On bilateral agreements: just a matter of matching*

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Abstract

This paper aims at assessing the importance of the initial technological endowments when firms decide to establish a technological agreement. We propose a Bertrand duopoly model where firms evaluate the advantages they can get from the agreement according to its length. Allowing them to exploit a learning process, we depict a strict connection between the starting point and the final result. Moreover, as far as learning is evaluated as an iterative process, the set of initial conditions that lead to successful ventures switches from a continuum of values to a Cantor set.

Keywords: Bertrand Competition, Duopoly, Learning, Firm agreements.

JEL classification: D21, D43, L1.

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1 Introduction

Particular attention is commonly addressed to the analysis of the benefits that firms may exploit by participating in bilateral agreements. The importance of this phenomenon should be directly related not only to the incentives inducing firms to engage in a bilateral agreement, but also to the perspectives that these agreements entail.

According to Harrigan (1986), firms engage in a few basic R&D partnerships to exploit knowledge in new applications, to enter in new fields etc. Indeed, these ventures allow to share research costs, to save on assets, and to avoid to replicate laboratories and testing periods. Agreements often involve dissimilar partners and in such deals, usually the partner with better technology exchanges it against retail access in new markets. In those situations contract agreements are preferred to joint ventures, since managers seem uncomfortable with long term ventures. Nevertheless, successful agreements or ventures do not last forever. Generally agreements do not last for more than ten years, particularly when they involve firms of unequal size.

Commercial agreements undoubtedly display more flexible features than joint ventures, mergers, or acquisitions. In agreements, both parts benefit from the advantages of their collaboration still keeping their own identity and still behaving independently in the market. From this viewpoint, the great challenge for the partners is to define as precisely as possible the object of the agreement itself.

In recent years, a few stylized facts have contributed to open a new line of analysis. In particular, the proliferation of partnerships and collaborations between transnational firms in Europe has fostered research to capture the rationale of the choice among such heterogeneous kinds of agreements. In the European context, we may easily realize that most of the actual agreements involve the development and exploitation of the so-called new technologies (see Leamer and Storper (2001).).

Note however that in addition to the traditional commercial/R&D agreements signed by firms endowed with similar technology, an increasing number of contracts involve specific agreements between very asymmetric firms. In particular, we observe a significant increase of agreements between firms in industrialized countries and in transition countries (see various issues by the EBRD). It is precisely the structural asymmetry between the partners involve in these agreements that calls for an analysis of the basic factors favoring the success of these deals.

There exists a wide range of contributions focusing on the elements supporting the creation of an agreement. In general, they study the conditions guaranteeing the stability of the agreement. This is particularly relevant in R&D settings where the problem of the appropriation of the profits plays a crucial role (cfr. Hinloopen (1997)). The interest of this topic is often related to the existing trade-off between R&D activity, in some sense external to a firm and other forms of collaborations such as joint ventures (Kabiraj and Mukherjee (2000)). In these models, the property rights dilemma and the exploitation of the benefits from the synergies stemming from the collaboration turn out to be the two factors that drive firms' decisions and ensure the positive outcome of the agreement. Indeed, according to some empirical results, the decision to continue or discontinue an agreement will depend on the balance between re-deployable information and specific assets. The conclusion of an agreement can be caused by the failure of the venture or the attainment of satisfactory results (Bureth et al. (1997)).

Another strand of literature tackles the problem of defining an optimal contract supporting a stable agreement between equal or different firms. This is addressed in Pérez-Castrillo and Sandonís (1996). In particular, they focus on the existence of incentive contracts making firms disclose and share their know-how in research joint ventures (RJVs) via technical uncertainty. They show that sometimes profitable RJVs do not start when the disclosure of know-how is not contractible. They also describe the incentive contracts when existing. If projects are advantageous, it is always possible to find contracts acceptable to both firms that give incentives to disclose knowledge to the more advanced firm. In addition Veugelers et al. (1994) prove that the emergence of a stable joint venture is directly related to the importance of the synergies between the two partners. The higher the synergies the partners expect, the less incentives firms have to cheat. They also show that even in an asymmetric case, a stable equilibrium may involve a loyal partner and a cheating one. Indeed, the dominant strategy for the loyal partner is to comply with the agreement as far as it earns more from the venture than from the own development of a new technology.

Differently from the previous contributions, we address neither the stability problem of the agreements nor the design of a optimal contract. Our analysis concentrates on the case of firm agreements concerning R&D or, more generally, technological cooperation between two firms. The main contribution of the paper is the modeling of the dynamic interaction between the partners to study the initial technological conditions allowing firms to join in profitable agreements. That is, we focus the attention not in the existence of optimal and stable contracts (we will assume the conditions ensuring their existence), but in the previous stage of selecting a proper partner. Our purpose is to examine whether firms' initial technological endowments are relevant in the successful completion of an agreement in a dynamic framework where we introduce a learning process in time.

We propose a duopoly model where firms compete à la Bertrand and share the market demand according to the degree of substitutability of goods. An original feature of our model is the introduction of a learning process throughout the length of the agreement. Bureth et al. (1997) show that learning is a key factor in the evolution of firms' collaboration. Indeed, a continuous collaborative interaction may influence the decision to continue or not the agreement. In out paper, learning turns out to be the crucial element in the cumulation of advantages stemming from the collaboration. It is the influence of the learning process that, at the end, allow for selecting the kind of initial endowments in technology leading to successful collaborations.

The paper is organized as follows. Section 2 presents the main building blocks of the theoretical setting. Section 3 studies the conditions for the completion of successful agreements with and without learning. In particular, a learning iterative process gives rise to a Cantor set of solutions. Section 4 concludes.

2 The model

Following Vives (1999) and Singh and Vives (1984), we consider a differentiated duopoly with two firms $(1, 2)$. They use a constant, but different marginal cost technologies without fix costs. Firms compete à la Bertrand. Market demand is linear and goods (respectively $1, 2$) produced by firms may be substitutes or complements.

2.1 Consumers' program

As in Singh and Vives (1984) we consider an economy composed of an oligopolistic sector and a competitive (numeraire) sector summarizing the rest of the economy. Consumers are all identical. They select a consumption bundle to maximize a separable utility function linear in the numeraire good. Thus, we can consider a representative consumer and concentrate in the (sub)utility function corresponding to the differentiated sector of the economy. Formally, the representative consumer selects a pair (q_1, q_2) solving

$$\max_{q_1,q_2} U(q_1, q_2) \text{ s.t. } R = p_1 q_1 + p_2 q_2, \tag{1}$$

where $q_i, \ i = 1, 2$ is the amount of good 1 consumed at price p_i, and R denotes the income devoted to the differentiated sector.

We consider a quadratic and strictly concave utility function,

$$U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{1}{2}(\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2), \tag{2}$$

where $\alpha_i > 0, \ \alpha_i \beta_j - \gamma \alpha_j > 0$, and $\beta_1 \beta_2 - \gamma^2 > 0$. Goods are substitutes, independent or complements according to γ greater than, equal to or less than zero. When $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2 = \gamma$, goods are perfect substitutes. Also, when $\alpha_1 = \alpha_2$ we can define an index of product differentiation as $\gamma^2/\beta_1\beta_2$. This index takes value zero for independent goods and one for perfect substitutes or complements. The solution of the problem (1) yields the following system of inverse demands,

$$p_1 = \alpha_1 - \beta_1 q_1 - \gamma q_2, \tag{3}$$
$$p_2 = \alpha_2 - \beta_2 q_2 - \gamma q_1.$$

Let $\delta = \beta_1\beta_2 - \gamma^2, c = \gamma/\delta, a_i = (\alpha_i\beta_j - \gamma\alpha_j)/\delta, b_i = \beta_j/\delta$ for $i \neq j$ and $i = 1, 2$, we can write the direct demand functions as:

$$q_1 = a_1 - b_1 p_1 + c p_2, \tag{4}$$
$$q_2 = a_2 - b_2 p_2 + c p_1.$$

A quick inspection of this demand system reveals that the demand for a single good is downward sloping in its price and increasing in the price of the competitor if goods are substitutes.

2.2 Firms' program

We consider an asymmetric duopoly, as described in Vives (1999). Firms compete à la Bertrand. They use constant but different marginal cost technologies given by,

$$C_i(q_i) = m\xi q_i, \quad i = 1, 2,$$

where $m > 0$ and $\xi_i \in [0, 1]$ is a parameter linked to the efficiency in the reduction of costs (see below). For simplicity we normalize $\xi_1 = 1$ and assume that $\xi_2 = \xi \leq 1$. Firms use the same technology if $\xi = 1$, while the lower ξ the more efficient is firm 2 with respect to firm 1.

Solving firms' profit maximization problems, we obtain the system of reaction functions:

$$p_1 = \frac{a_1 + mb_1 + cp_2}{2b_1}, \tag{5}$$
$$p_2 = \frac{a_2 + m\xi b_2 + cp_1}{2b_2}. \tag{6}$$

Following Singh and Vives (1984) and Vives (1999), we consider prices net of marginal costs. Thus, we define

$$\hat{p}_1 = p_1 - m; \qquad \hat{a}_1 = a_1 - b_1 m + cm\xi,$$
$$\hat{p}_2 = p_2 - m\xi; \qquad \hat{a}_2 = a_2 - b_2 m\xi + cm,$$

so that firm i's profit function is $\Pi_i = \hat{p}_i(\hat{a}_i - b_i\hat{p}_i + c\hat{p}_j)$ with $i \neq j$ and $i = 1, 2$. Let also $m = 1$ without loss of generality.

Equilibrium prices are,

$$\hat{p}_1^* = \frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2}; \quad \hat{p}_2^* = \frac{2b_1\hat{a}_2 + c\hat{a}_1}{4b_1b_2 - c^2}, \tag{7}$$

In the case of independent goods (i.e. $c = 0$), markets are separated and we obtain monopoly prices:

$$\hat{p}_1^m = \frac{a_1 + b_1}{2b_1}; \quad \hat{p}_2^m = \frac{a_2 + b_2\xi}{2b_2}. \tag{8}$$

From the equilibrium prices (7), we compute the associated equilibrium quantities,

$$q_1^* = b_1\hat{p}_1^*; \quad q_2^* = b_2\hat{p}_2^*. \tag{9}$$

Finally, equilibrium profits are given by,

$$\Pi_1 = b_1\left(\frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2}\right)^2; \quad \Pi_2 = b_2\left(\frac{2b_1\hat{a}_2 + c\hat{a}_1}{4b_1b_2 - c^2}\right)^2. \tag{10}$$

For future reference, monopoly profits are,

$$\Pi_1^m = \frac{1}{b_1}\left(\frac{a_1 - b_1}{2}\right)^2; \quad \Pi_2^m = \frac{1}{b_2}\left(\frac{a_2 - b_2\xi}{2}\right)^2. \tag{11}$$

3 The agreement

Let us now consider that firms decide to make an agreement. We can think of agreements between firms in industrialized countries that are technically similar and agreements where one firm is located in an industrialized area while the other belongs to a less developed country, like those between enterprises in western and eastern Europe. In either case, we can envisage two scenarios. On the one hand, (i) the agreement may be renewed period by period both in a finite and infinite period of time; on the other hand, (ii) the agreement contemplates a dynamic learning process in discrete time, when firms agree in keeping the collaboration for more than one period. In either scenario, the agreement aims at improving the technology, namely to reduce their production costs. We concentrate in the second kind of agreement. We assume that benefits stemming from the agreement do not allow the technological lagged firm to fill up the existing gap respect to the other firm. We are interested in studying the conditions under which this kind of agreement is sustainable stressing the role played by the benefits the collaboration.

Let us consider a parameter $\lambda \in [0, 1]$ representing a combination of the existing technologies ξ_1 and ξ_2 at the moment firms decide to collaborate. The index λ tends to one when ξ_1 is similar to ξ_2. It tends to zero in the opposite case. We follow some well known models in industrial organization literature, such as Mansfield (1961) or De Palma et al. (1991), where technological changes or the spreading of new technologies follow a diffusion process.

Looking at this diffusion process in the discrete time, one possible representation to transpose that idea to our situation is considering the following general dynamic process $F(\lambda) = \mu\lambda(1 - \lambda)$ so that (see May(1976) and Li-Yorke (1975))

$$\lambda_{t+1} = \mu\lambda_t(1 - \lambda_t), \tag{12}$$

where $\quad \lambda \in [0, 1],\ \lambda_0 > 0,\ \mu > 0,\ t = 0, 1...n.$

Equation (12) tells us that λ increases from one period to the next when it is small, while decreases when it is large. The parameter μ is a multiplier of this dynamics. It affects the steepness of the hump in the curve. Indirectly, this process also captures a learning or cumulation process that appears when the agreement lasts for several periods. In terms of our model, this process can be interpreted as follows. There exists a continuum of possible agreements that span from the case in which firms participating to the agreement display different technologies (λ small) to the case in which firms are very similar in technology (λ large). Formalizing an agreement allow them to improve their technology reducing production costs. Of course, the size of these advantages and expectations of benefits of these two extreme types of agreement are different. The maximum is reached at a point where although technologies are not identical they match in an optimal way. This is so because the law of motion of λ given by (12) is quadratic and concave in λ. Nevertheless, it worth noting that the two kinds of agreements deserve attention since each of them allows firms to improve their available technology.

Indeed, given that optimal contracts supporting such agreements exist (see Pérez-Castrillo and Sandonís (1996) and Veugelers and Kesteloot (1994)), our concern is to find the initial technological conditions allowing two firms to join the agreement leading to the optimal contract.

3.1 Static agreement

Let us start our analysis of the agreements that do not span in time and thus there is no learning process. We want to study the constellation of parameter values allowing firm top benefit from the agreement. As we have mentioned before, the agreement aims at developing a new cost reducing technology in the spirit of Veugelers et al. (1994).

From (12), and recalling that $m = 1$, the cost functions for each of the two firms are:

$$C_1 = \mu\lambda_0(1-\lambda_0)q_1; \quad C_2 = \mu\lambda_0(1-\lambda_0)\xi q_2.$$

Also, we can easily imagine that the degree of differentiation of the products supplied by the firms may range from independent goods (so that firms serve separate markets) to some level of substitutability, so that markets will be interrelated. We will consider both cases as well.

3.1.1 Separate markets

First, we consider the case where firms' markets are separated. That is firms produce products so differentiated that they hold monopoly status in their respective markets (i.e. $c = 0$). We find condition under which firms both with similar and very different technologies are willing to engage in an agreement.

Proposition 1. *When firms are local monopolies, at stage 1 they are willing to engage in an agreement for* $\mu > 4$ *when* $\lambda_0 \in \left[0, \frac{1}{2} - \frac{\overline{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\overline{\mu}}{2}, 1\right]$*, where* $\overline{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2} \in (0,1)$.

Proof. We start by computing the corresponding equilibrium prices, quantities and profits for both firms.

$$\tilde{p}_1^m = \frac{a_1 + \mu\lambda_0(1-\lambda_0)b_1}{2b_1}; \quad \tilde{q}_1^m = \frac{a_1 - \mu\lambda_0(1-\lambda_0)b_1}{2},$$
$$\tilde{\Pi}_1^m = \frac{1}{b_1}\left(\frac{a_1 - \mu\lambda_0(1-\lambda_0)b_1}{2}\right)^2, \tag{13}$$
$$\tilde{p}_2^m = \frac{a_2 + \mu\lambda_0(1-\lambda_0)b\xi}{2b_1}; \quad \tilde{q}_2^m = \frac{a_2 - \mu\lambda_0(1-\lambda_0)b_2\xi}{2},$$
$$\tilde{\Pi}_2^m = \frac{1}{b_2}\left(\frac{a_2 - \mu\lambda_0(1-\lambda_0)b_2\xi}{2}\right)^2. \tag{14}$$

Not surprisingly, equilibrium values are symmetric. Hence, we can concentrate on firm 1 and extend the conclusions to firm 2. Comparing profits firm 1 gets in (11) and in (13), it is easy to see that firm 1 will participate in the agreement if,

$$\frac{1}{b_1}\left(\frac{a_1 - \mu\lambda_0(1-\lambda_0)b_1}{2}\right)^2 > \frac{1}{b_1}\left(\frac{a_1 - b_1}{2}\right)^2,$$

that reduces to a quadratic function of λ_0,

$$b_1[1 - \mu\lambda_0(1-\lambda_0)] > 0. \tag{15}$$

Given $b_1 > 0$ by assumption, we need to verify that $[1 - \mu\lambda_0(1-\lambda_0)] > 0$. This inequality admits real roots for $\mu > 4$. These are $\lambda_{1,2} = \frac{1}{2} \pm \frac{\overline{\mu}}{2}$ with $\overline{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2}$. Note that $0 < \frac{1-\overline{\mu}}{2} < \frac{1+\overline{\mu}}{2} < 1$. Therefore, inequality(15) is fulfilled for $\lambda_0 \in \left[0, \frac{1}{2} - \frac{\overline{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\overline{\mu}}{2}, 1\right]$. □

Figure 1 summarizes the discussion.

3.1.2 Interrelated markets

Next, we consider the case where products are substitutes so that the two firms interact in the market. We have now two degrees of freedom to characterize the conditions under which firms may engage in an agreement. On the one hand the degree of substitutability given by c; on the other hand, the degree of technical similarity between firms given by λ. As the next proposition states, we obtain a qualitatively similar result to the previous case.

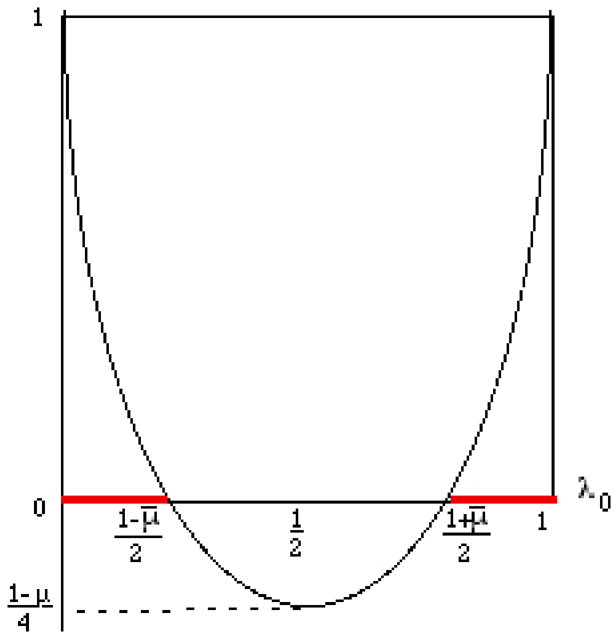

Figure 1: Range of solutions for a one-period iteration.

Proposition 2. *When markets interact, firms are willing to engage in an agreement for $\mu > 4$ and goods are either poor substitutes or close substitutes. Then, (i) for $c \to 0$ technological conditions making the agreement sustainable are described by $\lambda_0 \in [0, \frac{1-\overline{\mu}}{2}] \cup [\frac{1+\overline{\mu}}{2}, 1]$, and (ii) for values of c large enough the agreement is sustainable for $\lambda_0 \in (\frac{1-\overline{\mu}}{2}, \frac{1+\overline{\mu}}{2})$, where $\overline{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2}$.*

Proof. Now firms compete à la Bertrand in the market. The equilibrium prices, quantities, and profits are,

$$\bar{p}_1 = \frac{2b_2\overline{a}_1 + c\overline{a}_2}{4b_1b_2 - c^2}; \quad \bar{q}_1 = b_1\bar{p}_1; \quad \overline{\overline{\Pi}}_1 = b_1\left(\frac{2b_2\overline{a}_1 + c\overline{a}_2}{4b_1b_2 - c^2}\right)^2, \tag{16}$$

$$\bar{p}_2 = \frac{2b_1\overline{a}_2 + c\overline{a}_1}{4b_1b_2 - c^2}; \quad \bar{q}_2 = b_2\bar{p}_2; \quad \overline{\overline{\Pi}}_2 = b_2\left(\frac{2b_1\overline{a}_2 + c\overline{a}_1}{4b_1b_2 - c^2}\right)^2, \tag{17}$$

where $\overline{a}_1 = a_1 - \mu\lambda_0(1-\lambda_0)[b_1 - c\xi]$ and $\overline{a}_2 = a_2 - \mu\lambda_0(1-\lambda_0)[b_2\xi - c]$.

As before, given the symmetry of the problem we concentrate on the behavior of firm 1. Firm 1 evaluates the benefits it can get from the agreement comparing the level of profits with and without the agreement. That is, it compares profits in

(10) and (16). Participating in an agreement will be profitable if,

$$b_1\left(\frac{2b_2\overline{a}_1 + c\overline{a}_2}{4b_1b_2 - c^2}\right)^2 > b_1\left(\frac{2b_2\hat{a}_1 + c\hat{a}_2}{4b_1b_2 - c^2}\right)^2.$$

After some algebraic computations, the previous inequality reduces to,

$$[1 - \mu\lambda_0(1-\lambda_0)](2b_1b_2 - b_2c\xi - c^2) > 0. \tag{18}$$

Note that (18) differs from (15) in the term in brackets. Now for low enough values of c, the term $(2b_1b_2 - b_2c\xi - c^2)$ is positive, and inequality (18) behaves as (15). Thus, we obtain the same results as in the monopoly case. In contrast though, for large enough values of c, the term $(2b_1b_2 - b_2c\xi - c^2)$ is negative, so that the inequality is fulfilled when $[1-\mu\lambda_0(1-\lambda_0)] < 0$ that is, for $\lambda_0 \in (\frac{1-\overline{\mu}}{2}, \frac{1+\overline{\mu}}{2})$. □

Proposition 2 implicitely considers substitute goods only because this is the sensible case in our context. Nevertheless, let us stress that the case of complementary goods leads us back to the monopoly case. Now $c < 0$, so that the term $(2b_1b_2 - b_2c\xi - c^2)$ is always positive. Thus, inequality (18) holds if $[1 - \mu\lambda_0(1-\lambda_0)] > 0$, or $\lambda_0 \in [0, \frac{1-\overline{\mu}}{2}] \cup [\frac{1+\overline{\mu}}{2}, 1]$ as in proposition 1

3.2 Agreements and dynamic iteration

We may refine the results obtained in the previous subsection by introducing the time dimension and, as a consequence of that, a learning process. In other words, we assume that when a firm takes its decision, it is aware that the advantages it can get from the agreement follow an iterating dynamic process. This is possible because the benefits carried out by the agreement (12) is modeled as a dynamic equation and firms know the horizon of the agreement.

We will examine first whether there are combinations of technologies, embodied in the parameter λ, giving firms incentive to maintain their collaboration for n periods (lemma 1). Next, we will illustrate, by means of an example, how the set of solutions depends on the time horizon. We will conjecture that when the number of iterations is infinite, the set of solutions is a Cantor set. Finally, theorem 1 proves this conjecture.

Lemma 1. *Assume $\mu > 4$. There is a range of values of λ that in a n-period iterative learning process among firms allows them to improve their level of profits. It is given by $\lambda \in (\frac{3}{4}, 1]$.*

Proof. Let us consider the case of local monopolies. Given the structure of the iterative learning function, $\lambda_1 = \mu\lambda_0(1-\lambda_0), \ldots, \lambda_t = \mu\lambda_{t-1}(1-\lambda_{t-1}), \lambda_{t+1} = \mu\lambda_t(1-\lambda_t)$.

As a consequence, the sequence of profits for, say, firm 1 in every iteration t are,

$$\tilde{\Pi}_t^m = \frac{1}{b_1}\left(\frac{a_1 - \lambda_t b_1}{2}\right)^2, \ t = 1, 2, \ldots \tag{19}$$

Our local monopolies will be willing to extend the agreement from the time $t-1$ to the time t if,

$$\tilde{\Pi}_t^m > \tilde{\Pi}_{t-1}^m. \tag{20}$$

Note that from the expressions of profits it follows that $sign[\tilde{\Pi}_t^m - \tilde{\Pi}_{t-1}^m] = sign[\lambda_{t-1} - \lambda_t]$. Accordingly, inequality (20) reduces to studying the values of λ satisfying $\lambda_{t-1} - \lambda_t > 0$.

Given that $\lambda_t = \mu\lambda_{t-1}(1-\lambda_{t-1})$, the previous expression holds for $\lambda_{t-1} > 1 - \frac{1}{\mu}$. As we have seen before, for $\mu > 4$ so that firms will be willing to extend the agreement from period $t-1$ to t if $\lambda_{t-1} > \frac{3}{4}$. □

Lemma 1 tells us that two local monopolies will engage in an agreement as long as their technologies are sufficiently similar. Note that equation (12), describing the diffusion of the technological change, considers λ_0 as the initial (exogenous) condition. That is the description, before the agreement, of the technological differences between firms. Thus, the lemma has to be read as saying that, given some initial conditions, firms will maintain their collaboration period after period as long as the diffusion process maintain their technologies similar enough. Note also, that the degree of feasible similarity is increasing in time although the less efficient firm never ends up catching up with its partner. Moreover, according the expected length of the agreement, the magnitude of the benefits over the costs of production varies.

Let us illustrate the dynamics just described thinking of a local monopolist forecasting the impact on its profits period by period of signing an n-period agreement[1].

To make the example tractable, let us think of an agreement lasting for two periods. Firm 1 evaluates the profits it will get at the end of period two, according to the technology available at that time. Then, it compares these profits with the ones in absence of agreement. That is, firm 1 compares profits in (11) with profits given by (19)[2]. From the expressions of the profits above, it turns out that $\tilde{\Pi}_2^m > \Pi^m$ if $b_1(1-\lambda_2) > 0$, that is,

$$b_1\{1-\mu^2\lambda_0(1-\lambda_0)][1-\mu\lambda_0(1-\lambda_0)]\} > 0. \tag{21}$$

As displayed in figure 2, for $\mu > 4$, inequality (21) admits four strictly positive critical points ($0 < \lambda_{21} < \lambda_{22} < \lambda_{23} < \lambda_{24} < 1$), where

$$\lambda_{2i} = \frac{1}{2} \pm \frac{\sqrt{\mu^2 - 2\mu(1 \pm \overline{\mu})}}{2\mu}.$$

As before $\overline{\mu} = \left(\frac{\mu-4}{\mu}\right)^{1/2} \in (0,1)$, and $i = 1,2,3,4$ according to the combination of positive or negative signs of the square roots chosen. Therefore, (21) is satisfied for $\lambda_0 \in [0, \lambda_{21}] \cup [\lambda_{22}, \lambda_{23}] \cup [\lambda_{24}, 1]$.

Finally, combining the range of admissible values of λ_0 just obtained for period 2 with the corresponding ones in period 1 (see Proposition 1) we obtain the the range of values of λ_0 for which the two-period agreement is profitable.

$$\lambda_0 \in [0, \lambda_{21}] \cup \left[\lambda_{22}, \frac{1}{2} - \frac{\overline{\mu}}{2}\right] \cup \left[\frac{1}{2} + \frac{\overline{\mu}}{2},\ \lambda_{23}\right] \cup [\lambda_{24}, 1].$$

As it is well displayed by this example, and figure 3 illustrates, the different intervals of solutions shrink as far as the number of iterations increases, i.e. the length of the agreement expands.

[1] In general, this is the kind of cost-benefit analysis that firms carry out when they evaluate the convenience of joining an agreement. Firms look at the evolution of profits over a finite horizon from the actual situation by computing the present (discounted) value of the flow of future profits.

[2] This is so because we are assuming to be in the case of optimal long-term non renegotiable contracts.

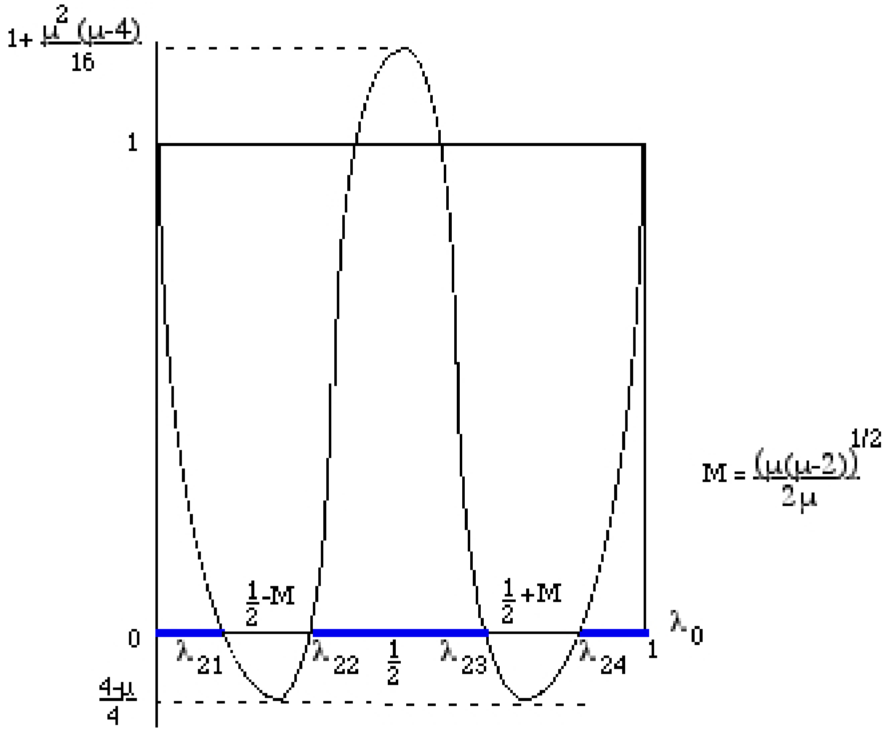

Figure 2: Range of solutions for a second period iteration.

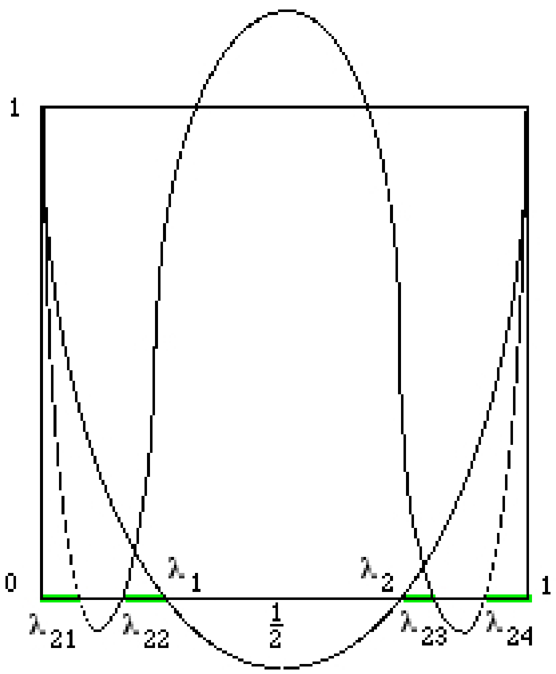

Figure 3: Range of solutions of a 2-period agreement.

At the limit, when $t \to \infty$ we obtain a infinite collection of points as the set of solutions. These points are precisely the (infinite) roots of the polynomial (of infinite degree) resulting from the comparison of profits between signing an infinite horizon agreement and no agreement at all. To clarify this argument, define A_t as the set of the points that escape from the interval $I = [0, 1]$ at iteration $t + 1$, that is, those points that were admissible at iteration t but are no longer solutions after iteration $t + 1$. Formally,

$$A_t = \{\lambda \in I | F_t(\lambda) < 0 \text{ and } F_\tau(\lambda) \in I,\ \tau < t\}.$$

As a consequence, the set of the solutions (Λ), in the case of an infinite number of iterations, reduces to:

$$\Lambda = I \setminus \bigcup_{t=0}^{\infty} A_t.$$

We will prove that Λ is a Cantor set, namely that it is a closed, perfect and totally disconnected subset of I. Before proceeding to the formal proof, we offer an intuitive argument.

Note that A_t are open sets. Thus, Λ is formed by (sequentially) suppressing from the interval I a collection of open sets that are disjoint intervals. In other words, Λ is the union of closed and disjoint intervals, and thus closed. Incidentally, note that Λ is not empty because at least contains the extreme points of the suppressed intervals,

Next, by definition, a set is perfect if it does not contain isolated points, that is, all its points are limit points. Let us assume, on the contrary, that $x \in \Lambda$ is an isolated point. Then x must be an extreme point common to two adjacent intervals. But as we have argued before, Λ is a collection of disjoint intervals. Hence, those adjacent intervals do not have points in common. Accordingly, x cannot be an isolated point.

Finally, a set is totally disconnected if it does not contain any open interval. Again let us proceed by contradiction. Assume that there exists an open interval $\delta \in \Lambda$. then δ has to be contained in one of the open intervals obtained in an iteration τ. But this is not possible since as $\tau \to \infty$, the length of the intervals tend to zero. Thus, at the limit Λ has infinitely many points (i.e. is a set of measure zero).

Theorem 1. Λ *is a Cantor set for* $\mu > 4$.

Proof. We will structure the proof in three steps keeping Devaney (1985) as guideline.

1. Λ **is a closed set.** Let us define $G = 1 - F$. By construction A_i is an open interval centered around 1/2 (see Figure 1 or 2). Let us focus on Figure 1 (one iteration), namely concentrating on A_0. In that case, the function G maps both the intervals $I_0 = [0, \lambda_1]$ and $I_1 = [\lambda_2, 1]$ monotonically onto I. Moreover, G is decreasing on the first interval and increasing on the second. Since $G(I_0) = G(I_1) = I$ there is a pair of intervals (one in I_0 and the other in I_1) which are mapped into A_0 by G. These intervals define the set A_1.

Next, let us consider $\Lambda_1 = I - (A_0 \cup A_1)$. This set consists of four closed intervals (see Figure 2) and G maps them monotonically onto either I_0 or I_1, but, as before, each of the four intervals contains an open subinterval which is mapped by G_2 onto A_0, i.e. the points of this interval escape from I after the third iteration of G. By applying this iterative process, we note that A_t consists of 2^t disjoint open intervals and $\Lambda_t = I - (A_0 \cup .. \cup A_t)$ consists of 2^{t+1} closed intervals. Hence, Λ is a nested intersection of closed intervals, and thus, a closed set.

2. Λ **is a perfect set.** Note that all endpoints of A_t, $(t = 1, ...)$ are contained in Λ. Such points are eventually mapped to the fixed point of G at 1, and they stay in I under iteration. If a point $x \in \Lambda$, were isolated, each nearby point must leave I under iteration and, hence, these points must belong to some A_t. Two possibilities arise. We can think of a sequence of endpoints of A_t converging to x. In this case the endpoints of A_t map to 1 and so, they are in Λ. Alternatively, all points in a deleted area nearby x are mapped out of I by some iteration of G. In this case, we may assume that G_τ maps x to 1 and all the other nearby points are mapped in the positive axis above 1. Then, G_τ has a minimum at x, i.e. $G'_\tau(x) = 0$. This iterative process ensures that it must be so for some $t < \tau$. Hence, $G_t(x) = 1/2$, but then $G_{t+1}(x) \notin I$ and $G_\tau(x) \to -\infty$, contradicting the fact that $G_\tau(x) = 1$.

3. Λ is a totally disconnected set. Let us focus in the first iteration and assume μ is large enough so that $|G'(x)| > 1$ for all $x \in I_0 \cup I_1$. For those values of μ, there exists $\gamma > 1$ such that $|G'(x)| > \gamma$ for all $x \in \Lambda$. Our iterative process yields $|G'_\tau(x)| > \gamma_\tau$. We want to prove that Λ does not contain any interval. Let us proceed by contradiction and assume that there is a closed interval $[x, y] \in \Lambda$, $x, y \in I_0 \cup I_1$, $x \neq y$. In this case, $|G'_\tau(z)| > \gamma_\tau$, for all $z \in [x, y]$. Choose τ so that $\lambda_\tau|y - x| > 1$. Applying the Mean Value Theorem, it follows that $|G_\tau(y) - G_\tau(x)| \geq \gamma_\tau \mid y - x \mid > 1$ implying that either $G_\tau(y)$ or $G_\tau(x)$ lies outside of I. But this contradicts our main hypothesis, hence Λ does not contain intervals.

It remains to determine the μ-values for which the previous argument holds. Finding the values of μ allowing $|G'(x)| > 1$ means to identify μ values for which $[-\mu\,(1 - 2x)]^2 > 1$. When $G = 0$, this inequality holds for $\mu > 2 + \sqrt{5}$. Thus we have proved that Λ is totally disconnected for $\mu > 2 + \sqrt{5}$. Recall that we have already imposed a condition on μ, namely $\mu > 4$. Hence, we need to verify whether Λ is also totally disconnected for $\mu \in (4, 2 + \sqrt{5}]$. We appeal to Kraft (1999) who proves that Λ is a Cantor set for $\mu > 4$. The idea behind the proof is that for $\mu \in (4, 2 + \sqrt{5}]$ it turns out that $|G'(x)| \lesseqqgtr 1$. Kraft argues that the iteration process shrinks some components of I, and stretches some others. His proof thus, consists in showing that in the interval $(4, 2+\sqrt{5})$ the stretching is dominated by the shrinking. He proves that Λ is an hyperbolic set, namely $|G'_\tau(x)| > k\delta_\tau > 1$ for $x \in \Lambda$, $k > 0$, $\delta > 1$.

$\square$

As we have seen, every iteration eliminates an open set of λ_0-values that were solutions in the previous iteration. The extreme points of those intervals remain in Λ though. It is important to bear in mind that a value of λ_0 that has been eliminated as a solution after an iteration, it remains out of Λ forever, i.e. it cannot be considered as solution again as the number of iterations increase.

Given the learning process we consider, as firms envisage longer and longer agreements, an increasing number of smaller intervals are excluded as solutions. In fact, in the limit as $t \to \infty$, we obtain a (countable) set of solutions with infinitely many points. Formally, at every iteration t, the admissible values of λ_0 supporting an agreement of length t is characterized by a polynomial of degree 2^t. The roots of the successive polynomials associated to every iteration always remain in Λ. As t increases the length of admissible intervals shrink, so that at the limit we have a polynomial of degree infinite characterizing intervals of measure zero. That is only the points corresponding to the infinite solutions remain in Λ as solutions of an agreement of infinite length.

To help to visualize the evolution of the set of solutions, think of an image where a firm willing to sign a short-term agreement can find a compatible partner almost effortless. As the commitment the firm is willing to engage in becomes deeper and deeper, the difficulty to find a suitable partner is also increasing. The reason behind this difficulty is *not* that there are less partners available (there are always infinite), but that getting to know about them and matching with the good one is increasingly hard. Some casual empiricism points a the longest contracts involving around ten years. Also, this casual empiricism suggests that the number of joint ventures decreases as the time span increases

In addition, conditions encountered for parameter λ in Lemma 1 imply that lasting agreements are those signed by firms displaying similar technological endowments (i.e. high values of λ). Nevertheless we need to keep in mind the meaning of this result. Knowing the length of the agreement, a firm evaluates the advantages it can get before signing it. According to the initial conditions (λ_0) it will be able or not to fulfill its expectations. Moreover, the iteration process we analyze imposes that firms need to pay as much attention as possible for choosing the proper final agreement, given the initial technology they dispose. In other words, if a firm wants to get the expected benefits from the agreement, needs to be extremely precise in choosing the right agreement (alias the right counterpart) allowing to fulfill its expectations. Put differently, with an infinite number of iterations, there is just a number of *discrete* points ensuring the success of the agreement. These correspond to the optimal combination of the initial technology available at firm level.

So far, in this section we have only consider firms operating in separate markets. Recall that in the previous section studying agreements that do not span in time, we obtained the same qualitative results for both the case of local monopolies and of firm interaction. The introduction of time in the analysis involves a learning process but it does not change the mechanics of the decision process of firms. Hence, we should not expect to obtain qualitatively different results either. That is, if firms operate in the same market, we should expect to obtain also a Cantor set of solutions as the number of iterations increase.

4 Conclusion

In this paper, we study the consequences that a given level of technological endowment may exert on the successfulness of the results of a firm agreement. Based on a duopoly setting in which firms compete *à la Bertrand*, we prove that not all initial technologies are suitable for getting advantages from such an agreement. Indeed, according to the expected length of the agreement, there exists just a particular and precise set of initial conditions (evaluated as the technology available at firm level at the moment they create the agreement) ensuring firms to benefit from all the advantages that agreement can carry out. The central issue of this analysis is related to the existence of a learning process throughout the length of the contract. as the number of iterations increase, an increasing number of smaller intervals of values of λ_0 are excluded as solutions. In the limit, when considering agreements lasting forever, we obtain a countable set of infinitely many points characterized as a Cantor set. According to the structure of our framework, this last outcome means that in the case of agreements lasting for long periods, firms can benefit as much as possible from the advantages issued by the agreement just in the case they succeed in finding the proper combination of technological initial conditions. Put differently, not all the agreements are suitable for all the firms. Of course, to get this result we assume that, *a priori* firms have perfect foresight of the status of the agreement from the initial period on. Indeed, it is this assumption that allow them to deal properly with the cost-benefit analysis of the agreement to detect the optimal combination of initial technological conditions.

Some extensions deserve attention. Accounting for uncertainty should complete the picture of present results. Also, an effort to give structure to λ is in order. In this paper we do not model it. Nevertheless, giving λ a particular structure, should help in stating some further details to focus better its connection with the successfulness of the final results engendered by the agreement.

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