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Competing for Ownership

Patrick Legros and Andrew F Newman

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Patrick Legros[†]and Andrew F. Newman[‡]

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Abstract

We develop a tractable model of the allocation of ownership and control in firms in competitive markets that permits study of how the scarcity of assets in the market translates into control allocations inside the organization. The model identifies a price-like mechanism whereby local liquidity or productivity shocks propagate and lead to widespread organizational restructuring. Firms will be more integrated when the terms of trade are more favorable to the short side of the market, when liquidity is unequally distributed among existing firms and following a uniform increase in productivity. Shocks to the first two moments of the liquidity distribution have multiplier effects on the corresponding moments of the distribution of ownership.

1 Introduction

In the neoclassical theory of the firm, market signals affect choices of products, factor mixes, and production techniques. If labor becomes scarce, wages

rise, and firms substitute machines for workers. Firm behavior, in turn, feeds back to the market, and through it, to other firms: if a labor-saving production process is introduced by some firms, wages will fall, and the other firms will reduce the capital intensity of their production. The neoclassical firm remains the backbone of much of economic analysis because it is so readily incorporated into the study of feedback effects like these.

The modern theory of the firm emphasizes contractual frictions and organizational design elements such as monitoring technologies, task allocations, asset ownership, and the assignment of authority and control. By augmenting economic analysis with this new set of variables, it has led to breakthroughs in our comprehension of institutions as different as the modern corporation and the sharecropped farm. But despite the theory's formative purpose – to understand the nature of firms in market economies – as well as evidence that firms restructure themselves in response to market conditions or the behavior of other firms,[1] there are few models that can take account of the effects of the neoclassical feedbacks on the modern variables of interest.

The purpose of this paper is to provide a simple framework for this kind of analysis. We focus on the structure of ownership and control, understood here, as in Grossman and Hart (1986), as an allocation of residual decision rights among a firm's stakeholders.[2] The model illuminates how scarcity in the market translates into control inside the firm and how changes in the fundamentals of some firms can spill over to economy-wide reorganizations.

The basic setup is a two-sided matching model, with the sides representing two types of production units, each one consisting of a manager and a collection of assets. Firms comprising one unit of each type form through

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[1]To mention just two examples, the wholesale restructuring of relations between US automakers and their suppliers in the 1980s was likely triggered by entry of Japanese firms into the US market; on a smaller scale, decision rights over the outfitting of truck cabs or the accompaniment of drivers by their spouses during hauls have recently shifted from trucking firms to their drivers in response to the growth of wages in the construction industry.

[2]This distinguishes the present paper from earlier work such as Calvo and Wellisz (1978) and Legros and Newman (1996), which focused on the general equilibrium determination of monitoring and incentives. See also Garicano and Rossi-Hansberg (2006).

a competitive matching process that determines, for each matched pair, a contract specifying its ownership structure.

After a firm has formed, a series of noncontractible management decisions, one for each asset, has to be taken, after which output is realized and the relationship ends. The organization must be designed to strike a compromise between productivity (managers share the firm's profit) and the private costs of managing, and because of the noncontractibility, this can only be accomplished by a (re-)allocation of the rights to own or control the various assets.[3]

In general the more assets a manager owns, the better off he will be, since he will be able to ensure that more decisions go in his preferred direction. But because these decisions will impose both profit and private cost externalities on the other manager, different organizational designs generate different *levels* of total surplus for the firm as well as different *divisions* of that surplus between its managers.

A crucial attribute of the environment we analyze is that liquidity – instruments such as cash that can be transferred costlessly and without any incentive distortions – is scarce. Managers have quasi-linear utility, so liquidity transfers are the preferred means of reallocating surplus between them. But when liquidity is in short supply, a large transfer of surplus will have to be done through an organizational distortion, i.e., a reassignment of control. This feature generates a key role for competitive analysis. The equilibrium outcome can no longer be identified with the surplus-maximizing allocation of ownership; instead, the market-determined division of the surplus is needed to pin down the organizational outcome.

The model highlights two distinct effects that arise from a change in fundamentals such as liquidity endowments or technology. The first is an "internal effect," various forms of which have been studied in the literature

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[3]The literature following Grossman and Hart (1986) and Hart and Moore (1990) tends to distinguish ownership from control by identifying ownership with a party's right to exclude others' access to an asset, whereas control is applied to most other decisions concerning its use. In the static environment without renegotiation we study here, there is little meaningful distinction between these concepts, and we use the terms interchangeably.

on ownership: the surplus that each partner obtains from a given contract is a function of the characteristics of the partners in a relationship, in particular the amount of liquidity they have and the production technology available. Specifically, in our model, more liquidity in the firm enlarges the set of feasible payoffs for the two managers by increasing transferability, though it does not enlarge their set of production possibilities, since there is no need to acquire productive assets from outside the partnership. Higher productivity not only enlarges the payoff sets by expanding production possibilities, but also increases transferability, because it induces managers to increase the weight of profit (which can be shared) relative to private cost (which cannot) in their decisions. Hence, a positive shock to a firm's liquidity or productivity will enable it to accomplish surplus division more efficiently and reduce organizational distortions.

But that same shock can have wider effects than on the firm that first experiences it. The internal effect implies that a manager has effectively a higher "ability to pay" for a partner after a positive shock than before. He may therefore bid up the terms of trade in the matching market: in order to meet the new price, firms which have not benefited from the shock will have to restructure. Thus the shock may have an *external effect*: "local" shocks may propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium of our model turns out to be amenable to a Marshallian supply-demand style of analysis, making the role of the external effect especially transparent. Suppose for instance, that one side of the market represents automobile manufacturers selling in the U.S. market and the other side represents their suppliers. An increase in the number of manufacturers due to entry from abroad will reduce the share of surplus accruing to the auto makers. This will entail a transfer of control to the suppliers, and many manufacturer-supplier relationships will become less integrated in the sense that a smaller fraction of the assets will be controlled by the auto maker's manager.

Furthermore, while the internal effects of positive shocks to liquidity and technology are similar – they both decrease integration – the external effects differ. A uniform increase in the liquidity level of all agents lowers the degree

of integration in all firms (the internal effect dominates the external effect). By contrast, a uniform shock to productivity increases the degree of integration in all firms (the external effect dominates the internal effect). These effects can be quite pronounced: there is an "organizational multiplier" effect of shocks, with, for instance, a unit change in mean liquidity producing a larger than unit change in the mean degree of integration. As we show in Section 3, the model can also capture the effects of more complex changes in the liquidity endowments or in productivity.

Our model of the determination of ownership structure is inspired by Grossman and Hart (1986). However, we depart from their analysis in three respects. First, as in Hart and Moore (1990), we allow for a richer set (in fact, a continuum) of ownership structures rather than the two (integration and non-integration) discussed by Grossman and Hart. This feature yields both tractability for competitive analysis, and the flexibility to capture the rich array of control allocations displayed by real firms (for examples, see Lerner and Merges, 1998 on biotechnology R&D alliances; Arruñada, Garicano, and Vázquez, 2001 on automobile dealerships; and Blair and Lafontaine, 2005 on fast food franchises). Second, as have a few recent papers (e.g. Hart and Holmström, 2002; Aghion, Dewatripont, and Rey, 2004; Baker, Gibbons, and Murphy, 2006), we abstract away from the hold up problem by dropping ex-ante investments and assuming instead that ex-post decisions are not contractible. Our purpose in doing so is to make the surplus transfer role of ownership especially transparent: the set of feasible decisions is unaffected by who owns an asset, and therefore awarding ownership of more assets to one manager unambiguously raises his payoff.

The third and most important departure is the assumption that liquidity is scarce. The corporate finance literature beginning with Aghion and Bolton (1992) has already highlighted what we have termed the internal effect of limited liquidity on the allocation of control: given the division of surplus, raising a contractual party's liquidity endowment will tend to give him more control and increase the efficiency of the relationship. What is new here is the identification and analysis of the external effect: limited liquidity implies that a firm may modify its control right allocation, at a possible efficiency

cost, in response to changes in the liquidity (or technology) of *another* firm. This effect would also be present for many other specific models of ownership and organizational design: all that is important is that the payoff frontier *not* reflect transferable utility, which in our formulation scarce liquidity helps to guarantee.

2 Model

We consider an economy in which there are two types of production units, indexed by $1, 2$. Each unit consists of a risk-neutral manager and a collection of assets that he will have to work with in order to produce. We have in mind competitive outcomes, and so we suppose that there is a large number of production units: each side of the market is a continuum with Lebesgue measure. The type 1's are represented by $i \in I = [0, 1]$ while the type 2's are represented by $j \in J = [0, n]$, where $n < 1$; thus, the 2's are relatively scarce. We assume that production units may operate on a stand-alone basis, in which case they earn an outside option (normalized to zero), or cooperate in pairs comprising one unit of each type, in which case they can generate strictly more than zero surplus.

Many interpretations are possible: the two types of manager might be supplier and manufacturer, and the assets plant and equipment; a chain restaurateur and franchising corporation (in which case some of the assets might be reputational); or as a firm and its workforce, for which the assets might be interpreted as tasks.

In an individual production unit, an asset's contribution to profit depends on a planning decision made by one of the managers, not necessarily the one who will have to operate it. Planning decisions are not contractible, but the right to make them can be allocated via contract to either manager. For simplicity we assume that planning choices (e.g., choosing the background music for a retail store) are costless. But while potentially beneficial for profits (some music is likely to induce consumers to make impulse purchases), those choices affect the private cost of later operations (such music may be unpleasant for the store's floor manager).

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The i-th type-1 manager will have at her disposal a quantity $l_1(i) \geq 0$ of cash (or "liquidity") which may be consumed at the end of the period and which may be useful in contracting with managers of the opposite type; for the type 2's, the liquidity endowment is $l_2(j)$. The indices i and j have been chosen in order of increasing liquidity.

When discussing a generic production unit or its manager, we shall usually drop the indices.

2.1 The Basic Organizational Design Problem

2.1.1 Technology and Preferences

Managers seek to maximize their expected income (including the initial liquidity) less the private costs of operating the enterprise.

The collection of assets in the type-1 production unit is represented by a continuum indexed by $k \in [0,1)$; the type-2 assets are indexed by $k \in [1,2)$. An asset's contribution to profit is proportional to the planning level $q(k)$, where $q(k) \in [0,1]$.

Planning decisions contribute to the firm's performance as follows. The firm either succeeds, generating profit $R > 0$, with probability $p(q)$; or it fails, generating 0, with probability $1 - p(q)$, where $q : [0,2] \to [0,1]$ are the planning decisions. The success probability functional is

$$p(q) = \gamma \int_0^2 q(k) dk,$$

where $\gamma < 1/2$ is a technological parameter. It is convenient to define $A = \gamma R$.

Either manager is capable of making planning decisions. There is no cost to making a plan, but there is a (private) operating cost to the manager who subsequently works with an asset: the 1-manager bears cost $c(q(k)) = \frac{1}{2}q(k)^2$ for $k \in [0,1)$, and zero for $k \in [1,2]$; similarly for 2, the cost is $c(q(k))$ on $[1,2)$ and zero on $[0,1]$. For brevity we write

$$C_1(q) = \int_0^1 c(q(k)) dk, \qquad C_2(q) = \int_1^2 c(q(k)) dk.$$

This is the cost externality we alluded to: the cost to the manager operating the asset is increasing in $q(k)$, whether or not he has chosen it.[4] For instance in a manufacturing enterprise, q could index choices of possible parts or material inputs, ordered by the value they contribute to the final product, while $c(q)$ could represent the cost of managerial attention devoted to overseeing assembly, supervising workers, and so on; we are supposing that higher value inputs require greater effort on the part of the manufacturer's management.

2.1.2 Contracts

We have already made the following contractibility assumptions:

Assumption   (i) The *right to decide* $q(k)$ is both alienable and contractible.
   (ii) The decisions q are never contractible.
   (iii) The costs $C_i(q)$ are private and noncontractible.

A contract (ω, t) specifies the allocation of ownership ω and liquidity transfers t made from 1 to 2 before any planning or production takes place. The liquidity levels of the two types being l_1 and l_2 respectively, we must have $t \in [-l_2, l_1]$. The ownership allocation ω is the fraction of assets re-assigned to one of the managers. The type-1 manager owns assets in $[0, 1-\omega)$, where $-1 < \omega \leq 1$, and the type-2 owns $[1-\omega, 2)$.

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[4]Note that we are assuming symmetry in the technology and cost between the two managers; any difference that emerges between the two sides will be only due to a difference in scarcity. One could extend the model to allow for asymmetries in cost, productivity or initial number of assets. For instance, if $C_2 \equiv 0$, a firm is basically a principal-agent model. If the type-2 is interpreted as "capital," the model could be viewed a static version of a financial contracting problem, like Aghion and Bolton (1992). Assuming that one type is more productive that the other allows one to to ask the kind questions addressed by Grossman and Hart (1986) and Hart and Moore (1990) concerning who should (as against who does) own the assets. For some applications, e.g. firms and workers, it might be appropriate to assume that one type (firms) initially owns and bears the cost from most of the assets.

Since we want to focus here on allocations of control rights, we will simplify matters by ignoring the effects related to variations in the sharing of profits. Instead, we simply assume that each manager gets half of the realized output, that is he gets $R/2$ if output is R and 0 if output is 0. This is a simple representation of the constraints faced by real firms in the use of incentive pay. Similar kinds of assumptions have been used elsewhere in the literature (e.g., Hart, 1983; Holmström and Tirole, 1998), and in Appendix I, we show that it can be derived as a consequence of a moral hazard problem.[5]

This leaves out a logical possibility:[6] the managers might use a third party "budget breaker" who will pay the firm if there is success and will be paid out of the liquidity available in the firm if there is failure. Using third parties in this way may improve efficiency, but only if the third party gets more when the firm fails than when it succeeds. Apart from the undesirable incentive problems this creates (the third party may want the firm to fail), this modification would not change the basic message of this paper.[7]

When $\omega = 0$, each manager retains ownership of his original assets, and, following the literature, we refer to this situation as *non-integration*. As ω increases beyond 0, we have an increasing degree of integration (a growing fraction of the assets are owned by 2), until with $\omega = 1$ we have full integration. (The symmetric cases with $\omega < 0$ correspond to 1-ownership; with scarce 2's and zero outside options for the 1's, ω will turn out to be posi-

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[5]One can also relax the assumption and allow for a rich set of budget-balancing sharing rules to yield predictions on the interplay between ownership allocations and profit shares. The modified model of the firm can easily be embedded in our framework, leading to only minor modification of the results in Section 3. See Legros and Newman (2007).

[6]There are three others. First, that the managers "swap" assets: in additon to ω, which indicates how many of 1's assets are shifted to 2, the contract would have an additional variable ψ indicating how many of 2's assets are shifted to 1. Second, that the managers pledge their liquidity to increase the total revenue available *after* the output is realized. Third that agents use external finance, i.e., sign debt contracts. We show in Appendix I that none of these possibilities can improve on contracts as we define them.

[7]It would, however, make the analysis more complex; in particular, we would lose the simple supply-demand analysis that we perform here. For an example of the use of third parties in the formation of firms when there are liquidity constraints see Legros and Newman (1996).

tive in equilibrium, and we focus on this case in what follows unless noted otherwise.) Since ω not only describes the ownership structure but also provides a scalar measure of the fraction owned by one party, we shall often refer to its (absolute) value as the *degree of integration* of the firm.

2.1.3   The Feasible Set for a Firm

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which $q(k) = A$ for all k) cannot be attained. For tasks $k \in [0, 1)$, when manager 1 makes the planning decision, he will underprovide q since he bears the full cost of the decision but gets only half of the revenue benefit. By contrast, if the plan is made by manager 2, that manager will overprovide q since by increasing q, expected output increases and 2 bears no cost.

Since the profit shares are fixed, without liquidity, the only remaining way to allocate surplus is to modify the degree of integration ω. Given a contract (ω, t) the two managers subsequently choose q noncooperatively to maximize their corresponding objectives:

$$u_1(\omega, t) = \max_{q(k) \in [0,1], k \in [0, 1-\omega]} \frac{A}{2} \int_0^2 q(k) dk - \frac{1}{2} \int_0^1 q(k)^2 dk - t$$

$$u_2(\omega, t) = \max_{q(k) \in [0,1], k \in (1-\omega, 2]} \frac{A}{2} \int_0^2 q(k) dk - \frac{1}{2} \int_1^2 q(k)^2 dk + t.$$

It is straightforward to see that manager 1 will set $q(k) = \frac{A}{2}$ on the assets $k \in [0, 1 - \omega)$ that he controls, and that manager 2 will choose $q(k) = 1$ for $k \in [1 - \omega, 1)$ and $q(k) = \frac{A}{2}$ for $k \in [1, 2)$. Then, the payoffs associated to a contract (ω, t) are

$$u_1(\omega, t) = \frac{3}{8} A^2 - \omega \frac{(2 - A)^2}{8} - t \tag{1}$$

and

$$u_2(\omega, t) = \frac{3}{8} A^2 + \omega \frac{A(2 - A)}{4} + t \tag{2}$$

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Because reallocating control rights does not affect the feasible set of planning decisions, a manager gaining control of additional assets cannot be worse off.[8]

Proposition 1 *A manager's payoff is nondecreasing in the fraction of assets he controls.*

Note that the Pareto frontier when there is no liquidity (so that $t = 0$) and when $v_2 \geq 3A^2/8$ is

$$v_2 = \begin{cases} -\alpha v_1 + (\alpha + 1)\frac{3}{8}A^2, v_1 \leq \frac{3}{8}A^2 \\ -\frac{1}{\alpha}v_1 + \left(\frac{1}{\alpha} + 1\right)\frac{3}{8}A^2, \frac{3}{8}A^2 \leq v_1 \end{cases} \tag{3}$$

where $\alpha = 2A/(2 - A) < 1$ measures the degree of payoff transferability. Observe that the total surplus generated by a contract ω, $u_1(\omega, t) + u_2(\omega, t)$, is maximal at $\omega = 0$ (nonintegration) provided

$$A < 2/3. \tag{4}$$

We shall focus on this case.[9]

When managers have no liquidity, $t = 0$ and as 1's payoff decreases, the number of assets 2 owns (weakly) increases. At the same time total surplus is decreasing; thus it is fair to say that here *reallocations of ownership are used to transfer surplus, not merely to generate it.* Notice as well that this mode of surplus transfer is less efficient than transferring cash; thus any liquidity that the managers have to spare will be used first to meet the surplus division demanded by the market before they transfer ownership.

When agents of types 1 and 2 have liquidity l_1 and l_2, the set of feasible payoffs they can attain via contracting is defined by (1) and (2), along with uncontingent transfers that do not exceed the initial liquidities. Given the risk neutrality of the managers, ex-ante transfers do not affect total surplus; in particular we have $u_1(\omega, t) = u_1(\omega, 0) - t$ and $u_2(\omega, t) = u_2(\omega, 0) + t$. Figure 1 illustrates a typical feasible set when agents have liquidity l_1 and l_2.

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[8]This invariance of the feasible set to transfers of control stems from the absence of investments made before q is chosen; in particular it extends to cases in which there are noncontractible investments ex post and/or in which sharing rules are flexible. See Legros-Newman (2007).

[9]When $A > 2/3$, the frontier is non-concave, and, absent lotteries, the most efficient organization will entail give nearly full control to one of the managers.
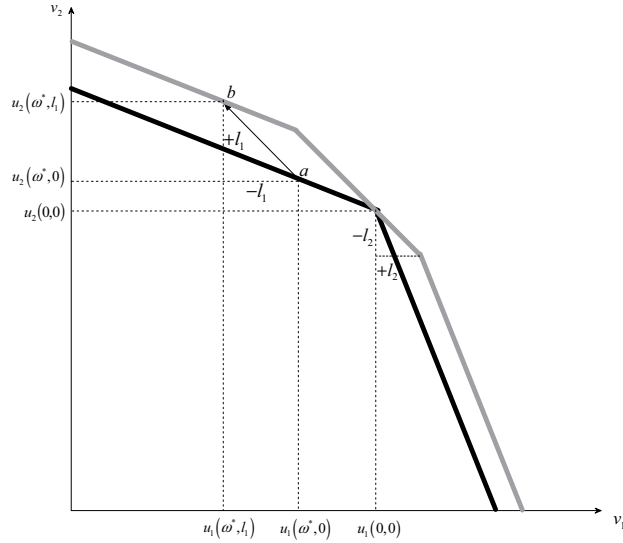
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Figure 1: Feasible Set

The dark segments represent the frontier in the absence of liquidity transfers. The surplus maximum occurs at the kink, where $\omega = 0$; we have indicated a point a on this frontier corresponding to a transfer ω^* of control to 2. Point b indicates the surplus levels to 1 and 2 after 1 also transfers all of his liquidity l_1; the gray segments trace the entire frontier available to this pair of managers.

2.2 Market Equilibrium

Market equilibrium is a partition of the set of agents into coalitions that share surplus on the Pareto frontier; the partition is stable in the sense that no new firm could form and strictly improve the payoffs to its members. The only coalitions that matter are singletons and pairs (which we call "firms") consisting of one type 1 production unit $i \in I$ and one type 2 production unit $j \in J$. Since there is excess supply of type 1 production units, there is at least a measure $1 - n$ of type 1 managers who do not find a match and who therefore obtain a surplus of zero. Stability requires that no unmatched type 1 manager can bid up the surplus of a type-2 manager while getting a

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positive surplus. Necessary conditions for this are that all type 2 managers are matched and that they have a surplus not smaller than $u_2(0,0) = \frac{3}{8}A^2$. As is apparent from the construction of the feasible set, when $v_2 > u_2(0,0)$, payoffs on the Pareto frontier are achieved by transferring the liquidity of type 1 only, that is, the 2's liquidity does not matter. Thus all 2's are equally good as far as a 1 is concerned and they must therefore receive the same surplus.[10]

This "equal treatment" property for the 2's is an important simplification relative to most assignment models in which there is heterogeneity on both sides of the market. Identify the set of firms F with the index of the type 1 manager in the firm "firm i" indicates that the firm consists of the i-th type 1 production unit and a type 2 manager.

Definition 1 *An equilibrium consists of a set of firms $F \subset I$ with Lebesgue measure n, a surplus v_2^* received by the type 2 managers, and a surplus function $v_1^*(i)$ for type 1 managers such that:*

(i) (feasibility) For all $i \in F$, $(v_1^*(i), v_2^*) \in U(l_1(i), 0)$. For all $i \notin F$, $v_1^*(i) = 0$.

(ii) (stability) For all $i \in I$, for all $j \in J$, for all $(v_1, v_2) \in U(l_1(i), l_2(j))$, either $v_1 \leq v_1^*(i)$ or $v_2 \leq v_2^*$.

2.2.1 Characterizing Market Equilibrium

Since the type-2 managers have the same equilibrium payoff, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2's. We construct the demand as follows. The amount of surplus a 1 is willing *and able* to transfer to a 2 depends on how much liquidity he has. The willingness to pay of type 1 is the value of

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[10]If in firm (i,j) type 2 j has a strictly larger surplus than type 2 j' in the firm (i',j'), the firm (i,j') could form and both i and j' could be better off since the Pareto frontier is strictly decreasing. Note that if the 1's have large enough outside options (or are more scarce than the 2's), their liquidity does not matter, while the liquidity of 2's does. It can also be shown that only 2 liquidities matter in case $A > 2/3$.

the problem

$$\max_{(\omega,t)} u_2\left(\omega,t\right)$$

$$u_1\left(\omega,0\right)\geq t$$

$$t\in\left[0,l_1\right].$$

In the contract (ω,t), the type 1 manager gets $u_1\left(\omega,t\right)+l_1$; the opportunity cost of the contract is to be unmatched and get l_1; hence the manager is willing to contract when $u_1\left(\omega,t\right)\geq 0$ which is equivalent to the condition stated since $u_1\left(\omega,t\right)=u_1\left(\omega,0\right)-t$. Simple computations show that the solution to this program is

$$\text{If } l_1 \geq \frac{3}{8}A^2, (\omega,t)=(0,\frac{3}{8}A^2), \tag{5}$$

$$\text{If } l_1 < \frac{3}{8}A^2, (\omega,t)=(\frac{3A^2-8l_1}{\left(2-A\right)^2},l_1).$$

The willingness of a type 1 manager to pay for matching with a type 2 manager is then

$$W\left(i\right)=\begin{cases} \frac{3}{4}A^2 & \text{if } l_1\left(i\right)\geq\frac{3}{8}A^2 \\ \frac{3}{8}A^2+\left(\frac{3}{4}A^2-2l_1\left(i\right)\right)\frac{A}{2-A}+l_1\left(i\right) & \text{if } l_1\left(i\right)\leq\frac{3}{8}A^2 \end{cases} \tag{6}$$

Since the frontier has slope magnitude less than unity above the 45°-line, and since $l_1\left(i\right)$ is increasing in i, the willingness to pay of i is nondecreasing in i. If type 2 agents must get a payoff of v_2, the type 1 agents who are willing and able to pay this price is

$$D\left(v_2\right)=1-\min\left\{i\in\left[0,1\right]:W\left(i\right)\geq v_2\right\}.$$

The supply is vertical at n, the measure of 2's. Equilibrium is at the intersection of the two curves: this indicates that n of the 1's are matched, as claimed above, and that the marginal 1 is receiving zero surplus.

Proposition 2 *The equilibrium set of firms is $F=\left[1-n,1\right]$ and the equilibrium surplus of type 2 managers is*

$$v_2^*=\min\{\frac{3}{4}A^2,W\left(\bar{l}_1\right))\},$$
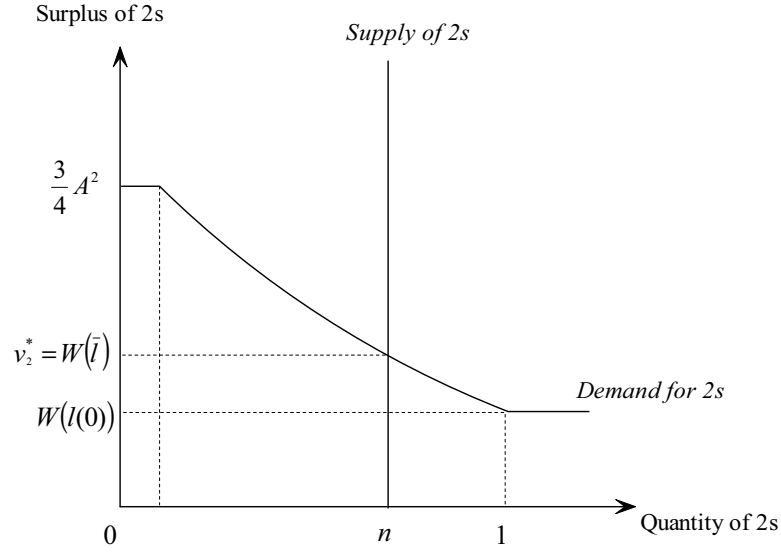
where $\bar{l}_1=l_1\left(1-n\right).$

Figure 2: The Market for Ownership

If $\bar{l}_1 \geq 3A^2/8$, efficiency is obtained since each matched type 1 is able to pay $3A^2/8$ to the type 2 manager; note that in this case the equilibrium surplus of all type 1 managers is zero. We will consider below situations in which $\bar{l}_1 < 3A^2/8$.

In this case, the equilibrium surplus of type 2 managers is $v_2^* = W\left(\bar{l}_1\right) < \frac{3}{4}A^2$. The marginal type 1 manager $1 - n$ has a surplus of 0, but the inframarginal type 1 managers with liquidity $l_1 > \bar{l}_1$ will be able to generate a positive surplus for themselves since they can transfer more liquidity than the marginal type 1. The surplus of an inframarginal type 1 with liquidity $l_1 \geq \bar{l}_1$ when the price is v_2^*, it the value of the problem

$$\max_{\omega} u_1\left(\omega, t\right) + l_1$$
$$u_2\left(\omega, 0\right) + t = v_2^*$$
$$t \leq l_1$$

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The solution to this problem is $\omega\left(v_2^*, l_1\right), t\left(v_2^*, l_1\right)$ where

$$\omega\left(v_2^*, l_1\right) = 0, \quad t\left(v_2^*, l_1\right) = v_2^* - \frac{3}{8}A^2, \text{ if } l_1 \geq v_2^* - \frac{3}{8}A^2 \qquad (7)$$

$$\omega\left(v_2^*, l_1\right) = 4\frac{v_2^* - \frac{3}{8}A^2 - l_1}{A\left(2 - A\right)}, \quad t\left(v_2^*, l_1\right) = l_1 \text{ if } l_1 \leq v_2^* - \frac{3}{8}A^2.$$

In this model, there is a piece-wise linear relationship between the liquidity, the degree of integration ω, the level of output and the managerial welfare. The fact that the degree of integration is a globally convex and decreasing function of liquidity and is an increasing function of the price v_2^* illustrates the internal and external effects we alluded to. The effect of a higher level of liquidity may be overcome by an increase in the price v_2^*. Of course, the price itself reflects the liquidity and the technology available in the economy. To study the effects of shocks systematically, we must take account of the fact that v_2^* itself is endogenous, which we do in the next section.

Lemma 3 *The degree of integration $\omega\left(v_2^*, l_1\right)$ is piece-wise linear: it is linear nondecreasing in v_2^*, nonincreasing in l_1 when $l_1 < v_2^* - \frac{3}{8}A^2$, it is equal to zero when $l_1 \geq v_2^* - \frac{3}{8}A^2$.*

3 Comparative Statics of Market Equilibrium

In equilibrium, there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, "richer" firms are less integrated and generate greater surplus for the managers.[11]

But more liquidity overall can also lead to *more* integration: if the marginal firm's liquidity v_2^* rises, possibly by more than an inframarginal firm's gain in liquidity. As a result, the inframarginal firm may become more inte-

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[11]Holmström and Milgrom (1994) emphasize a similar cross-sectional variation in organizational variables. In their model, the variation reflects differences in technology but not differences in efficiency relative to their potential, since all firms are surplus maximizing. Here by contrast, the variation stems from differences in liquidity and reflects differences in organizational efficiency.

grated, and indeed it is possible that the economy's average level of integration may increase via this external effect.

We shall consider three types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity, and changes in the parameter A.

3.1   Relative Scarcity

In order to isolate the "external effect" our first comparative statics exercise involves changes in the tightness of the supplier market, i.e., in the relative scarcities of 1's and 2's.

Suppose that the measure of 2's increases, for instance from entry of downstream producers into the domestic market from overseas. Then just as in the standard textbook analysis, we represent this by a rightward shift of the supply schedule: the price of 2's decreases. Indeed, as n increases the liquidity of the marginal type 1 decreases since $l_1 (1 - n)$ is decreasing with n. What of course is different from the standard textbook analysis is that this change in price entails (widespread) corporate restructuring.

Let $F(n)$ be the set of firms when there is a measure n of type 2 firms. As n increases to $\hat{n}$, there is an equilibrium set $F(\hat{n})$ where $F(n) \subset F(\hat{n})$; that is after the increase in supply, new firms are created but we can consider that previously matched managers stay together. The surplus of *all* type 1 managers in firms in $F(n)$ increases. Managers in a firm in $F(n)$ will restructure (decrease ω) in response to the reduction in the equilibrium value of v_2^*. The analysis is similar in the opposite direction: a decrease in the measure of 2's leads to an increase in v_2^*. Thus, we have

Proposition 4 *In response to an increase in the measure of 2's, the firms remaining in the market become less integrated.*

It is worth remarking that if the relative scarcity changes so drastically that the 2's become more numerous, then 1's get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have seen, with the role of 1's and 2's reversed. The point is that the owners

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of the integrated firm gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

However, this story is heuristic: increases in demand for the type 1 most likely emanate from entry of new firms (which in turn entails a change in the liquidity distribution among the active firms) and from increases in productivity (e.g., "skill-biased technical change"). Thus, a general analysis of the effects of changes in relative scarcity requires separate consideration of the effects of changes in liquidity and productivity; we provide this in the next two subsections.

3.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the interplay of the internal and external effects described above. The dependence of the ownership structure ω on the type-1 liquidity l_1 and the equilibrium surplus v_2^* was summarized in Proposition 2 and Lemma 3. Equipped with this result, we can derive some characterizations and simple comparative statics of the distribution of ownership structures.

First, if one is interested in minimizing the degree of integration in the economy (this maximizes the surplus), it is clear from (7), Proposition 2, and Lemma 3 that one wants the marginal liquidity as low as possible, so as to minimize the equilibrium price, and one wants to maximize the liquidity of the inframarginal firms. Because the function $\omega\left(v_2^*, l_1\right)$ is globally convex in l_1, $E\left[\omega\right]$ is minimal when all firms have the same level of liquidity; more generally, there is a simple description of the set of distributions that minimize average integration in the economy.

Proposition 5 *Let L be the average liquidity among the type 1 managers. The degree of integration is minimized when the marginal type 1 has zero liquidity and when the distribution of liquidity among the inframarginal type 1's has support in $[0, \alpha 3A^2/8]$ when $L < \alpha 3A^2/8$ and support in $[\alpha 3A^2/8, \infty)$ when $L > \alpha 3A^2/8$.*

We now consider how the distribution of ownership depends on the distribution of liquidity. To simplify, we restrict attention to liquidity distributions in which all type 1's are liquidity constrained and belong to firms with a positive ω. In the Appendix, we consider the general case in which a positive measure of type 1's are in decentralized firms.

Let $G(l)$ be the distribution of liquidity among the type 1's, $\bar{l}_1$ the marginal liquidity, and $\mu = \frac{1}{n} \int_{\{l \geq \bar{l}_1\}} l dG(l)$ and $\sigma^2 = \frac{1}{n} \int_{\{l \geq \bar{l}_1\}} (l - \mu)^2 dG(l)$ be the mean and variance of liquidity of the inframarginal type 1's. The linearity of the degree of integration in l implies a monotonic relationship between the first two moments of the distribution of liquidity and those of the distribution of ownership when all firms choose integration.

Proposition 6 *The mean and the variance of the degree of ownership are*

$$
\begin{aligned}
E[\omega] &= \omega_0 + a\bar{l}_1 - b\mu \\
Var[\omega] &= b^2 \sigma^2
\end{aligned}
$$

where $\omega_0 = \frac{3A^2}{(2-A)^2}$, $a = 4\frac{2-3A}{A(2-A)^2}$, $b = \frac{4}{A(2-A)}$.

The dependence of the mean degree of ownership on the liquidity of the marginal type reflects the external effect, since a higher liquidity at the marginal relationship implies a higher degree of integration in other firms. When all firms choose a positive ω, the variance of ω depends only on the variance of liquidity. As we show in the Appendix, when there is a positive measure of type 1 who are not liquidity constrained, the variance of ownership also depends on the marginal and mean liquidities as well as the variance.

The average level and degree of heterogeneity in ownership structure is sensitive to the distribution of liquidity. Since b is greater than 4, a unit increase in the mean liquidity leads to a fourfold decrease in the average level of integration; a unit increase in the variance of liquidity generates a sixteen-fold increase in the variance of integration. We are not aware that such a "multiplier effect" of fundamentals on organizational structure has been previously noted in the literature.

Equipped with this proposition, it is easy to compare outcomes for two distributions of liquidity G and H. Suppose that the marginal level of liquidity

is larger at H than at $G : G(\bar{l}_1^G) = H(\bar{l}_1^H) = 1 - n$ implies $\bar{l}_1^G < \bar{l}_1^H$. It follows that the price of type 2 is greater with H than with G; in fact from (6), $v_2^{*H} = v_2^{*G} + (1 - \alpha)(\bar{l}_1^H - \bar{l}_1^G)$. Hence each type 1 who is inframarginal with H uses a greater degree of integration than with G. However this is not incompatible with a decrease in the average degree of integration if the average liquidity increases enough: the internal effect must compensate for the external effect. This is formally stated below.

Proposition 7 *Consider two distributions of liquidity G and H for which all firms choose $\omega > 0$.*
(i) The mean degree of ownership is lower with H than with G if and only if

$$\underbrace{(1 - \alpha)(\bar{l}_1^H - \bar{l}_1^G)}_{\text{change in price}} < \underbrace{\mu^H - \mu^G}_{\text{change in average liquidity}}.$$

(ii) The variance of the degree of ownership is lower in H than in G if and only if the variance of liquidity is lower with H than with G.

A special case worth highlighting is that of *positive, nondecreasing shocks* to each type-1's liquidity. Note that a uniform shock in which every type 1 receives the same increase to his endowment is a special case, as is a multiplicative shock in which the percentage increase to the endowment is the same for all 1's. The shock will increase both the willingness to pay of the type 1's, which, via the internal effect, reduces the degree of integration, but also will increase the equilibrium surplus to 2, which, via the external effect, has the opposite impact.

However, it is a simple matter to demonstrate that in this case, the internal effect dominates: more liquidity implies less integration. For instance, if every type 1 has ε more in liquidity, the price increases by $(1 - \alpha)\varepsilon$ while the average liquidity increases by ε and the condition of the proposition holds since $\alpha < 1$. With nondecreasing shocks, if the marginal liquidity goes up by ε, the average liquidity increases by more than ε and therefore the condition of the proposition is satisfied. Of course, negative, nonincreasing shocks yield the opposite changes in surplus and organization.

Corollary 8 *Under positive, nondecreasing, shocks to the liquidity distribution of type 1 the aggregate degree of integration decreases.*

To maintain this conclusion, the proviso that the shocks are monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is higher overall degree of integration. Intuitively, if the positive shock hits only a small neighborhood of the marginal type 1, the price v_2^* will increase and the inframarginal unshocked firms will choose to integrate more in response to the increase in v_2^*.

Proposition 9 *There exist first order stochastic dominant shifts in the distribution of type-1 liquidity that lead to more integration.*

For the proof, see Appendix II.

3.3 Technology and Demand Shocks

The external effect outlined in the previous section offers a propagation mechanism whereby local shocks that affect only a few firms initially may nevertheless entail widespread reorganization. Empirically this implies that to explain why a particular reorganization happens, there is no need to find a smoking gun in the form of a change *within* that organization: instead the impetus for such change may originate elsewhere in the economy. The same logic applies to other types of shocks, most prominently among them innovating productivity shocks. These are often thought to be the basis of large-scale reorganizations such as merger waves (Jovanovic and Rousseau, 2002).

We model a (positive) productivity or technological innovation as an increase in A. This could come from an increase in the success probability parameter γ, or in the output generated when there is success; it could also be interpreted as a demand shock that raises the profit R via an increase in the product price (particularly if all firms experience an increase in A).

It is helpful (to facilitate the Marshallian analysis) to think of the technology as inhering in the type 1's. Suppose that in the initial economy, all

firms have the same technology; after a shock, a subset of them, an interval $[i_0, i_1]$, have access to a better technology (for them, $\hat{A} > A$). We restrict ourselves here to considering "small" shocks in the sense that $\hat{A} < 2/3$.

Raising A modifies the game that managers play given a contract ω : it is clear from (1) and (2) that both managers obtain a larger surplus from a given contract. Hence the feasible set expands and the type-1's willingness to pay also increases. What is perhaps less immediate is that there is also more transferability within the firm.

Lemma 10 *Let A be the initial productivity. After a positive productivity shock,*
(i) the feasible set expands.
(ii) For any $t < 3A^2/8$, the degree of integration solving $u_1(\omega, t) = 0$ increases.
(iii) there is more transferability in the sense that the slope of the frontier is steeper in the region $v_2 \geq v_1$ when A increases.

Proof. (i) From (4), differentiating (1) and (2) with respect to A, shows that for any contract (ω, t), both $u_1(\omega, t)$ and $u_2(\omega, t)$ are increasing in A. (ii) Use (5). (iii) The absolute value of the slope of the frontier in the region $v_2 \geq v_1$ is $\alpha = 2A/(2 - A)$ which is also increasing in A. ∎

The willingness to pay (6) depends on the technology available to the firm; since we assume that some firms have a different technology, we can make explicit the relationship between technology and willingness to pay:

$$W(i; A_i) = \min\left\{\frac{3}{4}A_i^2, \frac{3}{8}A_i^2 + \left(\frac{3}{4}A_i^2 - 2l_1(i)\right)\frac{A_i}{2 - A_i} + l_1(i)\right\}, \quad (8)$$

$$\text{with} \begin{cases} A_i = A & \text{if } i \notin [i_0, i_1] \\ A_i = \hat{A} & \text{if } i \in [i_0, i_1]. \end{cases}$$

Lemma 10(iii) implies that – for a *fixed* equilibrium surplus for 2 – a shocked firm integrates less since it is able to transfer surplus via ω in a more efficient way. Hence when the 2s' equilibrium surplus is fixed, positive technological shocks lead to less integration in the economy.

However, Lemma 10(ii) implies that when the marginal firm is shocked, the price will increase. Since by (iii) there is more transferability with ω, liquidity has less value: the inefficiency linked to the use of integration is lower and integration is a better substitute to liquidity transfers. This implies that type 1 agents find it more expensive, in terms of liquidity, to "buy" decision rights or reduce the degree of integration. Therefore, if the 2s' equilibrium surplus increases, there is a force toward more integration. Unshocked firms certainly integrate more; for shocked firms, we show below that while they benefit internally from the technological shock, the countervailing effect of an increase in the 2s' equilibrium surplus dominates. The net effect is towards more integration *for all firms in the economy* if the marginal firm is a shocked firm. Other results are contained in the following proposition:

Proposition 11 *(i) (Inframarginal shocks) If $i_0 > 1 - n$ the shocked firms become less integrated and the unshocked firms remain unaffected*
(ii) (Marginal shocks) If $1 - n \in (i_0, i_1)$ and $1 - n$ is still the marginal type 1 agent, the equilibrium price increases and all firms, shocked and unshocked, integrate more.
(iii) (Uniform shocks) If there is a uniform shock to the technology ($i_0 = 0, i_1 = 1$) each firm integrates more.

Thus the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in "rich" firms (case (i)), only the innovating firms are affected, and they become less integrated. But innovations that occur in "poor" firms (case (ii)) may affect the whole economy, and in the opposite direction: even firms that don't possess the new technology become more integrated.

Proposition 11 (iii) emphasizes that, in contrast to reduced integration after a positive uniform liquidity shock, a uniform positive productivity shock will have the *opposite* effect. In this sense the external effect of productivity shocks is more powerful than that for liquidity shocks.

If A inheres on type 1 with the same level of liquidity, and are differentiated, entry of type 2 production units will lead to a marginal relationship that is characterized by a smaller value of A than before entry. By Lemma

10, the price going to type 2 decreases and all previous type 2 will choose a lower level of ω. This argument can be generalized if type 1 are differentiated both by their liquidity endowment and their productivity A.

From (6), the willingness to pay of type 1 depends both on A and l_1. The market analysis can be extended by ordering the type 1 by their willingness to pay, the marginal type 1 being the agent having a measure n of type 1 with larger willingness to pay. A higher willingness to pay indicates a higher liquidity or a higher value of A but not necessarily of both. When type 1 have different productivity levels, it is actually possible that units that are more productive than the marginal unit *gain* control over type 2 assets. Since for a given A and l_1, the frontier is decreasing in the payoff going to type 1 and in ω, entry on the downstream market has the unambiguous effect to increase control flowing to previously active type 1. This generalizes Proposition 4 obtained when type 1 are differentiated by liquidity only.

Proposition 12 *Suppose that type 1 are differentiated by their liquidity and technology A. Then, entry of type 2 production units will lead to more control by original active type 1.*

4   Illustrations

4.1   Entry in Supplier and Product Markets: Automobiles

Until the 1980s, large U.S. automobile manufacturers maintained arms-length relationships with their suppliers, usually setting specifications for parts without involvement by suppliers and then awarding production contracts via competitive bidding. By contrast, Japanese automotive firms had long embraced a "partnership" model with their suppliers.

Following a wave of foreign direct investment by Japanese firms in the U.S., Chrysler started reorganizing its relationship with suppliers, eventually involving suppliers as almost equal partners in product and process development; other US manufacturers soon followed suit. This change in supplier

relations has been linked (see, for instance, Dyer 1996) to the threat posed by the entry of Japanese firms, their dominance on the market for small cars (which was the fast growing segment given the successive oil crises), and the comparatively greater quality of Japanese cars seemingly due to the close cooperation with suppliers for design and development.

In terms of our model, interpret type 2 as the car manufacturers, type 1 as the suppliers, and ω as the degree of control that car manufacturers have in their relationships with suppliers. A move from the old arms-length relationship to the partnership arrangement is characterized by a decrease in ω as the suppliers gain control over aspects of the design and production process. The entry of Japanese producers into the U.S. affected both the product market, corresponding to a fall in revenue parameter R (and therefore A) for all firms and, since the Japanese firms relied in part on local suppliers, to an outward shift of the supply of 2's in the supplier market (that there was not concomitant entry into the supplier side is suggested by the fact that the US automakers reduced the number of suppliers they dealt with as part of their reorganization).

The change in supplier relations in the US auto industry is consistent with our model. From Proposition 11(iii), our model predicts that the reduced profitability for the US automakers (a uniform decrease in R) leads to a decrease in ω for all US firms. The increased competition in the supplier market from the Japanese (rightward shift in the supply of 2's) will have the same effect (Proposition 4).

Observed that if one looked only at the relationship between one auto firm (Chrysler, say) and its supplier, assuming a fall in R due to Japanese competition would provide little guidance as to how ω would change. Indeed, from expression (7), a decline in A implies an *increase* rather than a fall in ω unless v_2^* falls enough; only the full "general equilibrium" analysis provided in Proposition 11(iii) tells us that v_2^* does fall enough to bring about the observed decline in ω.

4.2   Technological Shocks outside the Industry: Trucking

In the 1980s and 1990s the trucking industry in the US experienced a shift away from drivers who owned their own trucks toward employee drivers. This organizational change has been attributed to various technological developments, such as the introduction of "on-board computers" (OBCs), which offered both better monitoring of driver actions and greater flexibility in dispatching, permitting more efficient use of trucks (Baker and Hubbard, 2004).

By the early 2000s, the prevalence of owner operators and use of OBCs had stabilized. But the industry has begun to shift some control (in the form of "perks") back to drivers. Between 2004 and 2006, carriers began offering drivers the right to travel with spouses or to outfit their cabs with satellite televisions. Since drivers decide whether and when to exercise these rights, they constitute an increase in their control. The question is why there has been a shift of control allocations in trucking without an apparent technological shift.

A possible answer comes from the observation that an important alternative employment for truckers is construction, which experienced a boom in the early 2000s. Thinking of the drivers now as the type 2's, and construction-cum-trucking firms as the type 1's, the construction boom would raise A for the construction firms (considered to be the marginal ones). By Proposition 11(ii), our model predicts a rise in ω, i.e., an increase in the degree of control enjoyed by the drivers. The evidence suggests that participants in the industry understand this perfectly well: firms perceive a "shortage" of drivers (Nagarajan, Bander and White, 2000 – this justifies thinking of drivers as type 2's) and both kinds of participants attribute the need to offer perks to the boom in construction (Urbina 2006). The outside options of drivers in trucking firms increases, leading to a rise in ω, precisely as a result of the external effect generated by the increase in A in the construction sector.

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5   Discussion

If one asks the question "who gets organizational power in a market economy?," one is tempted to answer "to the scarce goes the power." There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) which ascribes power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore, 1990; Stole and Zweibel, 1996). Our results suggest that organizational power may emanate from scarcity *outside* the organization, i.e., from market power: agents on the short side of the market, those with the greatest wealth, or those with the highest skills will tend to get more control than other agents. How much they get will depend in part on the market price of partners and therefore on the distribution of resources among all agents in the economy, not just those in the organization. And the lesson has to be interpreted with some care: redistribution of a scarce resource may cause the recipient to *lose* power, via the external effect (think of an increase in productivity by the marginal manager, as in Proposition 11(ii)).

As we discussed, one empirical implication of the external effect is that it may account for organizational change that does *not* originate inside the organization. While it is clear that legal or regulatory change may influence a firm's ownership structure, the point is that external influences on a firm's organization are not limited to these but may include liquidity, technological or demand shocks in other firms or industries. We are not aware of attempts to quantify the real-world significance of external effects, but hope that models such as the present one will encourage empirical investigations in that direction.

We now discuss some other implications of the model.

5.1   Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or the technology available to firms. One can easily extend the model to allow for

liquidity that yields a positive return though the period of production. Because liquidity in this model is used only as a means of surplus transfer, and not as a means to purchase new assets, the effects of this can be somewhat surprising. Raising this interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before: formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and by Proposition 8, firms will integrate less if the interest rate increases, and will integrate more if the interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity, by lowering interest rates, may constitute a force for integration above and beyond that suggested by the example in Proposition 9. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward; whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

5.2   Product Market

If we imagine all the firms sell to a competitive product market, then the selling price inheres in A, which we have thus far viewed as exogenous. But if instead price is determined endogenously in the product market, then shocks to some firms will be transmitted to the others via the product market as well as the supplier market. In other words, more than just the very poorest firms in the economy may be "marginal." For instance, suppose that a number of perfectly nonintegrated firms innovate. With fixed prices, these firms produce more output, but nothing further happens. With endogenous prices, the increased output in the first instance lowers product price; all other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of "amplification" effect on organizational restructuring.

Moreover, organizational decisions may affect the quantity of goods produced and therefore the product price. For instance, if R is the price of

a single unit of output, then industry output is increasing in the degree of integration. As discussed in Legros and Newman (2006), the fact that the product market – even a competitive one – can be affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

6   Appendix I: Contracting

We have defined contracts by (ω, t) and equal sharing of the output ex-post. This definition could be restrictive because it ignores the following four potential extensions.

- *Contingent shares.* A contract could specify state contingent revenues $x_i(R), x_i(0)$ to $i = 1, 2$.

- *Debt contract.* Type 1 borrows B from a financial institution in exchange for a repayment of D after output is realized.

- *Ex-post transfers of liquidity.* The total liquidity available in the firm is $L = l_1 + l_2$. This liquidity can be transferred either ex-ante or added to the revenue of the firm ex-post.

- *Asset swapping.* This is a means of effectively committing the managers to high levels of q. This commitment is only worthwhile if productivity is sufficiently high relative to costs, which will not be the case given our parametric restriction. If assets are to be swapped, we can characterize the situation via two ownership parameters ψ and ω : manager 1 owns $k \in [0, 1 - \omega)$ and $k \in [2 - \psi, 2)$, and 2 owns the other assets.

We show that our definition of contracting is without loss of generality by introducing into the contracting model described in the text a moral hazard element. The incentive compatibility condition associated to this moral hazard problem will restrict the marginal revenue $x_i(R) - x_i(0)$ to be equal to $R/2$ for each agent. The result will then follows.

A manager has the opportunity to divert revenue R in the high state by choosing an effort $e \in [0,1]$: if the state is high, with probability e the perceived output in the firm will be R while with probability $1-e$ the perceived output in the firm is 0, in which case the manager diverts a share cR and $(1-c)R$ is lost; if the state is low, the perceived output in the firm will be 0 independently of e. Only one manager has the opportunity to divert (the identity of that manager being chosen by nature).

The ex-post revenue of the firm consists of two components: the risky component with realizations 0 and R and a non-risky component denoted by T, typically the amount of ex-ante liquidity than is pledged (in an escrow account) to the firm. By choosing e, the manager can "hide" R but not T.

Let $x_i(R)$ and $x_i(0)$ be the revenues to the manager if the perceived realization of the risky component is R and 0 respectively. Then, with $e=1$, the expected revenue to the manager is $px_i(R) + (1-p)x_i(0)$. With $e=0$, the expected revenue is $p(cR + x_i(0)) + (1-p)x_i(0)$. Hence $e=1$ is optimal when $x_i(R) \geq cR + x_i(0)$, or $x_i(R) - x_i(0) \geq cR$. Clearly if $c > 1/2$, both incentive compatibility constraints cannot hold. By choosing $c = 1/2$, we have

$$x_i(R) - x_i(0) = R/2 \tag{9}$$

as claimed. If $c < 1/2$, there is scope for unequal marginal revenues for the two agents, but it still remains true that there is no loss in assuming that $T = 0$ and that debt contracts are weakly dominated by non-debt contracts.

Suppose that (9) holds. A contract is $((\omega, \psi), (B, D), (x_1, x_2)(t_1, t_2))$, where we assume without loss of generality that only agent 1 engages in a debt contract. Let x_i^* be the state contingent revenue equal to $R/2$ in state R and 0 in state 0. We want to show that there exists a contract $((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\hat{t}_1, L - \hat{t}_1))$ that leads to payoffs that are weakly greater for both managers. We establish this result sequentially: first by showing that $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$ is weakly dominated by the contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*), (t_1 + x_1(0), t_2 + x_2(0)))$, where neither debt nor ex-post transfers of liquidity are used, second by showing that this contract is dominated by a contract in which only part of the assets of type 1 are reassigned to type 2 $((\hat{\omega}, 0), (0, 0), (x_1^*, x_2^*), (\tilde{t}, L - \tilde{t}))$.

Step 1. In a contract $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$, feasibility requires that $t_1 + t_2 \leq L + B$ and $t_i \geq 0$, $i = 1, 2$. We write $t = t_1 + t_2$ the total liquidity ex-ante and $T = L + B - t$ the liquidity that is pledged to the firm. Ex-post total revenues are then T and $T + R$. Managers get state contingent revenues $x_i(0), x_i(R)$ satisfying budget balancing and limited liability: $x_1(0) + x_2(0) = T$, $x_1(R) + x_2(R) = T + R$, $x_i(0) \geq 0$, $x_i(R) \geq 0$.

If there is a debt contract, manager 1 has to repay $\min\{D, x_1(0)\}$ in state 0 and $\min\{D, x_1(R)\}$ in state R. Since by (9), we need $x_2(R) - x_2(0) = R/2$, we have $x_1(R) - x_1(0) = R/2$, however since manager 1 has to repay the debt, his effective marginal compensation is

$$x_1(R) - x_1(0) - [\min\{D, x_1(R)\} - \min\{D, x_1(0)\}].$$

This is consistent with (9) only if $\min\{D, x_1(R)\} = \min\{D, x_1(0)\}$, or if $D \leq x_1(0)$. In this case, debt is not risky; the creditor makes a non-negative profit only if $D \geq B$, but then we need $x_1(0) \geq B$ and therefore $x_2(0) \leq L + B - t - B = L - t$. It follows that the initial contract $((\omega, \psi), (B, D), (x_1, x_2), (t_1, t_2))$ is weakly dominated by the contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*)(t_1 + x_1(0), t_2 + x_2(0)))$. Since $\sum_{i=1,2}(t_i + x_i(0)) = L$, there is no liquidity transferred ex-post.

Step 2. Finally we show that swapping of assets is dominated by no swapping of assets

Consider a contract $((\omega, \psi), (0, 0), (x_1^*, x_2^*), (t, L - t))$ consisting of a swap of assets and ex-ante transfers; we denote such contracts by $((\omega, \psi), t)$. We have the following Nash equilibrium payoffs:

$$u_1(\omega, \psi, t) = \frac{A}{2}\left((2 - \omega - \psi)\frac{A}{2} + \omega + \psi\right) - \frac{1}{2}\left(\omega + (1 - \omega)\frac{A^2}{4}\right) - t$$

$$u_2(\omega, \psi, t) = \frac{A}{2}\left((2 - \omega - \psi)\frac{A}{2} + \omega + \psi\right) - \frac{1}{2}\left(\psi + (1 - \psi)\frac{A^2}{4}\right) + t$$

Suppose without loss of generality that $t > 0$ and that $u_2(\omega, \psi, t) - t > u_1(\omega, \psi, t) + t$; then we must have $\omega > \psi$.

Let $\omega^0 = \omega - \psi A/(1 - A/2)$; since $A/(1 - A/2) < 1$ and $\omega > \psi$, $\omega^0 > 0$. Then, $u_1(\omega^0, 0, t) = u_1(\omega, \psi, t)$ while $u_2(\omega^0, 0, t) - u_2(\omega, \psi, t) =$

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$\psi (2 - A - A^2)/4 > 0$ since $A < 1$. By continuity there exists $\hat{\omega} < \omega^0$ such that the contract $((\hat{\omega}, 0), t)$ strictly Pareto dominates the contract $((\omega, \psi), t)$. If $u_2(\omega, \psi, t) - t < u_1(\omega, \psi, t) + t$, a similar argument applies by decreasing the value of ψ appropriately.

7 Appendix II: Proofs

7.1 Proof of Proposition 5

If $\bar{l}_1 = 0$, note that $v_2^* = W(0) = (1 + \alpha)\frac{3}{8}A^2$ and from (7), $\omega(v_2^*, l)$ has a kink at $l = \alpha\frac{3}{8}A^2$: for lower values the degree of integration is linear and for larger values it is zero; hence $\omega(v_2^*, l)$ is indeed globally convex in l (we suppress the subscript on l where there is no ambiguity).

Suppose that $L < \alpha 3A^2/8$. Let $\underline{L} = \int_{l < \alpha 3A^2/8} l dG(l)$ and $\bar{L} = \int_{l > \alpha 3A^2/8} l dG(l)$. Note that by (7), $\int_{l < \alpha 3A^2/8} \omega(v_2^*, l) dG(l) = \omega(v_2^*, \underline{L})$ and that $\int_{l > 3A^2/8} \omega(v_2^*, l) dG(l) = \omega(v_2^*, \bar{L})$. Hence, $E\omega = G(\alpha 3A^2/8) \omega(v_2^*, \underline{L}) + (1 - G(\alpha 3A^2/8)) \omega(v_2^*, \bar{L})$. However since $\omega(v_2^*, \bar{L}) = 0$, and since ω is globally convex, $L = G(\alpha 3A^2/8) \underline{L} + (1 - G(\alpha 3A^2/8)) \bar{L}$ implies that $E\omega > \omega(v_2^*, L)$. This shows that $\bar{L} = 0$ and that the support of G is contained in $[0, \alpha 3A^2/8]$. The same argument applies when $L > \alpha 3A^2/8$.

7.2 Proof of Proposition 6

We know from (7) and Proposition 2 that for a given distribution G the degree of integration is positive when l belongs to $\left[\bar{l}_1, v_2^* - \frac{3}{8}A^2\right)$. In this case we can write $\omega(v_2^*, l) = \omega_0 + a\bar{l}_1 - bl$ where $\omega_0 = \frac{3A^2}{(2-A)^2}$, $a = 4\frac{2-3A}{A(2-A)^2}$, $b = \frac{4}{A(2-A)}$, note that $a/b = 1 - \alpha$. Let $\kappa = G(v_2^* - \frac{3}{8}A^2) - G(l)$ be the measure of firms choosing a positive ω.

(i) Let $\mu = \frac{1}{\kappa} \int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} l dG(l)$ be the conditional mean among firms choos-

ing a positive ω. We have,

$$
\begin{aligned}
E\left[\omega\right] &= \int \omega\left(v_2^*, l\right) \frac{dG(l)}{n} \\
&= \frac{1}{n} \int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} \left(\omega_0 + a\bar{l}_1 - bl\right) dG\left(l\right) \\
&= \frac{\kappa}{n}\left(\omega_0 + a\bar{l}_1\right) - b \int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} l \frac{dG\left(l\right)}{n} \\
&= \frac{\kappa}{n}\left(\omega_0 + a\bar{l}_1 - b\mu\right).
\end{aligned}
$$

when all firms choose $\omega > 0$, $\kappa = n$, leading to the expression in the Lemma.

(ii) Let $\sigma^2 = \int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} \left(l - \mu\right)^2 \frac{dG(l)}{\kappa}$ be the variance of liquidity among the liquidity constrained type 1, that is those that will be in firms with $\omega > 0$. Direct computations show that the variance of ownership is

$$
\begin{aligned}
Var\left[\omega\right] &= \int \left[\omega\left(v_2^*, l\right) - E\left[\omega\right]\right]^2 \frac{dG(l)}{n} \\
&= \int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} \left[\omega\left(v_2^*, l\right)\right]^2 \frac{dG(l)}{n} - E\left[\omega\right]^2 \\
&= \frac{\kappa}{n}\left(1 - \frac{\kappa}{n}\right)\left(\omega_0 + a\bar{l}_1\right)\left(\omega_0 + a\bar{l}_1 - 2b\mu\right) \\
&\quad + \frac{\kappa}{n}b^2 \left(\int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} l^2 \frac{dG(l)}{\kappa} - \frac{\kappa}{n}\left(\mu\right)^2\right) \\
&= \frac{\kappa}{n}\left(1 - \frac{\kappa}{n}\right)\left(\omega_0 + a\bar{l}_1\right)\left(E\left[\omega\right] - b\mu\right) \\
&\quad + \frac{\kappa}{n}b^2 \left(\sigma^2 - (1 - \frac{\kappa}{n})\mu^2\right)
\end{aligned}
$$

Since the degree of ownership ω is positive only if the type 1 is liquidity constrained $(l < v_2^* - \frac{3}{8}A^2)$, the degree of heterogeneity of ownership will depend on the distribution among these constrained type 1 agents. When all type 1 are constrained, $\kappa = n$ and we have as in the Lemma, $Var\left[\omega\right] = b^2 \left(\int_{\bar{l}_1}^{v_2^* - \frac{3}{8}A^2} l^2 \frac{dG(l)}{n} - \mu^2\right) = b^2\sigma^2$.

7.3 Proof of Proposition 7

(i) It is immediate from Lemma 6 that if $\kappa^G = \kappa^H$, $\int \omega\left(v_2^{*H}, l\right) dH(l) < \int \omega\left(v_2^{*G}, l\right) dG(l)$ if and only if $a\bar{l}_1^H - b\mu^H < a\bar{l}_1^G - b\mu^G$ or if $(1 - \alpha)\left(\bar{l}_1^H - \bar{l}_1^G\right) <$

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$\mu^H - \mu^G$ since $\frac{a}{b} = 1 - \alpha$.

(ii) If $\kappa^G = \kappa^H = n$, the result is immediate from Lemma 6(ii).

7.4 Proof of Proposition 9

It is enough to provide an example. Suppose that liquidity is uniformly distributed on $[0, x]$, where $x < \frac{3}{8} A^2$, and suppose that $n = 1 - \epsilon$; then $\bar{l}_1^G = \epsilon x$ and the inframarginal mean liquidity is $\mu^G = \frac{x}{2}(1 + \epsilon)$. Suppose that all agents with liquidity in $[0, \delta]$, where $0 < \epsilon x < \delta \le x$, have a liquidity shock and their new liquidity is δ while other type 1's have the same liquidity as before. Then the new liquidity distribution is $H(l) = 0$ for $l < \delta$ and $H(l) = \frac{l}{x}$ for $x \ge l \ge \delta$.

The new marginal liquidity $\bar{l}_1^H$ is δ, while $\mu^H = \frac{x^2 + \delta^2 - 2\epsilon\delta x}{2x}$. The condition in Proposition 7 is violated when $(1 - \alpha)(\delta - \epsilon x) > \frac{\delta^2 - 2\epsilon\delta x - \epsilon x^2}{2x}$. In particular, if $\alpha < \frac{1}{2}$, integration increases even if every type 1 is given liquidity x.

7.5   Proof of Proposition 11

Let

$$\rho \; : \; [0, 1] \to [0, 1]$$
$$\rho(i) \; \ge \; \rho(\hat{\imath}) \Leftrightarrow W(i) \ge W(\hat{\imath}).$$

be a reordering of the indexes of type 1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type 1 agent is i_π such that the Lebesgue measure of the set $\{i : W(i) \ge W(i_\pi)\}$ is n and the set of equilibrium firms is $F = \{i : \pi(i) \ge \pi(i_\pi)\}$.

Let $v_2^*(A)$ be the equilibrium price in the initial situation and $v_2^*(\hat{A})$ the equilibrium price after the shock to the technology available to agents in $[i_0, i_1]$.

Remark 1 *Proposition 11 is concerned with situations where $i_\mu = 1 - n$. However, note that the marginal type may not be $1 - n$. This can happen in two cases.*

Case 1: A first possibility is $i_1 < 1 - n$, that is, shocked firms were not matched in the initial economy but because $W(i_1) > v_2^*(A)$, some of these firms will be matched. In this case, the set of "new entrants" are firms with $i \in [i_\pi, i_1]$ while the set of "old firms" are those with index $i \geq k$, where $k \geq 1 - n$ satisfies $i_1 - i_\pi = k - (1 - n)$ (hence firms $i \in [i_\pi, i_1]$ "replace" firms $i \in [1 - n, k]$). Since $W(i_\pi) > v_2^*(A)$, the degree of integration in old firms increases. For new firms, the question is whether the increase in price $W(i_\pi) - W(1 - n)$ is large enough to overcome the internal effect of technology shock pushing towards less integration.

Case 2: Another possibility is $1 - n \in (i_0, i_1)$ and $W(1 - n) > \lim_{\varepsilon \downarrow 0} W(i_1 + \varepsilon)$. Then there exists $k > i_1$ such that $W(k) = W(1 - n)$, and either $i_\pi \in (i_1, k]$ or $i_\pi \in [i_0, 1 - n)$. In either case, if $l_1(i_\pi)$ is low enough, the increase in equilibrium surplus to the 2 may be small enough that the internal effect dominates and shocked firms integrate less.

(i) (Inframarginal shocks) If $i_0 > 1 - n$, then $i_\pi = 1 - n$ and $W(i_\pi) = v_2^*(A)$, then the shocked firms become less integrated while the unshocked firms remain unaffected

This is a direct consequence of Lemma 10

(ii) (Marginal shocks) If $1 - n \in (i_0, i_1)$ is still the marginal type 1, the equilibrium price increases and all firms, shocked and unshocked, integrate more.

Note that $1 - n$ is still the marginal type if and only if $W(1 - n) \leq \lim_{\varepsilon \downarrow 0} W(i_1 + \varepsilon)$, for in this case, all agents $i > 1 - n$ have higher willingness to pay than $1 - n$.

From (8), $v_2^*(A) = W(1 - n)$ is increasing in A, hence $v_2^*(\hat{A}) > v_2^*(A)$ and it follows that all unshocked firms $[i_1, 1]$ integrate more.

If the firm $1 - n$ did not integrate before the shock (that is chose $\omega = 0$),then all $i > 1 - n$ firms also chose not to integrate since ω is decreasing in the liquidity of type 1. Hence, it is immediate that an increase in A can only lead to more integration.

Consider now the case where firm $1 - n$ integrated before, that is chose a contract with $\omega > 0$. If i_1 chose initially a contract $\omega = 0$, there exists

$k \in (1-n, i_1)$ such that all firms with $i < k$ integrate ($\omega > 0$) and all firms with $i \geq k$ do not integrate; firms with $i \geq k$ will necessarily integrate more after the shock. We have $v_2^* (A) = W(1-n; A)$, $v_2^* \left(\hat{A} \right) = W \left(1-n; \hat{A} \right)$, and from (7), (8), for all shocked firms $i \in [1-n, k)$, the difference in the degree of integration after and before the shock is

$$\frac{3\hat{A}^2 - 4\bar{l}_1}{\left(2 - \hat{A} \right)^2} - \frac{3A^2 - 4\bar{l}_1}{(2-A)^2} > 0.$$

(here $\bar{l}_1 = l_1 (1-n)$) and all firms integrate more as claimed.

(iii) If $i_0 = 0$ and $i_1 = 1$, the arguments for (ii) apply since $1 - n$ is still the marginal type 1 manager.

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