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ABSTRACT

This study develops a new model-free benchmark of implied volatility for the Japanese stock market similar in construction to the new VIX based on the S&P 500 index. It also examines the stochastic dynamics of the implied volatility index and its relationship with realized volatility in both markets. There is evidence that implied volatility is governed by a long-memory process. Despite its upward bias, implied volatility is more reflective of changes in realized volatility than alternative GARCH models, which account for volatility persistence and the asymmetric impact of news. The implied volatility index is also found to be inclusive of some but not all information on future volatility contained in historical returns. However, its higher out-of sample performance provides further support to the rationale behind drawing inference about future stock market volatility based on the incremental information contained in options prices.

INTRODUCTION

According to Heisenberg uncertainty principle in quantum theory, it is not possible to measure with perfect accuracy the position and momentum of a particle simultaneously. This argument may apply with equal force to financial markets where the level and volatility of asset prices cannot be measured with the same precision. The daunting difficulties associated with this observation problem derive from the fact that these parameters are complementary and the measurement of one parameter with greater accuracy is likely to be accompanied with some loss in measurement precision of the other. Whereas information on stock price indices is being disseminated at increasingly higher data frequency, measures

of future volatility with comparable accuracy and frequency are rather hardly available. Assuming that the second moment of return distribution exists, it is possible to estimate volatility as the variance or standard deviation from the history of returns. However, the volatility implicit in option prices can provide an alternative measure of future return variability. One of the merits of this implied volatility approach is that it results in a time-series of observations that reconciles differences in data frequency, albeit not necessarily in accuracy, of price level and volatility.

The construction of a time-series of implied volatility is important to enhance our understanding of how market volatility ebbs and flows in increasingly integrated financial markets. Stock market volatility constitutes indeed an essential ingredient not only in investment and risk-hedging but in market regulation and monetary policy-making as well. Market participants can view volatility quite differently and their anticipated reactions to economic events and the release of macroeconomic information may differ as well. The development of an implied volatility index is important for policymaking purposes insofar as it reveals the changing and perhaps stochastic beliefs of market participants. The importance of such measure of consensus expectations about short-term volatility for monetary policies is evident for instance, in the explicit reference by the Bank of England to the implied functions from options markets in its monetary policy meetings. There is evidence from Fornari (2004) that the implied volatility from swaption prices is responsive to macroeconomic announcements such as the release of US economic indicators. Furthermore, Neely (2005) shows that changes in the implied volatility from three-month eurodollar interest rates are associated with major events about monetary policy, stock markets and the real economy. Carr and Wu (2006) provide also evidence that the new VIX implied volatility index from S&P 500 option prices is reflective of the increasing uncertainty prior to monetary policy decisions about the Fed Fund target rate.

The new VIX index computed and disseminated by the Chicago Board of Options Exchange is a model-free benchmark in the sense that it does not depend on a particular option pricing model. This approach has the merit of avoiding measurement errors arising from model misspecification. This index aggregates information in the term structure of implied volatility and the nonlinear relationship with respect to exercise prices reflected by volatility smiles, sneers and smirks. However, the absence of implied volatility benchmarks for other international stock markets impedes the growth of international evidence on the usefulness of implied volatility for risk-hedging and policymaking purposes. It is the purpose of the present study to develop a new VIX benchmark for the Japanese stock market. The construction of this volatility benchmark is an acknowledgment of the fact that the Nikkei 225 index is the underlying asset of several financial derivatives traded, albeit under different contract specifications, on major Asia-Pacific derivatives markets.¹

This paper examines the stochastic dynamics of the forward-looking implied volatility index of Japanese and US equity markets. It assesses its relationship with realized volatility by addressing in particular, the classical question of whether the implied volatility index reveals incremental information beyond that contained in historical returns. It also examines the empirical question which naturally arises as to whether drawing inference about future stock market volatility based on implied volatility is justified by higher out-of-sample forecasting ability. Based on a novel measure of forecasting performance proposed by Blair, Poon and Granger (2001), this study assesses the forecasting ability of implied volatility indices relative to alternative GARCH models. Thus, in addition to the development of the new VIX index for the Japanese equity market, this study differs also

¹ Futures on Nikkei 225 stock average are traded on three derivatives markets, namely the Singapore Exchange Derivatives Trading Division, formerly the Singapore International Monetary Exchange since September 3, 1986, on the Osaka Securities Exchange since September 3, 1988 and on the Chicago Mercantile Exchange since September 25, 1990.

from the existing literature as it uses a sample period covering fifteen years of daily options prices, and spanning the Asian financial crisis in 1997, the Russian debt default and the Long-Term Capital Management (LTCM) crisis in 1998, the burst of the information technology bubble in 2000, the Latin American debt crisis in 2002 and the 1990s decade-long recession of the Japanese economy.

The remainder of the paper is structured as follows. The next section presents a brief review of the literature on implied volatility. Section 3 examines the distributional properties of implied volatility and the underlying benchmark returns. Section 4 assesses the informational content of implied volatility by modelling its relationship with realized volatility. Section 5 examines the forecasting performance of implied volatility relative to GARCH models. Section 6 concludes the paper.

2. REVIEW OF LITERATURE

The literature on market volatility implicit in option premium seems to extend along several strands of studies. The first class of research explores various issues related to the numerical difficulties encountered in the determination of implied volatility. There are convergence problems which arise from the failure of the iterative process to equate the option's market price with its theoretical premium and the absence of closed-form solutions to option pricing models. The literature examines the observed properties of implied volatility function and their implications for option pricing and proposes some nonlinear numerical approaches. There is evidence of 'smile' patterns in implied volatility functions based on Black-Scholes (1973) option pricing model, where deep-in-the-money or out-of-the-money options are associated with higher implied volatility than at-the-money options. Pena, Rubio and Serno (1999) attribute such systematic volatility smiles to transaction costs and find a significant inverse relationship between the degree of curvature and time remaining until expiration. As noted by Hentschel (2003), the implied volatility functions derived from

options away from the money are sensitive to various sources of measurement errors.

Arguably however, the observed functions are inconsistent with Black-Scholes option pricing model, which assumes constant volatility that is independent of exercise prices and time. In order to address these inherent inconsistencies and recover unbiased estimates of implied volatility, approximation techniques have been advocated in several studies such as Corrado and Miller (1996) and Chambers and Nawalkha (2001). A “model-free” approach is also proposed by Britten-Jones and Neuberger (2000) who adjust arbitrary volatility processes to option prices, drawing upon the standard practice of fitting interest rate processes to bond prices. The modelling of implied volatility as function of exercise prices such as Ait-Sahalia and Lo (1998), can also be based on polynomial smoothing, interpolation or splines smoothing of the pricing function. Furthermore, the modelling of option prices under GARCH process by Duan, Gauthier and Simonato (1999), and Ritchken and Trevor (1999) is an attempt to capture the path-dependence of volatility and its negative correlation with return.

It is also possible as in Hull and White (1987), and Heston (1993) *inter alia*, to substitute the constant volatility with the entire joint probability distribution of returns and changes in volatility. Furthermore, the modelling of stochastic volatility is found by Ball and Roma (1994), to be consistent with implied volatility smiles. However, this approach based on the assumption of volatility being a deterministic function of asset price and time faces difficulties of its own. Indeed, the dynamics of the market price of risk are hard to estimate. Furthermore, Dumas, Fleming and Whaley (1998) provide evidence that the predictive power and hedging performance of the deterministic volatility function are not higher than Black-Scholes implied volatility. It is also shown that the estimated deterministic function is itself unstable over time and Black-Scholes hedge-ratios are rather more reliable.

The second class of studies in the literature on implied volatility focuses on its relationship with realized volatility, its information content and significance for forecasting purposes.² Day and Lewis (1992) provide evidence that volatility implied by S&P 100 index options contains useful information for forecasting market volatility. However, it is also found to be inefficient in the sense that the conditional volatility based on GARCH modeling contains incremental information beyond that reflected by implied volatility. Further evidence of inefficiency is provided by Lamoureux and Lastrapes (1993) using volatilities implicit in individual stock option prices. The empirical results from Canina and Figlewski (1993) are also suggestive of the absence of correlation between S&P 100 implied volatility and future volatility. In contrast, the empirical evidence, from Jorion (1995) and Amin and Ng (1997), indicates that implied volatility provides efficient, albeit biased, estimates of future volatility in foreign exchange markets. The implied volatility from S&P 100 option prices is also found by Fleming (1998) to constitute despite its upward bias, a reliable estimate of future volatility. There is evidence that implied volatility is efficient with respect to its past forecast errors and that such errors are orthogonal to parameters usually embedded in ARCH models.

Arguably, the assessment of the relationship between implied volatility and realized volatility is a joint test of the option pricing model and market efficiency. But, the difficulties in reconciling this body of conflicting evidence may also stem from various sources of measurement errors such as model misspecification and the failure to account for dividend payments. As noted by Christensen and Prabhala (1998), there are also “maturity mismatch” problems associated with early studies, which may arise from the assessment of the predictive power (one-day or week-ahead horizons) of implied volatilities calculated

² An interesting review of the literature on volatility forecasting including studies on the predictive performance of implied volatility is provided by Poon and Granger (2003).

from options with much longer time remaining to expiration. Using monthly observations of non-overlapping volatility, they explain away part of the bias in S&P 100 implied volatility with regime shifts in options pricing around the October 1987 stock market crash. The empirical results suggest also that implied volatility is associated with higher predictive power than historical volatility. Using an index of implied volatility based on the S&P 100 options, Blair, Poon and Taylor (2001) find also that implied volatility is more accurate for out-of-sample forecasting than past realized volatility. There is also evidence from Whaley (2000) that this implied volatility index is indicative of the extent of investor fear and stress, thereby reveals useful information about market sentiment. More recently, there is evidence from Giot (2005) that implied volatility indices contain useful information for Value-at-Risk modelling.

In light of the widely documented errors in implied volatility estimation, it is important to reduce measurement errors and to take into account the term structure and nonlinear relation between implied volatility and exercise prices. The new implied volatility index, which provides an aggregate estimate of future volatility, is based on a model-free methodology in order to avoid misspecification problems. The next section briefly describes this model-free approach and examines its distributional properties in the Japanese and US equity markets.

3. DISTRIBUTIONAL PROPERTIES OF IMPLIED VOLATILITY

The new VIX index represents a model-free approximation of a thirty-day return variance swap rate.³ It is based on a hypothetical at-the-money option with fixed expiration and with exercise price equal to the futures on the underlying index. The model-free methodology is

³ Unlike this model-free new VIX index, the original CBOE-VIX index based on the OEX bid-ask quotes on S&P100 American options is calculated using Black-Scholes pricing model. Carr and Wu (2006) provide an insightful analysis of the rationale underlying the calculation of these indices and their major differences.

aimed at gathering information contained not only in the market prices of at-the-money or near-the-money options but in the volatility structure across the prices of a portfolio of in-the-money and out-of-the-money options as well. The implied volatility index is estimated by interpolating the implied volatilities from the nearest and next-term options, which are calculated as follows.

$$v_{i,t}^2 = \frac{2}{\tau_i} \sum_j^{N_i} \frac{\Delta X_{ij,t}}{X_{ij,t}^2} e^{r_f \tau_i} G(X_{ij,t}) - \frac{1}{\tau_i} \left(\frac{F_t}{X_{i0,t}} - 1 \right)^2$$

where τ_i is the time expressed in minutes until the i^{th} maturity of near or next-term options, r_f is the risk-free interest rate until expiration, F is the forward equity index determined from option prices and X_0 is the exercise price immediately below F . For each strip of N_i out-of-the-money options, the exercise price X_j of the j^{th} option corresponds to a call if $X_j > F$ and a put otherwise. The spread between exercise prices ΔX_j is equal to the average difference between the strike prices nearest to X_j . For the highest (lowest) exercise price, ΔX_j is equal to the difference between that exercise price and the next lower (higher) strike price. The closing price $G(\cdot)$ of each option is used to calculate its contribution $\frac{\Delta X_{ij}}{X_{ij}^2} e^{r_f \tau_i} G(X_{ij})$ to the implied volatility index. The contribution to implied volatility is an increasing function of the exercise price for puts and a decreasing function of the strike price of calls. The index uses the nearest and next-term expirations to span the fixed period of 30 days to expiration. As expiration draws near, the rollover to the second and third contract months takes place with eight days remaining to maturity in order to avoid pricing errors in options with imminent expiration. The interpolation process of the nearest and next-term implied volatilities results in a single measure of implied volatility, which is annualized to obtain the new VIX index.

The sample period extends from January 1990 through December 2004, resulting in daily observations and spanning 180 options expirations. There is evidence from Figure 1, which describes the behavior of daily time-series of implied volatility for the S&P 500 and Nikkei 225 indices that expectations of the future level of volatility vary over time and across markets. There is a tendency for implied volatility in the US market to be lower than in the Japanese market and to converge since the late 1990s. The peaks in implied volatility for the Japanese market in the early 1990s coincide with the onset of the economic recession and heightened uncertainty induced by lingering bad debt problems and the appreciation of the yen, among other factors. The similarities in the behavior of these indices are also reflective of major economic events including the Asian financial crisis in 1997 and the Russian debt and LTCM crises in 1998, among others. The burst of the information technology bubble in 2000 is not associated with as significant an increase in implied volatility as that associated with the Latin America debt crisis in 2002. There is a tendency for implied volatilities to increase sharply but decrease rather on monotonous basis. This volatility persistence suggests that shocks to the volatility process are likely to be long-lived.

The distributional moments reported in Table 1 indicate that the average sample implied volatility in the US market is indeed significantly lower than in the Japanese market. Expectations of higher Nikkei 225 volatility may be related to the stronger concerns over the Japanese economy during the post-bubble recession of the 1990s, which are reflected by negative stock market returns. It is noted however that though the means of implied volatilities are significantly different, their variances are not. The test of the null hypothesis for the equality of means indicates also that the magnitude of daily log changes in implied volatility are comparable across markets. In light of the distributional moments and normality tests, there is little evidence that stock market returns and volatility are normally

distributed.

The autocorrelations for the levels of implied volatility reported in Table 2 remain positive, suggesting that the autocorrelation function decays exponentially to zero. This suggests that the dynamics of implied volatility are stochastic and likely to be governed by a long-memory process. In contrast, the evidence of negative autocorrelations for stock market returns and changes in daily implied volatility suggests that the autocorrelation functions are more likely to decay rather in oscillatory patterns. Also, the results of unit-root tests indicate that all time-series of stock market returns and volatility are found to be stationary.

4. MODELLING THE RELATIONSHIP BETWEEN IMPLIED AND REALIZED VOLATILITIES

The standard deviations of returns reported in Table 1 can be converted into annualized volatility estimates which are rather close to the sample averages of implied volatilities (19.40% against 19.88% for the S&P 500 index and 28.22% against 24.76% for the Nikkei 225 index). In order to assess the significance of the long-term relationship between the forward-looking implied volatility index and actual volatility, an *ex post* estimate of realized volatility is calculated as the standard deviation of log returns. This measure is based on a rolling 30-day sample period, which is consistent with the fixed time-to-expiration of the new VIX index's hypothetical option

$$\sigma_{r,t} = \sqrt{\frac{1}{T_t} \sum_{s=1}^{T_t} (r_{t,s} - \bar{r}_t)^2} \quad (1)$$

where T_t is the number of calendar days until maturity, $r_{t,s}$ is the return on a given day s until expiry, and $\bar{r}_t = \frac{1}{T_t} \sum_{s=1}^{T_t} r_{t,s}$ denotes the average return over the monthly sample period. For comparative purposes, this measure of realized volatility is annualized.

These daily measures of realized volatility are plotted against implied volatility for the US and Japanese markets in Figures 2 and 3 respectively. Implied volatility does not

constitute a perfect forecast of realized volatility but the relationship between these *ex ante* and *ex post* measures of volatility tends to be positive. There is evidence from both stock markets that the relationship appears to weaken as the level of volatility increases. The conventional approach adopted by several studies that examine the significance of this relationship is to estimate the following linear regression model

$$\sigma_{r,t} = \gamma_0 + \gamma_1 \sigma_{it} + \xi_t \quad (2)$$

where $\sigma_{r,t}$ and σ_{it} denote the daily estimates of realized and implied volatility, respectively. Based on the sign and magnitude of the regression coefficients, it is possible to assess the extent of the informational content of implied volatility. In the case where $\gamma_0 = 0$ and $\gamma_1 = 1$, implied volatility is an unbiased forecast of realized volatility. It is also an efficient estimate of realized volatility if the residuals ξ_t are white noise.

The estimation results reported in Table 3 are based on white heteroskedasticity-consistent standard errors under the efficiency hypothesis. The slope coefficient γ_1 is found to be positive and significant suggesting that implied volatility contains some information about future volatility. However, it seems to be a biased estimate of subsequent volatility since the F-test rejects the joint hypothesis that the intercept γ_0 and slope γ_1 are significantly different from zero and unity, respectively. There is also evidence of serially correlated residuals, which suggests that implied volatility does not constitute an efficient estimate of actual volatility. Although the relationship between implied and realized volatility remains positive, the extent of unbiasedness and efficiency differ across markets. The regression line lies below the 45° line characteristic of unbiased forecasts for US market, and it is rather flatter for the Japanese market. As noted by Christensen and Prabhala (1998), the negative intercept for the US market may be at least partially, induced by the errors-in-variables problem associated with the implied volatility index.

The positive relationship may be the artifact of the measurement process resulting in overlapping observations of volatility. The error terms ξ_t are indeed likely to show signs of heteroskedasticity due to the serial dependence of the regression variables. In order to reduce measurement errors and inconsistent regression estimators from daily observations, the regression model is reexamined using nonoverlapping observations. The autoregressive process equation (3) which describes the behavior of monthly realized volatility includes implied volatility as an exogenous variable.

$$\sigma_{r,t} = \sum_k \gamma_k \sigma_{r,t-k} + \gamma_i \sigma_{i,t} + \xi_t \quad (3)$$

The two time-series of monthly measures of realized volatility until option expiry and implied volatility one-month ahead are obtained from all maturities of the S&P 500 and Nikkei 225 options from February 1990 to January 2005. The expiration dates differ across these options markets, but the sampling procedure results in 180 nonoverlapping observations for each volatility series.

The estimation results for monthly expirations reported in Table 3 indicate that the inclusion of additional lags in order to correct for serial correlation in the residuals induce a marginal improvement of the explanatory power of the regression models. For both markets, the parameters γ_1 for lagged values of realized volatility are positive but statistically insignificant whereas the γ_i coefficients for implied volatility are significant. These findings are consistent with early evidence that implied volatility does contain some useful information about subsequent volatility.

However, the F-statistics for Wald tests reject the joint hypothesis that $\gamma_k = 0$ and $\gamma_i = 1$ at the 1% level in the US market. Similar test results are obtained for the Japanese market at the 5% level with two lags of realized volatility included into the autoregressive process. Thus, there remains evidence of implied volatility being a biased forecast of future

volatility. Compared to the regression results with overlapping observations, there is however weaker evidence against the efficiency of implied volatility judging from the tests of serial correlation.

5. Forecasting performance of implied volatility against alternative GARCH models

5.1. In-sample forecasting performance

The empirical results reported thus far suggest that implied volatility does contain some information about future volatility but it remains biased and to some extent inefficient. It is important at this point to determine whether alternative GARCH models of market volatility, which have the merit of capturing the clustering properties of market volatility and its persistence over time, do also contain useful information that is not reflected by implied volatility. The conditional mean and variance equations for the standard GARCH-in-mean model can be written as follows

$$\begin{aligned} r_t &= \lambda_0 + \lambda_1 \sigma_{t-1}^2 + \varepsilon_t \\ \sigma_t^2 &= w_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \end{aligned} \quad (4)$$

where the conditional mean, variance and error terms ε_t are conditional on the information set \mathfrak{I}_{t-1} available at time $t-1$. In this standard GARCH-in-mean specification (model (1)), the conditional variance is positive definite and stationary when $\beta > 0$ and $\alpha + \beta < 1$. The persistence of the volatility process increases as approaches unity. It is noted that the variance term is included as a measure of risk into the return generating process in order to account for possible risk premium in stock markets.

Following Day and Lewis (1992), and Lamoureux and Lastrapes (1993), it is possible to assess the additional informational content of implied volatility by including implied variance into the conditional variance equation (model (2)).

$$\sigma_t^2 = w_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \pi_1 \sigma_{i,t-1}^2 \quad (5)$$

The test of significance of π provides evidence on whether implied volatility contains useful information on the dynamics of future volatility beyond that reflected by past volatility levels and innovations. In order to avoid underestimating the significance of implied volatility, a stronger test of information efficiency proposed by Amin and Ng (1997), can be performed by including both the contemporaneous and past implied variance into the conditional variance equation (model (3))

$$\sigma_t^2 = w_0 + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \pi_0 \sigma_{i,t}^2 + \pi_1 \sigma_{i,t-1}^2 \quad (6)$$

The test of the informational content of implied volatility relative to GARCH modelling is straightforward in the sense that if the GARCH terms α and β are both found to be statistically insignificant while π_0 and $\pi_1 = -\pi_0(\alpha + \beta)$ are, then implied volatility is sufficient to convey all past and current information about future volatility.

In addition to volatility persistence, it is important given the growing literature on leverage effects, to account for the asymmetric response of volatility to negative news. The GARCH-in-mean model proposed by Glosten, Jagannathan and Runkle (1993) allows for the volatility dynamics to depend not only on the magnitude of innovations but on their sign as well. This is achieved by estimating the following conditional variance equation (model (4)).

$$\sigma_t^2 = w_0 + \alpha \varepsilon_{t-1}^2 + \delta d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (7)$$

where d_{t-1} is an indicator variable which equals unity when $\varepsilon_{t-1} < 0$ and zero otherwise. Following this GJR approach, it is also possible to examine the hypothesis of whether implied volatility contains useful information beyond that conveyed by past returns by including past implied variance into the conditional variance equation (model (5)).

$$\sigma_t^2 = w_0 + \alpha \varepsilon_{t-1}^2 + \delta d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \pi_1 \sigma_{i,t-1}^2 \quad (8)$$

The significance of π_1 can be interpreted as evidence of the implied volatility conveying

useful information about future volatility beyond that contained in the history of stock returns even upon accounting for the asymmetric impact of shocks on the return-generating process.

The five competing models differ only in their specification of the conditional variance. The models (2), (3) and (5) include implied volatility as explanatory variable in order to assess its informational content relative to referring to the standard GARCH-M model (1) and GJR model (4). Table 4 presents the estimation results of these models with daily sample observations using the quasi-maximum likelihood method. The GARCH terms for the standard model (1) are positive and significant except for the negative parameter α for the Japanese stock market index. The sum of α and β parameters is close to unity, suggesting volatility clustering in both markets. There is however no evidence of a significant risk premium over the sample period.

The parameter π_1 associated with past implied volatility in model (2) is found to be positive and significant for both the US and Japanese markets. This evidence suggests that implied volatility provides incremental information about future volatility that is not contained in the history of returns as described by various GARCH-M models. The GARCH terms α and β are statistically insignificant for the US market whereas the magnitude of the parameter β decreases for the Japanese market. This evidence on information efficiency is consistent with the empirical findings by Day and Lewis (1992), Harvey and Whaley, (1992) and Lamoureux and Lastrapes (1993), inter alia. In order to avoid underestimating the significance of implied volatility, model (3) further includes the contemporaneous observations of implied variance as explanatory variable. The estimation results indicate that the parameter π_0 for current implied variances is positive and significant but the parameter π_1 for the lagged observations of implied variances becomes negative for both markets. It is interesting to note that GARCH effects remain significant in the presence of

past and contemporaneous implied volatility. These results suggest that implied volatility is reflective of some but not all information on future volatility contained in historical returns.

There is evidence based on the sign and magnitude of the δ parameters in the estimation of model (4), which accounts for the asymmetric effects of bad news on market volatility, that the leverage effects are significant in the US stock market but insignificant for Japanese equity. The incorporation by model (5) of past implied variances as explanatory variable has again the effect of altering the significance of GARCH terms. The significant parameter π_1 for past implied volatility is indeed associated with lower degree of volatility persistence for both markets. However, the asymmetric effects of bad news are found to be significant in the presence of past implied volatility. These empirical results suggest that even after allowing for leverage effects, implied volatility does still reveal some useful information about future volatility that is not reflected by historical returns. It does not reflect all information contained in the historical series though.

Judging from the estimates of the log-likelihood functions, it is the GARCH-M model (3) including past and current implied variances that performs better than alternative models for both markets. But among the estimation models, which do not include implied volatility, it is the GJR model accounting for potential leverage effects that performs marginally better than the standard GARCH-M model.⁴

5.2. Out-of-sample forecasting performance

The results reported so far imply within-sample forecasts which assume that parameters

⁴ The models of volatility dynamics were also estimated using monthly time-series for options expirations. The results, which are not reported here for the sake of brevity, are qualitatively similar to the estimation results from daily data, though the GJR models are no longer associated with an excess log-likelihood over GARCH-in-mean models for both the US and Japanese markets. This is possibly due to the lower monthly data frequency, which does not fully capture the impact of leverage effects on the volatility dynamics.

estimates are stable over time. The economic usefulness of the implied volatility index in forecasting future market volatility may be judged in light of out-of-sample tests of robustness. Furthermore, the forecasting performance of implied volatility can be assessed against alternative models describing volatility dynamics. With the focus being restricted to GARCH models where implied volatility is not included, it is the GJR model that is used for comparative purposes given its higher log-likelihood function relative to the standard GARCH-M model. It is noted that the forecasting performance of GARCH models may be affected when the underlying assumption of variance stationarity is violated. It can be also sensitive to small sample periods. Hence, the out-of-sample forecasting of volatility over the five-year period from 2000 to 2004 is performed using implied volatility as well as GJR model estimates over the entire 1990s decade.

There is evidence from forecasts based on implied volatility and GJR models shown in Figures 4 and 5 for the US and Japanese markets respectively, that both forecast series represent close approximations of realized volatility. There is also a tendency for implied volatility in the US market to remain at levels above realized volatility. This result is consistent with evidence from Fleming (1998) that the implied volatility from S&P 100 options is upwardly biased. The tendency for volatility in the US market since 2003 to remain below the 20% upper boundary is suggestive of regime switches in market volatility. The sharp increase in realized volatility associated with the Latin American debt crisis in 2002 is also reflected in the forecast series, albeit with some time lag. This constitutes evidence that implied volatility is able to capture not only the impact of domestic shocks but the volatility spillover from international stock markets as well.

Figure 6 shows the time-series of forecast errors relative to realized volatility, with positive errors indicating the overestimation of market volatility. There is evidence from both markets that the implied volatility and GJR forecasts fluctuate around zero, with a

more obvious upward bias for the US market since 2003. While the forecast errors are rather close for the US market, they seem to depart from each other for the Japanese series. The spread in forecast errors seems to be more significant when both series overestimate future volatility than when they underestimate it. Judging by the sharper increases in GJR forecasts, there is thus evidence that inference from GARCH models, which assume variance stationarity, are more sensitive to large deviations of stock returns from the long-run mean. In contrast, implied volatility seems to constitute a smoother forecast of future volatility. It is more reflective of patterns in realized volatility than the GJR conditional volatility model.

In order to evaluate the predictive accuracy of implied volatility relative to GJR forecasts, it is possible to compute the proportion of explained variability P following Blair, Poon and Taylor (2001)

$$P = 1 - \frac{\sum (\sigma_{r,t} - v_t)^2}{\sum (\sigma_{r,t} - \bar{\sigma}_r)^2} \quad (9)$$

where v_t denotes the time-series of implied volatility or GJR forecasts depending on the model being used, and $\bar{\sigma}_r$ represents the mean of realized volatility over the forecast period. This measure of predictive power estimates the sum of squared forecast errors relative to variations in actual volatility. It approaches unity when forecast errors are small and can take the negative sign when forecast errors have greater variability than actual volatility.

The estimated P measures of explained variability for the US market amount to 0.566 for implied volatility and 0.520 for GJR forecasts. For the Japanese market however, these measures take the lower estimates of 0.297 for implied volatility and -0.040 for GJR forecasts. This negative P value suggests that inferences from the GJR model are likely to show more variability than the actual fluctuations of market volatility. This is not a desirable feature of a forecasting model, and it can be due to the stronger sensitivity of GJR modelling to large shocks to equilibrium returns. The lower P-values for the Japanese market relative

to the US estimates may be related to the stronger tendency for options mispricing following the inception of trading on Osaka Securities Exchange. The existence of arbitrage opportunities has the potential of introducing a degree of bias in implied volatility that ultimately affects its forecasting performance. Inference from GJR models based on the shorter estimation period from 1995 to 1999 where both VIX indices tend to converge is associated with a lower P statistic of 0.462 for the US market but a positive estimate 0.038 for the Japanese market. The discrepancies in predictive power may also result from structural shifts in the level of uncertainty associated with the Japanese economic recovery.

Judging by the higher P measures of VIX indices relative to GJR models for both markets, the implied volatility index offers more accurate forecasts of future market volatility. This empirical evidence is also consistent with Blair, Poon and Taylor (2001) results based on the implied volatility from S&P 100 index options. Part of the explanation for the higher predictive performance of implied volatility may lie in, *inter alia*, the absence of restrictions on variance stationarity and the model-free methodological approach applied to aggregate expectations about future volatility. It is equally plausible that this evidence pertains to the forward-looking nature of implied volatility, the formation of expectations and the information efficiency of options markets. Addressing these issues is important as they relate to the determinants of implied volatility, but they lie beyond the scope of this paper.

6. CONCLUSION

The principal objective of this study was to examine from an international perspective, the stochastic properties and forecasting performance of stock market volatility implied in options prices. For this aim, we developed an index of implied volatility heretofore unavailable for the Japanese equity market. The Nikkei 225 implied volatility index was constructed following the methodology underlying the calculation of the new

VIX index for S&P 500 benchmark. These implied volatility indices are useful in providing a volatility measure with data frequency comparable to that of benchmark stock prices. This estimate of volatility may not be as accurate as the level of stock prices due to the complementarity of these parameters. Such measurement problems are however akin to the difficulties in quantum mechanics of ascertaining with accuracy both the location and velocity of objects.

The empirical analysis provided some insights into the stochastic dynamics and the relationship between implied volatility and realized volatility. Despite its upward bias, the implied volatility index is found to be reflective of changes in actual volatility. There is also evidence that implied volatility reveals some but not all information on future volatility contained in historical returns. The tests of out-of-sample performance indicate that implied volatility represents a smoother forecast of future volatility. This higher performance contrasts with inference from GARCH models, which assume variance stationarity and are more sensitive to large deviations of returns from the long-run mean. Judging from the measures of predictive power, the implied volatility index provides more accurate forecasts of future market volatility.

The empirical evidence and the development of the new implied volatility index for the Japanese equity market have some bearing for market regulation, policy-making and risk-hedging decisions and open interesting avenues for future research. The implied volatility index can be helpful in examining the impact of the release of macroeconomic information on the dynamics of implied volatility using for instance, event-study methodology. These studies can contribute to a better understanding of market sentiment, investor confidence, and the perceptions by market participants of economic uncertainty. Drawing parallel with the leverage effects in stock market returns, future research may also explore the asymmetric effect of bad news on the implied volatility index. Furthermore, the

economic significance of implied volatility forecasts can be also assessed in the VaR context of risk-hedging. Finally, by providing a model-free measure of uncertainty, the implied volatility indices can throw light on the debate over what constitutes excessive market volatility. It can thereby benefit research on the impact of financial crises and help bring a broader perspective to the issues and controversies related to margin regulation and stock market volatility.

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TABLES

TABLE 1. Distributional properties of implied volatility and stock market returns

	S&P 500 series			Nikkei 225 series		
	Stock returns	Implied volatility	Log IV differences	Stock returns	Implied volatility	Log VIX differences
Mean	0.0003	0.1988	-0.0001	-0.0003	0.2476	0.0002
Std. Dev.	0.0102	0.0635	0.0550	0.0148	0.0647	0.0642
Minimum	-0.0711	0.0931	-0.2751	-0.0723	0.0769	-0.6597
Maximum	0.0557	0.4574	0.4169	0.1243	0.6376	0.9382
Skewness	-0.1031	0.9063	0.6082	0.1961	0.6528	2.0064
Kurtosis	6.8986	3.7215	6.8647	6.3530	4.1171	40.1397
Jarque-Bera	2485.06	620.70	2676.43	1858.13	481.53	227517.89

Notes: The sample period extends from January 2, 1990 to December 31, 2004. JB is the standard Jarque-Bera normality test distributed as χ^2 on the null. The series of stock market returns and changes in implied volatility are calculated with log differences.

Table 2. Autocorrelation and stationarity tests of implied volatility and stock returns

	Autocorrelation function					Stationarity tests	
	φ_1	φ_2	φ_3	φ_{10}	φ_{25}	ADF	LB
S&P500 index							
Stock returns	-0.003	-0.019	-0.035	0.022	0.003	-37.904 ^b	0.222
Implied volatility	0.981	0.964	0.950	0.889	0.782	-4.220 ^a	0.401
Changes in VIX	-0.059	-0.074	-0.069	0.050	0.048	-23.007 ^c	1.000
Nikkei 225 index							
Stock returns	-0.020	-0.051	0.001	0.007	0.032	-46.979 ^c	0.790
Implied volatility	0.966	0.943	0.923	0.823	0.669	-5.937 ^b	0.121
Changes in VIX	-0.166	-0.044	-0.045	-0.002	0.053	-28.187 ^c	0.290

Notes: The sample period extends from January 2, 1990 to December 31, 2004. ADF are the Augmented Dickey-Fuller test statistics, using Schwarz information criterion with additional lags included to eliminate any remaining ARCH effects. The 1% critical values are -3.966, -3.435 and -2.566 for unit root tests with ^(a) trend and intercept, ^(b) with intercept only, and ^(c) with neither. LB refers to the p-values for Ljung-Box Q-statistics distributed as χ^2 on the null of no autocorrelation up to the 10th order.

Table 3 . Modelling the relationship between realized and implied volatility

Regression parameters	γ_0	γ_i	γ_1	γ_2	Wald Test	DW Test	LB(1)	LB(4)	Adjusted R^2
	Daily observations								
S&P 500	-0.0209 (0.000)	0.9916 (0.000)			526.882 (0.000)	0.091	3566 (0.000)	12310 (0.000)	0.583
Nikkei 225	0.0640 (0.000)	0.8070 (0.000)			108.689 (0.000)	0.068	3654 (0.000)	13196 (0.000)	0.266
	Monthly expirations								
S&P 500	-0.0205 (0.196)	0.9999 (0.000)			22.489 (0.000)	1.738	3.027 (0.082)	8.358 (0.079)	0.596
	-0.0191 (0.243)	0.9282 (0.000)	0.0733 (0.470)		15.071 (0.000)	1.868	0.765 (0.382)	4.097 (0.393)	0.598
Nikkei 225	0.0401 (0.087)	0.8808 (0.000)			3.072 (0.049)	1.472	12.450 (0.000)	13.222 (0.010)	0.364
	0.0472 (0.039)	0.9002 (0.000)	0.1446 (0.164)	-0.1938 (0.013)	2.839 (0.026)	1.779	1.036 (0.309)	3.644 (0.456)	0.394

Notes: Estimation of the regression model with white-heterostkedasticity consistent standard errors. The sample of daily observations includes rolling volatility series over the period January 1990 to December 2004. The sample of monthly observations includes 180 non-overlapping expirations covering the period from February 1990 to January 2005. Asymptotic p-values for regression coefficients reported in parentheses. F-statistics for Wald Test of the null of $(\gamma_0 = 0, \gamma_i = 1)$ and for regression models with additional lags, the null of $(\gamma_k = 0, \gamma_i = 1)$ with the associated p-values reported in parentheses. DW is the Durbin-Watson statistic for the test of first-order serial autocorrelation. LB(k) denotes Ljung-Box Q-statistics distributed as χ^2 on the null of no serial correlation up to lag k, with corresponding asymptotic p-values reported in parentheses.

TABLE 4. GARCH modelling of stock market volatility in the US and Japanese markets

Model Parameters	S&P 500 stock index					Nikkei 225 stock index				
	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)
λ_0	0.0260	0.0289	0.2221 ^a	0.0133	0.8448	-0.0776 ^c	-0.1002 ^b	0.0684 ^c	-0.0921 ^b	-0.1047 ^a
λ_1	3.7734	0.1575	-19.7920	1.8391	0.0197	4.6164 ^c	5.0157 ^b	-4.0209 ^c	2.9588	3.2893 ^c
w_0	0.0005	-0.0109 ^a	-0.0010 ^c	0.0010 ^a	-0.0013	0.0061 ^a	0.0052	0.0008 ^c	0.0050 ^a	0.0005
α	0.0538	-0.0076	0.0165 ^b	0.0059	-0.0355 ^a	0.0833 ^a	0.0974 ^a	0.0096 ^c	0.0220 ^b	-0.0236 ^a
β	0.9421 ^a	-0.0054	0.8856 ^a	0.9347 ^a	0.7231 ^a	0.8899 ^a	0.7136 ^a	0.9761 ^a	0.9016 ^a	0.7089 ^a
π_0			0.0038 ^a					0.0038 ^a		
δ				0.1020 ^a	0.1743 ^a				0.1069	0.2130 ^a
π_1		0.0025 ^a	-0.0035 ^a		0.0005 ^a		0.0005 ^a	-0.0037 ^a		0.0007 ^a
LB	0.225	0.338	0.507	0.279	0.290	0.956	0.837	0.878	0.967	0.781
LM	0.723	0.268	0.326	0.832	0.489	0.790	0.225	0.050	0.479	0.280
Log-L	12913.33	12994.84	13133.58	12955.37	13009.80	11200.90	11226.22	11304.84	11242.72	11280.27
Excess log-L		81.51	220.25	42.04	96.47		25.32	103.94	41.82	79.37

Notes: The sample period of daily observations extends from January 1990 to December 2004.

The model (1) refers to the GARCH-M model equation (4), model (2) to the GARCH-M with past implied volatility equation (5), model (3) to the GARCH-M with current and past implied volatility equation (6), model (4) to the GJR-GARCH-M equation (7) and model (5) to the GJR-GARCH-M with past implied volatility equation (8). The various GARCH models are estimated with Bollerslev-Woodridge robust standard errors. The parameters estimates λ_0 and w_0 are multiplied by 10^2 and 10^3 , respectively. Significance at the 1, 5 and 10% level is denoted by ^a, ^b and ^c, respectively. LB refers to the p-values associated with Ljung-Box test for serial correlation in squared residuals up to the 10th order. LM refers to p-values of the Lagrange-Multiplier test for neglected ARCH effects. Both tests are distributed as χ^2 on the null. LogL is the estimated log-likelihood function. Excess log-l is the excess log-likelihood with respect to model (1).

FIGURES

FIGURE 1. The behavior of implied volatility in the Japanese and US markets

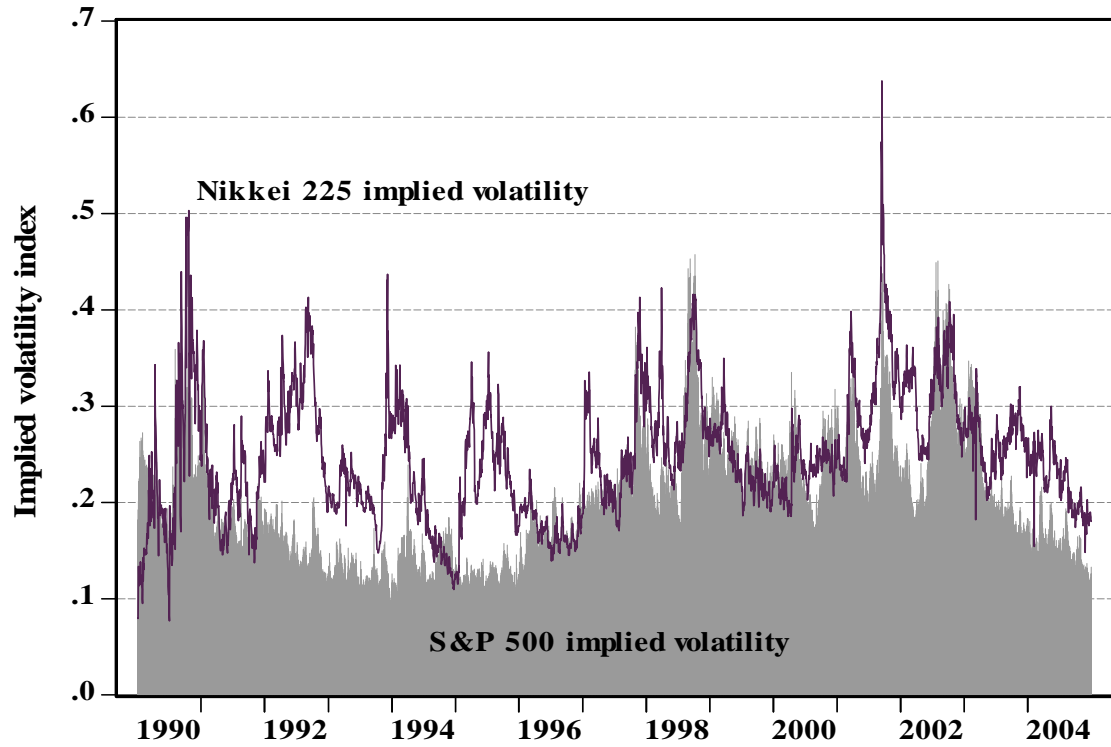


FIGURE 2. The relation between implied and realized volatilities in the US market

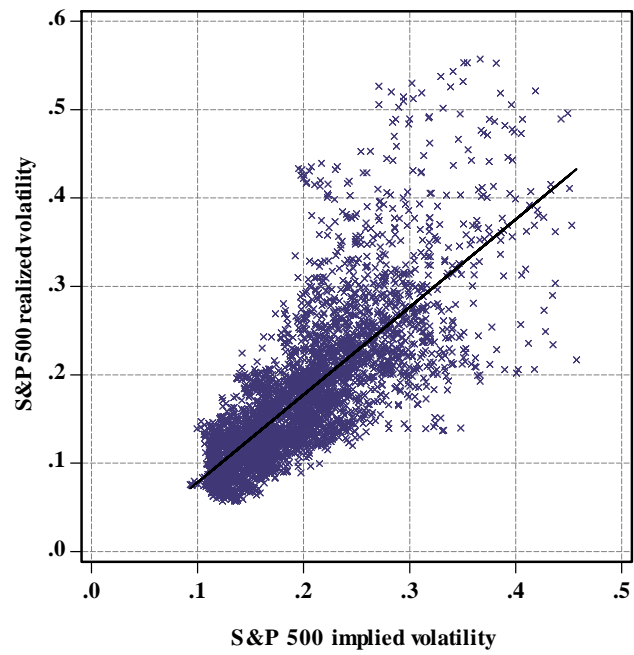


FIGURE 3. The relation between implied and realized volatilities in the Japanese market

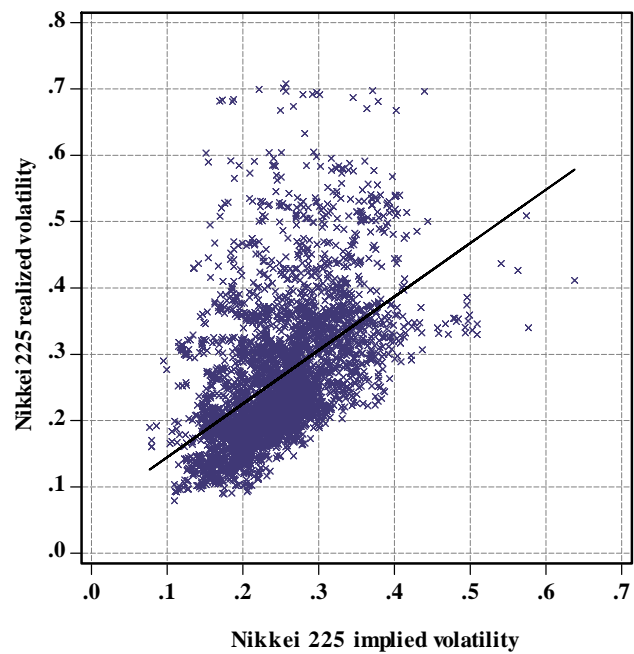


FIGURE 4. Out-of-sample forecasting of the US stock market volatility

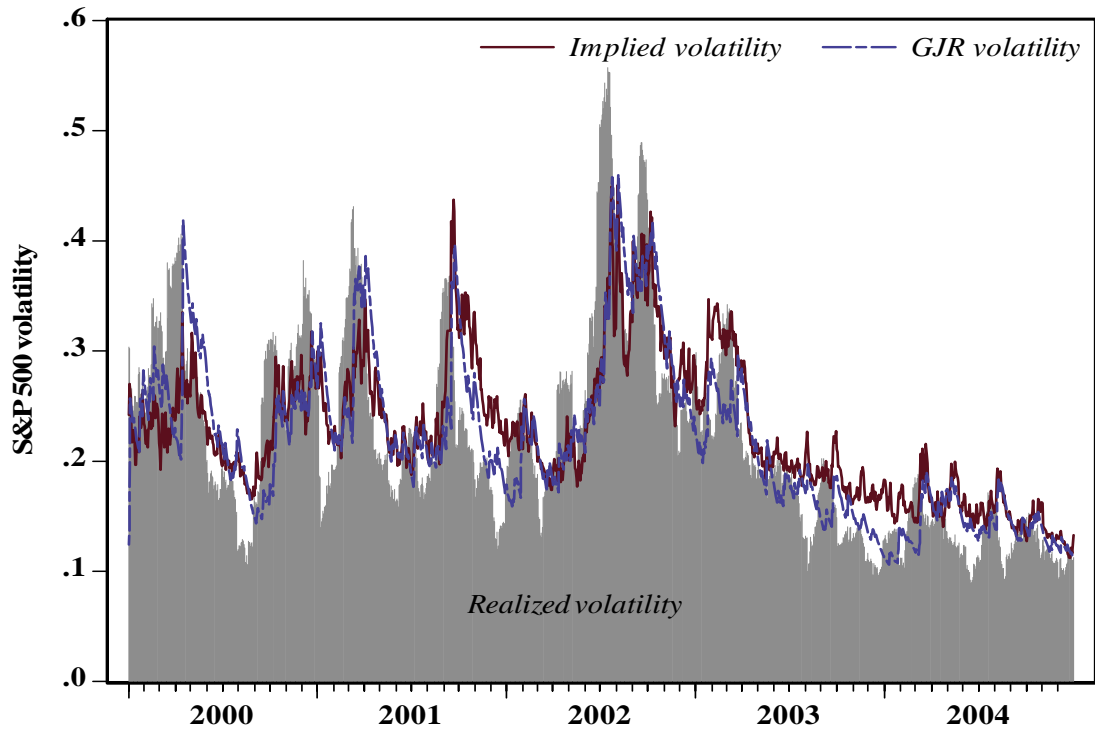


FIGURE 5. Out-of-sample forecasting of the Japanese stock market volatility

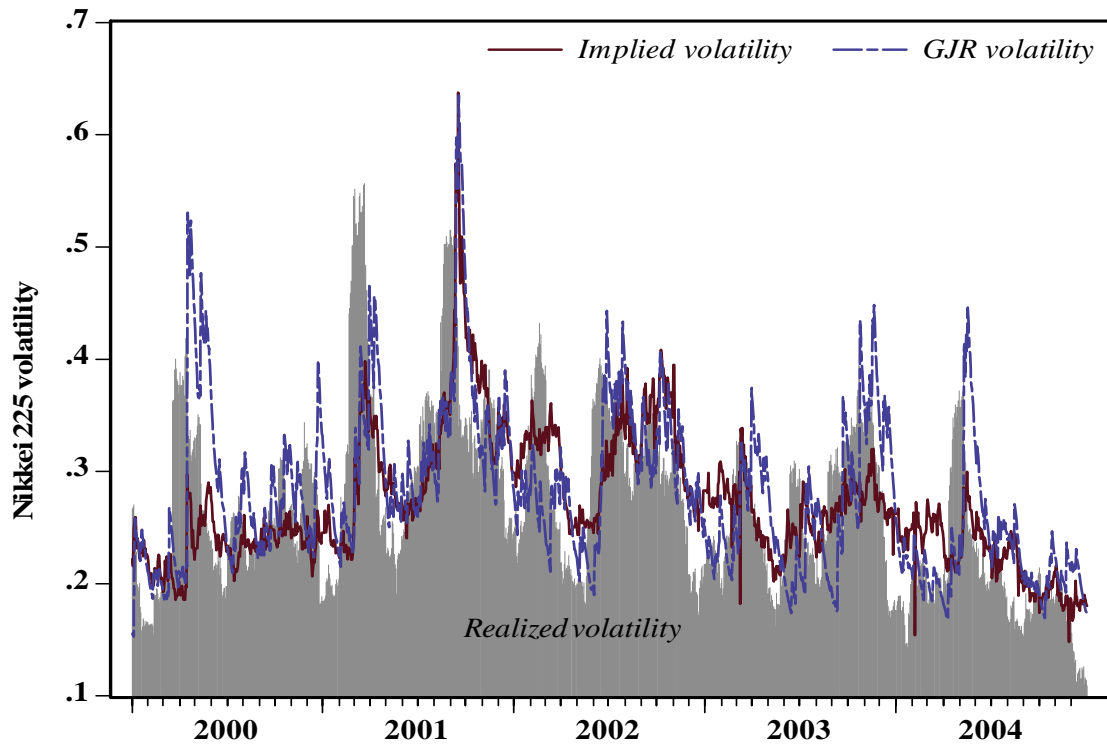


FIGURE 6. Out-of-sample forecast errors of stock market volatility

