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Love and Addiction: The Importance of Commitment

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LOVE AND ADDICTION: THE IMPORTANCE OF COMMITMENT[†]

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ABSTRACT. The bilaterally monopolistic nature of relationships between partners, combined with the addictive nature of love, which represents the emotion people feel during the course of a relationship, results in love growth when relationships are based on commitment. However, in relationships with less than perfect commitment, love will wither, and potentially die. In this paper, the path of love through a relationship is examined under the assumption that love is addictive and that partners may or may not be able to commit to a relationship. The differences in the results suggest that cohabiting partners may, by their very lack of commitment at the outset, be unsuited to successful marriages.

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1. INTRODUCTION

Recent trends in social attitudes (mirrored by the establishment of unilateral divorce laws, often referred to as no-fault divorce laws), and economic independence (shown by the reduction in the wage differential between males and females), have changed the relationship dynamics between potential suitors and couples in the marriage market. In particular, divorce is far more common than it was in the past. As suggested by Becker, Landes & Michael (1977), Sander (1985), and Sweezy & Tiefenthaler (1996), an increase in female earnings is associated with an increase in the probability of divorce; however, that result may be, as Becker et al. (1977) suggested, due to self-fulfilling expectations.[1] Research by Johnson & Skinner (1986), however, does not lend support to the self-fulfilling expectations hypothesis.[2]

Another possible reason for increases in the divorce rates is due to changes in the legal institutions surrounding marriage. The effect of the change in the divorce laws in many states, for example, has effectively reduced the cost of divorce, and, therefore, should have increased the divorce rates in those states. However, the empirical link has not been established, e.g., Peters (1986) and Gray (1998), at least for any extended period of time, e.g., Wolfers (2000). Recent research by Binner & Dnes (2001) has been the exception; they use time series data to show that the introduction of no-fault divorce laws has increased the rate of divorce in England and Wales.

Theoretically, the difficulty in establishing a link between easier divorce availability and increased divorce in the aggregate is not surprising. A reduction in the cost of exit should lead to an increase in the number of entrants, thus increasing the divorce rate denominator; therefore, the overall divorce rates may remain quite stable. However, that theoretical possibility does not square with data showing relatively stable marriage rates at the same time, although those marriages, on average, began at later ages than in the past, Schoen & Standish (2001).[3]

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[1]Females expecting a divorce may begin more earnest participiation in the labor market. Their subsequent reduction in home activities may, in turn, lead the male to instigate a divorce.

[2]Their simultaneous equations estimation shows that an increased future probability of divorce raises female labor supply, while increases in female labor supply do not lead to increases in the probability of divorce.

[3]They find that 83% of men and 88% of women will eventually marry, but men and women are getting married at later ages. Interestingly, marriages that begin at later ages are less likely to

A more likely possibility is that some substitution between formal marriage and cohabitation has taken place. The reduction in social stigma associated with cohabitation has made entry into less formal arrangements less costly relative to marriage. Simultaneously, the relative exit cost of a relationship will be lower for cohabitation, despite the reduction in divorce costs. The reduction in the relative entry costs combined with the increase in relative exit costs has probably kept the actual divorce rate relatively constant. However, empirical examinations do not readily support the relative cost of entry versus exit hypothesis, either. For example, Waters & Ressler (1999) find: a) higher divorce rates are associated with an increased likelihood of cohabitation, and b) higher rates of cohabitation are associated with increased divorce probabilities.

Despite the absence of direct empirical support, at this time, that relative entry and exit costs matter, there does seem to be anectdotal support for the hypothesis, especially concerning the reduction in the relative social cost associated with cohabitation. A survey of cohabitants, non-married couples living together, by Ressler & Waters (1995), revealed many who viewed their arrangement as a *trial marriage*.[4] According to Berrington & Diamond (2000), nearly 60% of first-partnership cohabitations in Britian become marriages by the time respondents reach age 33. Evidence presented by Cherlin (1992) and Bacrach, Hindon & Thomson (2000) lend further credence to an increase in the rate of cohabitation; however, they suggest that these cohabitants, most of whom will eventually marry each other, are more likely to enter into a marriage that will end in divorce.

The preceding data present a puzzle not easily explained by current models of marriage and households. Importantly, if cohabitation is meant to be a trial marriage, it would be reasonable to presume that only successful trials eventually lead to actual marriage, and those marriages ought to be more successful as well. However, the data suggests otherwise, i.e., divorce rates actually increase with rates of cohabititation. In other words, there may be a fundamental difference between cohabiting partners and marriage partners, such that the experience of cohabitation

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end in divorce, which may partly explain the leveling of the divorce rate at approximately 43% of marriages.
[4]Within the survey, 51% of the men and 56% of the women were cohabitating as a compatibility check.

cannot be easily transversed preceding the marriage, resulting in optimal behavior that is Pareto dominated.

Below, I present a model suggesting there is an important difference between cohabiting, non-committed couples, and married, committed partners. The difference, the commitment to provide love, leads to a growing love relationship for some, while a lack of commitment amongst cohabitants leads to opportunistic, using behavior amongst partners. Members within the cohabiting relationship, fully cognizant of the potential to be used, underinvest in their relationships. Because there is less love without commitment than with commitment, cohabiting partners may be less successful marriage partners than those whose partnership was initially based on a commitment, e.g., a marriage.[5] In Section 2, I continue by marrying, pun intended, the firm literature with the family formation literature. A simple two-period model is presented in Section 3 and analyzed in Section 4. The conclusion of the paper, in Section 5, discusses limitations and useful extensions of the present analysis.

2. LITERATURE

The literature on family formation can be traced to three pioneering papers: Becker's (1973) seminal paper on marriage, Becker's (1974) paper on the influence of market conditions on different marriage formations, and Becker et al.'s (1977) paper on marriage dissolution. Many contributions have been made to the understanding of families, because of these three papers. Marriages are most likely to occur between people with similar traits.[6] Furthermore, marriage in the Beckerian formulation is due to expectations of supermodularity in the household production function.[7] Poor realizations of the random variables, over which the expectations were originally made, are important determinants of dissolution of the couple.[8]

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[5] The distinction between marriage and cohabitation, based on level of commitment, cannot apply to all marriages and cohabitations, but is used for exposition purposes. This research does, however, beg another question, what will lead to perfect commitment? Thoughts on that will be discussed later.

[6] Nakosteen & Zimmer (2001) provide evidence of positive marital matching on the basis of similarities between earnings.

[7] The expectation is that combined production will be greater than the sumtotal of individual productivities due to economies of scale resulting from specialization in various household production activities, Shimer & Smith (2000).

[8] Support for the dissolution hypothesis has been provided by Boheim & Ermisch (2001). Their results suggest couples who experience an improvement in economic conditions are less likely to get a divorce than couples who do not experience an improvement.

The Beckerian approach does not, generally, distinguish between cohabitation and marriage; however, Becker, himself, alludes to long-term contracts and the importance of commitment, "Since married women have been specialized to child-bearing and other domestic activities, they have demanded long-term 'contracts' from their husbands to protect them from abandonment and other adversities."[9]

Along the preceding lines, Murphy (2002) has developed an institutional model of marriage, wherein marriage is a public signal and is costly to exit, while informal relationships are not costly to exit. Divorce, in his model, creates a costly tag, which makes it more difficult for men to be remarried. However, this theory does not accord well with the fact that men are far more likely to be remarried following a divorce than women, especially if there are children.[10]

The firm formation literature, on the other hand, can be traced to the seminal work of Coase (1937), who suggested that firms exist as an institution to reduce the transcactions costs associated with spot markets.[11] Although many different types of transactions costs exist, the focus of much of the research in this area was on the *ex post* hold-up problem.[12] Due to the hold-up problem, i.e., a firm takes advantage of an asset specific investment to charge another firm more than was originally agreed upon, the firm that is held-up may find it in their best interest to purchase the firm doing the holding-up, or vice-versa.

Applying the same logic to households, formations might occur in order to reduce the transactions costs associated with market transactions. In particular, reproduction may not lend itself well to the market process. According to Becker's (1973) theory, households form, in part, to take advantage of increased specialization possibilities and related economies of scale. It is likely that specialization,

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[9]See Becker (1991) p. 137.

[10]Divorced women with children are 25% less likely to remarry than women without children (Da Vonza & Rahman (1994) from Bergstrom (1996)). Divorced men remarry at a rate of about 78%, while, for women, the remarriage percentage is only 69% (Schoen & Standish (2001)).

[11]A plethora of research has resulted from Coase's (1937) initial insight. Much of the discussion within the "Nature of the Firm" research is beyond the scope of this paper, however, an excellent overview of the effect of Coase's insight can be found in Williamson & Winter (1993), Holmstrom & Tirole (1989), and Williamson (1989), where the reader is directed.

[12]The classic case study was that of the purchase of Fisher Body by General Motors in 1926, which was thought to have occurred due to the desire of GM to eliminate the contractual hold-up problems resulting from required asset specific investments. However, as Casadesus-Masanell & Spulber (2000) have discovered, an initial 60% purchase was made in 1919, and the remaining purchase was made to improve the alignment of production and inventories as well as to provide GM with access to the talents of the Fisher brothers.

within a household, could increase transactions costs, especially in the underinvestment or hold-up categories. The model in this paper takes advantage of the potential for hold-up in a relationship between two individuals that are specialized in their household production; one is the provider of love, while the other is the recipient of love.[13] Because love is complementary through time and a provider has within-match market power, the result of the model presented here is similar to the results presented in a number of papers concerning complementary monopoly.[14]

Under the assumption that commitments cannot be guaranteed, insufficient investment is undertaken. Therefore, in this model, the hold-up is in terms of love. After people meet in the marriage market, so affectionately called the meat market, they begin relationships. Those relationships require large investments in time and, specifically, love. Love is addictive, in the Iannaccone (1986) and Becker & Murphy (1988) sense, simlpified here as more love today leads to a greater desire for love in the future. Since the addicted individual is willing to 'pay' more for future love, the love monopoly provider has an incentive to cheat on future provisions of love by, for example, withholding love in an effort to withdraw even larger payments from the love addict. Given the incentive to cheat in later stages of the partnership, the partners will rationally choose to underinvest in love, initially. Therefore, marriage, or some other commitment device, as an institution, may Pareto dominate any other alternative, such as cohabitation.[15]

3. THE MODEL

Consider the behavior of one of the members of a couple, where the couples are assumed to be symmetric, and, therefore, the analysis of one partner is sufficient for understanding behavior within the partnership. There are two types of partnerships, commitment and non-commitment, which are meant to encompass marriage and cohabitation, respectively. It is assumed that the partners have already been matched; this model does not include a matching equilibrium. Rather, the model provides insight into the behavior of individuals within the match.

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[13]Of course, each individual in the match is assumed to be a provider and recipient; however, only one side of the match is analysed.

[14]A recent paper providing an excellent discussion is by Feinberg & Kamien (2001).

[15]Importantly, marriage and cohabitation are two terms meant to capture the level of commitment within a relationship, not necessarily the legal component of the relationship.

Following the formation of the couple, each partner has within-partnership market power, i.e., one partner is presumed to be the only person capable of providing the love sought by the other partner. Furthemore, love is an addictive commodity.[16] Because love is addictive, there are important intertemporal components, and, therefore, the match must be followed through time. For this reason, the match is examined over two periods.[17]

3.1. **Desire.** Each partner is assumed to desire love from the other partner. The love they desire is assumed to be addictive, i.e., current positive love experiences increase the desire for future love experiences. The addictive nature of love is dynamic, and, therefore, the dynamic demand for love is considered over two periods, the shortest allowable time period for the incorporation of dynamics. Generically, inverse addictive demand, based on the model by Becker & Murphy (1988), for love in any period t is given by

$$(1) \qquad p_t = p_t(q_{t-1}, q_t, q_{t+1}^e; \alpha),$$

where $\alpha < 1$ captures the degree of complementarity across time periods. Demand today depends on the past as well as the expectation of future love provision. The discussion below will focus on the inverse demands, p_1 and p_2, and, therefore, the quantities q_0 and q_3 need to be pinned down. The easiest interpretation is that individuals come into the relationship with an established amount of love for each other, q_0, and that their relationships end after two periods, so that q_3 is zero. However, in order to appeal to symmetry in the solutions below, love outside the match period is also assumed to be symmetric, so that $q_3 = q_0 = \bar{q}$.[18]

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[16]Iannaccone (1986) provides conditions on utility, called adjacent complementarity, and derived demand, which lead to addiction, as used here. Intertemporal complementarity has a long history in the economics literature dating back to Marshall's (1920) discussion of the likelihood that habits, or past consumption experiences affect current consumption decisions. In addition, a number of ideas that have appeared in the economics literature: habit formation, Pollak (1970); addiction, Becker & Murphy (1988); learning-by-doing, Arrow (1961), and consumer switching costs, Klemperer (1995) are all models involving intertemporal complementarity, Koch (2001).

[17]An interesting extension is the potential for match-breaking in each period. Presumably, that would reduce the monopoly power assumed in this formulation, thus decreasing the ability of one partner to use the other. On the other hand, one partner may have more options outside the match than the other, which could actually increase the market power of one partner within the match.

[18]Symmetry outside the match could be interpreted as a match market equilibrium. For example, in order for a match to occur, love, or the interest therein, can never be less than some amount $\bar{q}$. In fact, an assumption that no love is initially brought into the relationship so none can be taken away at the end, would be the easiest interpretation. For example, it could be assumed that

This desire curve, although simple, is enough to elicit the important differences between a monogamist, and a user. Throughout the paper, desire is assumed to be linear, so that,

(2) $$p_t(\cdot) = A + \alpha q_{t-1} - q_t + \alpha q_{t+1}.$$

Addiction or intertemporal complementarity is assumed to be symmetric with regard to future expectations about love, as well as past experiences with love. Unfortunately, interpretational difficulty lies with p, the price paid for love. Preferably, the willingness to pay for love represents the willingness of one partner to sacrifice their own pursuits for their mate, a standard opportunity cost.

3.2. **Supply.** Each partner is presumed to provide love to their partner. However, despite the potential for outside options in the partnering market, once the partnership has been formed, each of them has within-match market power; it is assumed that your mate is the only one allowed to provide love to you. Providers of love are not entirely selfish, since they receive felicity from their partner's happiness, which is a parameterized measure of the love provided, as well as the returns to the love provided. Therefore, the provider of love's felicity is described by

(3)
$$u(q_t) = \sigma v(q_t) + \pi(q_t)$$
$$= \sigma v(q_t) + (p_t - c)q_t$$

where $\sigma \cdot v(q_t)$, given $0 \leq \sigma \leq 1$, measures happiness from providing love to your partner, interpreted as the degree to which one cares about one's partner.[19] It is assumed that $v(q_t)$ is quadratic and concave, $v' > 0$, $v'' < 0$, and $v''' = 0$. However, $\sigma \cdot v'(0) \to c$ is required for the opportunity cost of love production to remain non-negative, i.e., for the match to exist. The opportunity cost of providing additional love, denoted by c, is assumed to be constant.[20] In order for love to be provided in the match, $A > c$ is assumed, where A represents the extent of love.

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love is not offered before the match, so that $q_0 = 0$, and the relationship ends in death after two periods, so that $q_3 = 0$.

[19]As $\sigma \to 0$, the supplier of love is completely selfish, while larger values of σ imply more selflessness.

[20]The inclusion of a fixed cost could be used to model outside options for the provider, but it would not affect behavior other than influencing actual participation in the match. For that reason, outside options (and fixed costs) are ignored in the following analysis.

Given the fact that current felicity depends on past and future love provision, it is necessary for the monogamist to consider happiness over the entire match,

$$(4) \qquad U = \sum_{t=1}^{2} \beta^{t-1} \sigma v(q_t) + [p_t - c]q_t,$$

where, under the assmption of perfect capital markets, $\beta = 1/(1+r)$ is the market determined valuation of future flows of happiness.

4. RESULTS

In this section, the properties of love provision are examined under two different circumstances, commitment and non-commitment. Although the commitment device is not defined, it is assumed to be perfect, i.e., the lover will not deviate from the optimal path of love. However, non-commitment is meant to imply the potential for deviation from the prescribed path.

The primary result from the analysis is that commitment matters. If love can be provided under commitment, the love within the relationship will blossom. However, if love cannot be provided through a perfect commitment device, the love within the relationship will wither. The growth in love through the relationship is due to the assumption that love is addictive. Love will wither, without commitment, despite the fact that love is addictive; instead, the addiction property leads lovers to oversupply initially and recoup their investments later. The other participant in the match, despite knowing the planned behavior of the user, cannot reverse the situation.

Another important result is that love within the commitment match will exceed the love in the non-commitment match. Much like the durable goods monopoly model, Coase (1972), an imperfect commitment device leads the monopolist to alter the production stream. In this model, unlike in the durable goods monopoly, the user wants to reduce future output. Rational partners will respond to the expected future reduction by demanding less today, which, because of addiction, means less future demand, and, therefore, less love, for all.

4.1. **Commitment.** Consider a monogamist, i.e., someone capable of committing to their partner over the entire period of relationship. Under commitment, the

monogamist is able to commit to a stream of love provision, i.e., choose the amount of love to provide at the beginning of the match and follow it until the end of the match. The monogamist's love provision problem is to maximize the utility associated with the provision of love over the entire life of the relationship, where the c superscripts denote commitment values.

$$
(5) \qquad \underset{q_1^c, q_2^c}{Max} \ \mathcal{L}^c = \sigma v(q_1^c) + q_1^c(p_1^c - c)
$$
$$
+ \beta \sigma v(q_2^c) + \beta q_2^c(p_2^c - c)
$$

Optimal love provision under perfect commitment is governed by the usual first order conditions. After rearrangement, those conditions become

$$
(6) \qquad \frac{\partial p_1^c}{\partial q_1^c} q_1^c + p_1^c + \beta \frac{\partial p_2^c}{\partial q_1^c} q_2^c = c - \sigma v'(q_1^c)
$$

and,

$$
(7) \qquad \frac{\partial p_1^c}{\partial q_2^c} q_1^c + \beta p_2^c + \beta \frac{\partial p_2^c}{\partial q_2^c} q_2^c = \beta(c - \sigma v'(q_2^c)).
$$

The conditions are exactly as expected, the marginal return to the provision of additional love, including the love addiction component from the adjacent (preceding or succeeding) period, is equated to the marginal opportunity cost of providing the addtional love. The marginal cost is subsidized according to the degree of selflessness.[21] However, condition (7) also points out the potential for future dynamic inconsistency, which will be more fully discussed below.

Intuitively, increases in demand will lead to increases in production for any profit maximizing firm, because they raise the marginal return from production. In this case, the intuition is that the provision of love within the match is increased when more love is desired. The necessary conditions, if further analysed,[22] would show that increases in either, α, A, q_0, or q_3, because those increases represent rising desires, will lead to an increase in marginal rewards in either of the match periods. It is also expected that any increase in cost will lead to a decreased willingness of a supplier to provide the product. In this model, an increase in cost is represented

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[21]Due to the assumption of downward sloping demand and increasing marginal cost, the sufficient conditions for a maximum are easily satisfied.

[22]A formal discussion is provided in Appendix A.1.

as either an increase in the explicit marginal product cost, c, or as a decrease in the implicit production subsidy, σ. Therefore, an increase in c or a decrease in σ, will lead to a decrease in the willingness to provide love.

The impact of time, however, is not as clear. An increase in the valuation placed on the future, as measured by an increase in β, measures the willingness to postpone current rewards in exchange for future rewards. Due to addiction, current love provision is a form of investment in the future desire for the other partner in the match. If delayed rewards have more value, more rewards will be delayed, and, therefore, the returns to today's love provision are greater. Due to addiction, therefore, the provision of love at the outset will increase if β increases. However, the provision of love in the future is decreasing in β. Given the fact that period two represents the end of the relationship, so there is no additional reason to invest in future love, an increase in the value of future rewards leads to a disproportionate rise in marginal costs relative to marginal returns.

A person who receives love will seek more love in the future. Because the desire for love in the future exceeds the current desire, and the provider is a mongamist, more love will be provided. If love were not addictive, the monogamist would provide the same amount of love in each period. The romantic expectation that love should grow throughout the partnership can obtain in this model, where love is assumed to be addictive and is provided by a committed partner. The result is formalized below. An illustration of the complete commitment equilibrium is provided in Figure 1.

Theorem 1. *Regardless of the degree of selflessness, if love production is addictive, $\alpha > 0$, and opportunities for love outside the match are constant, $q_0 = q_3 = \bar{q}$, love grows throughout the match duration, i.e., $q_2^c > q_1^c$.*

Proof. See Appendix A.2. □

As will be shown, the ability of love to grow in the relationship also depends upon the strength of the commitment device, i.e., whether or not the love provider can commit to the specified path of love provision over the entire relationship. If the commitment of the provider cannot be believed, the commitment equilibrium will collapse. Worse, love within the relationship will wither and, potentially, die

4.2. **Potential Defection from Commitment.** If the commitment device is not perfect, the provider has an incentive to renege on the original promise. The condition governing the commitment choice of future love provision, q_2^c, is influenced by the commitment level of preceding love production, q_1^c. Once time has passed, it cannot be recaptured. Therefore, any rewards accumulated from love provision in the past become sunk; those rewards no longer affect current decisions.

Consider the case in which the partner supplied the full-commitment level of love in the first period of the relationship. Once the future has been reached, the monopoly provider of love faces a utility maximization problem, which is partly determined by past behavior represented by q_1^c.

$$(8) \qquad \underset{q_2^d}{Max} \; \mathcal{L}^d|_{q_1^c} = \sigma v(q_2^d) + q_2^d(p_2^d - c),$$

The d superscript represents possible defection from the commitment equilibrium.

The solution to the maximization problem in equation (8) is governed by

$$(9) \qquad p_2^d + \frac{\partial p_2^d}{\partial q_2^d} q_2^d = c - \sigma v'(q_2^d).$$

As expected, discounting no longer applies.[23] In addition, a comparison of equation (9) with equation (7) shows a clear difference. The marginal effect of current, now period two, provision on past provision and happiness is no longer plausible. The period one result has been achieved, and, importantly, cannot be changed; therefore, the marginal effect of today's decision on yesterday's decision must be zero; past rewards are sunk. Due to the dynamic inconsistency in the problem, defection is guaranteed if the recipient in the relationship match is naïve.

Theorem 2. *If commitment cannot be guaranteed, naïve partners are used. Users defect by providing less than the scheduled commitment level of love in the final period of the match, i.e., $q_2^c > q_2^d$. In addition, less love is made available than was available in the past, i.e., $q_1^c > q_2^d$.*

Proof. See Appendix A.4. □

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[23]Further discussion of the properties of this first order condition are located in Appendix A.3.

Importantly, the desire to cheat is not entirely due to the fact that the model is only over two periods; rather, the reduction is due to the fact that past behaviour cannot be altered. The limited number of periods in the model does make the user want to cut production even more than if the model time line was more extensive, however, the user would still face the incentive to cut production below what was planned, in every period.

For comparison, in the original durable goods monopoly model, see for example, Coase (1972) and Gul, Sonnenshein & Wilson (1992), the monopoly defection is in the direction of extra production, due to the substitutability of the products through time. When the monopoly output in one time period is a substitute for the output in the next period, the initial purchaser of the durable product suffers a capital loss; the initial purchaser pays too much for the product having naïvely believed the monopolist would not produce more in the future. The externality, the monopolist does not bear the full cost of future output decisions, effectively subsidizes production so that the mononpolist will, in fact, produce more in the future than originally planned.

In this model, where love in one time period complements love in an adjacent time period, the naïve initial purchaser receives a capital gain; love is provided at a price that is below its value to the recipient. The externality, the potential monogamist cannot collect on the full benefits of current decisions, effectively taxes the provision of love, thus reducing provision below the amount initially planned.[24]

4.3. **Commitment Through Subgame Perfection.** If one mate is unable to guarantee commitment to the other, defection is the expected result. Therefore, it is not reasonable to believe that potential lovers will naïvely accept the commitment equilibrium amount of love, initially, since they should expect to be used in the future. Given the expectation of being used, the love recipient should reduce the initial desire for love to temper the potential for being used. Due to the addictive nature of love, the expectation of being used should induce reductions in current

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[24]In addition, there is an end game effect in this two period model. The recipient of love, because of addiction, is investing in future desires. The asset specificity of the investment, demand for future love within the match can only be provided by the same individual, leads to an ex post hold-up problem, where the user reduces supply to take advantage of that initial investment, which cannot be recouped anywhere else.

desires. Below, a Subgame Perfect Nash Equilibrium to this within-match two-period game of love is described. In equilibrium, love no longer grows; instead, love withers, and, therefore, less love is provided within the match in equilibrium. Equilibrium behaviour in the match is examined via backwards induction, beginning with the final period.

4.3.1. *The Final Period.* In the final period of the match, the potential user determines the amount of love to provide, given the love that was provided in the intial period of the relationship. Behaviour is based on the solution to the following optimization problem.

$$(10) \qquad \underset{q_2^s}{Max} \ \mathcal{L}^s|_{q_1} = \sigma v(q_2^s) + q_2^s(p_2^s - c),$$

Equation (10) is identical to the maximization problem given by equation (8), other than notation. Therefore, other than notation, the solution follows a necessary condition, which is identical to the condition in equation (9).

$$(11) \qquad p_2^s + \frac{\partial p_2}{\partial q_2^s} q_2^s = c - \sigma v'(q_2^s)$$

The s superscript connotes the subgame perfect solution. Without a specification on the function $v(q)$, there is no closed form solution to the problem. However, equation (11) represents an implicit function, $q_2^s(q_1; A, c, \alpha, \sigma, q_3)$, representing the choice of current love provision, q_2^s, given any possible level of past love, q_1.[25]

4.3.2. *The Initial Period.* If partners in a love relationship expect to be used, they will fully incorporate that potential into their decisions. Potential users, will also incorporate the expected optimal future behavior into their initial decision. Full use of the information provided by the optimal responses will allow recipients to receive love from a monogamist, where commitment is guaranteed, not by word or deed, but by self-interest. Despite the initial assumpton that partners are partially selfless, the provision of love will not be entirely selfless.

The selfless, but self-interested, love provider will maximize within-match utility, below. However, the choice of love to supply, especially in the future, is constrained

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[25]The properties of this implicit function are exactly the same as the properties of the defection quantity implicit function examined in Appendix A.3.

to be consistent, i.e., future love production must maximize future within-match utility. Substitution of the consistency constraint, or reaction function for future production into the within-match utility function, forces dynamic consistency.[26]

$$
\begin{aligned}
\underset{q_1^s}{Max}\ \mathcal{L}^s = & \sigma v(q_1^s) + q_1^s(p_1^s - c) \\
& + \beta\sigma v(q_2^s) + \beta q_2^s(p_2^s - c).
\end{aligned}
$$

(12)

The first period reaction function is found from the necessary condition for a maximum to the preceding problem; that condition is stated below.

(13)
$$
\frac{\partial p_1^s}{\partial q_1^s}q_1^s + p_1^s + \beta q_2^s \frac{\partial p_2^s}{\partial q_1^s} = c - \sigma v'(q_1^s)
$$

As seen in the preceding problems, marginal returns are equated to subsidized marginal costs in order for a maximum to exist. Also, the maximum is guaranteed by the downward sloping demand and rising marginal costs.

Theorem 3. *Regardless of the degree of selflessness, even if love is addictive, $\alpha > 0$, and love opportunities outside the match are constant, $q_0 = q_3 = \bar{q}$, love withers throughout the match duration, i.e., $q_1^s > q_2^s$.*

Proof. See Appendix B.3 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Figure 2, below, provides a full illustration of the Subgame Perfect Nash Equilibrium, and the properties surrounding the equilibrium. Further comparisons between the two different equilibria are provided in the next section.

4.4. **Commitment vs. Noncommitment.** As has already been shown, within-match love will grow, as long as commitment can be guaranteed in the relationship. However, if the relationship cannot be based on commitment, love will decay through the course of the match. Not surprisingly, the total availability of love under commitment is higher than it is without commitment. More importantly, a match between committed mates will exchange more love during every period in which they are matched than the couple that is not committed to their relationship.

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[26] Another solution involves maximization of the within-match utility based on the choice of q_1, effectively creating an implicit function for $q_1^{-1}(q_2^s)$, which, when combined with the implicit function for future output, $q_2^s(q_1; A, c, \alpha, \sigma, q_3)$, will yield a Subgame Perfect Nash Equilibrium path of love provision.

Intuitively, love, because there is an addictive component to it, requires investments for growth. If the buyer perceives a mate who cannot be trusted, that mate will not fully participate in the relationship. A reduction in participation at the outset means even less participation in the relationship in the future, due to the complementary nature of love, i.e., love must be fostered to grow. That intuition is formalized below, but not explicity proven.

Theorem 4. *The provision and receipt of love in the Subgame Perfect Nash Equilibrium is exceeded by the provision and receipt of love in the Commitment Equilibrium, i.e., $q_1^{s*} < q_1^{c*}$ and $q_2^{s*} < q_2^{c*}$.*

An outline of the proof is based on a simple fixed point theorem, and is discussed in Appendix B, where the two functions begin equidistantly from the origin, while committed optimal responses for future love provision responds more quickly to increases in initial love provision due to the fact that questionable motives are not plausible.

Figure 3, which is an overlay of Figure 2 on top of Figure 1, highlights the similarities and differences between the two equilibria. The main point of the illustration is that if the proposed future love cannot be believed, the mate will not desire as much love as could be made available, initially. The effect of the reduction in initial love, compounded by the fact that the non-committed partner has an incentive to reduce future expressions of love, yields a net result of love that fades, without commitment.[27]

4.5. **Commitment Devices.** To this point, actual commitment devices have not been modeled, because they are not the primary focus of this research. However, it is worth considering a few marriage traditions, in order to determine whether or not they elicit effective commitment.

In the case of the durable goods monopolist, for whom dynamic inconsistency involves a desire to increase production, appropriate commitment entails rental contracts, production disincentives, or planned obselescence. If the monopolist rents, instead of selling the product, then the monopolist does not have an incentive

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[27]The strength of this result is also a feature of the end of the game. If the game were not, yet about to end, there would still be some incentive left for the partner to invest further into the future, in order to reap the rewards of using.

to produce any extra, since extra production will reduce the rental price for all units. Disincentives to produce additional units in the future, such as rebates to previous purchasers, would reduce the desire of the monopolist to cheat, thus keeping future production in line with planned production. Finally, planned obselescence reduces the durability of the item, thus eliminating the intertemporal substitutability of the monopoly's output.[28]

With love, however, the commitment devices must differ from those in the durable goods monopoly example, due to the fact that love is intertemporally complementary, rather than substitutable. Due to intertemporal complementarity, defection entails a reduction in the amount of love that was promised within the match. Therefore, an appropriate commitment device will improve the likelihood of love provision in the future. Current marriage traditions, unfortunately, do not provide incentives to increase the provision of love in the future. For example, explaining the extensive use of engagement rings to represent commitment to the marriage is a puzzle. In our model, as in its definition, commitment must represent a future promise that can be kept. Engagement rings are only current sunk costs, and do not represent an investment in "love provision", or the capacity to love further. Other examples, such as dowries and bride prices, are also only current expenses, and are not associated with future love and happiness within the marriage.

Furthermore, the advent of no-fault divorce laws may have lessened the contractural component of the marriage. In particular, if one of the participants in the match does not face a breach penalty for not living up to their part of the bargain, then there will be less incentive to live up to their part of the bargain. In other words, there may very well be more cheating and using in current matches, then in past matches.

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[28]However, it should be noted that planned obselescence may not be dynamically consistent, either. The monopolist will only be willing to upgrade to a new product if expected profits from the new product exceed the profits to be made by maintaining the current technological level of the product.

5. Conclusions and Extensions

The model in this paper was used to examine the importance of commitment in a potential love relationship, where love was presumed to be addictive in nature. The primary result was that commitment was much better for the partners concerned. If a lover was able to commit at the beginning of the relationship to a path of love throughout the match, love within the match grew. However, if the commitment device was not perfect, the incentive in the model was to produce less than the prescribed amount.

The difference between commitment and non-commitment in the relationship examined in this paper is the difference between love growing through the course of the relationship and love faltering through the course of the match. Therefore, from the perspective of potential entrants into a relationship, some form of commitment must be embedded in the relationship. Whether the commitment is legal, social, or reputational does not matter; only the enforceability of the commitment matters.

In the paper, cohabitation was used to describe partnerships that were not initially based upon commitment; rather, as survey participants in other research have stated (see Ressler & Waters (1995), the cohabitations were viewed as trials. Furthermore, marriage was used to describe relationships that were based upon commitment. This research has shown that cohabiting relationships are prone to failure, through using behaviour; therefore, if people initially choose to cohabit rather than marry, they may not be able to regain the commitment necessary to make an actual marriage work at a later stage in their relationships.

The model does not endogenize the relationship behavior with the market, and, therefore, is unable to completely capture the effects of changes in the ratio of men relative to women, or the effects search costs have on how the behavior within the relationship might evolve. Extending the relationship model to multiple periods and allowing matches to begin or dissolve in any period, could result in different types of relationship paths. One type of relationshop could involve full commitment between two mates and love would grow throughout that relationship. Another type of relationship path could involve a series of non-commitment relationships between people that continually attempt to use each other. Further analysis of this

idea could prove valuable for further understanding the changes in family structure, which have recently occurred and may continue into the future.

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APPENDIX A. POPERTIES OF THE COMMITMENT EQUILIBRIUM AND DEFECTION

In this Appendix, the propoerties of the commitment equilibrium are examined. In addition, the potential for defection and the properties of the defection decision are examined.

A.1. **Commitment.** Equations (6) and (7) could be interpreted in the same fashion as reaction functions. The first equation measures the optimal response to a level of future output, while the second measures the optimal response to the amount of output in the preceding period. The solution to equation (6) is an implicit function, $q_1^c(q_2^c; A, c, \alpha, \beta, \sigma, q_0)$. The solution to equation (7) is also an implicit function, $q_2^c(q_1^c; A, c, \alpha, \beta, \sigma, q_3)$.

Lemma 1. *The inverse reaction function for q_1^c, $q_1^{-1}(q_2; A, c, \alpha, \beta, \sigma, q_0)$, is an increasing linear function of q_2. Also, the level of output q_1^c increases with increases in A, α, β, σ, and q_0, but decreases with increases in c.*

Proof. Taking total differentials of equation (6) results in the following equation.

(14)

$$dq_1^c(\sigma v''(q_1^c) - 2) + v'(q_1^c)d\sigma + dA + \alpha dq_0 + d\alpha(q_0 + (1+\beta)q_2^c) + \alpha q_2^c d\beta + \alpha(1+\beta)dq_2^c - dc = 0$$

One of the sufficient conditions for a maximum requires marginal costs to be rising faster than marginal revenues. Given linear demand and increasing marginal costs, the condition is easily satisfied. The condition, with regards to this problem states $\sigma v''(q_1^c) - 2 < 0$, which is obviously true since $\sigma \geq 0$ and $v''(q_1^c) < 0$.

The shape of the inverse reaction function is determined by looking at the slope of the reaction function in $q_1 - q_2$ space, and the derivative of that slope in the same space.

(15)
$$\frac{dq_2^c}{dq_1^c} = \frac{2 - \sigma v''(q_1^c)}{\alpha(1+\beta)} > 1$$

(16)
$$\frac{d^2 q_2^c}{(dq_1^c)^2} = \frac{-\sigma v'''(q_1^c)}{\alpha(1+\beta)} = 0$$

The differential measured in equation (15), which is the slope of the inverse reaction function, is positive; the denominator is positive by the assumption of positive parameter values, while the numerator is positive because it is the opposite of

the condition required for a maximum to exist. The shape of the inverse reaction function, measured by the derivative of the slope of the inverse reaction function in equation (16) is linear because $v''' = 0$, by assumption.

Other properties of the inverse reaction function can be found by discerning the effect of any differential on the q_1^c differenial. Defining $z_1 = [2 - \sigma v''(q_1^c)]^{-1} > 0$, yields the following relationships:

$$\frac{dq_1^c}{dA} = z_1 > 0$$

$$\frac{dq_1^c}{d\alpha} = z_1 \cdot (q_0 + (1 + \beta)q_2^c) > 0$$

$$\frac{dq_1^c}{d\beta} = z_1 \cdot (\alpha q_2^c) > 0$$

$$\frac{dq_1^c}{dq_0} = z_1 \cdot \alpha > 0$$

$$\frac{dq_1^c}{d\sigma} = z_1 \cdot v'(q_1^c) > 0$$

and

$$\frac{dq_1^c}{dc} = -z_1 < 0.$$

The signs all follow from the definition of z_1 and the assumption that all of the parameters are defined to be positive real numbers.   □

Furthermore, the provision of love in the final period can be examined.

Lemma 2. *The reaction function for q_2^c, $q_2^c(q_1; A, c, \alpha, \beta, \sigma, q_3)$ is an increasing linear function of q_1. In addition, the reaction function increases with increases in A, α, σ, and q_3, but decreases with increases in c, and β.*

Proof. Taking total differentials of equation (7) results in the following equation.

(17)
$$dq_2^c[\beta(\sigma v''(q_2^c) - 2)] + \beta v'(q_2^c)d\sigma + \beta dA + \alpha \beta dq_3 + d\alpha(\beta q_3 + (1 + \beta)q_1^c)$$
$$+ d\beta[A + \alpha(q_1^c + q_3) - 2q_2^c - c + \sigma v'(q_2^c)] + \alpha(1 + \beta)dq_1^c - \beta dc = 0$$

One of the sufficient conditions for a maximum requires marginal costs to be rising faster than marginal revenues. Given linear demand and increasing marginal costs,

the condition is easily satisfied. The condition, with regards to this problem states $\sigma v''(q_2^c) - 2 < 0$, which is obviously true since $\sigma \geq 0$ and $v''(q_2^c) < 0$.

The shape of the reaction function is determined by looking at the slope of the reaction function in $q_1 - q_2$ space, and the derivative of that slope in the same space.

$$(18) \qquad \frac{dq_2^c}{dq_1^c} = \frac{\alpha(1+\beta)}{2 - \sigma v''(q_2^c)} < 1$$

$$(19) \qquad \frac{d^2 q_2^c}{(dq_1^c)^2} = \frac{\alpha(1+\beta)[\sigma v'''(q_2^c)\frac{dq_2^c}{dq_1^c}]}{\beta(2 - \sigma v''(q_2^c))^2} = 0.$$

The differential measured in equation (18), which is the slope of the reaction function, is positive; the numerator is positive by the assumption of positive parameter values, while the denominator is positive because it is the opposite of the condition required for a maximum to exist. The shape of the reaction function, measured by the derivative of the slope of the reaction function in equation (19) is linear because $v''' = 0$, by assumption.

Other properties of the inverse reaction function can be found by discerning the effect of any differential on the q_2^c differenial. Defining $z_2^c = [2 - \sigma v''(q_2^c)]^{-1} > 0$, yields the following relationships:

$$\frac{dq_2^c}{dA} = z_2^c > 0$$

$$\frac{dq_2^c}{d\alpha} = \beta^{-1} z_2^c \cdot (\beta q_3 + (1+\beta)q_1^c) > 0$$

$$\frac{dq_2^c}{dq_3} = z_2^c \cdot \alpha > 0$$

$$\frac{dq_2^c}{d\sigma} = z_2^c \cdot v'(q_2^c) > 0$$

$$\frac{dq_2^c}{dc} = -z_2^c < 0$$

and

$$\frac{dq_2^c}{d\beta} = z_2^c \cdot [A + \alpha q_1^c + \alpha q_3 - 2q_2^c - c + \sigma v'(q_2^c)] < 0.$$

The signs all follow from the definition of z_2 and the assumption that all of the parameters are defined to be positive real numbers, with the exception of the last property. The numerator of the comparitive static with respect to β is the same as

the necessary condition for a maximum in equation (7) except for $\alpha q_1^c > 0$, which is missing. Therefore, if the necessary condition is zero, the necessary condition without a positive portion must be negative.

$\square$

Intuitively, increases in demand will lead to increases in production for any profit maximizing firm. In this case, the intuition is that the provision of love within the match is increased when love is more desired. For that reason, increases in either, α, A, q_0, or q_3, because those increases represent rising desires, will lead to an increase in either of the aforementioned reaction functions. It is also expected that any increase in cost will lead to a decreased willingness of a supplier to provide the product. In this model, an increase in cost is represented as either an increase in the explicit marginal product cost, c, or as a decrease in the implicit production subsidy, σ. Therefore, an increase in c or a decrease in σ, will lead to a decrease in both reaction functions.

An increase in the valuation of future profits, as measured by an increase in β, measures the willingness to postpone current rewards in exchange for future rewards. Due to addiction, current production and sales are a form of investment in future demand. If delayed rewards have more value, due to an increase in β, more rewards will be delayed, and, therefore, the provider of love is willing to provide more today. Due to addiction, therefore, the reaction function for love provision at the outset will increase if β increases. However, the period two reaction function is decreasing in β. Given the fact that period two represents the end of the relationship, so there is no additional reason to invest in future love, an increase in the value of future rewards leads to a disproportionate rise in marginal costs relative to marginal returns. The direct consequences of the preceding two lemmas are stated below in the following lemma.

Lemma 3. *The equilibrium quantities of full-commitment love production are increasing in the addiction parameter, α, the measure of selflessness, σ, the extent of the market, A, and out-of-relationship love, q_0 and q_3. However, the equilibrium commitment quantities decrease with increases in the cost of production, c, and*

change indeterminantly with respect to the producer's valuation of future rewards, β.

Proof. Combining the effects on each of the separate reaction functions is sufficient for proving the theorem. □

Although the properties of the equilibrium are now well understood, because the properties of the reaction functions are well understood, the actual equilibrium has not been pinned down. In order to complete the analysis, the intersection of the reaction functions must be obtained. A final solution, therefore, relies on a more complete analysis of the reaction functions. Under very general conditions, it is possible to determine the optimal level of output in any one time period, given no production in the adjacent time period. Supplementing the general conditions with an additional symmetry assumption forces some symmetry on the reaction functions. Furthermore, in order to determine the actual equilibrium values, it is necessary to pin down the reaction function intercepts. These intercepts are made explicit, below.

Lemma 4. *Assuming love possibilities outside the match are constant, $q_0 = q_3 = \bar{q}$, production in each period, given no production in the adjacent time period, will be equal, i.e., $q_1^{-1c}(q_2 = 0; A, c, \alpha, \beta, \sigma, \bar{q}) = q_2^c(q_1 = 0; A, c, \alpha, \beta, \sigma, \bar{q})$, i.e., reaction functions have the same axis intercept value.*

Proof. Rewriting the conditions in equations (6) and (7), which are necessary for a maximum, assuming $q_0 = q_3 = \bar{q}$, yields

$$(20) \qquad A + \alpha\bar{q} - 2q_1 = c - \sigma v'(q_1)$$

and

$$(21) \qquad \beta(A + \alpha\bar{q} - 2q_2) = \beta(c - v'(q_2))$$

The solution to the equation, although not of closed form, is such that q_1 does not depend upon q_2, while q_2 does not depend upon q_1. In addition, the solution to each equation must be the same, i.e., only $q_1^c(q_2 = 0) = q_2^c(q_1 = 0)$ will solve either equation, since each equation is functionally equivalent to the other equation. □

These lemmas are the basis for the proof of the main commitment equilibrium result, which is presented in the following Appendix subsection.

A.2. **Proof of Theorem 1.** The result of Lemma 4 is that the reaction function for q_1 starts as far to the right of the origin as the reaction function for q_2 starts above the origin. The result of Lemma 1 and Lemma 2 is that the reaction functions are linear and increasing. The combination of these three lemmas allows for the use of a geometric result. First, an intersection will occur as long as the lower function has a greater slope than the function above it, in this case, as long as the reaction function for q_1 has a greater slope than that for q_2. Second, the intersection will occur above the 45° line, on which $q_1 = q_2$, if the multiplication of the slopes exceeds one. The intersection will occur below the 45° line if the multiplication of the slopes yields a number less than one.

Multiplying the slopes of the two commitment reaction functions, equations (15) and (18), yields

$$
(22) \qquad \frac{dq_2}{dq_1^{-1}} \cdot \frac{dq_2^c}{dq_1} = \frac{2 - \sigma v''(q_1)}{\alpha(1 + \beta)} \cdot \frac{\alpha(1 + \beta)}{\beta(2 - \sigma v''(q_2))}
$$
$$
= \frac{1}{\beta} > 1.
$$

The final result is due to the fact that $2 - \sigma v''(q_2) = 2 - \sigma v''(q_1)$, because $v''' = 0$, since v is a quadratic function. The fact that the multiplied slopes exceed one results in equilibrium values above the 45° line, i.e., $q_1^{c*} < q_2^{c*}$.　　□

A.3. **Potential for Defection and Properties.** Now that the commitment equilibrium has been discussed, it is necessary to outline the impact of defection from that commitment. The result is described below.

Lemma 5. *The potential defection reaction function for the final period within match output, q_2^d, is increasing in A, α, q_3, q_1^c, and σ; however, the level of consumption is decreasing in c.*

Proof. Taking total differentials of equation (9) results in the following equation.

$$
(23) \qquad dq_2^s(\sigma v''(q_2^s) - 2) + v'(q_2^c)d\sigma + dA + \alpha dq_3 + d\alpha(q_3 + q_1^s) + \alpha dq_1^s - dc = 0
$$

One of the sufficient conditions for a maximum requires marginal costs to be rising faster than marginal revenues. Given linear demand and increasing marginal costs, the condition is easily satisfied. The condition, with regards to this problem states $\sigma v''(q_2^d) - 2 < 0$, which is obviously true since $\sigma \geq 0$ and $v''(q_2^d) < 0$.

The shape of the reaction function is determined by looking at the slope of the reaction function in $q_1 - q_2$ space, and the derivative of that slope in the same space.

$$(24) \qquad \frac{dq_2^d}{dq_1^c} = \frac{\alpha}{2 - \sigma v''(q_2^d)} < 1$$

$$(25) \qquad \frac{d^2 q_2^d}{(dq_1^c)^2} = \frac{\alpha[\sigma v'''(q_2^d)\frac{dq_2^d}{dq_1^c}]}{(2 - \sigma v''(q_2^d))^2} = 0.$$

The differential measured in equation (24), which is the slope of the reaction function, is positive; the numerator is positive by the assumption of positive parameter values, while the denominator is positive because it is the opposite of the condition required for a maximum to exist. The shape of the reaction function, measured by the derivative of the slope of the reaction function in equation (25), is linear because $v''' = 0$, by assumption.

Other properties of the inverse reaction function can be found by discerning the effect of any differential on the q_2^d differenial. Defining $z_2^d = [2 - \sigma v''(q_2^d)]^{-1} > 0$, yields the following relationships:

$$\frac{dq_2^d}{dA} = z_2^d > 0$$

$$\frac{dq_2^d}{d\alpha} = z_2^d \cdot (q_3 + q_1^c) > 0$$

$$\frac{dq_2^d}{dq_3} = z_2^d \cdot \alpha > 0$$

$$\frac{dq_2^d}{d\sigma} = z_2^d \cdot v'(q_2^d) > 0$$

and

$$\frac{dq_2^d}{dc} = -z_2^d < 0.$$

The signs all follow from the definition of z_2^d and the assumption that all of the parameters are defined to be positive real numbers, with the exception of the last property.

$\square$

The intuition behind the properties of the potential defection reaction function for the production of love in the final period mirror the properties of the commitment equilibrium reaction functions. The reaction function will rise if marginal revenue is increased, but will fall if marginal costs are increased. Increases in A, the extent of the market, α, the degree of addiction, q_1, preceding love production, and q_3, out of match love expectations, increase demand, subsequently increasing marginal revenue. On the other hand, an increase in c, the explicit constant cost of love production, and a decrease in σ, the degree of selflessness (an implicit subsidy to love production), will lead to an increase in effective marginal costs, and, subsequently, to a reduction in love provision.

Lemma 6. *Assuming love possibilities outside the match are constant, $q_0 = q_3 = \bar{q}$, production in each period, even in the case of potential defection, given no production in the adjacent time period, will be equal, i.e., $q_1^{-1}(q_2 = 0) = q_2^d(q_1 = 0)$.*

Proof. Rewriting the conditions in equations (6) and (9), which are necessary for a maximum, assuming $q_0 = q_3 = \bar{q}$, yields

$$(26) \qquad\qquad A + \alpha\bar{q} - 2q_1 = c - \sigma v'(q_1)$$

and

$$(27) \qquad\qquad \beta(A + \alpha\bar{q} - 2q_2) = \beta(c - v'(q_2))$$

The solution to the system, although not of closed form, is such that q_1 does not depend upon q_2, while q_2 does not depend upon q_1. In addition, the solution must be the same for each time period, i.e., only $q_1^c = q_2^d$ will solve the previous system of equations, since each equation in the system is functionally equivalent to the other equation in the system. $\square$

A.4. **Proof of Theorem 2.** From the text, the conditions governing q_2^c and q_2^d are, respectively,

$$(28) \qquad \frac{\partial p_1}{\partial q_2^c} + \beta p_2^c + \beta \frac{\partial p_2}{\partial q_2^c} q_2^c - \beta(c - \sigma v'(q_2^c)) = 0$$

$$(29) \qquad \beta p_2^d + \beta \frac{\partial p_2}{\partial q_2^d} q_2^d - \beta(c - \sigma v'(q_2^d)) = 0,$$

where equation (29) is the β multiple of equation (9). If $q_2^c > q_2^d$, then the substitution of q_2^c into equation (29) will yield a negative result, i.e.,

$$\beta p_2 + \beta \frac{\partial p_2}{\partial q_2^c} q_2^c - \beta(c - \sigma v'(q_2^c)) < 0.$$

Equation (28), when rearranged, yields

$$\beta p_2^c + \beta \frac{\partial p_2}{\partial q_2^c} q_2^c - \beta(c - \sigma v'(q_2^c)) = -\frac{\partial p_1}{\partial q_2^c}$$

$$= -\alpha$$

$$< 0$$

which is the required result. Therefore, q_2^c exceeds q_2^d. □

APPENDIX B. PROPERTIES OF SUBGAME PERFECT NASH EQUILIBRIUM

In this appendix the Subgame Perfect Nash Equilibriu is examined in detail, beginning with the final period.

B.1. **The Final Period.** In the final period, first period outcomes are taken as given.

Lemma 7. *The reaction function for final period within match output, q_2^s, is increasing in A, α, q_3, q_1, and σ; however, the level of production is decreasing in c.*

Proof. Taking total differentials of equation (7) results in the following equation.

$$(30) \qquad dq_2^s(\sigma v''(q_2^s) - 2) + v'(q_2^c)d\sigma + dA + \alpha dq_3 + d\alpha(q_3 + q_1^s) + \alpha dq_1^s - dc = 0$$

One of the sufficient conditions for a maximum requires marginal costs to be rising faster than marginal revenues. Given linear demand and increasing marginal costs,

the condition is easily satisfied. The condition, with regards to this problem states $\sigma v''(q_2^s) - 2 < 0$, which is obviously true since $\sigma \geq 0$ and $v''(q_2^s) < 0$.

The shape of the reaction function is determined by looking at the slope of the reaction function in $q_1 - q_2$ space, and the derivative of that slope in the same space.

$$(31) \qquad \frac{dq_2^s}{dq_1^s} = \frac{\alpha}{2 - \sigma v''(q_2^s)} < 1$$

$$(32) \qquad \frac{d^2 q_2^s}{(dq_1^s)^2} = \frac{\alpha[\sigma v'''(q_2^s)\frac{dq_2^s}{dq_1^s}]}{(2 - \sigma v''(q_2^s))^2} = 0.$$

The differential measured in equation (31), which is the slope of the reaction function, is positive; the numerator is positive by the assumption of positive parameter values, while the denominator is positive because it is the opposite of the condition required for a maximum to exist. The shape of the reaction function, measured by the derivative of the slope of the reaction function in equation (32), is linear because $v''' = 0$, by assumption.

Other properties of the inverse reaction function can be found by discerning the effect of any differential on the q_2^s differenial. Defining $z_2^s = [2 - \sigma v''(q_2^s)]^{-1} > 0$, yields the following relationships:

$$\frac{dq_2^s}{dA} = z_2^s > 0$$

$$\frac{dq_2^s}{d\alpha} = z_2^s \cdot (q_3 + q_1^s) > 0$$

$$\frac{dq_2^s}{dq_3} = z_2^s \cdot \alpha > 0$$

$$\frac{dq_2^s}{d\sigma} = z_2^s \cdot v'(q_2^s) > 0$$

and

$$\frac{dq_2^s}{dc} = -z_2^s < 0.$$

The signs all follow from the definition of z_2^s and the assumption that all of the parameters are defined to be positive real numbers, with the exception of the last property.

Comparing the statics concerning the degree of addiction from Appendix 2, with those from this appendix yields the following comparison.

$$\frac{dq_2^c}{d\alpha} = \beta^{-1} z_2^c \cdot (\beta q_3 + (1 + \beta) q_1^c) > 0$$

$$\frac{dq_2^s}{d\alpha} = z_2^s \cdot (q_3 + q_1^s) > 0$$

and, therefore, at any fixed level of q_2,

$$\frac{dq_2^s}{d\alpha} < \frac{dq_2^c}{d\alpha}.$$

In addition, a comparison of the statics on the slope of the reaction functions from each appendix yields a very similar conclusion.

$$\frac{dq_2^c}{dq_1^c} = \frac{\alpha(1 + \beta)}{2 - \sigma v''(q_2^c)} < 1$$

$$\frac{dq_2^s}{dq_1^s} = \frac{\alpha}{2 - \sigma v''(q_2^s)} < 1$$

and, therefore, at any fixed level of q_2,

$$\frac{dq_2^s}{dq_1^s} < \frac{dq_2^c}{dq_1^c}.$$

In other words, given any particular level of q_2, the slope of the commitment reaction function will exceed the slope of the subgame perfect reaction function. Also, at any level of q_2, the effect of a change in α on that q_2 will be smaller in the subgame perfect reaction function than in the commitment reaction function. □

The intution behind the properties of the non-commitment reaction function for the production of love in the final period mirror the properties of the commitment equilibrium reaction functions. The reaction function will rise if marginal revenue is increased, but will fall if marginal costs are increased. Increases in A, the extent of the relative desire for love, α, the degree of addiction, q_1, preceding love production, and q_3, out of match love expectations, increase demand, subsequently increasing marginal revenue. On the other hand, an increase in c, the explicit constant cost of love production, and a decrease in σ, the degree of selflessness (an implicit

subsidy to love production), will lead to an increase in effective marginal costs, and, subsequently, to a reduction in love provision.

B.2. **Initial Period.** The above condition defining the reaction function for love provision is exactly the same as the condition defining the reaction function for initial period love production under perfect commitment, given by equation (6), with the exception of the notation. Therefore, the following lemma is stated without proof, as the proof is exactly the same as the proof for the reaction function for q_1^c given in Lemma 1.

Lemma 8. *The inverse reaction function for q_1^s, $q_1^{-1}(q_2; A, c, \alpha, \beta, \sigma, q_0)$, is an increasing linear function in q_2. Also the level of output, q_1^s, increases in A, α, β, σ, and q_0, but decreases in c.*

The properties of the two equilibrium levels of love provisin are very similar as well. As was the case with the commitment equilibrium, either reaction function increases when marginal revenues increase or when marginal costs fall. The results from Lemma 8 and Lemma 7 suggests that increases in the extent of the market, the degree of addiction, and out-of-match love production, or an increase in the relative valuation of future profitability all represent increases in demand (or in the valuation of demand), and, therefore, will result in increases in love production within each peroiod of the match. Additionally, an increase in the implicit subsidy to love production or a decrease in the explicit marginal cost of love production, which are reductions in marginal costs, will result in increases in the amount of love produced in each of the epochs within the relationship. The following theorem formalizes the intuition.

Lemma 9. *The Subgame Perfect Nash Equilibrium path of within match love production is increasing in A, α, β, q_0, q_3, and σ, but decreasing in c.*

Proof. The proof is a direct result of the combination of the proofs of Lemma 8 and Lemma 7. $\square$

The characterization of the equilibrium, although important, is not the result of primary interest; instead, the actual level of equilibrium love production in each

period is the result to be considered. Simultaneously solving the two reaction functions, $q_1^{-1}(\cdot)$ and $q_2^s(\cdot)$, will yield the Subgame Perfect Nash Equilibrium levels of love production within the match. When commitment is self-enforcing, love provision within the match will decrease through time. In other words, love does not grow in the relationship, it decays.

Lemma 10. *Assuming love possibilities outside the match are constant, $q_0 = q_3 = \bar{q}$, production in each period, given no production in the adjacent time period, will be equal, i.e., $q_1^{-1}(0; A, c, \alpha, \beta, \sigma, \bar{q}) = q_2^s(0; A, c, \alpha, \sigma, \bar{q})$.*

Proof. Rewriting the conditions in equations (13) and (11), which are necessary for a maximum, assuming $q_0 = q_3 = \bar{q}$, yields

$$(33) \qquad\qquad A + \alpha\bar{q} - 2q_1 = c - \sigma v'(q_1)$$

and

$$(34) \qquad\qquad \beta(A + \alpha\bar{q} - 2q_2) = \beta(c - v'(q_2))$$

The solution to the either equation, although not of closed form, is such that q_1 does not depend upon q_2, while q_2 does not depend upon q_1. In addition, the solution to each equation must be the same, i.e., only $q_1 = q_2$ will solve either equation, since each equation is functionally equivalent to the other equation. $\square$

Lemma 11. *Regardless of the type of commitment, production of love in the last period of the match is the same if there is no initial production of love, i.e., $q_2^{s*}(0; A, c, \alpha, \sigma, q_3) = q_2^{c*}(0; A, c, \alpha, \beta, \sigma, q_3)$.*

Proof. See Appendix B.4 $\square$

B.3. **Proof of Theorem 3.** The result of Lemma 10 is that the reaction function for q_1^s starts as far to the right of the origin as the reaction function for q_2^s starts above the origin. The result of Lemma 1 and Lemma 7 is that the reaction functions are linear and increasing. The combination of these three lemmas allows for the use of a geometric result. First, an intersection will occur as long as the lower function has a greater slope than the function above it, in this case, as long as the reaction

function for q_1^c has a greater slope than that for q_2^s. Second, the intersection will occur above the 45° line, on which $q_1^s = q_2^s$, if the multiplication of the slopes exceeds one. The intersection will occur below the 45° line if the multiplication of the slopes yields a number less than one.

Multiplying the slopes of the two subgame reaction functions, equations (15) and (31), yields

(35)
$$\frac{dq_2}{dq_1^{-1}} \cdot \frac{dq_2^c}{dq_1} = \frac{2 - \sigma v''(q_1)}{\alpha(1 + \beta)} \cdot \frac{\alpha}{2 - \sigma v''(q_2)}$$
$$= \frac{1}{1 + \beta} < 1.$$

The final result is due to the fact that $2 - \sigma v''(q_2) = 2 - \sigma v''(q_1)$, because $v''' = 0$, or v is a quadratic function.

The fact that the multiplied slopes is less than one results in equilibrium values below the 45° line, i.e., $q_1^{s*} > q_2^{s*}$.   □

B.4. **Proof of Lemma 12.**

Lemma 12. *Regardless of the type of commitment, production of love in the last period of the match is the same if there is no initial production of love, i.e.,*
$q_2^{s*}(0; A, c, \alpha, \sigma, q_3) = q_2^{c*}(0; A, c, \alpha, \beta, \sigma, q_3).$

Rewriting the conditions in equations (13) and (11) assuming $q_1 = 0$ and substituting in for p_2 using equation (2), yields

(36)
$$\beta(A + \alpha q_3 - 2q_2^c) = \beta(c - v'(q_2^c))$$

and

(37)
$$\beta(A + \alpha q_3 - 2q_2^s) = \beta(c - v'(q_2^s))$$

The only difference between the two equations is the notation for q_2. Therefore, the solutions must be the same.   □

Proof. According to Lemma 12, the Subgame Perfect optimal response function in the second period starts at the same point as the Commitment optimal response function. According to Lemma 8 and Lemma 1, the optimal response functions

for the choice of initial production are: (a) the same, and (b) increasing in q_2. Therefore, the fact that the committed optimal response in the future is steeper than the Subgame Perfect best response function is enough to prove the Theorem (compare slopes in Lemma 2 with the slope calculated in Lemma 7).   □
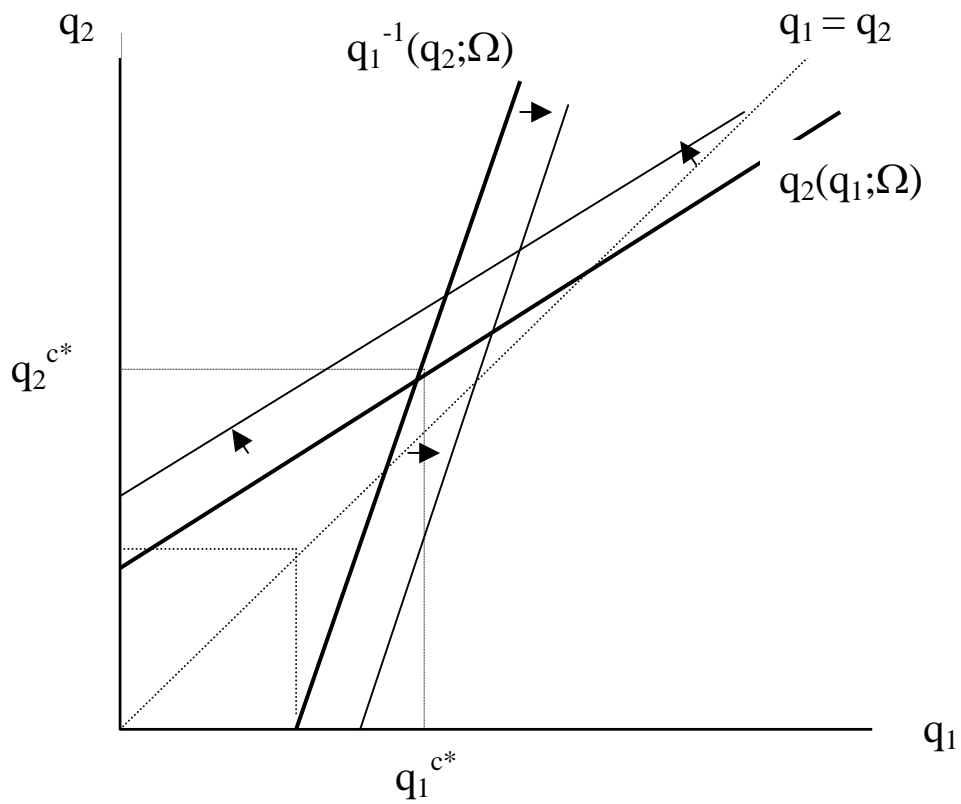
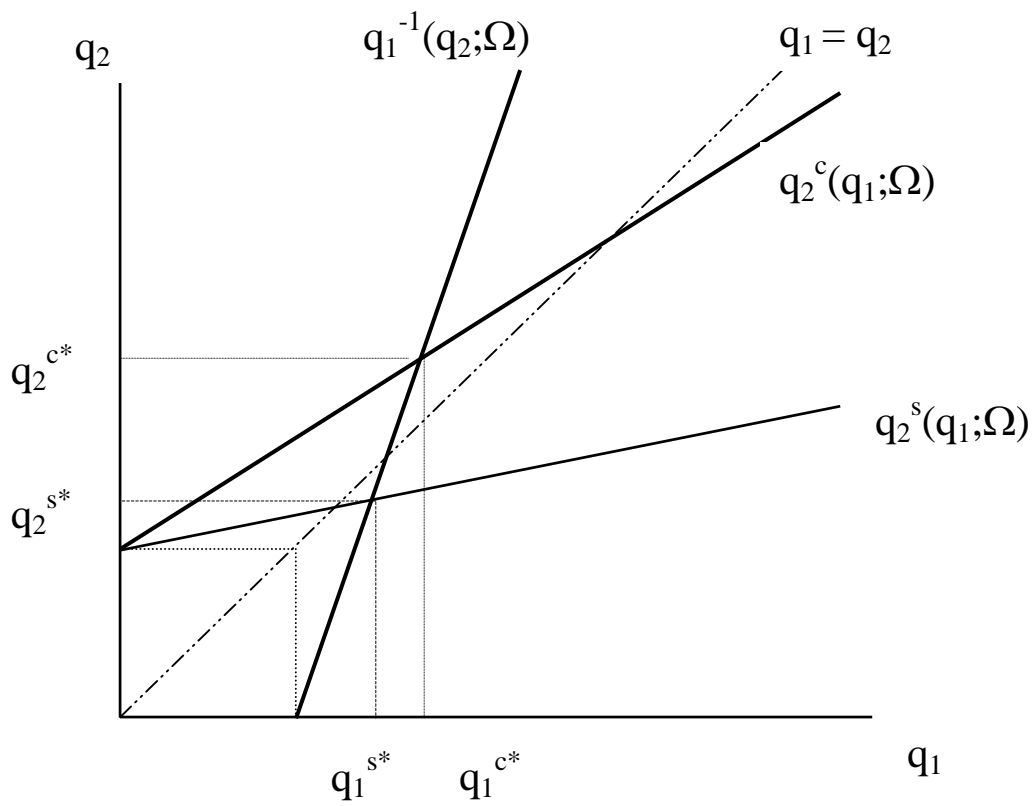For q_1, the increase is due to an increase in A, α, β, σ, and q_0; or a decrease in c. For q_2, the increase is due to an increase in A, α, σ, and q_3; or a decrease in β, or c.

Figure 1. Commitment Equilibrium
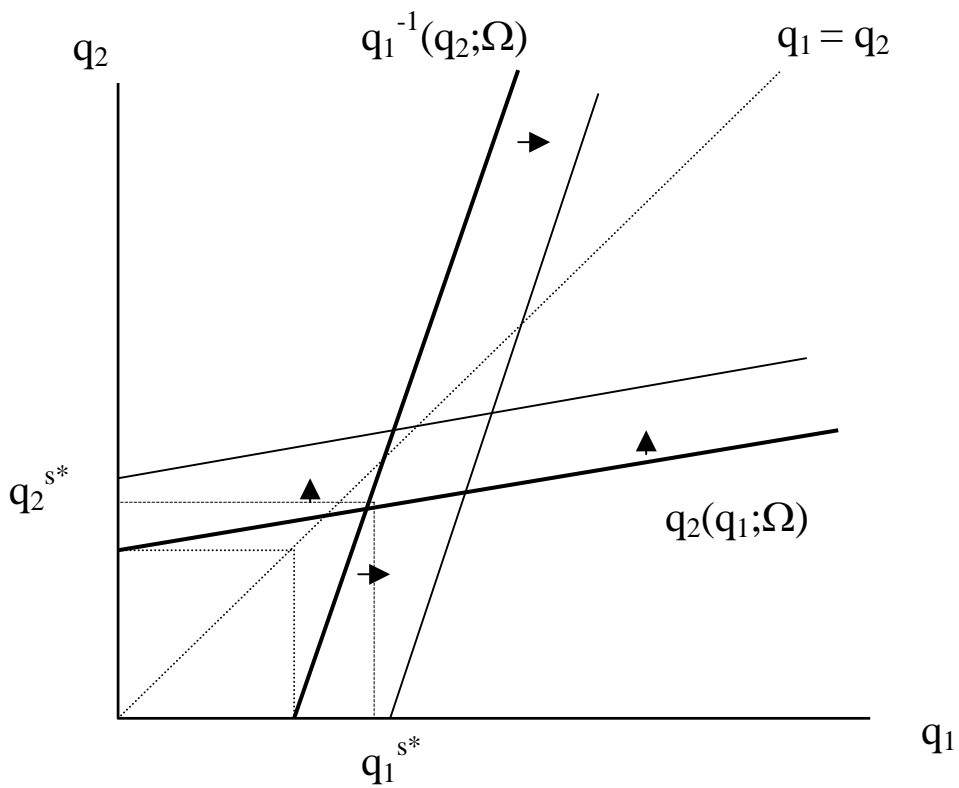
For q_1, the increase is due to an increase in A, α, β, σ, and q_0; or a decrease in c. For q_2, the increase is due to an increase in A, α, σ, and q_3; or a decrease in c.

Figure 2. Subgame Perfect Equilibrium

Figure 3. Commitment Equilibrium Compared to the Subgame Perfect Equilibrium