CEDI

Working Paper No. 08-20

Social Norms and Individual Savings in the Context of Informal Insurance

Zaki Wahhaj

October 2008

CEDI DISCUSSION PAPER SERIES

Centre for Economic
Development & Institutions
Brunel University
West London
http://cedi.org.uk

SOCIAL NORMS AND INDIVIDUAL SAVINGS IN THE CONTEXT OF INFORMAL INSURANCE

ZAKI WAHHAJ

Abstract. This paper develops a theory of informal insurance in the presence of an intertemporal technology. It is shown that when an insurance agreement suffers from enforcement problems, constraints on individual savings behaviour can enable the group to sustain greater cooperation. This result provides a motivation for a variety of social norms observed in traditional societies which discourage 'excessive' accumulation of wealth by individuals. The paper also shows that social norms that discourage savings are more likely to benefit poorer communities and thus, paradoxically, cause them to fall further behind even as it serves a useful purpose.

JEL Codes: D81, D91, O12, Z1

Corresponding Address for Author: Department of International Development, University of Oxford, 3 Mansfield Road, Oxford OX1 3TB, United Kingdom[1]. Email: zaki.wahhaj@qeh.ox.ac.uk

Date: October 2008

[1]This paper was written during my doctoral studies at the Massachusetts Institute of Technology and a postdoctoral fellowship at the Center for Research in the Economics of Development at the University of Namur. I am grateful to Abhijit Banerjee, Ethan Ligon, Jean-Philippe Platteau, and Jean-Marie Baland for their support, encouragement and advice in the project. Any remaining errors in the paper are mine only.

1. Introduction

The role of culture in the process of economic development has long been a subject of interest to social scientists. In recent years, in parallel to new developments in research on formal institutions, there has also been renewed interest in the question whether the varied economic paths of different societies through history can be traced to fundamental differences in their respective cultures, and the customs and social norms to which they give rise (see Platteau 2001 and Guiso, Sapienza and Zingales 2006 for surveys of this literature).

The discussion around social norms and economic development tends to distinguish between and concentrate on good and bad practices: on those that appear to encourage economic activity or provide an effective solution to a problem of social organisation and others that are unambiguously harmful for economic development. For example, Landes (1998) distinguish between cultural factors such as thrift, hard work, tenacity, honesty and tolerance that promote economic development, in contrast to xenophobia and religious intolerance that stifle enterprise. Greif (1994) highlights the merits of individualist cultural beliefs as opposed to collectivist cultural beliefs for the formation of efficient agency relations, in the context of medieval merchants; while Putnam (1993) attribute the greater success of modern political institutions in northern Italy to the pre-existence of a stronger civic culture.

This paper argues that a wide range of social norms may not easily fit into one or the other of these categories for a natural reason: where a cooperative agreement has to be self-enforcing, efficient norms take the form that ensures continued dependence of the individual on the service provided by the group; for it is precisely the need for this service which would induce the individual to keep his end of the bargain. In this manner, social norms appear both as constraining economic behaviour of agents at the individual level and as promoting cooperation at the level of the group. A practice that appears wasteful for an individual may be welfare-improving when the group and the surplus that it generates is taken into account.

The paper makes this argument formally in the context of self-enforcing mutual insurance agreements in an environment where agents have access to an intertemporal technology. In a self-enforcing contract, an agent is willing assist another member of the insurance network today only to the extent he values the promise of insurance from the network in the future. In the absence of an insurance network, own savings provide the agent an alternative means to smooth consumption in the face of adverse shocks. Therefore, increased saving on his part improves his ability to smooth consumption on his own, and thus lowers his need for the network. This means that if agents participating in an informal insurance network are discouraged from saving, greater demands can be made of them, to assist a fellow member in need, when such a situation arises. Consequently, in the constrained efficient agreement, individuals save below the level that is individually optimal, and the group achieves a higher level of insurance than would otherwise be possible. This is the main insight of the paper.

Thus, we obtain a theoretical result which shows how social norms that constrain the accumulation of wealth within a society can be welfare improving. Interestingly, the anthropological literature provides a variety of examples of precisely such social norms. Platteau (2000: Chapter 5) provides a survey of studies on tribal societies where social beliefs that associate the accumulation of excessive wealth to manifestations of witchcraft serve as severe deterrents to wealth accumulation by individuals. In addition, the obligation of marking important events in a household – birth, marriage or death – through extravagant feasts is common to many traditional societies[2]. The prevalence of such norms in traditional societies shows that it is often within the means of a social group to induce its members to engage in excessive consumption, as stipulated by the constrained efficient insurance agreement. This is not to argue that such social behaviour may have evolved specifically to ensure that households continue to depend on their social networks for insurance purposes. But more importantly, the paper provides a theoretical framework where it becomes apparent that such practices may have a positive effect on welfare, through their impact on group cooperation.

Interestingly, the type of social norms discussed here would tend to accentuate the differences between social groups that initially find it worthwhile to adopt them and those that do not. In the context of mutual insurance, we show that if there is some cost involved in forming an insurance network, they are more likely to arise in poorer communities, where individuals cannot effectively smooth consumption using own assets. Richer communities fail to form insurance networks and therefore also have no reason to adopt the social constraints which facilitate informal insurance in the presence of enforcement problems. In the absence of such constraints, they can opt for a higher growth path of consumption

[2]See, for example, Bloch, Rao and Desai (2004) for a discussion on excessive wedding expenditures in India; and Manuh (1995) for a discussion on exorbitant funerals in West Africa.

and wealth accumulation, compared to their counterparts who start off poor and do find it worthwhile to adopt the norms that constrain savings. Thus, the initial differences in wealth which makes the relevant social norms useful among some social groups and not in others would widen over time.

Thanks to a number of significant contributions in this area in recent years, the theoretical characteristics of self-enforcing mutual insurance agreements and their implications for empirical work are well-understood today. However, for the sake of analytical simplicity, the literature has concentrated, for the most part, on an environment where no intertemporal technology is available to the agents. An important exception in this regard is Ligon, Thomas and Worrall (2000), who also consider mutual insurance in an environment where a savings technology is available. In particular, they show that in the constrained efficient agreement, the consumption stream satisfies a modified Euler equation involving an additional term that depends on how saving affects the participation constraints. The key theoretical insight of this paper is to relate this term to the premium that agents are willing to pay to participate in the insurance agreement.

The remainder of this paper is organised as follows. Section 2 provides a more detailed survey of the theoretical and empirical literature on informal insurance. The model is presented in section 3, while sections 3.1 and 3.2 investigate self-enforcing insurance agreements and constrained efficient agreements respectively. Section 4 considers implications of the main results for communities that initially vary in the level of wealth, and section 5 concludes.

2. Related Literature

The idea of limited commitment as a basis of mutual insurance has been explored and developed extensively in the literature; it was first formalised in Kimball (1988) in the context of farm households in a rural community. Coate and Ravallion (1993) characterised the conditions under which the first-best insurance can be implemented under limited commitment. Kocherlakota (1996) provided a characterisation of constrained efficient agreements, and examined their long-run dynamics. Ligon, Thomas and Worrall (2002) showed that the constrained efficient agreements are characterised by a simple updating rule; specifically, that for each state of nature, there is a time-invariant interval for the ratio of marginal utilities; and in each period, the ratio of marginal utilities adjusts by the smallest amount necessary to bring it into the current interval.

Fafchamps (1999) argued that the theoretical characteristics of informal insurance under limited commitment correspond closely to the empirical evidence on gift-giving and informal credit in rural societies. For example, Udry (1994) finds that the terms of repayment of informal credit in rural Nigeria is affected by both shocks to the creditor and the debtor; which corresponds to the characteristics of informal insurance under limited commitment. Ligon, Thomas and Worrall (2002) test the limited commitment model using Indian village data and find that it can explain the consumption path of households more effectively than either the full insurance or the autarkic model.

The theoretical literature discussed thus far have generally assumed, for simplicity, that no intertemporal technology is available in the economy. An important exception to this literature is Ligon, Thomas and Worrall (2000), which investigates the properties of risk-sharing agreements in an environment where this assumption is relaxed. They show that in the constrained efficient agreement the consumption stream satisfies a modified Euler equation involving an additional term that depends on how saving affects the participation constraints. The key theoretical insight in the current paper is to relate this term to the premium that agents are willing to pay to participate in the insurance agreement. Ligon, Thomas and Worrall (2000) also provide an example where introduction of a savings technology diminishes welfare within a risk-sharing agreement.

Gobert and Poitevin (2006) study a related model where an agent who breaches a risk-sharing agreement also forfeits his savings. In this setting, saving functions like a collateral and always tends to slacken an agent's participation constraint. Therefore, in contrast to main result in this paper, the prescribed level of saving in a constrained efficient agreement is generally *higher* than that an individual would choose on his own.

The main result in this paper also has an interesting parallel in the literature on risk-sharing under private information. However, the mechanism at work in each case is quite different. Atkeson and Lucas (1992) study an environment where agents are subject to private idiosyncratic shocks and there is no scope for saving. They show that it is possible to induce agents to report their shocks truthfully by providing a 'front-loaded' consumption stream to those who report a bad shock in the current period, and a 'back-loaded' consumption stream to those who do not. Thus, some degree of risk-sharing is possible although shocks are unobserved. However, if agents have access to hidden storage, they would care only about the net present value of future transfers, not its timing. Then risk-sharing will break down since, under any insurance scheme, truthful reporting is no longer incentive-compatible (Cole and Kocherlakota 2001; see also Allen 1985). A social planner's ability to observe and constrain saving in this environment enables him to distinguish better between individuals with good and bad shocks for risk-sharing purposes[3].

By contrast, we show in the present paper that, when no borrowing is possible, constraints on saving diminishes an agent's ability to self-insure and therefore improves his commitment to a risk-sharing agreement. In line with the recent literature, we believe that the full information, limited commitment model may be more pertinent to the study of risk-sharing in village economies where formal enforcement institutions are absent but individuals generally have good information on the assets and incomes of their neighbours.

3. The Model

We consider an environment with n agents, represented by the set $\mathcal{I} = \{1, .., n\}$. Each agent faces a stochastic income stream, denoted $\{y^i(t)\}_{t=1,2...,\infty}$. In each period, there are $\mathcal{S}$ possible states of the world; and the probability of each state $s \in \mathcal{S}$ equals π_s, with $\sum_{s\in\mathcal{S}} \pi_s = 1$; i.e. the distribution of states is independantly and identically distributed over time. The income earned by person i when the realised state is s is denoted y^i_s. Each agent also has access to an intertemporal technology whereby 1 unit of the good stored at the end of period t is transformed into σ units in period $t+1$.

Individuals have the means to transfer some of their savings or income to each other in each period. The exact sequence of events within each period is as follows:

(i) the state of nature s is realised and each agent receives her output for the period, $\sigma k^i_{t-1} + y^i_s$;

(ii) each agent i publicly pledges amounts $\tau^{ij}_t \geq 0$ to transfer to each agent $j \neq i$, and k^i_t to save at the end of the period;

[3]Yet another parallel exists for dynamic principle-agent models. Here again a distinct mechanism is at work. An agent's saving acts as a buffer against any future punishment the principle would inflict in the event of poor performance. Therefore, constraints on saving induces the agent to put in more effort in the future to avoid punishment (Rogerson 1985; Kocherlakota 2004; see also Golosov, Kocherlakota, Tsyvinski 2003 for a generalisation of the result in Rogerson 1985).

(iii) agents choose whether to 'approve' or 'reject' all pledges made; if pledges are universally approved and are 'feasible' then the transfers and savings take place as pledged. If not, no transfers take place and individuals have a second opportunity to allocate autarkic resources between consumption and saving.

Given pledges, 'feasibility' means the following condition is satisfied for all agents:

$$\sigma k_{t-1}^{i} + y_{s}^{i} + \sum_{j \in \mathcal{I}} \left(\tau_{t}^{ji} - \tau_{t}^{ij} \right) \geq k_{t}^{i}$$

Thus, transfers and savings decisions are, in effect, collectively decided within the community. In particular, it is not possible for an agent to accept transfers from others in the community during a certain period, and then to choose a level of consumption that does not meet with their approval. This may be a reasonable approximation of reality given that, in a village community, transfers and consumption would take place continously through time, and the deviation by a community member from any consumption or saving rule, if publicly observed, can face immediate retaliation.[4].

Agent i's preferences over different consumption streams are given by the following expression:

$$E \sum_{t=1}^{\infty} \beta^{t-1} u^{i} \left(c_{t}^{i} \right)$$

where $u^i()$ is increasing and strictly concave. To ensure interior solutions for consumption, we also assume $\lim_{c \to 0} u^{i\prime}(c) = \infty$. Since $u^i()$ is concave, agents prefer to smooth consumption across time and over different states of the world. They have two means of doing so; by engaging in precautionary saving using the intertemporal technology, and by participating in a mutual insurance agreement.

Before considering mutual insurance agreements, we introduce some terms and notation that will be used later in the analysis. Let $h_t = (s_1, s_2, ..., s_t)$ denote a particular history of realised states up to and including period t. Let $\mathcal{H}_t$ denote the set of all possible histories of states in period t. We define an *agreement* $\mathcal{A}$ as follows:

Definition 3.1. *An agreement is a complete plan of all bilateral transfers made, as well as the level of savings chosen, by each agent, in each period, contingent on the history of states.*

[4]This setup is equivalent to that adopted in Ligon, Thomas and Worrall (2000). An alternative possibility for modelling purposes would be for agents to make and accept transfers before savings decisions are made. We do not explore this route in this paper for it adds considerably to the complexity of the analytical problem.

According to this definition, an agreement can be described by $\mathcal{A} = \left\{k_t^i(h_t), \left\{\tau_t^{ij}(h_t)\right\}_{j\neq i}\right\}$, $i \in \mathcal{I}, h_t \in \mathcal{H}_t, t = 1..\infty$[5]. Such an agreement would specify that if the history of realised states in period t is h_t, then an agent i should make transfers $\tau_t^{ij}(h_t)$ to, and receive transfers $\tau_t^{ji}(h_t)$ from, each agent j during the period and have savings of $k_t^i(h_t)$ at the end of the period. For the agreement to be feasible, we must have $\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{I}}\left(\tau_t^{ij}(h_t) - \tau_t^{ji}(h_t)\right) = 0, k_t^i(h_t) \geq 0$ for each period t and possible history h_t. An agreement implicitly defines a consumption stream if we assume that whatever assets are not saved at the end of a period will be consumed. Then the consumption stream $\left\{c_t^i(h_t)\right\}_{i\in\mathcal{I},h_t\in\mathcal{H}_t,t=1..\infty}$ is given by

$$c_t^i(h_t) = \sigma k_{t-1}^i(h_{t-1}) + y_s^i + \sum_{j\neq i}\left(\tau_t^{ji}(h_t) - \tau_t^{ij}(h_t)\right) - k_t^i(h_t)$$

where s is the realised state in period t.

Note that two agreements that prescribe the same levels of savings and *net* transfers after each history also imply the same consumption streams and are identical for analytical purposes. Also note that an agreement is not equivalent to a complete strategy profile because the transfers and savings prescribed by the agreement are not contingent on the history of past actions. However, the prescriptions of an agreement are sufficiently detailed to address the question what utility levels can be sustained through mutual insurance in a subgame perfect equilibrium. Both these assertions will become apparent in the following section when we provide a characterisation of subgame perfect equilibria.

We denote by $\mathcal{A}|h_t$ the continuation agreement implied by $\mathcal{A}$ after history h_t has been realised. Also let $\mathcal{A}_1|h_t$ denote the same continuation agreement except that all transfers in period t following history h_t equal to zero.. Define $V^i(z, \mathcal{A})$ as the maximum utility that person i can achieve in expectation with initial assets z when he participates in an agreement $\mathcal{A}$. Similarly, $U_a^i(z)$ is the maximum expected utility that person i can achieve in autarky using initial assets z. Let $I^i(z, \mathcal{A}_1|h_t)$ be the amount of money agent i would be willing to forego in the current period, given assets z, to participate in the continuation agreement $\mathcal{A}_1|h_t$. Then, assuming $z > I^i(z, \mathcal{A}_1|h_t)$, we have that $I = I^i(z, \mathcal{A}_1|h_t)$ solves the following equation:

$$V^i(z - I, \mathcal{A}_1|h_t) = U_a^i(z) \tag{1}$$

[5]For the sake of legibility, we suppress the notation for the range of values of i, h_t, t whenever writing an agreement in this form hereafter.

The condition $z > I^i(z, \mathcal{A}_1|h_t)$ is necessary to ensure that the expression $V^i(z - I^i(..), \mathcal{A}_1|h_t)$ is well-defined. The equation in (1) will frequently appear in our analysis since it will hold whenever the participation constraint is binding for person i for a particular allocation of resources. We will, in particular, be interested in the effect of additional saving by person i on a binding participation constraint, which is the subject of the following lemma (below, we denote by $\tilde{\tau}_t^i(\mathcal{A}|h_t)$, net transfers by agent i following history h_t).

Lemma 3.1. *Given a continuation agreement $\mathcal{A}|h_t$ if agent i has asset level $z > I^i(z, \mathcal{A}_1|h_t)$, and net transfers by agent i following history h_t equals $I^i(z, \mathcal{A}_1|h_t)$ in the agreement, then*

$$\frac{\partial V^i}{\partial z}(z, \mathcal{A}|h_t) - \frac{dU_a^i}{dz}(z) = -\frac{\partial V^i(z, \mathcal{A}|h_t)}{\partial \tilde{\tau}_t^i(\mathcal{A}|h_t)} \frac{\partial I^i(z, \mathcal{A}_1|h_t)}{\partial z}$$

Proof. We are given

$$\tilde{\tau}_t^i(\mathcal{A}|h_t) = I^i(z, \mathcal{A}_1|h_t)$$

Define $\mathcal{A}'(z)$ as an alternate agreement that corresponds to $\mathcal{A}$ for all t, h_t except that transfers following history h_t are a function of the parameter z: $\tau_t^i(\mathcal{A}'(z)|h_t) = I^i(z, \mathcal{A}_1|h_t)$, $\tau_t^{-i}(\mathcal{A}'(z)|h_t) = 0$. Therefore, by definition of $I^i(z, \mathcal{A}_1|h_t)$, we must have

$$V^i(z, \mathcal{A}'(z)|h_t) \equiv U_a^i(z)$$

Differentiating throughout with respect to z, we obtain

$$\frac{dV^i}{dz}(z, \mathcal{A}'(z)|h_t) - \frac{dU_a^i}{dz}(z) \equiv 0$$

$$\implies \frac{\partial V^i}{\partial z}(z, \mathcal{A}'(z)|h_t) + \frac{\partial V^i}{\partial \tau_t^i(.|h_t)}(z, \mathcal{A}'(z)|h_t)\frac{\partial I^i}{\partial z}(z, \mathcal{A}_1|h_t) - \frac{dU_a^i}{dz}(z) = 0$$

Note that we can replace $\mathcal{A}'(z)$ by $\mathcal{A}$ since, as per the initial condition, the net transfers by person i following history h_t in the latter agreement equals $I^i(z, \mathcal{A}_1|h_t)$. Therefore, we can write

$$\frac{\partial V^i}{\partial z}(z, \mathcal{A}|h_t) - \frac{dU_a^i}{dz}(z) = -\frac{\partial V^i}{\partial \tau_t^i(.|h_t)}(z, \mathcal{A}|h_t)\frac{\partial I^i}{\partial z}(z, \mathcal{A}_1|h_t)$$

□

Lemma 3.1 says that if, for a particular level of assets, an agreement provides an individual zero surplus over autarky, then the marginal effect on the surplus, of an increase in assets, equals the marginal change in the individual's valuation of the continuation agreement. This change corresponds to the effect of increased saving on a binding participation constraint in a constrained efficient agreement. As will be seen, this technical result provides the crucial step of reasoning for the propositions that follow.

3.1. **Self-Enforcing Agreements.** In this section, we characterise agreements that can be supported in a subgame perfect equilibrium. These characterisations will be used later to analyse constrained efficient agreements. The results in this section are based on the techniques developed in Abreu (1988) and closely follows the reasoning behind a similar proposition in Kocherlakota (1996).

Proposition 3.1. *An agreement* $\mathcal{A} = \left\{k_t^i\left(h_t\right), \left\{\tau_t^{ij}\left(h_t\right)\right\}_{j\neq i}\right\}$ *and the associated consumption streams* $\{c_t^i(h_t)\}$ *can be obtained in a subgame perfect equilibrium if and only if it satisfies the following conditions:*

$$u^i\left(c_t^i(h_t)\right) + \beta E_t \sum_{\epsilon=t+1}^{\infty} u^i\left(c_\epsilon^i(h_\epsilon)\right) \geq U_a^i(\sigma k_{t-1}^i(h_{t-1}) + y_s^i) \quad i \in \mathcal{I}, \forall h_t \in H_t, t = 1..\infty \tag{2}$$

where $c_t^i\left(h_t\right) = \sigma k_{t-1}^i\left(h_{t-1}\right) + y_s^i + \sum_{j\neq i}\left(\tau_t^{ji}\left(h_t\right) - \tau_t^{ij}\left(h_t\right)\right) - k_t^i\left(h_t\right)$.

The conditions ensure that for each person i, after each possible history, the expected utility obtained from the consumption path specified by the allocation is at least as large as the maximum expected utility that can be obtained under autarky. The reasoning behind the proposition is briefly sketched here. The proof can be found in the appendix. If a particular allocation is obtained in a subgame perfect equilibrium, then it must satisfy the conditions above; if it does not hold for some individual after some history, she could improve her expected utility by deviating to the autarkic strategy in that subgame. Conversely, if an allocation satisfies the conditions specified above, then a strategy profile along the following lines would be subgame perfect: each individual, after each possible history, chooses transfers and savings as specified by the allocation; if any individual deviates at a point in time, then he receives no transfers thereafter. As the cost of deviation is autarky, which cannot be utility-improving by construction, this strategy profile is subgame perfect.

Note that the conditions under which an agreement is subgame perfect, and the level of welfare it provides to each agent, are fully determined by the consumption and saving levels implied by the agreement. The consumption levels, in turn, depends on the *net* transfers and savings made by each agent under each contingency, but not on the bilateral transfers among agents. Thus, in terms of constrained efficiency, two agreements that prescribe the same level of net transfers and savings, for each agent, under each contingency, are identical. Therefore, to simplify the analysis of constrained efficient agreements, we make use of the variable net transfers made by an agent following some history:

$$\tilde{\tau}_t^i(h_t) = \sum_{j\in\mathcal{I}} \left\{\tau_t^{ij}(h_t) - \tau_t^{ji}(h_t)\right\}$$

Since all transfers are made to, or received from, members of the community, the total sum of net transfers should equal zero: $\sum_{i\in\mathcal{I}} \tilde{\tau}_t^i(h_t) = 0$.

3.2. **Constrained Efficient Agreements.** Among the allocations that can be obtained in a subgame perfect equilibrium, we concentrate, as in the related literature, on those that are constrained efficient; i.e. on allocations where agents exploit all the potential surplus that is available from mutual insurance in the face of the constraints imposed by their limited commitment.

Let $P\left(k^1, .., k^n, \underline{U}^1, .., \underline{U}^{n-1}\right)$ be the maximum expected utility that agent n can obtain in a subgame perfect equilibrium, when the level of utility to be received by the other $n-1$ agents are at least $\underline{U}^1, .., \underline{U}^{n-1}$, and the initial wealth levels of the n agents are given by $k^1, .., k^n$ respectively. Then, using Proposition 3.1, we know that $P\left(k^1, .., k^n, \underline{U}^1, .., \underline{U}^{n-1}\right)$ is given by the following programme:

$$P\left(k^1, .., k^n, \underline{U}^1, .., \underline{U}^{n-1}\right) = \max_{\substack{\left\{\tilde{\tau}_t^i(h_t), k_t^i(h_t)\right\} \\ h_t\in\mathcal{H}, t=1..\infty, i=1..n}} E_0 \sum_{t=1}^{\infty} \beta^{t-1} u^n\left(c_t^n(h_t)\right) \tag{3}$$

$$\text{subject to } E_0 \sum_{t=1}^{\infty} \beta^{t-1} u^i\left(c_t^i(h_t)\right) \geq \underline{U}^i,\ i = 1..(n-1) \tag{4}$$

$$u^i\left(c_t^i(h_t)\right) + \beta E_t \sum_{\varepsilon=t+1}^{\infty} \beta^{\varepsilon-t-1} u^i\left(c_\varepsilon^i(h_\varepsilon)\right) \geq U_a^i\left(\sigma k_{t-1}^i(h_{t-1}) + y_s^i\right) \tag{5}$$

$$k^i(h_t) \geq 0,\ i \in 1..n \tag{6}$$

$$\sum_{j\in\mathcal{I}} \tilde{\tau}_t^j(h_t) = 0 \text{ for } \forall h_t \in \mathcal{H}_t, t = 1..\infty \tag{7}$$

where $c_t^i(h_t) = \sigma k_{t-1}^i(h_{t-1}) + y_s^i - \tilde{\tau}_t^i(h_t) - k_t^i(h_t)$. The expression in (3) is optimised over the saving levels and net transfers only since, as mentioned in the previous section, these terms fully describe the consumption levels and outside options implied by an agreement following each history. The conditions in (4) ensure that each agent other than n receives his promised level of utility in the contract. The conditions in (5) are the participation constraints implied by Proposition (3.2) for self-enforcing agreements. The constraints in (6) says that agents cannot, at any time, hold negative assets, while (7) equates total transfers made and received within the community after each history. For each set of values $\underline{U}^1, .., \underline{U}^{n-1}$, the solution to this problem corresponds to a constrained efficient allocation. As noted in the previous literature, the fact that the maximisation problem includes a constraint for each possible history in each time period, and that the number of possible histories rise exponentially with each period makes it inconvenient to analyse constrained efficient allocations using such a programme.

Fortunately, there exists an alternative formulation of the maximisation problem, introduced by Thomas and Worrall (1988) and Kocherlakota (1996) that makes use of the fact that all continuation agreements of a constrained efficient agreement must also be constrained efficient[6]. Furthermore, given that all the constraints are forward-looking, and the game involves an infinite horizon, the problem of determining a constrained efficient continuation agreement is the same as the original problem whenever the asset levels and the promised utilities are the same. Therefore, the programme in (3) can also be written as a dynamic programming problem:

$$
\begin{aligned}
(8)\, P\left(k^1, .., k^n, \underline{U}^1, .., \underline{U}^{n-1}\right) \quad &= \quad \max_{\left\{\tilde{\tau}_s^i, k_s^i, U_s^i\right\}} E\left[u^n\left(c_s^n\right) + \beta P\left(k_s^1, .., k_s^n, U_s^1, .., U_s^{n-1}\right)\right] \\
\text{subject to} \quad &: \\
(9) \qquad \lambda^i \quad &: \quad E\left[u^i\left(c_s^i\right) + \beta U_s^i\right] \geq \underline{U}^i \text{ for } i = 1..n-1 \\
(10) \qquad \pi_s \theta_s^i \quad &: \quad u^i\left(c_s^i\right) + \beta U_s^i \geq U_a^i\left(\sigma k^i + y_s^i\right) \text{ for } i = 1..n-1 \\
(11) \qquad \pi_s \theta_s^n \quad &: \quad u^n\left(c_s^n\right) + \beta P\left(k_s^1, .., k_s^n, U_s^1, .., U_s^{n-1}\right) \geq U_a^n\left(\sigma k^n + y_s^n\right) \\
(12) \qquad \pi_s \omega_s^i \quad &: \quad k_s^i \geq 0 \text{ for } i = 1..n \\
(13) \qquad \zeta_s \quad &: \quad \sum_{i \in \mathcal{I}} \tilde{\tau}_s^i = 0 \\
\text{for each } s \quad &\in \quad \mathcal{S}
\end{aligned}
$$

where $c_s^i = \sigma k^i + y_s^i - \tilde{\tau}_s^i - k_s^i$. Here, the conditions in (9) correspond to the promise keeping constraints in (4). The conditions in (10) and (11) ensure that the allocation of

[6]The reasoning is provided by Ligon, Thomas & Worrall (2002). Consider a subgame perfect agreement starting at $t = 0$ that involves a continuation agreement, following some history h_t, that is not constrained efficient. Then there exists at least one other continuation agreement which is subgame perfect and Pareto superior. Replacing the original continuation agreement by the new agreement weakly relaxes the participation constraints and weakly raises expected utilities in the agreement starting at $t = 0$. Then the original agreement could not have been constrained efficient.

utilities in the first period of the agreement and the promised utilities hereafter satisfy the participation constraints for the first period. Following the reasoning provided by Ligon, Thomas and Worrall (2002) in a similar setting, we can show that $P(.)$ is decreasing and concave in each $\underline{U}^i$.[7]

The basic characteristics of the constrained efficient agreement can be inferred from the first-order conditions and the Envelope equations of the problem. From the first-order condition with respect to transfers, we obtain the ratio of marginal utilities of consumption between any two agents in any state of nature:

$$\frac{u^{j\prime}\left(c_s^j\right)}{u^{i\prime}\left(c_s^i\right)} = \frac{\left(\lambda^i + \theta_s^i\right)}{\left(\lambda^j + \theta_s^j\right)} \text{ for } i, j = 1..n, s \in \mathcal{S} \tag{14}$$

where $\lambda^n = 1$. When the participation constraints of both agents are slack, $\theta_s^i = \theta_s^j = 0$, and therefore the ratio of marginal utilities is the same across all states where this condition holds. Whenever the constraint binds for one agent and not the other, the ratio shifts in favour of the first.

Using the Envelope theorem, we obtain

$$\frac{\partial P}{\partial \underline{U}^i} = -\lambda^i,\ i = 1..n \tag{15}$$

This is the marginal rate of substitution between agents i and n. Since the function $P(.)$ is concave in each $\underline{U}^i$, we have λ^i increasing in $\underline{U}^i$. Therefore, given the promised utilities for all agents except i and n, and the initial level of assets of each, λ^i uniquely identifies the promised utility to agent i, and a higher λ^i corresponds to a higher level of utility.

The first-order condition with respect to U_s^i yields

$$\lambda^i + \theta_s^i = -\left(1 + \theta_s^n\right)\frac{\partial P}{\partial U_s^i} \tag{16}$$

[7]Consider two constrained efficient contracts $\mathcal{A}_1$ and $\mathcal{A}_2$ that award some agent i expected utilities equal to V_1^i and V_2^i respectively; while agent j receives V^j in each contract for $j = 1, 2..n-1, j \neq i$. Denote by $\left\{c_1^k(h_t)\right\}$ and $\left\{c_2^k(h_t)\right\}$, $k = 1..n$, the consumption streams corresponding to the two agreements. Any convex combination of these allocations would then produce consumption streams of the form $\left\{\alpha c_1^k(h_t) + (1-\alpha)c_2^k(h_t)\right\}$, $k = 1..n$, where $\alpha \in (0,1)$. Since the per-period utility functions are concave, such a consumption stream would provide agent i at least the level of utility $\alpha V_1^i + (1-\alpha)V_2^i$, and to agent n a higher utility than the linear combination of the utilities obtained from $\mathcal{A}_1$ and $\mathcal{A}_2$. Therefore, we have

$$P\left(k^1..k^n, V^1..\alpha V_1^i + (1-\alpha)V_2^i..V^{n-1}\right) \geq \alpha P\left(k^1..k^n, V^1..V_1^i..V^{n-1}\right) + (1-\alpha)P\left(k^1..k^n, V^1..V_2^i..V^{n-1}\right)$$

Therefore, $P(.)$ is concave in each V^i.

From (15), we can conclude that $\frac{\partial P}{\partial U_s^i} = -\lambda_s^i$ where λ_s^i is defined as the Lagrange multiplier on person i's promise-keeping constraint one period into the future when the realised state is s. Substituting for $\frac{\partial P}{\partial U_s^i}$ into (16), we obtain

$$\lambda_s^i = \frac{\lambda^i + \theta_s^i}{1 + \theta_s^n} \tag{17}$$

This last equation shows how person i's Pareto weight and therefore his promised utility evolves over time. Whenever his participation constraint binds, this tends to raise his promised utility since U_s^i is increasing in λ_s^i, and the opposite is true when the constraint is binding for some other agent. However, note that whether his utility actually rises or not also depends on the total assets available to the group in the new period.

From the first-order condition with respect to k_s^i, the Envelope condition with respect to k^i and Lemma 3.1, we obtain the following equation (the precise steps are shown in the proof of Proposition 3.2).

$$\frac{1}{\sigma} u^{i\prime}\left(c_s^i\right) = \beta E u^{i\prime}\left(c_{sr}^i\right) - \beta \sum_{r \in \mathcal{S}} \pi_r \frac{\theta_{sr}^i}{\lambda_s^i} \frac{\partial V^i\left(z, \mathcal{A} | h_t\right)}{\partial \tau_t^i\left(\mathcal{A} | h_t\right)} \frac{\partial I^i\left(z, \mathcal{A}_1 | h_t\right)}{\partial z} - \frac{\omega_s^i}{1 + \theta_s^n} \tag{18}$$

where θ_{sr}^i is the Lagrange multiplier on person i's participation constraint following the history of shocks (s, r). Ignoring the second and third terms on the right-hand side, this equation is equivalent to the standard Euler condition which equates marginal utility in the current period to expected marginal utility in the following period after adjusting for the discount factor and the rate of return on capital. The third term on the right-hand side appears because of the non-negativity constraint on savings. As for the second-term, remember that $I^i\left(z, \mathcal{A}_1 | h_t\right)$ is the payment that leaves an agent with wealth z indifferent between the continuation agreement $\mathcal{A}_1$ and autarky. Therefore the sign of the term $\frac{\partial I^i}{\partial z}$ depends on how an agent's valuation of the continuation agreement changes with wealth. Given that the term $\frac{\partial V^i}{\partial \tau_t^i}$ is always negative, if richer agents attach less value to an insurance agreement, then the second term on the right-hand side of (18) will also be negative. Then the level of savings prescribed by the constrained efficient agreement will be lower than that which is obtained from the standard Euler equation. We thus obtain the following result:

Proposition 3.2. *In the constrained efficient agreement, if an agent's valuation of an insurance agreement is decreasing in wealth, and his participation constraint is binding* *after some history, then his level of prescribed savings in the preceding period is below that which is individually optimal.*

Under what conditions is an agent's valuation of an insurance agreement decreasing in wealth? In the absence of a mutual insurance agreement, an agent can protect his own consumption against adverse income shocks by spending a part of his savings. However, a poorer agent is less disposed to do this for he runs a greater risk of running his stock completely dry through a succession of misfortunes, leaving him with no means to cope with future shocks. Therefore, in autarky, the rich would smooth consumption to a greater extent than the poor. Consequently, even if aversion to risk is constant in the level of wealth, an agent with a higher level of wealth places lower value on a mutual insurance agreement. This result would be further reinforced if the Bernoulli utility function exhibits decreasing absolute risk aversion.

Proposition 3.2 highlights the inherent tension between mutual insurance under lack of commitment and the ability to self-insure. Such an insurance network is most valuable for poor households that have few assets to smooth consumption on their own in the face of adverse shocks. The constrained efficient agreement would provide the household insurance against such shocks but also ensures that the household remains poor and thus dependent on the agreement so that it is, in turn, willing to provide assistance to others in the group as needed. This is done by requiring the household to save below the level that is individually optimal given its future stream of income and transfers. Thus, the agreement serves two functions: to enable poor households to smooth consumption and to ensure that they remain poor and therefore dependent on the agreement for consumption smoothing.

It is clear from the previous discussion that the level of assets of each individual or household is an important determinant of the extent of insurance that can be sustained in the community. The wealthier is a household, the better it is able to self-insure using its own assets, and therefore the less dependent it is on a social network that would provide support during hard times. This reasoning suggests that the poor are better insurance partners than the rich. We can verify, using the present theoretical framework, that this reasoning is indeed true.

Specifically, we consider the question how does replacing a poor agent in an insuring group by a richer agent affect the surplus from mutual insurance to the other $n-1$

agents. Since there will, in general, be more than one agreement that is constrained efficient, each corresponding to a different division of the surplus, we need to make an assumption about how the surplus from insurance is divided within the group when the rich agent replaces the poor agent in the contract. To assume that the richer agent would receive the same *level* of surplus as the poor agent he replaces is not satisfactory: since the richer agent would place a lower value on any insurance agreement, providing him the same level of surplus would necessarily reduce the surplus available for the remaining agents.

We consider instead, as a benchmark, the case where the share of the rich agent in the surplus is equal to that he would have received from the agreement that is implemented when the poor agent is part of the insuring group. If this condition does not hold, it means that when a rich agent replaces a poor agent in an insurance agreement, the former must transfer a part of his extra wealth *unconditionally* to others in the insurance network. An agreement where the rich agent is required to do so may well be feasible, but, as a starting point, it is useful to consider the level of surplus from an insurance agrement to the other participants when the richer agent is not required to make such a transfer and thus may enjoy the full benefit of his extra wealth.

Using this assumption, we can show that at least one of the remaining agents participating in the insurance agreement is made worse off when the richer agent replaces the poorer household in the agreement, and receives a level of utility according to the rule above. Formally, we state this result as follows:

Proposition 3.3. *In a constrained efficient insurance agreement, increasing the initial level of assets of any one agent, and raising his level of expected utility sufficiently so that he does not prefer the old agreement to the new one, means that one or more of the remaining agents are worse off*[8].

The intuition for this result is straightforward. Raising an agent's initial level of assets and his expected utility accordingly, tightens his participation constraint and also means that no one else can benefit from his increased wealth in the group. Then, one or more of the other agents must be worse off. If the rich agent is able to negotiate a higher level

[8]Equivalently, lowering the initial level of assets of any one agent, and lowering his level of expected utility from the agreement till the point where he is indifferent between the old and the new agreements, means that all of the remaining agents can be made better off.

of surplus for himself than that assumed above, this will reinforce the result obtained in Proposition 3.3.

Note that the richer agent is himself receiving a smaller surplus under the assumptions made in Proposition 3.3. The reason is that although the new agreement provides him the same level of utility as he would have received from the agreement offered to the poorer individual, such an agreement is itself less valuable to him since he is richer and better able to smooth consumption in autarky. It follows that when we raise the starting level of wealth of any one agent in the community, there is no new agreement which provides every agent at least the level of surplus they received in the original agreement. This result also holds true when the first-best agreement can be implemented but it is reinforced if there are binding participation constraints after some history. Formally, we state the result in terms of a corollary to Proposition 3.3.

Corollary 3.4. *: Given a constrained efficient insurance agreement, increasing the initial level of assets of any one agent means that one or more agents must receive a lower surplus in the revised agreement.*

This result has important consequences for the formation of informal insurance groups. It means that any group of individuals (or households) would benefit more from including a poor individual in their insurance network than a rich individual, irrespective of their own wealth levels. If there are costs involved in developing an insurance network – say in gaining the trust of one's neighbours and developing the rules and norms that would determine the obligations of the participants under different contingencies – then an individual in a poor community is more likely to find such an effort worthwhile, than an individual *with the same level of wealth* in a rich community. Reasoning thus, one may conclude that, *ceteris paribus*, insurance networks are more likely to develop in poorer communities. This idea is formalised in the next section.

4. Divergence of Communities with Differing Social Norms

In the preceding section, we learn, first, that in a constrained efficient mutual insurance agreement, an individual saves below the level that is individually optimal; and second, that the poor are better insurance partners than the rich. These two results together suggest that within a population where the means of consumption smoothing are limited to informal insurance groups and the use of own assets, the rate of consumption growth should be higher for the rich versus the poor. With a very few extensions to the model, we can demonstrate this result formally.

Consider two communities, A and B, each consisting of n agents, situated sufficiently far apart that there is no scope of mutual insurance across the communities. At $t = 0$, the agents within each community decide whether or not to form an insurance group. Suppose that joining an insurance group involves a cost δ to an agent in both communities. From the Corollary to Proposition 3.3, it follows that if the initial level of assets in a community are sufficiently high, then there is no feasible agreement which yields a surplus δ to each participating agent. Suppose that this is the case for community B, while agents in community A are poor enough that at least one such agreement is possible. Then, assuming that efficient bargaining can take place, an insurance network would develop in community A but not in B.

In community B, agents would then make consumption and savings decisions in autarky. Therefore, assuming positive savings, their consumption path would follow the standard Euler equation:

$$u^{j\prime}\left(c^{j}\left(t\right)\right) = \beta\sigma E u^{j\prime}\left(c^{j}\left(t+1\right)\right)$$

Rearranging this equation, we obtain the expected rate of growth of consumption as measured by the ratio of marginal utilities between two successive time periods:

$$E\frac{u^{j\prime}\left(c^{j}\left(t+1\right)\right)}{u^{j\prime}\left(c^{j}\left(t\right)\right)} = \frac{1}{\beta\sigma}$$

In community A, if agents opt for an insurance agreement that is constrained efficient, then, as we have already seen, the consumption path would satisfy a modified Euler equation with an extra term on the right-hand side that is negative:

$$\frac{1}{\sigma}u^{i\prime}\left(c_{s}^{i}\right) = \beta\left(1+r\right)Eu^{i\prime}\left(c_{sr}^{i}\right) - \sum_{r\in\mathcal{S}}\frac{\theta_{sr}^{i}}{\lambda_{s}^{i}}\frac{\partial V^{i}\left(z,\mathcal{A}|h_{t}\right)}{\partial\tau_{t}^{i}\left(\mathcal{A}|h_{t}\right)}\frac{\partial I^{i}\left(z,\mathcal{A}_{1}|h_{t}\right)}{\partial z} - \frac{\omega^{i}}{1+\theta_{s}^{n}}$$

Assume, as for community B, that the non-negativity constraint on saving is not binding. We obtain

$$E\frac{u^{i\prime}\left(c^{i}\left(t+1\right)\right)}{u^{i\prime}\left(c^{i}\left(t\right)\right)} > \frac{1}{\beta\sigma}$$

We have thus established the following result.

Proposition 4.1. *If participation in an insurance agreement carries a positive cost for an agent, then no mutual insurance agreement will develop in a community for sufficiently high levels of initial wealth. Whenever the non-negativity constraints on saving do not bind, the growth rate of consumption in this community, as measured by the expected* *period t ratio of marginal utilities of consumption between periods t and $t+1$, will be weakly greater than in a community where a constrained-efficient insurance agreement is adopted.*

Proposition 4.1 states, first, that if there are costs involved in setting up a mutual insurance agreement, then they will not appear in rich communities. The level of savings will be individually optimal, and the expected growth in consumption will be that implied by the standard Euler equation. By contrast, households in communities that are sufficiently poor will find it in their interest to put in the time and effort required to create a mutual insurance agreement. If they opt for the agreement that is constrained efficient, then households in these communities will weakly save below the level that is individually optimal, and correspondingly, they will experience a lower consumption growth path[9]. The theoretical implication of this result is that the efficient 'social norm' that arises out of a need for mutual insurance reinforces the gap between communities where such a need initially exists and where it doesn't.

Applying this result to the real world should, however, be done with caution. It involves the implicit assumption that social norms that discourage saving arise specifically when and where it can lead to improved insurance within a group, a claim which is beyond the scope of this paper. Nevertheless, it is worthwhile to consider such a possibility for it carries an important implication for empirical analysis.

In a long-term comparison between two social groups, one where the social constraints are adopted and another where they are not, the negative effects of the constraints should be more readily observable than the positive ones. The constraints on individual behaviour imposed by the social norm would be reflected in the widening gap in consumption and wealth between the two communities. The fact that the norm also enables the poorer community to achieve greater consumption smoothing through insurance would be less apparent, for it is precisely because individuals in the richer community were able to achieve at least the same level of consumption smoothing in autarky that the same norms did not develop there. It would appear that the social norm acts, primarily, to impede economic development when, in fact, it also plays a positive role for the community.

[9]Note, however, that this comparison of consumption growth paths across communities is valid only if the non-negativity constraints on saving are not binding. If the individually optimal level of saving equals zero, then there are no additional disincentives to save in the constrained efficient agreement.

5. Conclusion

A common assumption in the theoretical literature on informal insurance under lack of commitment is the absence of an intertemporal technology. This assumption may not be innocuous since, as pointed out by Besley (1995), an intertemporal technology enables an individual to engage in consumption smoothing outside of a group insurance contract. This paper attempts to fill this gap in the literature by investigating the interactions between savings behaviour and self-enforcing mutual insurance agreements when an intertemporal technology is available to agents.

The theoretical analysis shows that in the constrained efficient agreement, the accumulation of wealth by individuals will be constrained so that they remain dependent on the insurance network for consumption smoothing, and therefore more willing to satisfy any future demands for assistance from within the group. This feature of the constrained efficient agreement have strong parallels with anthropological observations relating to social norms in traditional societies where a variety of cultural values and social beliefs serve to deter the accumulation of 'excessive' wealth by individuals.

This theoretical result should not be interpreted as providing a causal motive for a certain type of social norm in traditional societies. As noted by Banerjee (2002) and Greif (2006), a game-theoretic framework provides limited scope for positive analysis of social norms or institutions, given the usual presence of multiple equilibria. The exercise in this paper reveals not so much when social norms that deter savings by individuals will arise, but more significantly, that when they do, they will have a positive effect on the scope for group cooperation, an effect that will be missed in an economic analysis restricted to individual behaviour.

As mentioned in the introduction, the relation between self-enforcing mutual insurance agreements and savings behaviour investigated in this paper points to a more general argument which we reiterate here: where a cooperative agreement has to be self-enforcing, efficient norms take the form that ensures continued dependence of the individual on the service provided by the group; for it is precisely the need for this service which would induce the individual to keep his end of the bargain.

This last statement suggests that the efficient social norm would tend to accentuate – or at least lead to persistence in – differences between social groups where the need for the service provided through group cooperation initially existed and where it did not. Proposition 4.1 makes this statement formally in the context of mutual insurance agreements. Herein lies the paradox of 'efficient' social norms: it enables the social group to come together to take care of an existing need among its members, and yet also encourages behaviour by individuals that would lead to the persistence of this need in the future.

6. Appendix

Proof. of Proposition 3.1: First note that, following any history (h_{t-1}, s), an individual who pledges zero transfers and the autarkic savings level in each period thereafter can guarantee himself the autarkic utility level $U_a^i(\sigma k_{t-1}^i(h_{t-1}) + y_s^i)$ in the continuation game. Therefore, if an allocation $\mathcal{A}$ is obtained in a subgame perfect equilibrium, the conditions in (2) must be satisfied. If it is not satisfied in some period t, after history h_t for person i, then person i would obtain a higher utility by deviating to the autarkic strategy for the continuation game, which contradicts the definition of subgame perfection. Conversely, if an agreement $\mathcal{A}$ satisfies the conditions in (2), then we can construct a strategy profile as follows. Each individual, after each possible history, pledges transfers and savings as specified in the agreement and approve of all pledges that correspond to this agreement, if all previous actions in the game follow this rule (the cooperation phase); after any deviation, each individual adopt autarkic strategies for the continuation game (the punishment phase). Then, the conditions in (2) ensure that, in the cooperation phase, a deviation in step (ii) of the stage game, when pledges are made, or in step (iii) of the game, when pledges are approved or rejected, cannot improve welfare. Likewise, in the punishment phase, a deviation cannot improve welfare because the autarkic strategies are subgame perfect. Therefore, the outlined strategy profile is also subgame perfect. □

Proof. of Proposition 3.2: From the first-order condition with respect to k_s^i, we obtain

$$(1 + \theta_s^n)\,\beta \frac{\partial P}{\partial k_s^i} = \left(\lambda^i + \theta_s^i\right) u^{i\prime}\left(c_s^i\right) + \omega_s^i \tag{19}$$

Using the Envelope theorem, we obtain the following expression for $\frac{\partial P}{\partial k_s^i}$.

$$\frac{\partial P}{\partial k_s^i} = \sigma \sum_{r \in \mathcal{S}} \pi_r \left[\lambda_s^i u^{i\prime}\left(c_{sr}^i\right) + \theta_{sr}^i \left\{u^{i\prime}\left(c_{sr}^i\right) - U_a^{i\prime}\left(\sigma k_s^i + y_s^i\right)\right\}\right] \tag{20}$$

where θ^i_{sr} is the Lagrange multiplier on person i's participation constraint following the history of shocks (s, r). Combining (19) and (20), and substituting for the term $\frac{1+\theta^n_s}{\lambda^i+\theta^i_s}$ using (17), we obtain

$$(21)\quad \frac{1}{\sigma}u^{i\prime}\left(c^i_s\right) = \beta E u^{i\prime}\left(c^i_{sr}\right) + \beta\sum_{r\in\mathcal{S}}\pi_r\left[\frac{\theta^i_{sr}}{\lambda^i_s}\left\{u^{i\prime}\left(c^i_{sr}\right) - U^{i\prime}_a\left(\sigma k^i_s + y^i_r\right)\right\}\right] - \frac{\omega^i_s}{\lambda^i_s\sigma\left(1+\theta^n_s\right)}$$

Let $\mathcal{A}$ be the constrained efficient agreement that corresponds to the solution to (8) and denote by U^i_{sr} the promised utility to person i from the agreement following the history of shocks (s, r). By construction, we can write

$$V^i\left(\sigma k^i_s + y^i_r, \mathcal{A}|\left(s,r\right)\right) = u^i\left(c^i_{sr}\right) + \beta U^i_{sr}$$

Let $\frac{\partial V^i}{\partial z}$ be the partial derivative of $V^i(.)$ with respect to its first argument. Recall that the expression $\frac{\partial V^i}{\partial z}\left(\sigma k^i_s + y^i_r, \mathcal{A}|\left(s,r\right)\right)$ represents the increase in utility to person i from a marginal increase in initial assets, assuming that he participates in the same continuation agreement $\mathcal{A}|\left(s,r\right)$. An increase in initial assets must translate into an equivalent increase in consumption in the initial period, for savings and consumption levels in all subsequent periods are determined by the continuation agreement $\mathcal{A}|\left(s,r\right)$. Therefore, we must have

$$\frac{\partial V^i}{\partial z}\left(\sigma k^i_s + y^i_r, \mathcal{A}|\left(s,r\right)\right) = u^{i\prime}\left(c^i_{sr}\right)$$

Therefore, we can rewrite (21) as

$$\frac{1}{\sigma}u^{i\prime}\left(c^i_s\right) = \beta E u^{i\prime}\left(c^i_{sr}\right) + \beta\sum_{r\in\mathcal{S}}\pi_r\left[\frac{\theta^i_{sr}}{\lambda^i_s}\left\{\frac{\partial V^i}{\partial z}\left(\sigma k^i_s + y^i_r, \mathcal{A}|\left(s,r\right)\right) - U^{i\prime}_a\left(\sigma k^i_s + y^i_r\right)\right\}\right] - \frac{\omega^i_s}{\lambda^i_s\sigma\left(1+\theta^n_s\right)}$$

Then, using Lemma 3.1, we obtain the result

$$\frac{\partial V^i}{\partial z}\left(z, \mathcal{A}|\left(s,r\right)\right) - U^{i\prime}_a(z) = -\frac{\partial V^i\left(z, \mathcal{A}|\left(s,r\right)\right)}{\partial \tau^i_t\left(\mathcal{A}|\left(s,r\right)\right)}\frac{\partial I^i\left(z, \mathcal{A}_1|\left(s,r\right)\right)}{\partial z}$$

where $z = \sigma k^i_s + y^i_r$. Therefore, assuming that the non-negatively constraint on saving is not binding, we can write

$$(22)\qquad \frac{1}{\sigma}u^{i\prime}\left(c^i_s\right) = \beta E u^{i\prime}\left(c^i_{sr}\right) - \beta\sum_{r\in\mathcal{S}}\pi_r\frac{\theta^i_{sr}}{\lambda^i_s}\frac{\partial V^i\left(z, \mathcal{A}|\left(s,r\right)\right)}{\partial \tau^i_t\left(\mathcal{A}|\left(s,r\right)\right)}\frac{\partial I^i\left(z, \mathcal{A}_1|\left(s,r\right)\right)}{\partial z}$$

The term $\frac{\partial V^i\left(z,\mathcal{A}|\left(s,r\right)\right)}{\partial \tau^i_t\left(\mathcal{A}|\left(s,r\right)\right)}$ is always negative. Therefore, if the value of the continuation agreement is decreasing in available assets z, then the expression involving the summation sign on the right-hand side is also negative. Therefore, equation (22) prescribes a level of saving below that which solves the standard Euler equation. □

Proof. of Proposition 3.3: Consider the constrained efficient agreement that provides agents $1..(n-1)$ utility levels $\underline{U}^1, .., \underline{U}^{n-1}$ given initial wealth levels $k^1, .., k^n$. The utility available to agent n then equals $\underline{U}^n = P\left(k^1, .., k^n, \underline{U}^1..\underline{U}^{n-1}\right)$. We consider the effect on the constrained efficient agreement of a small increase δ in the initial wealth level of some agent $i \neq n$ when agent i is awarded the same utility in the new agreement as he would have obtained from the old agreement at his now higher wealth level. Denote by $\tilde{U}^i(\delta)$ the target utility level for agent i. We have

$$\tilde{U}^i(\delta) = EV^i\left(\sigma\left(k^i + \delta\right) + y_s^i, \mathcal{A}\right)$$

where $\mathcal{A}$ is the original agreement. Then the increase in utility to agent i from the agreement following a marginal increase in wealth equals

$$\begin{aligned} \tilde{U}^{i\prime}(0) &= \frac{d}{dk^i} EV^i\left(\sigma k^i + y_s^i, \mathcal{A}\right) \\ &= \sigma E \frac{\partial V^i}{\partial z}\left(\sigma k^i + y_s^i, \mathcal{A}\right) \\ &= \sigma E u'\left(c_s^i\right) \end{aligned}$$

where $\frac{\partial V^i}{\partial z}$ is the partial derivative of $V^i(.)$ with respect to its first argument. The effect of a marginal increase in the initial wealth of agent i on the utility of agent n from the constrained efficient agreement, keeping fixed the utility of all agents other than i and n, equals

$$\frac{dP}{dk^i}\left(k^1, .., k^n, \underline{U}^1, .., \tilde{U}^i(0), .., \underline{U}^n\right) = \frac{\partial P}{\partial k^i} + \frac{\partial P}{\partial \underline{U}^i}\tilde{U}^{i\prime}(0)$$

From previous analysis, we know that

$$\begin{aligned} \frac{\partial P}{\partial k^i} &= \sigma \sum_{s \in \mathcal{S}} \pi_s \left[\lambda^i u^{i\prime}\left(c_s^i\right) + \theta_s^i \left\{\frac{\partial V^i}{\partial z}\left(\sigma k^i + y_s^i, \mathcal{A}\right) - U_a^{i\prime}\left(\sigma k^i + y_s^i\right)\right\}\right] \\ \frac{\partial P}{\partial U^i} &= -\lambda^i \end{aligned}$$

Therefore, we obtain

$$\begin{aligned} \frac{\partial P}{\partial k^i} + \frac{\partial P}{\partial \underline{U}^i}\tilde{U}^{i\prime}(0) &= \sigma \sum_{s \in \mathcal{S}} \pi_s \left[\lambda^i u^{i\prime}\left(c_s^i\right) + \theta_s^i \left\{\frac{\partial V^i}{\partial z}\left(\sigma k^i + y_s^i, \mathcal{A}\right) - \frac{\partial U_a^i}{\partial z}\left(\sigma k^i + y_s^i\right)\right\}\right] \\ &\quad -\lambda^i \sigma \sum_{s \in \mathcal{S}} \pi_s u'\left(c_s^i\right) \\ &= \sigma \sum_{s \in \mathcal{S}} \pi_s \theta_s^i \left[\frac{\partial V^i}{\partial z}\left(\sigma k^i + y_s^i, \mathcal{A}\right) - \frac{\partial U_a^i}{\partial z}\left(\sigma k^i + y_s^i\right)\right] \end{aligned}$$

Using Lemma 3.1, we obtain the expression

$$-\sigma \sum_{s \in \mathcal{S}} \pi_s \theta_s^i \frac{\partial V^i\left(\sigma k^i + y_s^i, \mathcal{A}\right)}{\partial \tau_t^i(\mathcal{A})} \frac{\partial I^i\left(\sigma k^i + y_s^i, \mathcal{A}_1\right)}{\partial z}$$

If the valuation of the insurance agreement is decreasing in wealth, then this expression is negative. Therefore, increasing the initial wealth of agent i, providing him the compensation described above, and holding fixed utility of all agents other than i and n leads to a decrease in the level of utility for agent n in the agreement. Therefore, at least one agent must be worse off in the agreement following an increase in the initial wealth of some agent and an increase in utility that leaves him indifferent between the old and new agreements. □

Proof. of Corollary to Proposition 3.3: In Proposition 3.3, we show that if there is a small increase in the initial wealth of agent i, then a new agreement that provides him the same utility that he would receive from the old agreement at his new wealth level would make at least one of the remaining agents worse off. It follows that at least one of the remaining agents would receive a smaller surplus than in the original agreement. The effect on the surplus of agent i in adopting the new agreement equals

$$\begin{aligned} & \frac{d}{dk^i}\left[EV^i\left(\sigma k^i + y_s^i, \mathcal{A}\right) - EU^i\left(\sigma k^i + y_s^i\right)\right] \\ = \; & E\left[\frac{d}{dk^i}\left\{V^i\left(\sigma k^i + y_s^i, \mathcal{A}\right) - U^i\left(\sigma k^i + y_s^i\right)\right\}\right] \end{aligned}$$

Applying Lemma 3.1 to substitute for the term within the curly brackets, we obtain an expression that is negative. Therefore, agent i receives a smaller surplus in the new agreement. Since the new agreement is constrained efficent, this implies that, following the increase in wealth of agent i, there is no feasible agreement in which each agent receives the same surplus as they had received in the original agreement. □

References

[1] Abreu, D., 1988. "On the Theory of Infinitely Repeated Games with Discounting", Econometrica 56(2), pp 383-396.

[2] Allen, F., 1985, "Repeated Principal-Agent Relationships With Lending and Borrowing", Economic Letters 17, pp 27-31.

[3] Atkeson, A. and R.E. Lucas, 1992. "On Efficient Distribution With Private Information", Review of Economic Studies, 59, pp 427-453.

[4] Banerjee, A. V., 2002. "The uses of economic theory: Against a purely positive interpretation of theoretical results", BREAD Working Paper No. 7, MIT, Bureau for Research in Economic Analysis of Development, Cambridge, MA.

[5] Besley, T., 1995. "Savings, credit and insurance", in *Handbook of Development Economics*, vol. 3A. Elsevier, Amsterdam, pp 2123-2207

[6] Bloch, F., V. Rao, and S. Desai, 2004. "Wedding Celebrations as Conspicuous Consumption: Signaling Social Status in Rural India", Journal of Human Resources 39(3), pp 675-695

[7] Coate, S. and M. Ravallion, 1993. "Reciprocity without commitment: Characterisation and performance of informal risk-sharing arrangements", Journal of Development Economics 40, pp 1-24
[8] Cole, H. L. and N. Kocherlakota, 2001. "Efficient Allocations with Hidden Income and Hidden Storage", Review of Economic Studies 68(3), pp 523-542.
[9] Fafchamps, M., 1999. "Risk sharing and quasi-credit", Journal of International Trade and Economic Development 8(3), pp 257-278
[10] Gauthier, C., M. Poitevin and P. Gonzàlez, 1997. "Ex-Ante Payments in Self-Enforcing Risk-Sharing Contracts", Journal of Economic Theory 76, pp 106-144
[11] Gobert, K. and M. Poitevin, 2006, "Non-commitment and savings in dynamic risk-sharing contracts", Economic Theory 28(2), pp 357-272.
[12] Golosov M., N. Kocherlakota, and A Tsyvinski, 2003. "Optimal Indirect and Capital Taxation", Review of Economic Studies 70(3), pp 569-587.
[13] Greif, A., 1994. "Cultural Beliefs and the Organization of Society: A Historical and Theoretical Reflection on Collectivist and Individualist Societies", Journal of Political Economy 102(5), pp 912-950.
[14] Greif, A., 2006. *Institutions and the Path to the Modern Economy: Lessons from Medieval Trade*, Cambridge University Press, Cambridge.
[15] Guiso, L., P. Sapienza and L. Zingales, 2006. "Does Culture Affect Economic Outcomes?", Journal of Economic Perspectives 20, pp 23-48.
[16] Kimball, M., 1988. "Farmers' cooperatives as behaviour toward risk", American Economic Review 78, pp 224-232.
[17] Kocherlakota, N., 1996. "Implications of Efficient Risk Sharing without Commitment", Review of Economic Studies 63
[18] Kocherlakota, N., 2004. "Figuring out the impact of hidden savings on optimal unemployment insurance", Review of Economic Dynamics 7, pp 541-554.
[19] Landes, D., 1998. *The Wealth and Poverty of Nations*, Norton and Company, Inc., New York.
[20] Ligon, E., J. P. Thomas, and T. Worrall, 2000. "Mutual Insurance, Individual Savings and Limited Commitment", Review of Economic Dynamics 3(3), pp 1-47.
[21] Ligon, E., J. Thomas, and T. Worrall, 2002. "Informal insurance arrangements with limited commitment: Theory and evidence from village economies", Review of Economic Studies 69.
[22] Manuh, T., 1995. "Changes in Marriage and Funeral Exchanges Among the Asante" in *Money Matters: Instability, Values and Social Payments in the Modern History of West African Communities*, ed. J. Guyer.
[23] Platteau, J.P., 2000. *Institutions, Social Norms, and Economic Development*, Harwood Academic Publishers.
[24] Putnam, R. D., 1993. *Making Democracy Work*. Princeton University Press, Princeton, New Jersey.
[25] Rogerson, W.P., 1985. "Repeated Moral Hazard", Econometrica 53(1), pp 69-76.
[26] Thomas, J., and T. Worrall, 1988. "Self-Enforcing Wage Contracts", Review of Economic Studies 55(4), pp 541-553.
[27] Udry, C., 1994. "Risk and Insurance in a Rural Credit Market: An Empirical Investivation in Northern Nigeria", Review of Economic Studies 61(3), pp 495-526.

University of Oxford