productivity is constant, or $\phi_t = 1$ for all $t$, and that the stocks of money in locations 1 and 2 in period 0 are $\gamma M_0$ and $M_0$, respectively, where $\gamma > 0$. After date 0, suppose that there are no transfers, so that the aggregate money stock is constant for all time. In typical monetary models, $\gamma$ would have no effect on real aggregate variables; that is, money would be neutral. Here, $n_{1t} = n_1$ for all $t$, where $n_1$ is the solution to (45a) with $\phi = 1$. However, $\gamma$ matters for the determination of $\{n_{2t}\}_{t=0}^{\infty}$, as from (40) and (41) $\gamma$ will affect $z_{2t}$ for each $t = 1, 2, \ldots$, which will, in turn, affect $\{\psi_t\}_{t=0}^{\infty}$ and $\{n_{2t}\}_{t=0}^{\infty}$ from (48) and (49). I will not go into detail here concerning the qualitative and quantitative nonneutralities of money in this model, as I want to focus in this section on optimal monetary policy, but these nonneutralities are potentially very interesting and worthy of study.

An optimal allocation is very easy to characterize in this model, as there is a single representative agent, and the optimization problem that an omniscient social planner would solve in this environment is a very simple static problem. That is, optimal $n_{it}$, for $i = 1, 2$, solves

$$\max_{n_{it}}\left[u(\phi_t n_{it}) - v(n_{it})\right].$$

Then, the first order condition for an optimum gives

$$\phi_t u'(\phi_t n_t^*) - v'(n_t^*) = 0. \quad (50)$$

Clearly, from (45a) and (50), employment is optimal in location 1, but employment will in general be suboptimal in location 2. From (48) through (50), the optimal allocation can be supported as a competitive equilibrium if $\{z_{2t}\}_{t=1}^{\infty} = \{z_{2t}^*\}_{t=1}^{\infty}$ where

$$z_{2,t+1}^* = \beta\frac{v'(n_{t+1}^*)n_{t+1}^*}{v'(n_t^*)n_t^*}. \quad (51)$$

The optimal allocation is then achieved in an equilibrium where

$$\psi_t = v'(n_t^*)n_t^* \text{ and}$$

$$p_{1t} = \frac{1}{n_t^*\phi_t}.$$

Now, if

$$-c\frac{u''(c)}{u'(c)} < 1, \tag{52}$$

so that the substitution effect dominates the income effect on labor supply for the household, then from (50), $n_t^*$ is increasing in $\phi_t$. Therefore, since $v'(n)n$ is increasing in $n$, the optimal money growth rate at location 2 in period $t$ is increasing in $\phi_t$ and decreasing in $\phi_{t-1}$. That is, the key monetary policy variable is the growth rate of the money stock in location 2, since location 2 is where transactions are conducted with outside money. At the optimum, monetary policy needs to correct for intertemporal price distortions due to (1) a suboptimal long-run rate of return on money and (2) the distortions introduced because output fluctuates in response to fluctuating productivity. To correct the first distortion, the money stock will tend to grow at the rate of time preference; note from (51) that if $\phi_t$ is constant for all $t$, then $z_{2,t+1}^* = \beta$ for all $t$. To correct the second distortion, since the price level will tend to be low when productivity and output are high (assuming (52)), money growth should be high when productivity is high.

The optimal money growth rule specified by (51) is typical of the optimal Friedman rules implied by representative-agent type monetary models in common use in macroeconomics. Friedman's (1969) prescription was to conduct monetary policy so that the nominal interest rate is zero in all states of the world. Though I have so far ignored the determination of nominal interest rates in this section, a standard approach to pricing a nominal bond would yield a zero nominal interest rate, given (51). Thus, so far nothing seems surprising about the implications for optimal monetary policy coming out of this model. However, the money growth rates specified by equation (51) are for the growth rates of the money stock in location 2 only. Note that the government controls these money growth rates only indirectly, through monetary intervention in location 1. It would be useful to see what (51) implies for the behavior of the money stock in location 1. Using (40), (41), and (51), the optimal money growth rates in location 1 are given by

$$z_{1t}^* = \left(\frac{z_{2,t+1}^* + \pi - 1}{z_{2t}^* + \pi - 1}\right) z_{2t}^*. \tag{53}$$

The optimal money growth rule given by (51) and (53) is more complicated than the simple Friedman rule in (51). This is because the indirect control of the money stock at location 2 through money injections and withdrawals at location 1 requires that the monetary authority take account of how the pattern of transactions diffuses money through the economy. In particular, note the role of $\pi$ in determining the optimal money growth rate in location 1, where $\pi$ governs the speed of diffusion of money through the economy. If $\pi = \frac{1}{2}$, then diffusion occurs in one period, while there is no diffusion if $\pi = 0$ or

$\pi = 1$. The speed of diffusion increases with $\pi$ for $0 < \pi < \frac{1}{2}$ and decreases with $\pi$ for $\frac{1}{2} < \pi < 1$.

**Aggregate Uncertainty**

It proves to be quite easy to generalize the optimal monetary rule given by (51) and (53) to the case where $\phi_t$ is an arbitrary first order Markov process. Here I want to consider how the monetary authority should react to unanticipated shocks to productivity that may be serially correlated. As in the previous subsection, a social optimum is $n_{it} = n_t^*$, for $i = 1, 2$, where $n_t^*$ is the solution to (50). Then, from (46) and (47), an optimal money growth rule is given by

$$z_{2,t+1}^* = \beta \frac{E_t \left[ v'(n_{t+1}^*) n_{t+1}^* \right]}{v'(n_t^*) n_t^*}, \tag{54}$$

and, as before, given the optimal money growth factor for location 2 from (54), the optimal money growth factor for location 1 is specified by (53).

Similar to the previous subsection, the optimal money growth rule specified by (54) and (53) has features similar to a standard Friedman rule in that the money stock at location 2 grows at the rate of time preference, modified by the corrections necessary for anticipated optimal growth in real output. Also, the optimal rate of growth in the money supply at location 1 follows a much more complicated rule for the same reasons as discussed in the previous subsection.

It may seem puzzling that the monetary authority can manipulate the money supply to achieve an optimal allocation, in spite of the fact that production and consumption occurs in two locations and the monetary authority can intervene directly only in one location. Critical to this result is that monetary exchange occurs only at location 2, that the important monetary variable is next period's money growth rate at location 2, and that the monetary authority can control that variable perfectly through current transfers at location 1. An interesting extension of this framework, which relates to my current research, is to allow for monetary exchange in both locations. In that case, the prices at which money trades for goods will in general differ across locations, and Friedman rules cease to be optimal monetary policy. This extension is much harder to study but could be potentially very fruitful for thinking about the role of monetary policy in actual economies.

## 4.   CONCLUSION

While limited participation asset-pricing models such as the one studied by Lucas (1990) provide an explanation for the liquidity effect of monetary policy on nominal interest rates, these models do not provide a rationale for central

banking or any guidance as to how a central bank should behave. Extensions of these models, such as in Fuerst (1992), that allow for nonneutralities of money lack plausibility, as they constrain firms to use cash in situations where their real-world counterparts use credit. In the latter part of this article I explored an extension of limited participation models that takes seriously the idea that monetary policy matters in the short run through its effects on the distribution of wealth across the population.

In ongoing research, I intend to explore further the qualitative and quantitative implications of a related class of limited participation models for monetary policy. These models represent a serious alternative to the sticky-price and sticky-wage Keynesian models that have been popular in recent policy analysis.

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