An Error-Correction Model of the Long-Term Bond Rate

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ost recent studies of long-term interest rates have emphasized term structure relations between long and short rates. They have not, however, looked behind these relations to find the basic economic factors that affect the overall level of interest rates.¹ In this article, I examine empirically the role of economic fundamentals in explaining changes in the long-term U.S. Treasury bond rate.

The economic determinants of the bond rate are identified by building on the loanable funds model used by Sargent (1969), among others.² The bond rate equation estimated here, however, differs from the one reported in Sargent in two major respects. First, it uses the federal funds rate rather than the money supply to capture the influence of monetary policy actions on the real component of the bond rate. As is now widely recognized, financial innovations and the deregulation of interest rates have altered the short-run indicator properties of the empirical measures of money. Hence, it is assumed that the impact of monetary policy actions on the real bond rate is better captured by changes in the real funds rate than in the real money supply. Second, it uses cointegration and error-correction methodology, which is better suited to distinguish between the short- and long-run economic determinants of the bond rate than the one used in Sargent and elsewhere.

The views expressed are those of the author and not necessarily those of the Federal Reserve Bank of Richmond or the Federal Reserve System.

¹ The main reason for this neglect is that the studies in question have been interested primarily in testing the validity of the expectations theory of the term structure of interest rates. One recent exception is the study by Goodfriend (1993), who has attempted to look at fundamentals. In Goodfriend, the long bond rate is viewed as an average of expected future short rates, the latter in turn depending on monetary policy actions and the expected trend rate of inflation. Goodfriend then uses a narrative approach to discuss the interactions between the bond rate and its economic determinants, including monetary policy and expected inflation. He does not, however, formally test for or estimate the impact of these economic determinants on the bond rate.

 $^{^{2}}$ For example, see Echols and Elliot (1976) and Hoelscher (1986), who have employed variants of this model to study the behavior of the long-term bond rate.

The empirical work presented here suggests several results. First, inflation rather than the deficit appears to be the major long-run economic determinant of the bond rate. The long-run deficit-interest rate link found here in the data is fragile.³ Second, monetary policy actions measured by the real funds rate have substantial short-run effects on the real bond rate. Third, the bond rate equation estimated here is consistent with the bond rate's actual, long-run behavior from 1971 to 1993. Nevertheless, it fails to explain some large, short-run upswings in the bond rate that have occurred during the subperiod 1979Q1 to 1993Q4. Those upswings in the bond rate are most likely due to short-run swings in its major long-run economic determinant—expected inflation—and hence may be labeled as reflecting inflation scares as in Goodfriend (1993).

The plan of this article is as follows. Section 1 presents the model and the method used in estimating the bond rate equation. Section 2 presents empirical results, and Section 3 contains concluding observations.

1. THE MODEL AND THE METHOD

A Discussion of the Economic Determinants of the Bond Rate: A Variant of the Sargent Model

The economic determinants of the nominal bond rate are identified here by specifying a loanable funds model employed by Sargent (1969), among others. In this model, the nominal interest rate is assumed to be composed of a real component, a component reflecting inflationary expectations, and a component reflecting the influence of monetary policy actions on the real rate. In particular, consider the identity (1) linking real and nominal components:

$$Rn_{(t)} = Re_{(t)} + [Rm_{(t)} - Re_{(t)}] + Rn_{(t)} - Rm_{(t)},$$
(1)

where Rn is the nominal interest rate, Re is the equilibrium real rate, and Rm is the market real rate. The nominal interest rate equation estimated here is based on hypotheses used to explain each of the three terms on the right-hand side of (1).

The first term, Re, is the real rate that equates ex ante saving with investment and the government deficit. Assume that savings (S) and investment (I) depend upon economic fundamentals as in (2) and (3):

$$I_{(t)} = g_0 + g_1 \Delta y_{(t)} - g_2 R_{e(t)}$$
(2)

$$S_{(t)} = s_0 + s_1 y_{(t)} + s_2 R_{e(t)},$$
(3)

³ This result is consistent with the Ricardian hypothesis that neither consumption nor interest rates are affected by the stock of government debt or by the deficit. In an extensive survey, Seater (1993) also concludes that the Ricardian hypothesis is approximately consistent with the data.

where y is real income. Equation (2) is an accelerator-investment equation with interest rate effects, while equation (3) is a standard Keynesian savings function. In equilibrium, the government deficit must be covered by an excess of savings over investment. Hence, the equilibrium real rate is the rate that solves equation (4):

$$RDEF_{(t)} = S_{(t)} - I_{(t)},$$
 (4)

where RDEF is the real government deficit. Substituting (2) and (3) into (4) yields the following expression for the equilibrium real rate:

$$R_{e(t)} = \frac{1}{s_2 + g_2} [(g_0 - s_0) + g_1 \Delta y_t - s_1 y_t + RDEF_{(t)}].$$
(5)

The deficit and increases in the rate of growth of real income raise the demand for funds and hence drive up the equilibrium real rate. In contrast, a higher level of output generates a larger volume of savings and hence reduces the equilibrium real rate.

The second term on the right-hand side of (1) is the deviation of the market real rate from the equilibrium real rate. This interest rate gap arises in part from monetary policy actions. The Federal Reserve can affect the real rate by changing the supply of high-powered money. In the loans market, such changes in the supply of money have effects on the demand and supply curves for funds and hence the market real rate as in (6):

$$Rm_{(t)} - Re_{(t)} = -h_i[\Delta r M_{(t)}],$$
 (6)

where rM is the real supply of money. A rise in real money supply drives the market rate downward with respect to the equilibrium real rate.

The third term on the right-hand side of (1) is the gap between the nominal and real market rates of interest. Such a gap arises as a result of anticipated inflation and is expressed as in (7):

$$Rn_{(t)} - Rm_{(t)} = \beta \dot{p}_t^e, \tag{7}$$

where \dot{p}^e is anticipated inflation. Substituting (5), (6), and (7) into (1) produces (8), which includes the main potential economic determinants of the bond rate suggested in Sargent (1969).

$$Rn_{(t)} = d_0 + d_1 \dot{p}^e_{(t)} + d_2 RDEF_{(t)} - d_3 y_{(t)} - d_4 \Delta r M^s_{(t)} + d_5 \Delta y_{(t)}$$
(8)

Equation (8) says that the nominal bond rate depends on anticipated inflation, the deficit, changes in real money supply and income, and the level of income.

An Alternative Econometric Specification

Sargent (1969) estimates equations like (8) for one-year and ten-year bond yields using annual data from 1902 to 1940. The bond rate equations estimated here, however, differ from those reported in Sargent in two major respects. In

Sargent, changes in real money supply capture the impact of monetary policy actions on the equilibrium real rate. As is now widely recognized, financial innovations and the deregulation of interest rates have altered the short-run indicator properties of the empirical measures of money.⁴ However, the nominal federal funds rate has been the instrument of monetary policy. Therefore, the impact of monetary policy actions on the real rate is captured by including the real funds rate in the bond rate equation.⁵ Secondly, the bond rate equation here is based on cointegration and error-correction methodology, which is better suited to distinguish between the short- and long-run economic determinants of the bond rate than the one used in Sargent and elsewhere.

The nominal bond rate equation estimated here consists of two parts: a long-run part and a short-run part. The long-run part that specifies the potential, long-run determinants of the level of the bond rate is expressed in (9).

$$Rn_{(t)} = a_0 + a_1 \dot{p}^e_{(t)} + a_2 RFR_{(t)} + a_3 RDEF_{(t)} - a_4 \ln ry_{(t)} + a_5 \Delta \ln ry_{(t)} + U_{(t)},$$
(9)

where *RFR* is the real federal funds rate, *RDEF* is the real deficit, $\ln ry$ is the logarithm of real income, and *U* is the disturbance term. Equation (9) describes the long-run responses of the bond rate to anticipated inflation, the real federal funds rate, the real deficit, changes in real income, and the level of real income. The coefficients a_i , i = 1 to 5, measure the long-run responses in the sense that they are the sums of coefficients that appear on current and past values of the relevant economic determinants. The term $a_1\dot{p}^e$ in (9) captures the inflation premium in the bond rate, whereas the remaining terms capture the influence of other variables on the equilibrium real component of the bond rate. If the nominal bond rate and anticipated inflation variables are nonstationary but cointegrated as in Engle and Granger (1987), then the other remaining long-run impact coefficients (a_2 , a_3 , a_4 , and a_5 in [9]) may all be zero.

Equation (9) may not do well in explaining short-run movements in the bond rate for a number of reasons. First, it ignores the short-run effects of fundamentals. Some economic factors, including those measuring monetary policy actions, may be important in explaining short-run changes in the bond rate, even though they may have no long-run effects. Second, the long-term bond equation (9) completely ignores short-run dynamics. The presence of expectations and/or adjustment lags in the effects of economic fundamentals on the bond rate may cause the bond rate to differ from the value determined in (9). Hence, in order to explain short-run changes in the bond rate;

⁴ See Hetzel and Mehra (1989) and Feinman and Porter (1992) for evidence on this issue.

⁵ Goodfriend (1993) also uses the funds rate to measure the impact of monetary policy actions on the real component of the bond rate.

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$$\Delta Rn_{(t)} = c_0 + c_1 \Delta \dot{p}_{(t)}^e + c_2 \Delta RFR_{(t)} + c_3 \Delta RDEF_{(t)} + c_4 \Delta \ln ry_{(t)} + c_5 \Delta^2 \ln ry_{(t)} + \sum_{s=1}^n c_{6s} \Delta Rn_{(t-s)} + c_7 U_{(t-1)} + \epsilon_{(t)}, \qquad (10)$$

where $U_{(t-1)}$ is the lagged residual from the long-run bond equation (9), Δ^2 is the second-difference operator, and other variables are as defined before. Equation (10) is the short-run bond rate equation, and the coefficients c_i , i = 1 to 5, capture the short-run responses of the bond rate to economic determinants suggested here. The coefficients that appear on lagged first differences of the bond rate, c_{6s} , s = 1 to n, capture short-run dynamics. The equation is in an error-correction form, indicating that the bond rate will adjust in the short run if the actual bond rate differs from its long-run value determined in (9), i.e., if $U_{(t-1)}$ is different from zero in (10). The coefficient c_7 that appears on the lagged error-correction residual in (10) thus captures the short-run influence of long-run dynamics on the bond rate.

Data and Definition of Variables

The main problem in estimating (9) or (10) is that long-run anticipated inflation is an unobservable variable. The empirical work here first uses actual inflation as a proxy for long-run anticipated inflation. In this case, the coefficient a_1 that appears on actual inflation in the long-run bond equation (9) measures the bond rate's response to anticipated inflation, where the latter is modeled as a distributed lag on current and past inflation rates. Hence, this specification is similar in spirit to the one used in Sargent (1969), who had employed an infinite (geometric) distributed lag as a proxy for inflationary expectations. I, however, also examine results using one-year-ahead inflation rates from the Livingston survey to proxy for long-run anticipated inflation.

The empirical work uses quarterly data from 1955Q1 to 1993Q4.⁶ The bond rate is the nominal yield on 30-year U.S. Treasury bonds. Inflation is measured by the behavior of the consumer price index. The real federal funds rate is the nominal federal funds rate minus the actual, annualized quarterly inflation rate. The real deficit variable is included in ratio form as federal government deficits scaled by nominal GDP.⁷ Real income is real GDP. Hence, the empirical specifications considered here are given in (11) and (12).

⁶ The data on the Livingston survey are provided by the Philadelphia Fed. All other data series are from the Citibank data base.

⁷ This specification reflects the assumption that in a growing economy higher deficits result in higher interest rates only if the deficit rises relative to GDP. Hence, the deficit is scaled by GDP. This specification amounts to the restriction that the coefficients a_3 and a_4 in (9) are equal in magnitude but opposite in sign. However, none of the results here qualitatively change if the deficit (*RDEF*) and real GDP (ln *ry*) enter separately in regressions.

$$R30_{(t)} = a_0 + a_1 \dot{p}_{(t)} + a_2 RFR_{(t)} + a_3 (DEF/y)_{(t)} + a_4 \Delta \ln r y_{(t)} + U_{(t)}$$
(11)

$$\Delta R30_{(t)} = c_0 + c_1 \Delta \dot{p}_{(t)} + c_2 \Delta RFR_{(t)} + c_3 \Delta (DEF/y)_{(t)} + c_4 \Delta^2 \ln ry_{(t)} + c_5 U_{(t-1)} + \epsilon_{(t)}, \qquad (12)$$

where *R*30 is the bond rate, \dot{p} is actual inflation, and (DEF/y) is the ratio of deficits to GDP. Equation (11) is the long-run bond rate equation and equation (12) the short-run equation. The alternative specification investigated here replaces $\dot{p}_{(t)}$ in (11) and (12) with $\dot{p}_{(t)}^e$, where \dot{p}^e is the Livingston survey measure of inflationary expectations.

Estimation Issues: The Long-Run Bond Rate Equation

The stationarity properties of the data are important in estimating the longrun bond equation. If empirical measures of economic determinants including the bond rate are all nonstationary variables but cointegrated as in Engle and Granger (1987), then the long-run equation (11) can be estimated by ordinary least squares. Tests of hypotheses on coefficients that appear in (11) can then be carried out by estimating Stock and Watson's (1993) dynamic OLS regressions of the form

$$R30_{(t)} = a_0 + a_1\dot{p}_{(t)} + a_2RFR_{(t)} + a_3[DEF_{(t)}/y_{(t)}] + a_4\Delta\ln ry_{(t)}$$
$$+ \sum_{s=-k}^{k} a_{4s}\Delta\dot{p}_{(t-s)} + \sum_{s=-k}^{k} a_{5s}\Delta RFR_{(t-s)}$$
$$+ \sum_{s=-k}^{k} a_{6s}\Delta[DEF_{(t-s)}/y_{(t-s)}] + \sum_{s=-k}^{k} a_{7s}\Delta^2\ln ry_{(t-s)} + \epsilon_{(t)}.$$
 (13)

Equation (13) includes, in addition to current levels of economic variables, past, current, and future values of changes in them.

In order to determine whether the variables have unit roots or are mean stationary, I perform both unit root and mean stationarity tests. The unit root tests are performed by estimating the augmented Dickey-Fuller regression of the form

$$X_{(t)} = m_0 + \rho X_{(t-1)} + \sum_{s=1}^k m_{1s} \Delta X_{(t-s)} + \epsilon_{(t)}, \qquad (14)$$

where X is the pertinent variable, ϵ is the random disturbance term, and k is the number of lagged first differences of X necessary to make ϵ serially uncorrelated. If $\rho = 1$, X has a unit root. The null hypothesis $\rho = 1$ is tested using the t-statistic. The lag length (k) used in tests is chosen using the procedure given in Hall (1990), as advocated by Campbell and Perron (1991).⁸

The Dickey-Fuller statistic tests the null hypothesis of unit root against the alternative that *X* is mean stationary. Recently, some authors including DeJong et al. (1992) have presented evidence that the Dickey-Fuller tests have low power in distinguishing between the null and the alternative. These studies suggest that it would also be useful to perform tests of the null hypothesis of mean stationarity to determine whether the variables are stationary or integrated. Thus, tests of mean stationarity are performed using the procedure advocated by Kwiatkowski, Phillips, Schmidt, and Shin (1992). The test, hereafter denoted as the KPSS test, is implemented by calculating the test statistic

$$\hat{n}_u = \frac{1}{T^2} \sum_{t=1}^T S_{(t)}^2 / \hat{\sigma}_k^2,$$

where $S_{(t)} = \sum_{i=1}^{t} e_i, t = 1, 2, \ldots, T$, e_t is the residual from the regression of $X_{(t)}$ on an intercept, $\hat{\sigma}_k$ is a consistent estimate of the long-run variance of X, and T is the sample size.⁹ The statistic \hat{n}_u has a nonstandard distribution and its critical values have been provided by Kwiatkowski et al. (1992). The null hypothesis of stationarity is rejected if \hat{n}_u is large. Thus, a variable $X_{(t)}$ is considered unit root nonstationary if the null hypothesis that $X_{(t)}$ has a unit root is not rejected by the augmented Dickey-Fuller test and the null hypothesis that it is mean stationary is rejected by the KPSS test.

The test for cointegration used is the one proposed in Johansen and Juselius (1990). The test procedure consists of estimating a VAR model that includes differences as well as levels of nonstationary variables. The matrix of coefficients associated with levels of these variables contains information about the long-run properties of the model. To explain the model, let Z_t be a vector of time series on the bond rate and its economic determinants. Under the hypothesis that the series in Z_t are difference stationary, one can write a VAR model as

$$\Delta Z_{t} = \Gamma_{1} \ \Delta Z_{(t-1)} + \dots + \Gamma_{(k-1)} \ \Delta Z_{(t-k-1)} + \prod Z_{(t-k)} + \epsilon_{(t)}, \quad (15)$$

$$\hat{\sigma}_k = \frac{1}{T} \sum_{t=1}^T e_t^2 + \frac{2}{T} \sum_{s=1}^T b(s,k) \sum_{t=s+1}^T e_t e_{t-s},$$

where *T* is the sample size, the weighing function $b(s, k) = 1 + \frac{S}{1+k}$, and *k* is the lag truncation parameter. The lag parameter was set at k = 8.

⁸ The procedure is to start with some upper bound on k, say k max, chosen a priori (eight quarters here). Estimate the regression (14) with k set at k max. If the last included lag is significant, select k = k max. If not, reduce the order of the estimated autoregression by one until the coefficient on the last included lag (on ΔX in [14]) is significant. If none is significant, select k = 0.

⁹ The residual e_t is from the regression $X_t = a + e_t$. The variance of X_t is the variance of the residuals from this regression and is estimated, using the Newey and West (1987) method, as

where Γ_i , i = 1, 2, ..., k - 1, and Π are matrices of coefficients that appear on first differences and levels of the time series in Z_t .

The component $\prod Z_{t-k}$ in (15) gives different linear combinations of levels of the time series in Z_t . Thus, the matrix \prod contains information about the long-run properties of the model. When the matrix's rank is zero, equation (15) reduces to a VAR in first differences. In that case, no series in Z_t can be expressed as a linear combination of other remaining series. This result indicates that there does not exist any long-run relationship between the series in the VAR. On the other hand, if the rank of Π is one, then there exists only one linear combination of series in Z_t . That result indicates that there is a unique, long-run relationship between the series.

Two test statistics can be used to evaluate the number of the cointegrating relationships. The trace test examines the rank of Π matrix and the hypothesis that rank (Π) $\leq r$, where *r* represents the number of cointegrating vectors. The maximum eigenvalue statistic tests the null that the number of cointegrating vectors is *r*, given the alternative of r + 1 vectors. The critical values of these test statistics have been reported in Johansen and Juselius (1990).

OLS estimates are inconsistent if any right-hand explanatory variable in the long-run bond equation (11) is stationary. In that case, the long-run bond equation (11) can be estimated jointly with the short-run bond equation (12). To do so, solve (11) for $U_{(t-1)}$ and then substitute for $U_{(t-1)}$ into (12) to yield (16).

$$\Delta R30_{(t)} = (c_0 - c_5 a_0) + c_1 \Delta \dot{p}_{(t)} + c_2 \Delta RFR_{(t)} + c_3 \Delta (DEF_{(t)}/y_{(t)}) + c_4 \Delta^2 \ln ry(t) - c_5 R30_{(t-1)} - c_5 a_1 \dot{p}_{(t-1)} - c_5 a_2 RFR_{(t-1)} - c_5 a_3 DEF_{(t-1)}/y_{(t-1)} - c_5 a_4 \Delta \ln ry_{(t-1)} + \epsilon_t$$
(16)

Equation (16) is the short-run bond rate equation that includes levels as well as differences of the relevant economic determinants. The long-run coefficients a_i , i = 1, 2, 3, can be recovered from the reduced-form estimates of equation (16).¹⁰ The equation can be estimated by ordinary least squares,¹¹ or by instrumental variables if contemporaneous right-hand variables are correlated with the disturbance term.

¹⁰ The long-run coefficient on inflation (*a*₁) is the coefficient on $\dot{p}(t-1)$ divided by the coefficient on $R30_{(t-1)}$; the long-run coefficient on deficit (*a*₃) is the coefficient on $DEF_{(t-1)}/y_{(t-1)}$ divided by the coefficient on $R30_{(t-1)}$; and the long-run coefficient on the real funds rate is the coefficient on $RFR_{(t-1)}$ divided by the coefficient on $R30_{(t-1)}$. The intercept *a*₀, however, cannot be recovered from these reduced-form estimates.

¹¹ Since lagged levels of economic determinants appear in (16), ordinary least squares estimates are consistent if some variables on the right-hand side of (16) are in fact stationary.

2. ESTIMATION RESULTS

Unit Root Test Results

Table 1 presents test results for determining whether the variables R30, \dot{p} , \dot{p}^e , and (DEF/y) have a unit root or are mean stationary. As can be seen, the t-statistic $(t_{\hat{p}})$ that tests the null hypothesis that a particular variable has a unit root is small for all these variables. On the other hand, the test statistic (\hat{n}_u) that tests the null hypothesis that a particular variable is mean stationary is large for R30, \dot{p} , \dot{p}^e and (DEF/y), but small for *RFR*. These results thus indicate that R30, \dot{p} , \dot{p}^e , and (DEF/y) have a unit root and are thus nonstationary in levels. The results are inconclusive for the *RFR* variable.

As indicated before, a variable has a unit root if $\rho = 1$ in (14). In order to indicate the extent of uncertainty about the point-estimate of ρ , Table 1 also contains estimates of ρ and their 95 percent confidence intervals. As can be seen, the estimated intervals contain the value $\rho = 1$ for levels of these variables. However, these intervals appear to be quite wide: their lower limits are close to .90 for the series shown. These results indicate that the variables may well be mean stationary. Hence, I also derive results treating all variables as stationary.

Table 1 also presents unit root tests using first differences of *R*30, \dot{p} , \dot{p}^e , *RFR*, ln *ry* and (*DEF*/y). As can be seen, the t-statistic for the hypothesis $\rho = 1$ is fairly large for all these variables. The point-estimates of ρ also diverge away from unity. These results indicate that first differences of these variables are stationary.

Cointegration Test Results

Treating the bond rate, inflation, the real funds rate, and government deficits as nonstationary variables, Table 2 presents test statistics for determining whether the bond rate is cointegrated with any of these variables.¹² Change in real income ($\Delta \ln ry$) is not considered because it is a stationary variable. Trace and maximum eigenvalue statistics, which test the null hypothesis that there is no cointegrating vector, are large for systems ($R30, \dot{p}$), ($R30, \dot{p}^e$), (R30, DEF/y), ($R30, \dot{p}F/y$) and ($R30, \dot{p}^e, DEF/y$), but are very small for the system (R30, RFR). These results indicate that the bond rate is cointegrated with inflation (actual or expected) and deficits, but not with the real funds rate. That is, the bond rate stochastically co-moves with inflation and the deficit variable, but not with the real funds rate.

 $^{^{12}}$ The lag length parameter (k) for the VAR model was chosen using the likelihood ratio test described in Sims (1980). In particular, the VAR model initially was estimated with k set equal to a maximum number of eight quarters. This unrestricted model was then tested against a restricted model, where k is reduced by one, using the likelihood ratio test. The lag length finally selected in performing the JJ procedure is the one that results in the rejection of the restricted model.

		F T Ur	Panel B Tests for Mean Stationarity		
Series X	ρ	t _ô	k	Confidence Interval for ρ	\hat{n}_u
R30	.97	-1.65	5	(.93, 1.03)	1.31*
<i>p</i>	.87	-2.74	7	(.84, 1.01)	.53*
\dot{p}^e	.97	-1.78	2	(.92, 1.02)	1.02*
RFR	.85	-2.38	2	(.87, 1.02)	.39
DEF/y	.93	-2.46	1	(.87, 1.02)	1.42*
$\Delta R30$	02	-5.47*	8		
$\Delta \dot{p}$	70	-5.09*	8		
$\Delta \dot{p}^e$.38	-6.33*	1		
ΔRFR	-1.50	-5.52^{*}	7		
$\Delta DEF/y$	80	-6.18^{*}	8		
$\Delta \ln ry$.20	-4.83^{*}	7		

 Table 1 Tests for Unit Roots and Mean Stationarity

* Significant at the 5 percent level.

Notes: R30 is the 30-year bond rate; \dot{p} is the annualized quarterly inflation rate measured by the behavior of consumer prices; \dot{p}^e is the Livingston survey measure of one-year-ahead expected inflation; *RFR* is the real federal funds rate; and *DEF*/y is the ratio of federal government deficits to nominal GDP. Δ is the first-difference operator. The sample period studied is 1955Q1 to 1993Q4. ρ and t-statistics ($t_{\hat{\rho}}$) for $\rho = 1$ in Panel A above are from the augmented Dickey-Fuller regressions of the form

$$X_{(t)} = a_0 + \rho X_{(t-1)} + \sum_{s=1}^k a_s \Delta X_{(t-s)},$$

where X is the pertinent series. The series has a unit root if $\rho = 1$. The 5 percent critical value is -2.9. The number of lagged first differences (k) included in these regressions are chosen using the procedure given in Hall (1990), with maximum lag set at eight quarters. The confidence interval for ρ is constructed using the procedure given in Stock (1991).

The test statistics \hat{n}_u in Panel B above is the statistic that tests the null hypothesis that the pertinent series is mean stationary. The 5 percent critical value for \hat{n}_u given in Kwiatkowski et al. (1992) is .463.

Table 3 presents the dynamic OLS estimates of the cointegrating vector between the bond rate and its long-run determinants, inflation and the deficit. Panel A presents estimates with actual inflation (\dot{p}) and Panel B with expected inflation (\dot{p}^e). In addition, the cointegrating vector is estimated under the restriction that the bond rate adjusts one for one with inflation in the long run. In regressions estimated without the above-noted full Fisher-effect restriction, the right-hand explanatory variables have their theoretically predicted signs and are statistically significant. Thus, the bond rate is positively correlated with inflation and deficits in the long run. The coefficient that appears on the

System	k^{a}	Trace Test	Maximum Eigenvalue Test
(<i>R</i> 30, <i>p</i>)	8	23.7*	21.2*
$(R30,\dot{p}^e)$	8	20.6*	17.6*
(<i>R</i> 30, <i>RFR</i>)	5	12.1	8.9
(<i>R</i> 30, <i>DEF</i> /y)	5	30.4*	27.5*
$(R30, \dot{p}, DEF/y)$	8	46.8*	35.6*
$(R30, \dot{p}^e, DEF/y)$	8	48.3*	31.9*

Table 2 Cointegration Test Results

^a The lag length k was selected using the likelihood ratio test procedure described in footnote 12 of the text.

* Significant at the 5 percent level.

Notes: Trace and maximum eigenvalue tests are tests of the null hypothesis that there is no cointegrating vector in the system. For the two-variable system, the 5 percent critical value is 17.8 for the trace statistic and 14.5 for the maximum eigenvalue statistic. Critical values are from Johansen and Juselius (1990). (For the three-variable system, the corresponding 5 percent critical values are 31.2 and 21.3.)

Table 3	Cointegrating	Regressions;	Dynamic	OLS
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(Leads, Lags)	Without the Full Fisher- Effect Restriction	With the Full Fisher- Effect Restriction		
	Panel A: (R30, <i>p</i> ,	DEF/y)		
(-4, 4)	$R30_t = 2.0 + .61\dot{p}_t + .73(DEF/y)_t$ (.03) (.03) (.04)	$R30_t = 1.3 + 1.0\dot{p}_t + .30(DEF/y)_t$ (.10) (.03) (.05)		
(-8, 8)	$R30_t = 2.0 + .60\dot{p}_t + .78(DEF/y)_t$ (.12) (.03) (.08)	$R30_t = 1.3 + 1.0\dot{p}_t13(DEF/y)_t$ (.10) (.03) (.03)		
	Panel B: (<i>R</i> 30, <i>p</i> ^{<i>e</i>} ,	DEF/y)		
(-4, 4)	$R30_t = 2.6 + .77\dot{p}_t^e + .46(DEF/y)_t$ (.13) (.03) (.03)	$R30_t = 2.1 + 1.0\dot{p}_t^e + .13(DEF/y)_t$ (.11) (.03) (.03)		
(-8, 8)	$R30_t = 2.6 + .72\dot{p}_t^e + .57(DEF/y)_t$ (.15) (.06) (.13)	$R30_t = 2.8 + 1.0\dot{p}_t^e + .00(DEF/y)_t$ (.14) (.00) (.03)		

Notes: All regressions are estimated by the dynamic OLS procedure given in Stock and Watson (1993), using leads and lags of first differences of the relevant right-hand side explanatory variables. Parentheses contain standard errors corrected for the presence of moving average serial correlation. The dynamic OLS regressions also include leads and lags of the real federal funds rate.

inflation variable ranges between .6 and .8 and is less than unity, indicating that the bond rate does not adjust one for one with inflation in the long run. The coefficient that appears on the deficit variable ranges between .4 and .8, indicating that a one percentage point increase in the ratio of deficits to GDP raises the bond rate by 40 to 80 basis points.¹³ However, the coefficient that appears on the deficit variable is sensitive to the restriction that there is a full Fisher effect. If the cointegrating regression is reestimated with this restriction, then the deficit variable coefficient becomes small and even turns negative in some cases (see Table 3).

The full Fisher-effect restriction is in fact rejected by the data, indicating that it should not be imposed routinely on the bond regression. Nevertheless, it is a reasonable restriction to consider if one wants to carry out the sensitivity analysis. The finding that the long-run deficit-interest rate link weakens when the restriction is imposed indicates that the deficit may be proxying the information that is already in inflation. The deficit appears to raise the long rate because of its positive effect on anticipated inflation rather than on the real component of the bond rate. Hence, these results imply that inflation is the main, long-run economic determinant of the bond rate.

The Short-Run Bond Rate Equation

Since unit root test results are inconclusive for some series, the short-run bond equation is estimated jointly with the long-run part as in (16), which includes lagged levels of the series. If the variables are stationary in levels, OLS estimates will still be consistent.

Table 4 presents instrumental variable estimates of the bond equation (16).¹⁴ Panel A there reports regressions with actual inflation (\dot{p}), and Panel B regressions with expected inflation (\dot{p}^e). In addition, I estimate the equation with and without the constraint that the bond rate adjusts one for one with inflation in the long run (compare equations in columns A.1 and B.1 versus A.2 and B.2, Table 4). As can be seen, the coefficients that appear on various economic variables have their theoretically predicted signs and in general are statistically significant. The results there indicate that in the short run the bond rate rises if inflation increases, or if the real federal funds rate rises. Changes

¹³ These estimates are close to those reported in Hoelscher (1986). Hoelscher uses the tenyear bond rate and the Livingston survey measure as proxies for long-term expected inflation. He estimates the bond regression from 1953 to 1984, using annual data. The coefficients that appear on his inflation and deficit variables are .84 and .42, respectively. Hoelscher does not, however, examine the sensitivity of results to the restriction that the bond rate adjusts one for one with inflation in the long run.

¹⁴ I use instrumental variable estimates because contemporary values of changes in the funds rate, inflation, and real income variables may be correlated with the disturbance term. For example, the evidence in Mehra (1994) indicates that the Fed has responded to the information in the bond rate about long-run expected inflation. Hence, the change in the funds rate may be contemporaneously correlated with the disturbance term.

Explanatory	Par Regressi Actual Inf	nel A ons Using flation Data	Panel B Regressions Using Livingston Survey Inflation Data		
Variables	A.1	A.2	B.1	B.2	
constant	.55 (2.1)	01 (0.1)	1.80 (2.4)	.59 (4.5)	
$R30_{t-1}$	29 (4.7)	18 (4.6)	59 (3.2)	30 (6.2)	
\dot{p}_{t-1}	.20 (4.9)	.18 (4.6)			
\dot{p}_{t-1}^e			.43 (4.2)	.30 (6.2)	
$(DEF/y)_{t-1}$.18 (3.2)	.06 (2.2)	.24 (1.8)	.02 (0.8)	
RFR_{t-1}	.19 (3.6)	.13 (2.9)	.31 (2.9)	.15 (4.1)	
$\Delta \dot{p}_t$.40 (3.7)	.32 (3.3)	, ,		
$\Delta \dot{p}_t^e$. ,	.61 (2.1)	.26 (1.8)	
ΔRFR_t	.35 (4.6)	.24 (4.0)	.31 (2.5)	.14 (2.5)	
$\Delta \ln r Y_t$	01(0.2)	.05 (1.4)	10(1.2)	.03 (1.1)	
$\Delta R30_{t-1}$	10(1.1)	24(3.2)	.31 (1.4)	01(0.1)	
$\Delta R30_{t-2}$.16 (1.7)	.09 (1.0)	.10 (0.8)	.03 (0.4)	
SER	.451	.434	.709	.504	
DW	2.0	1.84	2.0	1.95	
Q(36)	35.3	37.6	54.9	46.3	
n(p, RFR, DEF/y)	(.7, .7, .6)	(1.0, .7, .3)			
$n(\dot{p}^e, RFR, DEF/y)$			(.7, .5, .4)	(1.0, .5, .1)	

Table 4 Error-Correction Bond Rate Regressions

Notes: All regressions are estimated by instrumental variables. The instruments used are a constant, one lagged value of the bond rate, inflation, the real federal funds rate, and the ratio of deficits to GDP and two lagged values of changes in inflation, the real funds rate, real GDP, and the bond rate. Regressions in columns A.2 and B.2 above are estimated under the restriction that coefficients on $R30_{t-1}$ and \dot{p}_{t-1} (\dot{p}_{t-1}^e) sum to zero (there is complete Fisher-effect), while those in columns A.1 and B.1 are without this restriction. SER is the standard error of regression, DW is the Durbin-Watson statistic, and Q(36) is the Lung-Box Q-statistic based on 36 autocorrelations of the residuals. $n(x_1, x_2, x_3)$ indicates the long-run (distributed) responses of the 30-year bond rate to x_1 , x_2 , and x_3 , respectively.

in real GDP do not have much of an impact on the bond rate.¹⁵ The coefficients that appear on contemporaneous values of these variables range from .3 to .6 for inflation and from .2 to .4 for the real funds rate. Thus, a one percentage point increase in the rate of inflation raises the bond rate between 30 to 60 basis points, while a similar increase in the real funds rate raises it by 14 to 35 basis points in the short run.¹⁶

¹⁵ First differences of the deficit variable and second differences of real GDP when included in regressions were generally not significant.

¹⁶ The point-estimates of the short-run, monetary policy impact coefficient found here are close to those found or assumed in some other studies. For example, the empirical work presented in Cook and Hahn (1989) indicates that a one percentage point rise in the funds rate target raises the long rate by 10 to 20 basis points, whereas in Goodfriend (1993) such an increase in the funds rate is assumed to raise the long rate by 25 basis points.

As indicated before, the bond rate's long-run distributed-lag responses to economic determinants here can be recovered from the reduced-form estimates of the short-run bond equation presented in Table 4. As can be seen, the long-run coefficients that appear on these variables range from .7 to 1.0 for inflation, .5 to .7 for the real funds rate, and .1 to .6 for the deficit variable. Moreover, as before, the long-run coefficient on the deficit variable becomes small and is statistically insignificant if the full Fisher-effect restriction is imposed on the data (see equations A.2 and B.2 in Table 4). The long-run coefficient that appears on the real funds rate, however, remains quite large and is statistically significant. This result indicates that (stationary) movements in the real funds rate can have substantial effects on the bond rate in the short run.^{17,18}

Predictive Ability of the Bond Rate Equation

I now examine whether bond rate regressions presented in Table 4 can explain the actual behavior of the bond rate. In particular, I examine one-year-ahead dynamic forecasts of the bond rate from 1971Q1 to 1993Q4, using regressions A.2 and B.2 of Table 4. Recall that regression A.2 uses actual inflation as a proxy for long-run inflationary expectations and regression B.2 uses one-yearahead expected inflation as a proxy. Since the forecast performance of these two regressions is similar, I discuss results only for the former.

Figure 1 charts the quarterly values of the bond rate, actual and predicted. As can be seen, the regression captures fairly well broad movements in the bond rate from 1971Q1 to 1993Q4. The mean prediction error is small, only 6 basis points, and the root mean squared error is .74 percentage points. This regression outperforms a purely eight-order autoregressive model of the bond rate. For the time series model, the mean prediction error is 13 basis points and the root mean squared error is 1.2 percentage points.

I evaluate further the predictive performance from 1971Q1 to 1993Q4 by estimating regressions of the form (17).

¹⁷ If all variables are stationary, then the long-run coefficient that appears on the funds rate in the short-run bond equation measures the sum of coefficients associated with current and past values of changes in the funds rate. Since permanent movements in the funds rate have no permanent effects on the bond rate, this long-run coefficient in fact measures the short-run response of the bond rate to changes in the funds rate.

¹⁸ The long-run coefficient that appears on the funds rate in the bond rate regression may be an upwardly biased estimate of the impact of monetary policy actions on the real component of the bond rate. The main source of this potential bias is the absence of the relevant long-run expected inflation variable in these regressions. If the Fed responds to variables that have information about long-run expected inflation, then the funds rate may be picking up the influence of expected inflation on the bond rate rather than the influence of monetary policy actions on the real component of the bond rate. Some evidence that favors this view emerges in Table 4. As can be seen, the magnitude of the long-run coefficient that appears on the funds rate declines from .7 to .5 if one-year-ahead expected inflation (Livingston) data are substituted for actual inflation in the regression.



Figure 1 Actual and Predicted 30-Year Bond Rate

Note: Predicted values are generated using the regression with actual inflation (regression A.2 in Table 4).

$$A_{(t)} = d_0 + d_1 P_{(t)},\tag{17}$$

where *A* is the actual quarterly value of the bond rate and *P* is the value predicted by the bond rate regression. If $d_0 = 0$ and $d_1 = 1$, then regression forecasts are unbiased. The coefficients d_0 and d_1 take values .3 and .97, respectively, for regression A.2¹⁹ and 1.7 and .8, respectively, for the time series model. The hypothesis $d_0 = 0$, or $d_1 = 1$, is rejected for the time series model, but not for the economic models.²⁰

Unpredictable, Short-Run Upward Swings in the Bond Rate: Inflation

A look at Figure 1 indicates that the bond rate regression estimated here fails to predict some large, short-run movements in the bond rate that have occurred during the post-1979 period.²¹ Table 5 presents quarterly changes in the bond rate from 1979Q1 to 1994Q2. It also presents changes predicted by the bond

¹⁹ For regression B.2 of Table 4, $d_0 = .13$ and $d_1 = 1.0$.

²⁰ For regression A.2, the relevant Chi-squared statistics that test $d_0 = 0$ and $d_1 = 1$ are .3 and .2, respectively. The relevant statistics are .03 and 0.0 for regression B.2 of Table 4. For the time series model, the relevant Chi-squared statistics take values 3.9 and 4.1. Each Chi-squared statistic is distributed with one degree of freedom. The 5 percent critical value is 3.84.

²¹ Such large, short-term increases in the bond rate did not occur during the pre-1979 period.

Year/Qtr.	Actual	Predicted	Error	Year/Qtr.	Actual	Predicted	Error
1979Q1	.15	.07	.07	1987Q1	.18	.20	02
1979Q2	11	.06	17	1987Q2	1.02 ^a	.16	.85
1979Q3	.25	.37	12	1987Q3	1.02 ^a	07	1.09
1979Q4	.95	.49	.46	1987Q4	47	22	24
1980Q1	2.22 ^a	.55	1.66	1988Q1	49	25	24
1980Q2	-2.53	69	-1.84	1988Q2	.37	.25	.12
1980Q3	1.53 ^a	.74	.79	1988Q3	.06	07	.13
1980Q4	1.06	1.09	03	1988Q4	05	.22	27
1981Q1	.29	63	.92	1989Q1	.16	.44	27
1981Q2	.27	1.32	-1.06	1989Q2	90	.03	93
1981Q3	1.71 ^a	32	2.03	1989Q3	12	10	01
1981Q4	-1.22	14	-1.07	1989Q4	25	.12	37
1982Q1	.08	.44	36	1990Q1	.66	.55	.11
1982Q2	.39	.53	14	1990Q2	10	31	.21
1982Q3	-1.85	69	-1.15	1990Q3	.57	01	.58
1982Q4	-1.53	.01	-1.54	1990Q4	79	80	.01
1983Q1	.09	.42	33	1991Q1	.05	67	.72
1983Q2	.30	.48	18	1991Q2	.18	47	.65
1983Q3	.70 ^a	30	1.00	1991Q3	52	50	02
1983Q4	.25	.06	.18	1991Q4	25	64	.39
1984Q1	.50	.13	.36	1992Q1	.27	36	.63
1984Q2	1.06 ^a	.01	1.05	1992Q2	13	45	.32
1984Q3	-1.15	35	79	1992Q3	50	50	.00
1984Q4	77	37	40	1992Q4	.10	17	.27
1985Q1	.29	11	.40	1993Q1	62	60	02
1985Q2	-1.36	59	77	1993Q2	01	51	.29
1985Q3	.16	.17	01	1993Q3	81	51	29
1985Q4	-1.07	36	70	1993Q4	.25	.32	07
1986Q1	-1.58	.50	-2.10	1994Q1	.35	43 ^b	.78
1986Q2	39	.41	.03	1994Q2	.80 ^a	.01 ^b	.78
1986Q3	.05	13	.18				
1986Q4	25	03	21				
Mean Error							004
RmSE							.74

Table 5 Actual and Predicted Quarterly Changes in the Bond Rate1979Q1 to 1994Q2

 a This significant increase in the bond rate is not predicted by the bond rate regression A.2 of Table 4 (the prediction error is at least as large as the root mean squared error).

^b This forecast assumes that during the first and second quarters the ratio of deficits to GDP equals the value observed in 1993Q4.

Notes: The predicted values are generated using the bond rate regression A.2 of Table 4.

regression. If we focus on quarterly increases in the bond rate that are significantly underpredicted by the regression (that is, magnitudes of prediction errors either equal or exceed the root mean squared error), the results then indicate that the bond rate rose 2.2 percentage points in 1980Q1, 1.53 in 1980Q3, 1.71 in 1981Q3, .7 in 1983Q4, 1.1 in 1984Q2, 2.1 in 1987Q2 to 1987Q3, and .8 in 1994Q2 (see Table 5). Except for the latest episode, most of these short-run upswings in the bond rate have been subsequently reversed, so that for the period as a whole the bond rate is well predicted by the regression.

The bond rate equation here attempts to explain changes in the bond rate using actual, not long-run anticipated, values of economic fundamentals. In the long run, actual values of fundamentals may move with anticipated values, but that may not be so in the short run. Hence, if the bond rate in fact responds to anticipated fundamentals, then the bond rate regressions estimated here may not explain very well short-run movements in the bond rate. These considerations suggest one possible explanation of some unpredictable short-run upswings in the bond rate that have occurred since 1979: namely, short-term movements in its anticipated fundamentals. Since, as indicated by cointegration test results, inflation, rather than the deficit or the real funds rate, is the main long-run economic determinant of the bond rate, the short-run increases in the bond rate may then be due to short-run movements in its long-run determinant-anticipated inflation.²² Thus, the bond rate may rise with anticipated inflation in the short run even as actual inflation remains steady. Such upswings, however, are likely to be reversed if they are not substantiated by the behavior of actual inflation. As can be seen in Table 5, that in fact has been the case.

Following Goodfriend (1993), the periods during which large, unpredictable increases in the bond rate have occurred can be labeled as inflation scares. Goodfriend uses a narrative approach to discuss the interactions among the bond rate, the federal funds rate, and economic determinants such as inflation and real growth. He assumes that inflation is the bond rate's main long-run determinant and that changes in the funds rate have minor short-run effects on the bond rate. Hence, he calls a significant bond rate rise in the absence of an aggressive funds rate tightening an inflation scare. The results from a more formal bond rate equation here are in line with those in Goodfriend (1993).

²² Short-run changes in anticipated monetary policy actions and deficits cannot explain the big increases in the bond rate either. As noted before, the bond rate is unrelated to short-term changes in the deficit. Furthermore, the magnitudes of future funds rate increases needed to explain the current increases in the bond rate are too big to be consistent with past Fed behavior. In the past, the Fed has moved the funds rate in small increments most of the time.

3. CONCLUDING OBSERVATIONS

Using cointegration and error-correction methodology and building on the loanable funds model of interest rate determination given in Sargent (1969), this article identifies the main long- and short-run economic determinants of the bond rate. In the cointegrating regression, inflation and fiscal deficits appear as two potential long-run economic determinants of the bond rate. That regression indicates that the bond rate is positively correlated with inflation and the deficit and that the bond rate does not adjust one for one with inflation in the long run. However, if that regression is reestimated under the restriction that the bond rate does in fact adjust one for one with inflation, then the long-run deficit-interest rate link found here weakens. Those results imply that the positive effect of the deficit on the real component of the bond rate found here is suspect. Hence, inflation emerges as the main economic determinant of the long rate.

The results here also indicate that changes in the real federal funds rate have substantial short-run effects on the bond rate, even though long-run stochastic movements in the bond rate are unrelated to the real funds rate. In addition, the bond rate rises if inflation accelerates. Surprisingly, current changes in real GDP do not have much of an effect on the bond rate.

The bond rate regressions estimated here are broadly consistent with the actual behavior of the bond rate from 1971 to 1993. However, these regressions fail to predict some large, short-run upswings in the bond rate that have occurred during the subperiod 1979Q1 to 1994Q2. One possible explanation of these results is that actual inflation may be a poor proxy for the long-run expected rate of inflation, the main long-run economic determinant of the bond rate. Hence, the bond rate may rise significantly in the short run if long-run anticipated inflation increases, even though actual inflation may have been steady.

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