Using the Permanent Income Hypothesis for Forecasting

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Personal consumption expenditures grew by almost 2 percent during 1993 in real, per-capita terms. Real disposable income per capita, meanwhile, actually fell slightly. By definition, households draw down their savings when consumption grows faster than income. In fact, the figures for consumption and income just mentioned underlie a decline in the personal savings rate from over 6 percent in the fourth quarter of 1992 to only about 4 percent in the fourth quarter of 1993.[1]

One popular interpretation of these data starts with the idea that reductions in the savings rate cannot be permanently sustained. Eventually, households must rebuild their savings by cutting back on consumption; to the extent that lower consumption leads to lower income, income must fall as well. Thus, in *U.S. News & World Report*, David Hage (1993–94) used the behavior of consumption, income, and savings to forecast that the economy would slow in 1994: "[A] slowdown in consumer spending is likely, and it could trim an additional 0.6 percentage point off growth" (p. 43). Around the same time, Gene Epstein (1993) of *Barron's* quoted economist Philip Braverman as saying that "consumers don't have the wherewithal to keep up the current spending pace. The prevailing euphoria will get knocked for a loop" (p. 37). Similarly, in DRI/McGraw-Hill's *Review of the U.S. Economy*, professional forecaster Jill Thompson (1993) wrote: "All is not rosy, of course. Consumers went out on a limb to give the economy a needed jump-start. . . . They have pushed the saving rate very low and incurred more debt. . . . Consumption must slow" (pp. 16 and 18).

In light of this conventional wisdom, which suggests that a decline in savings presages a slowdown in economic growth, the continued strength of

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[1] Appendix B describes all data and their sources.

the U.S. economy in 1994 came as a surprise, raising the question of whether an alternative framework can better reconcile the recent behavior of consumption, income, and savings. This article considers one such alternative: Milton Friedman's (1957) permanent income hypothesis. This hypothesis implies that households save less when they expect future income to rise. Thus, according to the permanent income hypothesis, a decline in savings like that experienced during 1993 signals that faster, not slower, income growth lies ahead.

Although developed in detail by Friedman in his 1957 monograph, the permanent income hypothesis has its origins in Irving Fisher's (1907) theory of interest. Thus, the article begins by reviewing Fisher's graphical analysis and by indicating how this analysis extends to a full statement of the permanent income hypothesis. The article goes on to show how Robert Hall (1978) derives the permanent income hypothesis from a mathematical theory that has very specific implications for the joint behavior of consumption, income, and savings. Following John Campbell (1987), it draws on Hall's version of the hypothesis to formulate a simple econometric model that exploits data on savings to forecast future income growth. Estimates from this model show that the U.S. data conform not to conventional wisdom, but to the intuition provided by the permanent income hypothesis: historically, declines in savings have preceded periods of faster, not slower, income growth. Finally, the article uses the model to generate forecasts for the U.S. economy.

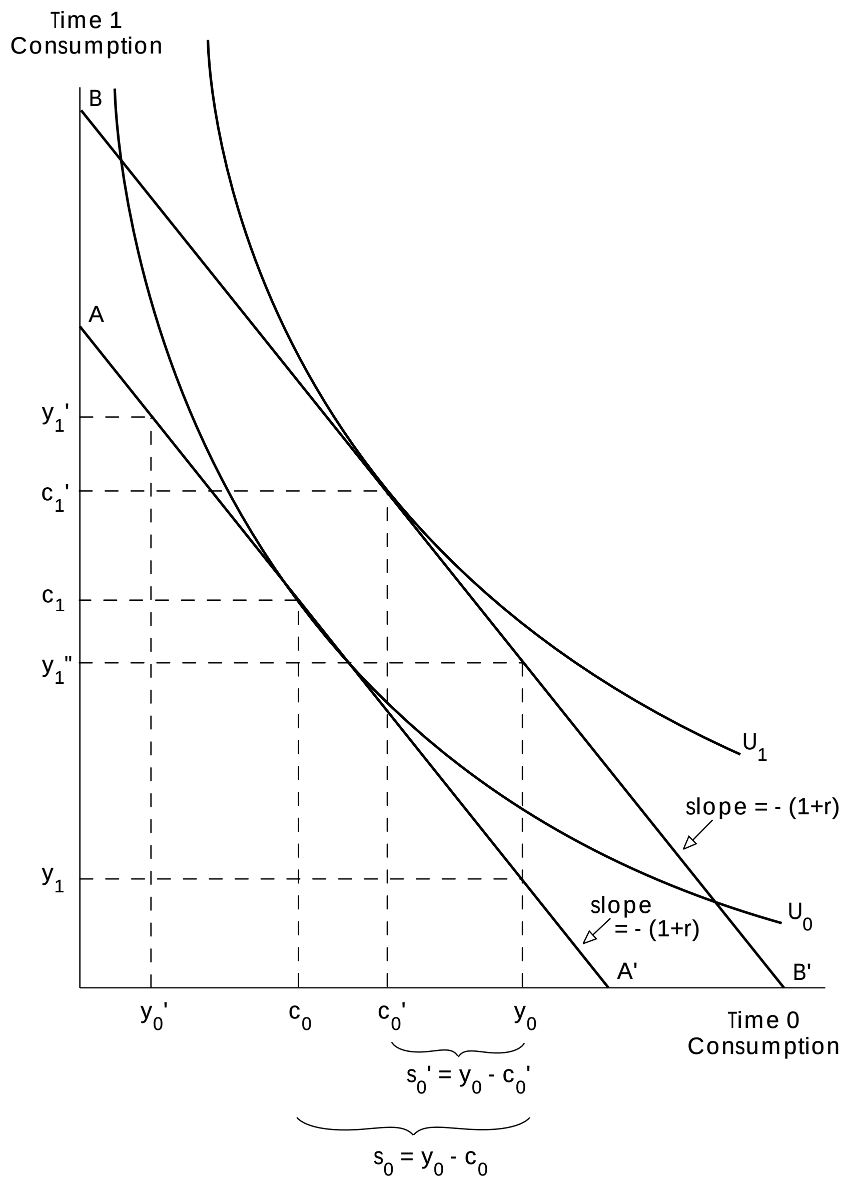
1. FISHER'S THEORY OF INTEREST AND THE PERMANENT INCOME HYPOTHESIS

In presenting his theory of interest, Irving Fisher (1907) uses a graph like that shown in Figure 1 to illustrate how a household makes its consumption and savings decisions. To simplify his graphical analysis, Fisher considers only two periods. His horizontal axis measures goods at time 0, and his vertical axis measures goods at time 1. Fisher's representative household receives income y_0 during time 0 and y_1 during time 1.

The representative household faces the fixed interest rate r, which serves as an intertemporal price. It measures the rate at which the market allows the household to exchange goods at time 1 for goods at time 0. In particular, if the household lends one unit of the good at time 0, it gets repaid $(1 + r)$ units of the good at time 1. Similarly, if the household borrows one unit of the good at time 0, it must repay $(1 + r)$ units of the good at time 1. Thus, in Figure 1, the household's budget constraint AA$'$, which passes through the income point (y_0, y_1), has slope $-(1 + r)$.

The household's preferences over consumption at the two dates are represented by the indifference curves U_0 and U_1, each of which traces out a set of consumption pairs that yield a given level of utility. Utility increases with

Figure 1 Fisher's Diagram



consumption in both periods; hence, $U_1 > U_0$. The slope of each indifference curve is determined by the household's marginal rate of intertemporal substitution, the ratio of its marginal utilities of consumption at dates 0 and 1, or the rate at which it is willing to exchange goods at time 1 for goods at time 0.

To maximize its utility, the household chooses the consumption pair (c_0, c_1), where the indifference curve U_0 is tangent to the budget constraint AA$'$. At (c_0, c_1), the household's marginal rate of intertemporal substitution equals the gross rate of interest $(1 + r)$. The household saves amount $s_0 = y_0 - c_0$.

Now suppose that the household's income pair changes to (y_0', y_1'). Since this new income point lies on the same budget constraint as (y_0, y_1), the household continues to select (c_0, c_1) as its optimal consumption combination. In fact, the household chooses (c_0, c_1) starting from any income point along AA$'$. Since all income points along AA$'$ have the same present value, equal to

$$PV = y_0 + \frac{y_1}{(1+r)}, \tag{1}$$

this example illustrates the first implication of Fisher's theory: the household's consumption choice depends only on the present value of its income pair (y_0, y_1), not on y_0 and y_1 separately.

Next, hold y_0 constant and suppose that the household's income at time 1 increases to y_1''. This change increases the present value of the household's income pair. It shifts the budget constraint out from AA$'$ to BB$'$ and leads the household to choose the preferred consumption pair (c_0', c_1'). Since $c_0' > c_0$, the increase in time 1 income allows the household to reduce its time 0 savings from $s_0 = y_0 - c_0$ to $s_0' = y_0 - c_0'$. This example illustrates the second implication of Fisher's theory: the household saves less when it expects future income to be high. Conversely, the household saves more when it expects future income to be low.

This second implication makes Fisher's model useful for forecasting. It suggests, in particular, that data on household savings help forecast future income. A low level of savings today indicates that households expect higher income in the future. A high level of savings today signals that households expect lower income in the future. Note that both of these predictions contradict the conventional wisdom, which indicates that low savings predate lower income.

Milton Friedman's (1957) permanent income hypothesis generalizes Fisher's analysis to a model in which there are more than two periods and the representative household may be uncertain about its future income prospects. Thus, Friedman also derives the result that a representative household's consumption depends not on its current income but on the present value of its future income. With an infinite number of periods and uncertainty, this present value can be written

$$PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1+r)^t}, \tag{2}$$

where Ey_t denotes the household's expected income at time t.

Friedman defines the household's permanent income y^p as the constant income level that, if received with certainty in each period t, has the same present value as the household's actual income path. That is, y^p satisfies

$$\sum_{t=0}^{\infty} \frac{y^p}{(1+r)^t} = PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1+r)^t}. \tag{3}$$

In light of the formula

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r}, \tag{4}$$

equation (3) simplifies to

$$y^p = \frac{r}{1+r} PV. \tag{5}$$

Thus, the first implication of the permanent income hypothesis is that the household's consumption at date 0 can be written as a function of its permanent income:

$$c_0 = f(y^p). \tag{6}$$

Similarly, Friedman generalizes the second implication of Fisher's analysis. Friedman's representative household borrows to increase consumption today when it anticipates higher income in the future. In other words, it saves less when it expects future income to be high. Conversely, the household uses additional savings to buffer its consumption against expected declines in income; it saves more when it expects future income to be low. Thus, like Fisher's theory of interest, Friedman's permanent income hypothesis suggests that data on savings help forecast future income.

2. HALL'S VERSION OF THE PERMANENT INCOME HYPOTHESIS

Robert Hall (1978) develops a mathematical version of the permanent income hypothesis that makes the relationship between savings and expected future income identified by Fisher and Friedman more precise. In fact, the details of Hall's model indicate exactly how data on savings can be used to forecast future changes in income.

Hall, like Friedman, assumes that there are many time periods $t = 0, 1, 2, \ldots$ and that the representative household is uncertain about its future income prospects.[2] Hall's infinitely lived representative household has expected utility

$$E \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{7}$$

where E once again denotes the household's expectation, $u(c_t)$ measures its utility from consuming amount c_t at time t, and the discount factor β lies between zero and one.

The household begins period t with assets of value A_t. It earns interest on these assets at the constant rate r; its capital income during period t is therefore $y_{kt} = rA_t$. The household also receives labor income y_{lt} during period t.

At the end of period t, the household divides its total income $y_t = y_{kt} + y_{lt}$ between consumption c_t and savings $s_t = y_t - c_t$. It then carries assets of value

$$A_{t+1} = A_t + s_t = (1 + r)A_t + y_{lt} - c_t \tag{8}$$

into period $t + 1$.

The household is allowed to borrow against its future labor income at the interest rate r; because of borrowing and the associated accumulation of debt, its assets A_t may become negative. Its borrowing is constrained in the long run, however, by the requirement that

$$\lim_{t\to\infty} \frac{A_t}{(1 + r)^t} = 0. \tag{9}$$

To see how equation (9) limits the household's borrowing, note that equation (8) is a difference equation in the variable A_t. Using equation (9) as a terminal condition, one can solve equation (8) forward to obtain

$$A_t = \sum_{j=0}^{\infty} \frac{c_{t+j} - y_{lt+j}}{(1 + r)^{j+1}}. \tag{10}$$

Equation (10) shows that the household must repay any debt owed today ($-A_t$) by setting future consumption c_{t+j} below future labor income y_{lt+j}. Equation (10) also implies that

$$A_t = \sum_{j=0}^{\infty} \frac{E_t c_{t+j}}{(1 + r)^{j+1}} - \sum_{j=0}^{\infty} \frac{E_t y_{lt+j}}{(1 + r)^{j+1}}, \tag{11}$$

where E_t denotes the representative household's expectation at time t. This condition states that the household's current level of assets A_t must be sufficient to cover any discrepancy between the present value of expected future consumption and the present value of expected future labor income.

[2] This presentation of Hall's model follows Sargent (1987, Ch. 12) quite closely.

The representative household chooses consumption c_t and asset holdings A_{t+1} for all $t = 0, 1, 2, \ldots$ to maximize the utility function (7) subject to the constraints (8) and (9). The solution to this problem dictates that

$$u'(c_t) = \beta(1 + r)E_t u'(c_{t+1}). \tag{12}$$

Equation (12) simply generalizes the optimality condition shown in Figure 1 as the tangency between the household's indifference curve and its budget constraint. It indicates that the household sets its expected marginal rate of intertemporal substitution, the ratio of its marginal utility of consumption at time t to its expected marginal utility of consumption at time $t+1$, $u'(c_t)/\beta E_t u'(c_{t+1})$, equal to the gross rate of interest $(1 + r)$.

Assume that the interest rate r is related to the household's discount factor β via $\beta = 1/(1+r)$. Assume also that the household's utility is quadratic, with $u(c) = u_0 + u_1 c - (u_2/2)c^2$ for some positive constants u_0, u_1, and u_2.[3] Under these additional assumptions, equation (12) reduces to

$$c_t = E_t c_{t+1}. \tag{13}$$

Equation (13) states Hall's famous result that under the permanent income hypothesis, consumption follows a random walk.

Equation (13) implies that $E_t c_{t+j} = c_t$ for all $j = 0, 1, 2, \ldots$. Substituting this result into equation (11) yields

$$c_t = rA_t + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{lt+j}}{(1+r)^j}. \tag{14}$$

The right-hand side of equation (14), equal to current capital income plus $r/(1 + r)$ times the present value of expected future labor income, defines the representative household's permanent income in Hall's model. Equation (14), like equation (6), states the first main implication of the permanent income hypothesis: consumption is determined by permanent income.

Using $y_{kt} = rA_t$ and $s_t = y_{kt} + y_{lt} - c_t$ and denoting the change in labor income by $\Delta y_{lt} = y_{lt} - y_{lt-1}$, one can rearrange equation (14) to reveal the second main implication of the permanent income hypothesis:

$$s_t = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{lt+j}}{(1+r)^j}. \tag{15}$$

According to equation (15), the household's current savings s_t equals the present value of expected future declines in its labor income. Thus, equation (15) states that the household saves less when it expects future gains in income, that is,

[3] These assumptions, while restrictive, greatly simplify the analysis. Hansen and Singleton (1982) derive and test the implications of Hall's (1978) model under more general assumptions about r and $u(c)$.

positive values for Δy_{lt+j}. Conversely, the household saves more when it anticipates future declines in income, that is, negative values for Δy_{lt+j}. Once again, this second implication of the permanent income hypothesis suggests that data on savings help forecast future changes in income.

3. THE PERMANENT INCOME FORECASTING MODEL

John Campbell (1987) shows exactly how Hall's version of the permanent income hypothesis can be used to formulate an econometric forecasting model for the U.S. economy. Since the permanent income hypothesis implies that data on savings will help forecast future changes in labor income, Campbell starts with a bivariate vector autoregression (VAR) for Δy_{lt} and s_t of the form

$$\begin{bmatrix} \Delta y_{lt} \\ s_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta y_{lt-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \tag{16}$$

where, for example, $a(L) = a_1 + a_2 L + a_3 L^2 + \cdots + a_p L^{p-1}$, L is the lag operator, and u_{1t} and u_{2t} are serially uncorrelated errors.[4] Campbell then shows how the relationship (15) between savings and future labor income identified by Hall's model translates into a set of parameter restrictions on the VAR (16).

Campbell works through the series of linear algebraic manipulations outlined in Appendix A. First, he uses the VAR (16) to compute the expected future declines in labor income $-E_t \Delta y_{lt+j}$ that appear on the right-hand side of equation (15). Next, he demonstrates that these expected future declines depend on the coefficients of the lag polynomials $a(L)$, $b(L)$, $c(L)$, and $d(L)$. In particular, if the present value of the expected future declines in income are to equal the current value of savings s_t, as required by equation (15), the following parameter restrictions must hold:

$$a_1 = c_1, \ \ldots, \ a_p = c_p, \ d_1 - b_1 = (1+r), \ b_2 = d_2, \ \ldots, \ b_p = d_p. \tag{17}$$

Equation (17) gives the restrictions imposed by Hall's version of the permanent income hypothesis on the VAR (16).

4. PERFORMANCE OF THE PERMANENT INCOME FORECASTING MODEL

Quarterly data, 1959:1–1994:3, are used to estimate the VAR in equation (16) both with and without the permanent income restrictions (17). The specification (16) assumes that Δy_{lt} and s_t have zero mean; in practice, adding constant terms to the VAR removes each variable's sample mean. The estimated models include six lags of each variable on the right-hand side.

[4] See Sargent (1987, Ch. 9) for details about the lag operator.

Panel (a) of Table 1 shows the unconstrained equation for labor income growth. The negative sum of the coefficients on lagged savings indicates that a decrease in savings translates into a forecast of faster income growth, exactly as implied by the permanent income hypothesis. Moreover, an F-test easily rejects the hypothesis that the savings data do not help to forecast future income growth; again as predicted by the permanent income hypothesis, the coefficients on the lags of s_t are jointly significant at the 0.00037 level.

Panel (b) of Table 1 displays the equation for labor income when the permanent income constraints (17) are imposed on the VAR. The estimates assume that $r = 0.01$, which corresponds to an annual real interest rate of 4 percent. The coefficients of the constrained equation closely resemble those of the unconstrained equation, indicating once again that the data are consistent with the permanent income hypothesis.

Table 1 Estimated Labor Income Equation from the Permanent Income Forecasting Model

(a) Unconstrained Model				
$\Delta y_{lt} = 88.9$	$+\ 0.107\Delta y_{lt-1}$	$-\ 0.0286\Delta y_{lt-2}$	$+\ 0.236\Delta y_{lt-3}$	$-\ 0.111\Delta y_{lt-4}$
	(0.118)	(0.121)	(0.118)	(0.121)
	$+\ 0.0261\Delta y_{lt-5}$	$-\ 0.0636\Delta y_{lt-6}$	$-\ 0.379 s_{t-1}$	$+\ 0.218 s_{t-2}$
	(0.120)	(0.0876)	(0.102)	(0.143)
	$-\ 0.140 s_{t-3}$	$+\ 0.172 s_{t-4}$	$-\ 0.109 s_{t-5}$	$+\ 0.170 s_{t-6}$
	(0.144)	(0.142)	(0.145)	(0.110)
(b) Constrained Model				
$\Delta y_{lt} = 106$	$+\ 0.0448\Delta y_{lt-1}$	$-\ 0.0734\Delta y_{lt-2}$	$+\ 0.168\Delta y_{lt-3}$	$-\ 0.0417\Delta y_{lt-4}$
	(0.109)	(0.111)	(0.109)	(0.111)
	$+\ 0.0115\Delta y_{lt-5}$	$-\ 0.0742\Delta y_{lt-6}$	$-\ 0.375 s_{t-1}$	$+\ 0.235 s_{t-2}$
	(0.111)	(0.0806)	(0.0942)	(0.131)
	$-\ 0.109 s_{t-3}$	$+\ 0.0899 s_{t-4}$	$-\ 0.0840 s_{t-5}$	$+\ 0.159 s_{t-6}$
	(0.132)	(0.131)	(0.133)	(0.101)

Note: Standard errors are in parentheses.

A statistical test rejects the constraints (17) at the 99 percent confidence level. As noted by King (1995), however, formal hypothesis tests seldom fail to reject the implications of detailed mathematical models such as Hall's.[5]

[5] Moreover, as indicated in footnote 3, Hall's model makes very restrictive assumptions about the interest rate and the household's utility function. The statistical rejection of the constraints (17) may therefore reflect the failure of one of these additional assumptions to hold in the data,

Ultimately, the permanent income hypothesis must be judged on its ability to forecast the data better than alternative models.

Thus, Table 2 reports on the forecasting performance of the permanent income model. First, the constrained VAR is estimated with data from 1959:1 through 1970:4 and is used to generate out-of-sample forecasts for the total change in labor income one, two, four, and eight quarters ahead. Next, the sample period is extended by one quarter, and additional out-of-sample forecasts are obtained. Continuing in this manner yields out-of-sample forecasts for 1971:1 through 1994:3.

The table computes the permanent income model's mean squared error at each forecast horizon. It expresses each mean squared error as a fraction of the mean squared error from a univariate model for labor income growth with six lags. Thus, figures less than unity in Table 2 indicate that the VAR's mean squared forecast error is smaller than the univariate model's.

The table shows that the permanent income forecasts improve on the univariate forecasts at all horizons. The gain in forecast accuracy exceeds 10 percent at horizons longer than one quarter. The permanent income model is especially valuable for forecasting at the annual horizon, where it reduces the univariate forecast errors by 25 percent.

Table 2 Performance of the Permanent Income Forecasting Model

Horizon (Quarters Ahead)	Improvement Over Univariate Model	Improvement Over Unconstrained VAR
1	0.95	1.00
2	0.87	0.97
4	0.75	0.92
8	0.89	0.78

Note: Performance is measured by the mean squared forecast error from the permanent income model expressed as a fraction of the mean squared forecast error from two alternative models: a univariate model for labor income and an unconstrained vector autoregression for savings and labor income.

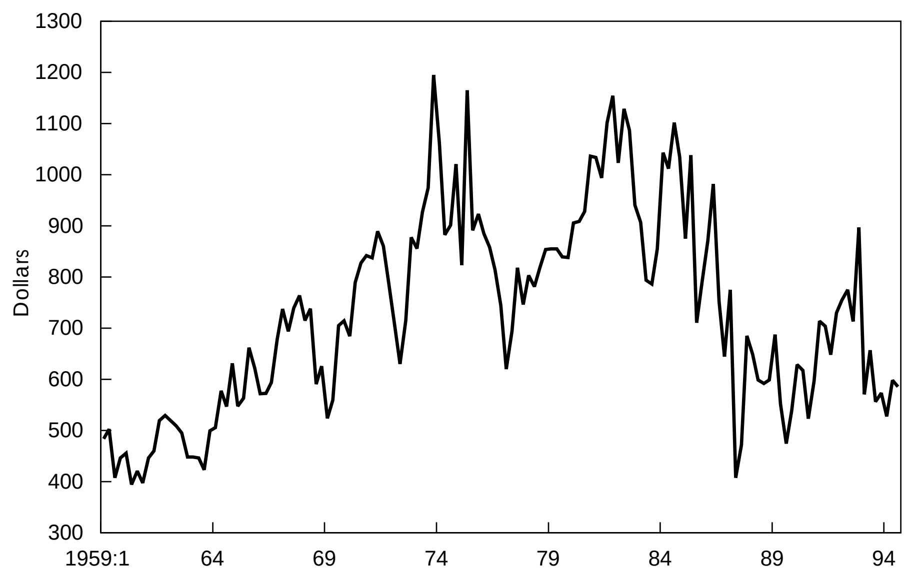
Table 2 also compares the forecasting performance of the constrained VAR to the performance of the VAR when the permanent income constraints (17) are not imposed. Once again, the figures less than unity indicate that the permanent income forecasts have lower mean squared error than the unconstrained forecasts. The improvement is most dramatic at longer horizons. Thus, Table 2 shows that the permanent income constraints help improve the model's out-of-sample forecasting ability relative to both a univariate model for labor income growth and an unconstrained VAR.

rather than a more general failure of the permanent income hypothesis itself.

Figure 2 plots the data for real personal savings per capita. It shows that savings increased from 1987 through the end of 1992, but have fallen since then. According to the permanent income hypothesis, this recent decline in savings indicates that households expect future gains in income.

Indeed, forecasts from the permanent income model reflect these expectations. When estimated with data through 1994:3, the constrained VAR predicts growth in real disposable labor income per capita of $181 for 1995. Since real disposable labor income now stands at about $12,300 and the population is growing at an annual rate of about 1 percent, this figure translates into a gain in aggregate real labor income of 2.5 percent. Thus, the permanent income model predicts that the U.S. economy will continue to expand in 1995.

Figure 2 Real Personal Savings
1987 Dollars per Capita



5. CONCLUSION

Conventional wisdom suggests that the recent decline in personal savings cannot be sustained. Eventually, households will have to reduce their consumption, causing economic growth to slow. The permanent income hypothesis, however, contradicts this conventional wisdom. According to this hypothesis, households reduce their savings when they expect future income to be high; a low level of savings indicates that faster, not slower, income growth lies ahead.

This article uses a mathematical version of the permanent income hypothesis to formulate a simple econometric forecasting model for the U.S. economy. Estimates from the model reveal that the data are broadly consistent with the hypothesis' implications. Most important, the data indicate that declines in savings typically precede periods of faster, rather than slower, growth in income.

The results show that the permanent income model improves on univariate forecasts for annual labor income growth by 25 percent. The model also improves on the forecasting ability of an unconstrained vector autoregression for savings and labor income. In light of the recent decline in savings, the permanent income model forecasts continuing growth in personal income for 1995.

APPENDIX A: DERIVATION OF THE PERMANENT INCOME RESTRICTIONS

Campbell (1987) rewrites the vector autoregression (16) as

$$z_t = Az_{t-1} + v_t, \tag{18}$$

where

$$z_t = [\Delta y_{lt} \; . \; . \; . \; \Delta y_{lt-p+1} s_t \; . \; . \; . \; s_{t-p+1}]', \tag{19}$$

$$A = \begin{bmatrix} a_1 & \cdot & \cdot & \cdot & a_p & b_1 & \cdot & \cdot & \cdot & b_p \\ 1 & & & & & & & & & \\ & \cdot & & & & & & & & \\ & & \cdot & & & & & & & \\ & & & 1 & & & & & & \\ c_1 & \cdot & \cdot & \cdot & c_p & d_1 & \cdot & \cdot & \cdot & d_p \\ & & & & & 1 & & & & \\ & & & & & & \cdot & & & \\ & & & & & & & \cdot & & \\ & & & & & & & & 1 & \end{bmatrix}, \tag{20}$$

and

$$v_t = [u_{1t} \; 0 \; . \; . \; . \; 0 \; u_{2t} \; 0 \; . \; . \; . \; 0]'. \tag{21}$$

Equation (18), along with the conditions $E_t v_{t+j} = 0$ for all $j = 1, 2, 3, \ldots,$ makes it easy to write the time t expectation of the vector z_{t+j} as

$$E_t z_{t+j} = A^j z_t. \tag{22}$$

Equation (22) implies, in particular, that

$$E_t \Delta y_{lt+j} = e_1 A^j z_t, \tag{23}$$

where e_1 is a row vector consisting of a one followed by $2p - 1$ zeros.

Equation (23) shows that expected changes in future labor income depend on the coefficients of the lag polynomials $a(L)$, $b(L)$, $c(L)$, and $d(L)$, which are contained in the matrix A. The current value of savings, meanwhile, is just

$$s_t = e_{p+1} z_t, \tag{24}$$

where e_{p+1} is a row vector consisting of p zeros followed by a one and $p - 1$ zeros. Equation (23) can be substituted into the right-hand side of equation (15), and equation (24) can be substituted into the left-hand side of equation (15), so that Hall's (1978) relationship between savings and income becomes

$$e_{p+1} z_t = -\sum_{j=1}^{\infty} (1 + r)^{-j} e_1 A^j z_t. \tag{25}$$

Equation (25) must hold if, as required by equation (15), the current value of savings is to equal the discounted value of expected future declines in labor income.

Campbell uses the matrix analog to equation (4), which implies

$$\sum_{j=1}^{\infty} (1 + r)^{-j} A^j = (1 + r)^{-1} A [I - (1 + r)^{-1} A]^{-1}, \tag{26}$$

to rewrite (25) as

$$e_{p+1} z_t = -e_1 (1 + r)^{-1} A [I - (1 + r)^{-1} A]^{-1} z_t, \tag{27}$$

or, more simply,

$$e_{p+1} [I - (1 + r)^{-1} A] = -e_1 (1 + r)^{-1} A, \tag{28}$$

where I is an identity matrix of the same size as A. The definition of A given by equation (20) implies that this last condition is equivalent to equation (17). Thus, equation (17) restates Hall's permanent income condition (15) as a set of parameter constraints that can be imposed on the VAR (16).

APPENDIX B: DATA SOURCES

All data used in this article come from the National Income and Product Accounts, as reported in the DRI/McGraw-Hill Data Base. The underlying quarterly series, 1959:1–1994:3, are the following:

WSD = Wage and Salary Disbursements
YOL = Other Labor Income
YENTADJ = Proprietors' Income
YRENTADJ = Rental Income
DIV@PER = Personal Dividend Income
YINTPER = Personal Interest Income
V = Transfer Payments
TWPER = Personal Contributions for Social Insurance
TP = Personal Tax and Nontax Payments
YD = Disposable Personal Income
C = Personal Consumption Expenditures
INTPER = Interest Paid by Consumers to Business
VFORPER = Personal Transfer Payments to Foreigners
YD87 = Disposable Personal Income, 1987 Dollars
NNIA = Population

From these series, disposable personal labor income (YDL) and disposable personal capital income (YKL) are constructed as

$$\begin{aligned}\text{YDL} = {} & \text{WSD} + \text{YOL} + (2/3) * \text{YENTADJ} + \text{V} - \text{TWPER} \\ & -(2/3) * \text{TP} - \text{VFORPER}\end{aligned}$$

and

$$\begin{aligned}\text{YDK} = {} & (1/3) * \text{YENTADJ} + \text{YRENTADJ} + \text{DIV@PER} + \text{YINTPER} \\ & -(1/3) * \text{TP} - \text{INTPER}.\end{aligned}$$

When converted into real, per-capita terms using the deflator for disposable personal income (YD/YD87) and the population NNIA, the series YDL, YKL, and C correspond to y_{lt}, y_{kt}, and c_t in the text. Real savings per capita is defined by $s_t = y_{lt} + y_{kt} - c_t$.

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