

# THE MAKING OF INTERNATIONAL ENVIRONMENTAL AGREEMENTS\*

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## Abstract

We examine in this paper the formation and the stability of international environmental agreements when cooperation means to commit to a minimum abatement level. Each country decides whether to ratify the agreement and this latter enters into force only if it is ratified by a number of countries at least equal to some ratification threshold. We analyze the role played by ratification threshold rules and provide conditions for international environmental agreements to enter into force. We show that a large typology of agreements can enter into force among the one constituted by the grand coalition.

KEYWORDS: International Environmental Agreement, abatement bound, self-enforcing agreement, coalition formation, ratification threshold.

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# 1 Introduction

After the Montreal Protocol on the protection of the ozone layer, the Oslo Protocol on the regulation of transboundary air pollution, the Kyoto Protocol on greenhouse gases entered into force last February.<sup>1</sup> The entry into force of those international environmental agreements (IEA) constitutes an achievement that many political skeptics had dismissed as impossible.<sup>2</sup> Absent a supranational authority able to bind countries to build an IEA, the making of and the participation to agreements is indeed at the entire discretion of each country. Entry into force is all the more difficult that when countries eventually elaborate a protocol, this latter to enter into force needs to comply with ratification requirements. Signatories rejecting domestically the protocol by not ratifying weaken therefore the IEA.<sup>3</sup>

Explaining the making of a self-enforcing IEA has been and remains an ongoing research topic. Theoretical contributions tend indeed to prove that IEA made of more than three countries are unstable since most countries would prefer to do nothing and rely on the efforts made by counterparts —e.g., Carraro and Siniscalco (1993), Barrett (1994), or more recently Diamantoudi and Sartzetakis (2005). Sanctions, side payments and issue linkage were first explored in order to explain cooperation. While sanctions to deter free riding were proved to be non credible (Hoel (1992), Barrett (1994, 1999)) side payments and issue linkage proved to help the making of cooperation but under restrictive conditions —Carraro and Siniscalco (1993), Barrett (1997, 2001), Hoel and Schneider (1997) or Conconi and Peroni (2001). Lately, the focus is mostly put on representation of asymmetries between countries. Intuition tells indeed that asymmetries might be a sufficient condition to deter the free riders bearing important damages. We

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<sup>1</sup>For an exhaustive list of the international protocol which entered into force see <http://fletcher.tufts.edu/multi/multiabout.html>.

<sup>2</sup>In the text, we will consider the terms “protocol”, “agreement” and “treaty” as synonymous.

<sup>3</sup>For an overview of ratification thresholds stipulated —see for instance Article 25 of the Kyoto protocol (1997), Article 16 of the Ozone Layer Montreal protocol (1987), Article 15 of the Oslo Sulphur protocol on acid rain (1994), the NOX protocol (1988) or the VOC protocol (1991).

propose in this paper an alternative explanation of the making of IEA keeping the symmetric assumption but modifying slightly the rule of the game. More precisely, we characterize the range of feasible stable IEA, keeping the simple payoff formulation of Barrett (1994) but taking into account two specific aspects of the IEA game, namely the nature of the abatement bound commitment and the ratification threshold ruling the entry into force of the agreement.

The first aspect we focus on is the nature of the abatement target. According to the United Nations terminology, a protocol is an instrument with substantive obligations that implements the general objectives of a previous Convention. As such, participation to an environmental protocol on gas pollution translates in a commitment to abate at least *some* amount of emission. That is, participation to a protocol means that abatement levels cannot be below some minimal bound.<sup>4</sup> Furthermore, it is implicitly assumed in the IEA literature that abatement target is an optimum. Cooperating countries are supposed to choose an abatement target that maximizes their joint welfare given emissions of the IEA outsiders, while it appears that in most protocols the target is not meant to be an optimum but simply aims at ensuring a large participation (see Grubb *et al.*, 1999). We propose therefore to abandon the welfarist approach and to study instead the minimum abatement bound commitment which allows for the making of a *stable* agreement. In fact, the only requirement is that the abatement bound commitment needs to be higher than abatement in the absence of IEA. Committing countries then choose their own abatement levels, which might well be higher than the agreed bound. The idea is hence to look at the problem the other way around and to focus on participation rather than on efficiency. The setting we consider is purely non cooperative. Countries are concerned solely by their own abatement target and associated welfare rather than by the abatement level and welfare of a possible cooperative coalition. Transfers or flexibility mechanisms are therefore not permitted.<sup>5</sup>

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<sup>4</sup>For instance, Article 3 of the Kyoto protocol stipulates that countries are bounded to reduce their *overall* emissions of greenhouse gases by at least 5% below the 1990 levels by 2008-2012.

<sup>5</sup>Note however that if flexibility mechanisms are realized ex-post to the agreement,

The second aspect we focus on is the dynamics of agreement formation. We pay specific attention to the condition for an agreement to enter into force. Recall that the making of an IEA is made of three successive stages. The first is the elaboration and the signature of a protocol. It consists of agreeing on a target (i.e., an abatement bound) *and* on an entry into force rule (i.e., a ratification threshold). The second is the ratification of that protocol. National authorities endorse it and legally commit to respect the protocol target as soon as the entry into force rule is fulfilled. The third is the entry into force. If the threshold rule is fulfilled, countries set their abatement levels given the abatement bound they committed to. Existing literature on self-enforcing agreements pays little attention to that dynamics, focusing on the last stage and implicitly assuming that an IEA enters into force if endorsed by at least two countries.<sup>6</sup> An IEA is then formed as soon as it is stable meaning that no country has an incentive to get in or out of the IEA. Distinctly, we assume that a country's third stage decision has to comply with the prescriptions of the protocol only if it has ratified the agreement and if the number of countries having done so is at least as high as the ratification threshold. If the number of signatories ratifying is below the threshold, the agreement is null and void and even ratifying countries quit their commitment.<sup>7</sup> As a consequence, we do not model explicitly the as stipulated for instance in the Kyoto Protocol, these last would be necessarily Pareto improving.

<sup>6</sup>These models consider that the set of countries entering in the agreement choose their abatement levels so as to maximize the sum of their utilities (note that this implies that target is determined endogenously by the number of countries participating to the agreement). When there is only one country, its choice coincides with his best-reply, which implies that the abatement level is the same as the one without agreement.

<sup>7</sup>Note that the use of minimal participation threshold to implement public good has already been studied in the literature. For instance, Bagnoli and Lipman (1989) show that the use of such participation threshold is sufficient to implement Core allocations. In the same vein and applying it to IEA negotiation, Currarini and Tulkens (2003) analyse the allocation rule allowing the existence of the making of efficient agreement given that ratification threshold constraint is to be fulfilled. Kohnz (2004) analyses the IEA game in a non cooperative setting. The approach followed is however distinct than our since she studies the problem under the angle of contract theory considering asymmetric information between agents with linear utilities.

first stage but focus on the interplay between the second and the third stage. More precisely, we analyze the relation between threshold and abatement target in order to characterize the set of all IEA that can enter into force. Given an agreement (a minimal abatement level and a ratification threshold), we consider a two-stage game where countries decide first to ratify or not and second to choose an abatement levels higher or equal to the abatement bound they committed to. We study the subgame perfect equilibria of this game, which turn out to be equivalent to the internal-external stability concept usually employed in this literature. The introduction of ratification mechanism proves not only to constitute a sufficient condition to explain the making of a large IEA, but also provides a natural justification for the use of the internal-external stability concept adopted by Carraro and Siniscalco (1993), Barrett (1994) and their successors. Hence, the ratification rule which is usually interpreted as a constraint to cooperation enforcement is shown to have positive feedbacks on this latter. Besides ensuring the entry into force of the protocol, the ratification threshold also turns out to determine the size of the cooperative coalition. We characterize the set of IEA that can enter into force and give conditions under which the maximization of the welfare (of the signatories of the IEA) coincides with the maximization of total abatement level.

The paper is organized as follow. We present in Section 2 the model. In Section 3 we describe the ratification game and provide conditions on the abatement bound and the ratification threshold for an agreement to enter into force. In Section 4 we analyze the interplay between the countries' welfare and the total abatement level, and conclude in Section 5. Most of the proofs are relegated to the Appendix.

## 2 The Model

### 2.1 Preliminaries

We consider a (finite) set of  $n$  identical countries,  $N = \{1, \dots, n\}$ , where each country  $i$  has to choose an abatement level of stock of pollutant,  $q_i$ . We call this situation the *abatement game*. For the sake of simplicity, we

assume that for each country  $i \in N$  the range of possible abatement levels is defined by  $X_i = [0, \infty)$ . We denote by  $\mathbf{q}$  the vector of abatement levels, i.e.,  $\mathbf{q} = (q_i)_{i \in N}$  and by  $Q$  the aggregate abatement level,  $Q = \sum_{j \in N} q_j$ .

We assume abatement is a public good with congestion. In other words, abatement allows to avoid global environmental damages and therefore benefits each country symmetrically but whenever drastic levels are attained, it also annihilates the functioning of the international economy. The benefit from avoiding damage can thus be partially overcome by a negative effect on trade.<sup>8</sup> Precisely, when the aggregate abatement level is  $Q$ , country  $i$  gets

$$P_i(Q) = aQ - \frac{1}{2}Q^2.$$

where  $a$  is a positive parameter.

Abatement is also individually costly in the sense that each country pays the cost of its own abatement level. The more a country abate the more it will be marginally costly. The cost function of country  $i$  is therefore

$$C_i(q_i) = \frac{c}{2}q_i^2.$$

where  $c$  is a positive parameter.

For a given vector of abatement the net payoff of country  $i$  is then given by the following equation,<sup>9</sup>

$$u_i(\mathbf{q}) = P_i(Q) - C_i(q_i) = aQ - \frac{1}{2}Q^2 - \frac{c}{2}q_i^2. \quad (1)$$

As usual, we write  $q_{-i}$  to denote the  $(n-1)$ -dimensional vector  $(q_h)_{h \in N \setminus \{i\}}$ . The abatement game has a unique equilibrium, in which each country chooses the same abatement level,  $q_0$ ,

$$q_0 = \frac{a}{n+c}. \quad (2)$$

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<sup>8</sup>Note, however, that most of our results carry over if we model total abatement as a public good without congestion.

<sup>9</sup>A similar formulation was proposed initially by Barrett (1994). Alternatively we could consider a model in which countries choose their level of emissions of pollutant. Diamantoudi and Sartzetakis (2003) show that these two approach are equivalent for the class of payoffs considered in this paper.

This yields the following utility level

$$u_0 = u_i(\mathbf{q}_0) = \frac{a^2(n^2 + 2nc - c^2)}{2(n + c)^2}. \quad (3)$$

where  $\mathbf{q}_0 = (q_0, \dots, q_0)$ .

## 2.2 The restricted abatement game

Traditionally, the literature on environmental agreements defines an IEA as a set of countries choosing jointly their abatement levels. This contrasts with most international treaties where an IEA is better considered as a target *and* an entry into force mechanism. In this paper we shall consider this latter approach. For expositional ease however, we restrict in this section to the definition usually employed in the literature, i.e. where an IEA is just a proposal of an abatement level. Later in the paper we shall broaden our definition of an IEA introducing a ratification threshold. Restricting to the target of an IEA and not considering the entry into force mechanism allows us to focus on some basic characteristics of IEA such as countries best response and IEA's effectiveness.

Because we assume that countries are symmetric, imposing the same lower bound on abatement levels for countries participating to the IEA is a natural assumption. Hence, we shall consider throughout the paper only values of  $\alpha$  such that  $\alpha > q_0$ .<sup>10</sup>

The main issue is then whether countries will follow the recommendations of the IEA. From a strategic point of view, the participation to an IEA consists of an alteration of one's strategy set in the abatement game, in which case we shall talk about the *restricted abatement game*. More precisely, the possible levels of abatement of a country  $i$  participating to an IEA will be  $X_i^{IEA}$ ,

$$X_i^{IEA}(\alpha) = [\alpha, \infty),$$

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<sup>10</sup>As explained in the Introduction, we consider the choice of the value  $\alpha$  is a secondary issue. Our main purpose in this paper is to characterize the different levels of  $\alpha$  that allows for the existence of a stable IEA. Later in the paper we use our characterization of the set of admissible values of  $\alpha$  to give some insights about which value is more likely to be chosen, depending on the objective of the signatories of an IEA, i.e., maximizing the total abatement level or the welfare of the signatories of the IEA.

and if it does not participate, its attainable abatement levels are unchanged, i.e.,  $X_i = [0, \infty)$ . Whenever no confusion can occur we shall omit the term  $\alpha$  in the strategy set of a country member of the IEA and write  $X_i^{IEA}$  instead of  $X_i^{IEA}(\alpha)$ .

While strategy sets are affected when there is an IEA (for the participant countries only), the payoffs are not affected. Observe that from a formal point of view, the situation with an IEA defines a different game than the situation without an IEA presented at the beginning of this section.<sup>11</sup> To keep things simple we shall abuse notation and use the same payoff function described in equation (1) to denote the payoffs of the game with an IEA and without an IEA.

Since the participation to an IEA bounds the choices available to a country, its strategic behavior will be affected. First, observe that the payoff function of each country  $i \in N$  is continuous in  $\mathbf{q}$  and strictly quasi-concave in  $q_i$ . It follows that each country's best-reply is continuous and single valued. For a country not participating to the IEA, its best reply is defined as follows,

$$BR_i(q_{-i}) = \{q_i \in X_i : u_i(q_i, q_{-i}) \geq u_i(q'_i, q_{-i}) \text{ for all } q'_i \in X_i\}.$$

Because  $q_i$  must belong to  $X_i$  for all  $i \in N$ , we thus have

$$BR_i(q_{-i}) = \max \left\{ \frac{a - \sum_{j \neq i} q_j}{1 + c}; 0 \right\}.$$

Consider now the case of a country, say  $i$ , participating to an IEA. In this case the best-reply, denoted  $br_i$ , is defined as follows,<sup>12</sup>

$$br_i(x_{-i}) = \{q_i \in X_i^{IEA} : u_i(q_i, q_{-i}) \geq u_i(q'_i, q_{-i}) \text{ for all } q'_i \in q_i\}.$$

As shown by Bade *et al.* (2005), the function  $br_i$  can be easily characterized from the function  $BR_i$  and the bound  $\alpha$ .<sup>13</sup>

<sup>11</sup>The main difference between the case without an IEA is the domain of the payoff functions. Without an IEA, the domain of each country's payoff function is  $\prod_{i \in N} X_i$  while with an IEA the domain becomes  $\prod_{i \in S} X_i^{IEA} \times \prod_{i \in N \setminus S} X_i$ .

<sup>12</sup>Note that since the domain of the best-reply  $br_i$  depends on the set of strategies of all other countries, and thus on the set of countries participating to the IEA, we should write  $br_i^{X^{IEA}}$  instead of  $br_i$ .

<sup>13</sup>Bade, Haeringer and Renou (2005) give a characterization of the restricted best reply

**Lemma 1 (Bade, Haeringer and Renou (2005))** *Let  $u_i$  be continuous and strictly quasi-concave in  $q_i$  for all  $i \in N$ . Let  $X_i = [0, \infty)$  and  $X_i^{IEA} = [\alpha, \infty)$ . Then,*

$$br_i(q_{-i}) = \begin{cases} \alpha & \text{if } BR_i(q_{-i}) < \alpha \\ BR_i(q_{-i}) & \text{if } \alpha \leq BR_i(q_{-i}) \end{cases}$$

Lemma 1 simply says that whenever a country's best response is to choose an abatement level higher than  $\alpha$ , the country is free to do so. However, if the best response consists of choosing an abatement level lower than  $\alpha$ , then the country chooses an abatement level equal to  $\alpha$ . While this result seems obvious, it is worth to note that it may not hold if the payoff functions are not strictly quasi-concave. Because we look at a model in which countries choose an abatement level, only a minimal abatement level is relevant —see Bade *et al.* for a statement of this result when a country has a minimal and a maximal abatement level.

Given an IEA, the main issue we address in this paper is to find out how many countries will follow the IEA's recommendation. To answer this question, we first characterize the equilibria of the restricted abatement games, for any number of countries participating to the IEA. Denoting by subscript  $s$  the signatory countries and by subscript  $ns$  the non-signatory countries, we have the following proposition.

**Proposition 1** *Let  $\alpha > q_0$  and let  $S$  be a coalition of countries that follow the IEA's recommendation, i.e., restrict their attainable abatement levels to  $[\alpha, \infty)$ , and let  $s = \#S$ . Then there is a unique equilibrium in the abatement game in which each signatory country chooses an abatement level equal to  $q_s(s) = \alpha$  and each non-signatory country chooses  $q_{ns}(s)$  where,*

$$q_{ns}(s) = \begin{cases} \frac{a - s\alpha}{c + n - s} & \text{if } s \leq \frac{a}{\alpha}, \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

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whenever the original and the constrained strategy sets are both compact and convex subsets of the real line (which includes for instance the case when the IEA also imposes an upper bound) and payoff functions are strictly quasi-concave. The Lemma presented here can be easily deduced from the Lemma presented in Bade *et al.*

By Proposition 1, all countries participating to an IEA (resp. not participating) can be treated symmetrically. It follows that all what matters when computing the payoffs of a country is the size of the IEA and whether or not the country participates to the IEA. For the sake of simplicity, we shall then write  $u(s, 1)$  to denote the payoff of a country participating to an IEA with  $s$  countries,  $u(s, 0)$  the payoff when the country is not participating to an IEA which contains  $s$  countries. When there is no IEA the payoff of a country will be simply denoted  $u(0)$ , and we write  $u(n)$  to denote a country's payoff when the IEA involves all countries.

Perhaps the most basic question regarding an IEA is its effectiveness, that is, whether an IEA increases the total abatement level. When an IEA forms, all countries following the IEA's recommendation will abate more than what they would abate without the IEA ( $\alpha > q_0$  by assumption). However, according to Proposition 1, countries not participating to the IEA will abate  $q_{ns}(s)$  which is always below  $q_0$ . In spite of this, we can show that as long as the abatement level of each countries form an equilibrium (i.e., abatement levels are given by Proposition 1), an IEA always increases the total abatement level. This result holds irrespective of the stability of the IEA. To see this, let  $\alpha$  be a minimal abatement level imposed by the IEA and suppose that there are  $t$  countries following the IEA's recommendation. Suppose first that  $t < a/\alpha$ . According to Proposition 1, the total abatement is  $Q(t)$ , where

$$Q(t) = (n - t)q_{ns}(t) + t\alpha = \frac{a(n - t) + \alpha tc}{c + n - t}.$$

When there is no IEA, the total abatement level is  $Q(0)$ ,

$$Q(0) = nq(0) = \frac{na}{n + c}.$$

It suffices then to compare both  $Q(t)$  and  $Q(0)$ . We then obtain

$$\frac{a(n - t) + \alpha tc}{c + n - t} > \frac{an}{c + n} \Leftrightarrow \alpha > \frac{a}{c + n}.$$

By assumption the second inequality is always satisfied. If, on the contrary we have  $t \geq a/\alpha$  then  $q_{ns}(t) = 0$ , and thus  $Q(t) \geq a$ . Because  $an > a/(n+c)$ , we then have the following result,

**Proposition 2** *For any IEA, the total abatement is higher than that obtained without IEA.*

### 3 The IEA game

#### 3.1 The game

We have seen in the previous section that given a minimal abatement level  $\alpha$  and a number of countries committing not to abate less than  $\alpha$ , abatement levels and payoffs are uniquely determined. What remains to be done in order to explain the making of IEA is to know how many countries will commit to a minimal bound on their abatement levels.

To this end, we consider the following two-stage game with perfect information between each stage. The first stage consists of the *ratification* stage, where countries choose simultaneously between two actions,  $R$ , for ratifying, and  $NR$ , for not ratifying. In the second stage, all countries choose simultaneously an abatement level.

The ratification of an IEA by a country can be interpreted here as a conditional commitment from the country to participate to the IEA. This conditionality comes from the presence of a *ratification threshold*,  $t$ , which consists of the minimal number of countries ratifying the agreement for the IEA to enter into force. It is the combination of each country's decision in the first stage and the ratification threshold that will determine which restricted abatement game is played in the second stage. If  $i$  is a country choosing  $NR$  in the first stage then its second stage action is  $X_i = [0, \infty)$ . Suppose now that country  $i$  chooses  $R$  in the first stage, and denote by  $T$  the set of all countries, including  $i$ , that choose  $R$  in the first stage. The action set of country  $i$  in the second stage is defined as follows:

$$X_i = \begin{cases} X_i^{IEA} = [\alpha, \infty) & \text{if } \#T \geq t, \\ X_i = [0, \infty) & \text{otherwise.} \end{cases}$$

Ratification of the agreement by a country is then a binding decision because if a country has ratified the IEA it cannot choose an abatement level below  $\alpha$ . Yet, this decision is conditional on the fact that at least  $t - 1$

other countries also ratified the agreement. We then say that the agreement *enters into force* if it has been ratified by at least  $t$  countries. We can now give a formal definition of an IEA,

**Definition 1** *An International Environmental Agreement (IEA) is a proposal to put a lower bound  $\alpha$  on countries' abatement levels, where  $\alpha > q_0$ , and a ratification threshold  $t$ .*

Without ratification threshold, a country ratifying the agreement is indeed restricting its abatement levels to be higher or equal to  $\alpha$ . This implies that the absence of a ratification threshold is equivalent to setting this latter to be equal to 1.

### 3.2 Stability

A natural equilibrium concept to use given our framework is subgame perfection. Because for any number of countries participating to the IEA the equilibrium in the restricted abatement game always exists and because countries' first stage action sets are finite we easily deduce that a subgame perfect equilibrium always exists.

Traditionally, the literature has focused on a stability concept originally introduced by d'Aspremont *et al.* (1983) in the literature on cartels. This stability concept is the combination of two stability requirements, called respectively internal and external stability. According to this concept, a coalition  $S$  is said to be *internally stable* if no country in  $S$  has an incentive to leave the coalition, and it is said to be *externally stable* if no country outside  $S$  has an incentive to join the coalition. A coalition  $S$  is *stable* if it is both internally and externally stable. Subgame perfection in our framework turns out to be equivalent to internal and external stability if one defines as the coalition the set of countries participating to the IEA. Indeed, at a subgame perfect equilibrium, no country ratifying wants to change its first-stage action by deciding not to ratify. Similarly, a country not ratifying does not have an incentive to change its first-stage action by ratifying the IEA. In other words, the choice of ratifying *vs.* not ratifying translates into

a choice of staying in or staying out of the coalition of countries respecting the IEA's recommendation.

It could be argued that the use of the internal-external stability concept is not well justified when the issue is that of making an environmental agreement. While the requirement that no country participating to an IEA wants to withdraw seems natural (the internal stability), imposing that non-signatories cannot join the IEA is difficult to justify (the external stability). Since the entry of another country increases the total abatement level, why should we impose external stability? As a comparison, we can observe that international organizations like the WTO have the objective to integrate as many countries as possible, i.e., the entry of a new member is considered as a desirable outcome. In this paper we take a different route. Starting from a sequential game (induced by our ratification mechanism), we analyze subgame perfect equilibria, the most sensible concept for such type of games. Because our model establishes an equivalence between the internal-external stability concept and the first stage actions of a subgame perfect equilibrium, the ratification mechanism we propose can therefore also be seen as a strategic justification for the use of the internal-stability concept.

We should note, however, that the existence of a subgame perfect equilibrium (and therefore of a stable coalition) is not sufficient to ensure that the IEA will enter into force. An IEA also needs to be ratified by at least  $t$  countries, i.e. ratification threshold requirement. For this reason, by a stable coalition (or stable IEA) we shall always refer to a group of countries choosing  $R$  in the first stage such that its size is greater or equal to the ratification threshold  $t$ .

### **3.3 Stability without ratification threshold**

We first consider the case when no ratification threshold is mentioned in the protocol. Note that the absence of ratification clause is in fact equivalent to the case in which the ratification threshold is set to 1, for a country decision to follow the IEA's recommendation is binding, independently of the decision of the other countries. To begin with, we show that it is not possible to have a stable IEA including all countries.

**Proposition 3** *For any  $\alpha > q_0$ , there is no stable IEA with  $n$  members.*

This result is not surprising. Indeed, since an IEA induces participants to choose a non-Nash abatement level, there is always one country who wishes to deviate. Since by Proposition 1 the abatement level chosen by non-deviating participant will be constrained (i.e., remain  $\alpha$ ), the deviating country will be the only one able to fully adjust his abatement level, and attain his best-reply. Perhaps more surprising is that the same results also holds for any number of countries ratifying the IEA, Indeed, if there are already some countries not participating to the IEA, the abatement levels of these countries will change if one participating country decides to withdraw from the IEA. In this case, the abatement level chosen by a country  $i$  withdrawing from the IEA will not be its best response against the abatement levels of all other countries when  $i$  was still participating to the IEA. Rather, it will be the outcome of a new equilibrium.

**Proposition 4** *For any  $s \in \{1, \dots, n\}$ , there is no stable IEA with  $s$  countries.*

Without a stabilization mechanism, Proposition 4 shows that there is no hope to obtain a stable IEA, thereby suggesting that an international agreement should not only consists of a target about abatement levels but also a mechanism regulating entry into force.<sup>14</sup> We show in the next section that imposing a ratification threshold greater than 1 is a natural mechanism to ensure entry into force of stable IEA's.

### 3.4 Stability with ratification threshold

We now consider the case of a non-trivial ratification threshold, i.e.,  $t \geq 2$ . We first look for conditions under which there is a threshold  $t^*$  such that a stable agreement exists. To do so, we study conditions for the payoff

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<sup>14</sup>This result can appear striking to the readers familiar with that literature. One could indeed intuitively expect that a coalition of at least two countries is eventually stable. Remind however that alternatively to the papers in the vein of Barrett (1994), we assume that countries participating to the IEA do not consider the join payoff of the coalition but simply their own payoff.

obtained by a country ratifying the IEA when it has also been ratified by  $t^* - 1$  other countries is greater than the payoff when the agreement is not ratified, i.e., studying whether the set  $Z(\alpha) = \{s \leq n \mid u(s, 1) - u(0) \geq 0\}$  is non-empty. It turns out that the existence of stable IEA's is greatly facilitated by observing that there is never over-ratification,

**Proposition 5** *Suppose that  $(t, \alpha)$  is a stable IEA. Then there is no equilibrium in which more than  $t$  countries ratify the IEA.*

This result is a direct consequence of Proposition 4. More precisely, suppose that  $t'$  countries ratify the IEA and  $t' > t$ , where  $t$  is the ratification threshold. Consider now a country ratifying, and hold the strategy of the other countries fixed. In this case, this country receives a payoff equal to  $u(t', 1)$ . If it decides not to ratify, then it will receive a payoff equal to  $u(t' - 1, 0)$ . According to Proposition 4, we must have that  $u(t', 1) < u(t' - 1, 0)$ . Hence, whenever there is over-ratification some ratifying countries have an incentive to withdraw from the agreement. On the contrary, when there are just  $t$  countries ratifying, a country participating to the IEA compares  $u(t, 1)$  with  $u_0$ , where  $u_0$  is the Nash equilibrium payoff of the abatement game without IEA. This is so because if the country opts for not ratifying the number of countries ratifying will be below the threshold  $t$  and thus we reach the situation when there is no IEA. The next result gives a necessary and sufficient condition on the minimal abatement level  $\alpha$  to ensure the existence of a stable IEA for some threshold  $t$ .

**Proposition 6** *There exists a threshold  $t^*$  such that a stable IEA exists if and only if  $\alpha \leq \bar{\alpha}$ , where  $\bar{\alpha} = q_0\sqrt{1+c}$ .*

We say that a ratification threshold  $t$  is *admissible* if there exists an abatement level  $\alpha$  such that  $t \in Z(\alpha)$ . Similarly, a minimal abatement level  $\alpha$  is *admissible* if  $\alpha \in (q_0, q_0\sqrt{1+c}]$ .<sup>15</sup>

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<sup>15</sup>It is important to note that the maximal value of abatement given by Proposition 6 does not yield, in principle, to a stable IEA. The reason is that the bound  $\bar{\alpha} = q_0\sqrt{1+c}$  may not be sufficiently high to ensure that there is an *integer* such that  $u(t, 1) \geq u(0)$  — although we know that when considering the trivial abatement level  $\alpha = q_0$  there exists a

Another observation we can make about the set of stable IEA's is that it is not necessarily unique, although its size is uniquely determined by the ratification threshold. That is, if  $t$  is an admissible threshold then there exists  $n!/(n-t)!t!$  distinct stable coalitions. It is worth to note however that for a given value of abatement level  $\alpha$  there might exist several admissible thresholds. If  $\alpha \leq q_0\sqrt{1+c}$  then any threshold between  $\underline{t}(\alpha)$  and  $\bar{t}(\alpha)$  is admissible, where  $\underline{t}(\alpha)$  and  $\bar{t}(\alpha)$  are the solution of the following programs,

$$\underline{t}(\alpha) = \arg \min_{t \in \{0, \dots, n\}} u(t, 1) - u(0) \quad (5)$$

$$\text{such that } u(t, 1) - u(0) \geq 0$$

$$\bar{t}(\alpha) = \arg \max_{t \in \{0, \dots, n\}} u(t, 1) - u(0) \quad (6)$$

$$\text{such that } u(t, 1) - u(0) \geq 0$$

It follows that whenever  $\alpha \leq q_0\sqrt{1+c}$  any threshold  $t \in [\underline{t}(\alpha), \bar{t}(\alpha)]$  is admissible. An extreme case is when  $t$  is set to  $n$ . In this case, we obtain the obvious result that any Pareto improvement is attainable in a stable IEA involving *all* countries.<sup>16</sup> One may then wonder why some agreements do not fix an abatement level that maximizes the collective welfare and set a ratification threshold equal to  $n$ . First, one can assume that requiring unanimous ratification could be too demanding, thereby decreasing the likelihood that the IEA enters into force. Second, Proposition 4 shows that there are strong incentives to free ride. That is, for any abatement level, each country would prefer to have a ratification threshold strictly below  $n$  and be one of the countries not ratifying. Hence, although a stable IEA involving all countries is theoretically plausible, the negotiation may lead to a threshold lower than  $n$ .

Finally, it is worth mentioning that the abatement level maximizing the welfare of the grand coalition (when the threshold is set to equal  $n$ ) is lower than the maximal abatement level  $\bar{\alpha}$ . To see this, note that when  $t = n$  the welfare of the grand coalition is maximized when  $\alpha = an/(n^2 + c)$ , which can be shown to be lower than  $\bar{\alpha}$ .

stable IEA for any ratification threshold  $t$ . For the sake of simplicity, we will consider in the remaining of the paper that  $\bar{\alpha}$  is such that there is an integer  $t^*$  such that  $u(t, 1) > u(0)$ .

<sup>16</sup>By *any* Pareto improvement we mean any choice of  $\alpha$  such that all countries choosing the abatement level  $\alpha$  makes them better off compared to the Nash equilibrium level  $q_0$ .

## 4 Welfare

The question of welfare improvement of an IEA is also quickly eluded. Let  $t^*$  be an admissible ratification threshold. By Proposition 5, any stable IEA contains  $s^* = t^*$  countries. By the stability of the IEA, we then have

$$u(t^*, 1) \geq u_0.$$

Hence, all countries forming part of the IEA are better off compared to the situation without the IEA. Furthermore, since the benefit of abatement is shared by all countries and only the cost of abatement is country-specific, we have,

$$\forall t = 1, \dots, n, \quad u(t, 0) > u(t, 1).$$

Combining this two previous observations we then have the following result,

**Proposition 7** *A stable IEA is always welfare improving for all countries, for any admissible ratification threshold.*

Let us now focus on the interplay between the welfare of signatories of an IEA, the ratification threshold and the equilibrium total abatement level. While an IEA benefits the environment and the welfare of all countries, it turns out that the maximization of the environmental impact (i.e. minimization of the damage) is not necessarily equivalent to the maximization of the signatories' welfare. It appears that this friction between these two objectives comes from the choice of the ratification threshold.

We first characterize the agreements that maximizes the total abatement level. Let  $\alpha$  be an admissible minimal abatement level. It follows that for any threshold  $t \in [\underline{t}(\alpha), \bar{t}(\alpha)]$  there is an equilibrium in which only  $s$  countries ratify, yielding the total abatement  $Q(s)$ ,

$$Q(s) = (n - s) \frac{a - s\alpha}{c + n - s} + s\alpha.$$

To facilitate the analysis, let  $g(s)$  be the differentiable mapping from  $\mathbb{R}$  to  $\mathbb{R}$  such that  $g(s) = (n - s) \frac{a - s\alpha}{c + n - s} + s\alpha$ .<sup>17</sup> Differentiating we obtain

$$g'(s) = \frac{c(\alpha(n + c) - a)}{(-c - n + s)^2}$$

Since  $\alpha > a/(n + c)$ ,  $g'(s) > 0$  for any  $s$ . We then have the following Proposition,

**Proposition 8** *Let  $\alpha$  be an admissible level of abatement. The total abatement is maximized when the ratification threshold is set to  $\bar{t}(\alpha)$ .*

Note that if the negotiated threshold is chosen so as to maximize the environmental impact, i.e. set to  $\bar{t}(\alpha)$ , then the welfare of the signatories is very close to their welfare without IEA.<sup>18</sup> It turns out that for a given abatement level  $\alpha$ , the ratification threshold that maximizes the welfare of signatories (at equilibrium, and provided that the IEA enters into force) is not necessary the same as the one maximizing total abatement.

**Proposition 9** *Let  $\alpha$  be an admissible level of abatement. The welfare of ratifying countries is maximized when the ratification threshold is set to  $\hat{t}(\alpha)$ , where*

$$\hat{t}(\alpha) = \begin{cases} \frac{a}{\alpha} & \text{if } n \geq \frac{c}{\sqrt{1+c}-1}, \\ n & \text{otherwise.} \end{cases}$$

The combination of Propositions 8 and 9 shows that, given a minimal abatement level  $\alpha$ , the maximization of the welfare of the signatories is not necessarily at odds with the maximization of the total abatement. This is the case whenever  $n < \frac{c}{\sqrt{1+c}-1}$ . Indeed, from the definition of  $\bar{t}(\alpha)$ , this can only be the case when  $\hat{t}(\alpha) = n$ . Whenever  $n$  is too large,  $\hat{t}(\alpha) \neq n$ , the maximal global abatement level is not a corollary of the maximization of the signatories' welfare. Finally, observe that since  $a/\alpha$  is decreasing in  $\alpha$ , the higher is the minimal abatement level required by the IEA, the less

<sup>17</sup>Since  $Q(s)$  is defined over integers we need to define a new function, which "coincide" with  $Q(s)$ , that is differentiable. Another way, albeit more tedious, would be to compute  $Q(s) - Q(s - 1)$ .

<sup>18</sup>Recall that by definition,  $\bar{t}(\alpha)$  is the highest integer such that  $u(\bar{t}(\alpha), 1) \geq u(0)$ .

likely it is that the maximization of the signatories' welfare coincides with the maximization of the total abatement.

Yet, whenever  $\bar{t}(\alpha) \neq \hat{t}(\alpha)$  we get the surprising result that whenever the ratification threshold is set to maximize the signatories' welfare, the total abatement in equilibrium is independent of the minimal level  $\alpha$ . To see this, note that  $\bar{t}(\alpha) \neq \hat{t}(\alpha)$  implies that  $\hat{t}(\alpha) = a/\alpha$ . Setting the ratification threshold to be equal to  $\hat{t}(\alpha)$ , the total abatement in equilibrium is then

$$\frac{a(n - \hat{t}(\alpha)) + \alpha \hat{t}(\alpha)c}{c + n - \hat{t}(\alpha)} = \frac{a(n - \hat{t}(\alpha) + c)}{c + n - \hat{t}(\alpha)} = a.$$

If  $c$  is high enough,  $\bar{\alpha}$  is such that there are as many stable and welfare maximizing IEA's as there are integers between  $a/q_0$  and  $a/\bar{\alpha}$ . The difference between each of them is the distribution of the burden. Since global abatement of the coalition is unchanged, a low  $\alpha$  would allow large coalition to be stable while a high  $\alpha$  would make difficult the entry into force of the agreement.

## 5 Conclusion

We show in this paper that far to hurt cooperation, a ratification mechanism allows for the entry into force of an IEA. Since it is always worth for a country to be the outsider of an existing IEA, ratification threshold rule binds countries at the margin to join the agreement. Indeed if an IEA enters into force, its size equals the threshold. While this enables us to give a sharp prediction regarding the number of countries participating in the IEA, it also shows how fragile are such agreements. Any country ratifying the IEA becomes pivotal, meaning that its non-ratification is sufficient to collapse the entire IEA. We show nevertheless that a large typology of IEA can enter into force. In particular, a set of welfare maximizing IEA as well as a set of environmentally maximizing IEA are characterized. It appears that according to the abatement bound and the threshold rule bargained, the IEA which will enter into force will stick to that bound and that threshold. Put differently, abatements will equal the bound and size will equal the threshold. It follows that the bound and the threshold bargained fully characterize the

IEA which eventually enters into force.

An aspect rather new in the paper regards the meaning of cooperation. By cooperating, we assume that countries commit simply to abate at the least an agreed target which does not need to maximize the joint welfare of the cooperative coalition. In that respect, our approach is purely non cooperative since countries do only care about their own welfare.<sup>19</sup> This assumption, besides its accuracy with regards to the IEA which entered into force in the past, admits the advantage of providing a simpler game formulation than what had been done so far. Henceforth, the game we deliberately enunciate as simple as possible by keeping the symmetry assumption, is easily tractable with asymmetries in the benefit and cost functions. One can also easily conceive to solve that game considering asymmetric abatement targets, a task currently underway.

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<sup>19</sup>Such assumption does not prevent the ex-post introduction of efficiency mechanisms such as joint implementation or tradeable permits. Such mechanisms if introduced later will eventually be pareto improving.

## Appendix

**Proof of Proposition 1** Let  $S$  be the set of countries restricted by the IEA. Suppose first that  $(s, \alpha)$  is such that  $a - s\alpha \geq 0$ . We first show that for all  $i \in S$ , the unique equilibrium is such that  $q_i = \alpha$ . Let  $\hat{q} = (\hat{q}_h)_{h \in N}$  be an equilibrium of the abatement game with  $s$  countries signing the IEA and suppose there is a country participating to the IEA, say  $i$ , such that  $\hat{q}_i \neq \alpha$ . It follows by Lemma 1 that  $\hat{q}_i = BR_i(\hat{q}_{-i})$ . Let  $j$  denote any country not participating to the IEA, i.e.,  $\hat{q}_j = BR_j(\hat{q}_{-j})$ . Let  $\hat{Q}_{-ij} = \sum_{h \neq i, j} \hat{q}_h$ . We then have

$$\hat{q}_i = \frac{a - \hat{Q}_{-ij} - \hat{q}_j}{1 + c} \quad (7)$$

$$\hat{q}_j = \frac{a - \hat{Q}_{-ij} - \hat{q}_i}{1 + c} \quad (8)$$

Solving (7) and (8) we then find that  $\hat{q}_i = \hat{q}_j$ . Repeating this for all countries not participating to the IEA, and because  $\hat{q}_i \geq \alpha$ , we have

$$\hat{q}_h \geq \alpha, \forall h \in N. \quad (9)$$

Let  $j$  be any country not participating to the IEA. Because  $\hat{q}_{-j} \geq (n-1)\alpha$ , we have

$$\hat{q}_j = \frac{a - \hat{q}_{-j}}{1 + c} < \frac{a - (n-1)\alpha}{1 + c}. \quad (10)$$

Moreover, observe that  $\alpha > a/(n+c)$  implies that

$$\alpha > \frac{a - (n-1)\alpha}{1 + c}. \quad (11)$$

Combining Eqs. (10) and (11) we obtain that country  $j$ 's best reply is to choose an abatement level strictly lower than  $\alpha$ , a contradiction with Eq. (9).

Hence, all countries participating to the IEA choose an abatement level equal to  $\alpha$ . The abatement game reduces then to a game with  $n-s$  players, where the payoffs functions are given by

$$\forall i \in N_-, u_i(q) = (a(s\alpha + Q_{ns}) - \frac{1}{2}(s\alpha + Q_{ns})^2) - \frac{c}{2}q_i^2, \quad (12)$$

where  $N_-$  is the set of countries not participating to the IEA, and  $Q_{ns} = \sum_{h \in N_-} q_h$ . It is easy to see that this game admits a unique Nash equilibrium in which all countries (not participating to the IEA) would choose an

abatement level equal to  $q_{ns}(s) = (a - s\alpha)/(c + n - s)$ . Because  $s\alpha < a$ ,  $q_{ns}(s) \in X_i$ , for all  $i \in S$ .

Suppose now that  $(s, \alpha)$  is such that  $a - s\alpha < 0$ . It follows that  $q_{ns}(s) = 0$ . We claim that in this case the unique equilibrium is then  $q^*$  such that  $q_i^* = \alpha$  for all  $i \in S$  and  $q_i^* = 0$  if  $i \notin S$ . Because  $q_i \in [\alpha, \infty)$  for  $i \in S$ , we have  $\sum_{i \in S} q_i \geq s\alpha$ , and thus  $Q_{-j} = \sum_{i \in N \setminus j} q_i \geq s\alpha$  for all  $j \notin S$ . Recall that the best reply of a country  $j \notin S$  is  $q_j^* = \max\{(a - Q_{-j})/(1 + c); 0\}$ , which implies that  $q_j^* = 0$ . Consider now  $i \in S$ , and suppose that  $q_i^* \neq \alpha$ . It follows by Lemma 1 that  $q_i^* = BR_i(q_{-i}^*)$ . Since for all  $j \in S$  we have  $q_j \geq \alpha$ , country  $i$ 's original best reply is bounded, i.e.,  $BR_i(q_{-i}^*) \leq (a - (s - 1)\alpha)/(1 + c)$ . To show that we must have  $q_i^* = \alpha$  it suffices to show that  $BR_i(q_{-i}^*) \leq \alpha$ , and thus that  $(a - (s - 1)\alpha)/(1 + c) < \alpha$ . This inequality is equivalent to  $\alpha > a/(c + s)$ . Since  $\alpha > q_0 = a/(c + n)$  and  $s \leq n$ , the result follows. ■

**Proof of Proposition 3** Let  $S$  be the coalition of countries constrained by the IEA. By Proposition 1 the abatement level chosen by each participant is equal to  $\alpha$ . Suppose now that one country, say  $i$ , decides to withdraw from the IEA. Again by Proposition 1, the abatement level chosen by the remaining participants will be equal to  $\alpha$ , and that of country  $i$  will be equal to  $(a - (n - 1)\alpha)/(1 + c)$ , i.e.,  $BR_i((n - 1)\alpha)$ . Since  $(q_h = \alpha)_{h \in N}$  is not a Nash equilibrium of the (unrestricted) abatement game, we have  $u_i(BR_i((n - 1)\alpha), (n - 1)\alpha) > u_i(\alpha, (n - 1)\alpha)$ . That is, country  $i$  is strictly better off withdrawing from the IEA. ■

**Proof of Proposition 4** If there is no stable *IEA* with one or more countries then it must be that the IEA game has a unique subgame perfect equilibrium in which all countries choose *NR* in the first stage. Hence, it suffices to show that for any  $\alpha > q_0$  and  $s = 1, \dots, n$ ,  $u(s, 1) < u(s - 1, 0)$ .

Consider first the case where  $s$  and  $\alpha$  are such that  $s\alpha < a$ . From Proposition 1 it follows that  $q_{ns}(s) > 0$ . Let  $\Delta(\alpha, s) = u(s - 1, 0) - u(s, 1)$ , which yields,

$$\Delta(\alpha, s) = -\frac{c}{2(c + n - s)2(1 + c + n - s)2}(\alpha c - a + \alpha n)(A + \alpha B) \quad (13)$$

Where

$$\begin{aligned}
A &= a(n^2 - 2sn - c^2 - c + s^2) \\
B &= n^3 + 3cn^2 - 4n^2s + 2n^2 - 6cns + 3cn + 3c^2n + 5s^2n \\
&\quad - 4sn + c^2 + 3s^2c - 2sc + 2s^2 - 2c^2s + c^3 - 2s^3.
\end{aligned}$$

Observe that  $\Delta$  is a polynomial in  $\alpha$  of degree 2, whose roots are

$$r_1 = \frac{a}{c+n}, \quad r_2 = -\frac{A}{B}.$$

Computing  $r_1 - r_2$  we obtain,

$$r_1 - r_2 = \frac{A(c+n) + aB}{(c+n)B} = \frac{(s-n)(s-c-n)(s-c-n-1)}{(c+n)B}$$

Because  $s \leq n$ ,  $r_1 > r_2$  if and only if  $B < 0$ . Moreover, the coefficient of  $\alpha^2$  in  $\Delta$  is equal to  $B(c+n)$ , implying that  $\Delta$  is convex (resp. concave) if  $B > 0$  (resp.  $B < 0$ ). Since  $\alpha > r_1$ ,  $\Delta(\alpha, s) > 0$  for any  $s$  whenever  $B > 0$ . If  $B < 0$  then  $r_1 < r_2$  and  $\Delta(\alpha, s) > 0$  for any  $\alpha \in (r_1, r_2)$ , which completes the proof of the Proposition.

Suppose now that  $s\alpha > a$ , and let  $i$  be a country that does not ratify the IEA. Let  $Q(s) = (n-s)q_{ns}(s) + (s-1)\alpha$  and  $Q(n-s) = (n-s-1)q_{ns}(s-1) + (s-1)\alpha$ . From Proposition 1  $q_s(s) = q_s(s-1) = \alpha$ , and  $q_{ns}(s-1) = 0$  or  $q_{ns}(s-1) = (a-s\alpha)/(c+n-s)$ .

Suppose first that  $q_{ns}(s-1) = 0$ . Hence, it must be the case that  $(s-1)\alpha \geq a$ , which implies that  $Q(s) > q(s-1) \geq \alpha$ . Therefore, we have  $aQ(s-1) - \frac{1}{2}Q(s-1)^2 > aQ(s) - \frac{1}{2}Q(s)^2$ . Adding the cost of abatement,  $q_{ns}(s-1)$  and  $\alpha$  respectively, we obtain  $aQ(s-1) - \frac{1}{2}Q(s-1)^2 > aQ(s) - \frac{1}{2}Q(s)^2 - \frac{c}{2}\alpha^2$ , which is tantamount to  $u(s-1, 0) > u(s, 1)$ , the desired result.

Consider now the case when  $q_{ns}(s-1) > 0$ . It follows that  $(s-1)\alpha < a$ , and thus  $s \in [a/\alpha, a/\alpha + 1]$ . We now show that  $u(s-1, 0)$  is minimized when  $s = a/\alpha$ . First, observe that the total abatement  $Q(s-1) < a$  if and only if  $(s-1)\alpha < a$ . Hence,  $Q(s-1)$  is increasing in  $s$  and takes its lowest value when  $s = a/\alpha$ . The abatement of a country not ratifying  $q_{ns}(s-1)$  is also a decreasing function of  $s$ , which implies that  $c/2q_{ns}(s-1)^2$  is maximized when

$s - 1 = a/\alpha - 1$ . Therefore,  $u(s - 1, 0) \geq u(a/\alpha, 0)$ . Furthermore,  $s > a/\alpha$  implies that  $Q(s) \geq a$ , and thus we have  $u(s, 1) \leq u(\alpha, 1) = \frac{1}{2}(a2 - \alpha 2)$ . Computing  $\Delta = u(s - 1, 0) - u(s, 1)$ , we obtain

$$\Delta \geq \frac{c\alpha 2(a2 - \alpha(2a(n + c + 1) - \alpha((c + n)2 + c + 2n)))}{2(a - c\alpha - n\alpha - \alpha)2}$$

To show that  $\Delta > 0$ , it suffices to show that the following holds true,

$$\alpha > \frac{2a(n + c + 1)}{(c + n)2 + c + 2n}. \quad (14)$$

Consider now the following inequality,

$$\frac{a}{c + n} > \frac{2a(n + c + 1)}{(c + n)2 + c + 2n}. \quad (15)$$

Because  $\alpha > a/(c + n)$ , Eq. (15) being true implies that (14) is true as well. Now, it is readily verified that Eq. (15) always holds true (the above inequality simplifies to  $n > 0$ ), and thus we have  $u(s - 1, 0) > u(s, 1)$ , which completes the proof.  $\blacksquare$

**Proof of Proposition 6** Let  $\Delta = u(s, 1) - u(0)$ . Suppose first that  $q_{ns}(s) \neq 0$ , i.e., non-ratifying countries are not constrained. Since  $\alpha > q_0$ , it is convenient to pose  $\alpha = \beta q_0$  with  $\beta$  being a parameter strictly greater than 1. Simplifying we then obtain:

$$\Delta = -\frac{a^2 c}{2(c + n - s)^2(n + c)^2}(-X + Y + \beta(\beta X - Y)) \quad (16)$$

where

$$X = 2nc + cs^2 - 2sc + n^2 + s^2 - 2ns + c^2$$

$$Y = 2c^2s + 2cns$$

Consider now the function  $f(s) : \mathbb{R} \rightarrow \mathbb{R}$  with  $f(s) = -X + Y + \beta(\beta X - Y)$ . Observe that the sign of  $f(s)$  is the opposite of the sign of  $\Delta$ . Furthermore, the mapping  $f(s)$  is a polynomial of degree 2, such that the coefficient of  $s^2$  is equal to  $(1 + c)(\beta 2 - 1)$ , which implies that  $\Delta > 0$  whenever  $s \in (s_1, s_2)$  where  $s_1$  and  $s_2$  are the two roots of  $f(s)$ ,

$$s_1 = \frac{(1 + c + \beta - \sqrt{c + c^2 - c\beta^2})(n + c)}{\beta c + \beta + c + 1} \quad (17)$$

$$s_2 = \frac{(1 + c + \beta + \sqrt{c + c^2 - c\beta^2})(n + c)}{\beta c + \beta + c + 1} \quad (18)$$

$s_1, s_2 \in \mathbb{R}$  if and only if  $\beta \leq \sqrt{1+c}$ , the desired result.

Suppose now that  $q_{ns}(s) = 0$ , i.e.,  $s > a/\alpha$ . In this case the total abatement is equal to  $a$  if  $s$  countries ratify, which gives the following payoff,

$$u(s, 1) = \frac{1}{2}(a2 - \alpha 2),$$

Subtracting  $u(0)$  to  $u(s, 1)$  we obtain

$$\Delta = \frac{c(ca2 - \alpha^2 n2 - 2c\alpha^2 n - c2\alpha 2 + a2)}{2(n+c)2}$$

$\Delta$  is positive if and only if  $\alpha \in (-q_0\sqrt{1+c}, q_0\sqrt{1+c})$ , which completes the proof of the Proposition. ■

**Proof of Proposition 9** Consider the difference  $u(s, 1) - u(0)$  given by Eq. (16). Abusing notation, let  $d(s)$  be the differentiable mapping from  $[0, n]$  to  $\mathbb{R}$  such that  $d(s) = -\frac{a^2 c}{2(c+n-s)^2(n+c)^2} f(s)$ , where  $f(s)$  is the polynomial function defined in the proof of Proposition 6. Differentiating by  $s$  we obtain,

$$d'(s) = -\frac{c^2(a - n\alpha - c\alpha)(a - s\alpha)}{(c + n - s)^3}$$

$d'(s)$  is thus strictly positive if and only if  $s < \frac{a}{\alpha}$ . Given that  $d(s)$  is continuous, the function reaches its maximum when  $s = \frac{a}{\alpha}$ .

It remains to show that there exists some admissible values of  $\alpha$  such that  $a/\alpha \leq n$  or, equivalently,  $\alpha \geq a/n$ . From Proposition 6,  $\alpha$  is admissible only if  $\alpha \leq \frac{a}{c+n}\sqrt{1+c}$ . Hence, it suffices to show that

$$\frac{a}{c+n}\sqrt{1+c} \geq a/n.$$

Simplifying we obtain the following condition,

$$n \geq \frac{c}{\sqrt{1+c} - 1}. \tag{19}$$

Hence, whenever Eq. (19) holds true, the threshold maximizing the welfare is equal to  $\frac{a}{\alpha}$ . Otherwise, the threshold maximizing the welfare is equal to  $n$ . ■

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