Efficient Bidding with Externalities*

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February 10, 2005

Abstract

We implement a family of efficient proposals to share benefits generated in environments with externalities. These proposals extend the Shapley value to games with externalities and are parametrized through the method by which the externalities are averaged. We construct two slightly different mechanisms: one for environments with negative externalities and the other for positive externalities. We show that the subgame perfect equilibrium outcomes of these mechanisms coincide with the sharing proposals.

JEL Classification numbers: D62, C71.

Keywords: Implementation, Externalities, Bidding, Shapley Value.

*The authors gratefully acknowledge the financial support from BEC 2003-01132. Macho-Stadler and Pérez-Castrillo also acknowledge the finantial support of Generalitat de Catalunya (2001 SGR-00162 and Barcelona Economics, CREA).

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1 Introduction

Achieving cooperation and sharing the resulting benefits are central issues in any form of organization, particularly in economic environments. These issues are often difficult to resolve especially in environments with externalities, where the surplus generated by a group of agents depends upon the organization of agents outside the group. Common examples are the struggle to achieve international agreements or discussions regarding the allocation of wealth among family members.

One fruitful approach for solving the questions raised above is to proceed axiomatically and propose sharing methods (sometimes known as values) that extend the well-known Shapley value (Shapley [9]), which is defined for games with no externalities. This is the direction taken in the papers by Myerson [5], Bolger [1], and Macho-Stadler *et al.* [3]. Their proposals satisfy the desirable properties (axioms) of efficiency, anonymity (symmetry), linearity, and the "null" player property that states that players which have no effect on the outcome should neither receive nor pay anything.

Macho-Stadler *et al.* [3] strengthened the symmetry property through the strong symmetry axiom capturing the idea that players with "identical power" should receive the same outcome. This axiom is equivalent to adopting an average approach to the problem of sharing. This approach is quite intuitive: it yields to a player in a game with externalities the Shapley value of an average game with no externalities. The average game is obtained from the original game by assigning to each coalition its (weighted) average payoff. The average approach generates an attractive family of sharing methods.[1]

A well-known shortcoming of all the sharing methods discussed above is their vulnerability to strategic behavior on part of the group members. This problem is generally addressed by looking for non-cooperative games whose equilibria yield the desired outcomes. In this paper we focus on the average approach introduced in Macho-Stadler *et al.* [3], and solve this problem for environments with either negative or positive externalities.

This paper belongs to the literature that deals with the implementation of value concepts, particularly the implementation of the Shapley value; see Winter [12], Dasgupta

[1]By including an additional property Macho-Stadler *et al.* [3] derived a unique sharing method belonging to the family of averaging methods.

and Chiu [2], Pérez-Castrillo and Wettstein [7], Vidal-Puga and Bergantiños [10], and Vidal-Puga [11]. Our paper is also related to Maskin [4], who addresses the issue of coalition formation and value in environments with externalities. He considers a sequential process of coalition formation and characterizes the resulting sharing method.

We devise two slightly different families of mechanisms, one to be used in games with positive externalities and the other in games with negative externalities. These mechanisms are extensions of the multi-bidding procedure proposed by Pérez-Castrillo and Wettstein [7] and [8], for environments with no externalities. For each type of externality, the family of mechanisms is parametrized by the weighted averages (non-negative weights) used to obtain the sharing method. We show that the Subgame Perfect Equilibrium outcomes of these mechanisms coincide with the outcome prescribed by the particular sharing method. Thus we implement the family of values defined by the average approach in Subgame Perfect Equilibrium.

In the next section, we present the environment and the values to be implemented. In Section 3, we consider environments with negative externalities and we construct a family of mechanisms to implement the corresponding values. In Section 4, we modify these mechanisms to handle environments with positive externalities. Section 5 concludes and offers further directions for research.

2 The environment and the values

We denote by $N = \{1, ..., n\}$ the set of players. A coalition S is a non-empty subset of N, $S \subseteq N$. An embedded coalition is a pair (S, P), where S is a coalition and $P \ni S$ is a partition of N. An embedded coalition hence, specifies the coalition as well as the structure of coalitions formed by the other players. Let $\mathcal{P}$ denote the set of all partitions of N. The set of embedded coalitions is denoted by ECL and defined by:

$$ECL = \{(S, P) \mid S \in P,\ P \in \mathcal{P}\}.$$

We denote by (N, v) a game in partition function form, where $v : ECL \to \mathbb{R}$ is a characteristic function that associates a real number with each embedded coalition. Hence, $v(S, P)$ with $(S, P) \in ECL$, is the worth of coalition S when the players are organized according to the partition P. The partition P is always taken to include the empty set $\varnothing$ and, for notational convenience, when describing a partition we only list the non-empty coalitions. We assume that the characteristic function satisfies $v(\varnothing, P) = 0$.

A sharing method, or *a value*, is a mapping φ which associates with every game (N, v) a vector in R^n that satisfies $\sum_{i \in N} \varphi_i(N, v) = v(N, N)$. A value determines the payoffs for every player in the game and, by definition, it is always *efficient* since the value of the grand coalition is shared among the players. Note that we assume that all the players end up together since we have in mind economic environments where forming the grand coalition is the most efficient way of organizing the society. Formally, $v(N, N) \geq \sum_{S \in P} v(S, P)$ for every partition $P \in \mathcal{P}$.

A player $i \in N$ is called a *null player* in the game (N, v) if and only if $v(S, P) = v(S', P')$ for every $(S, P) \in ECL$ and for any embedded coalition (S', P') that can be obtained from (S, P) by changing the affiliation of player i. Hence, a null player alone receives zero for any organization of the other players. Also a null player has no effect on the worth of any coalition S. In games in partition function form, this also means that if the null player is not a member of S, changing the organization of players outside S by moving this player around will not affect the worth of S.

The *addition* of two games (N, v) and (N, v') is defined as the game $(N, v + v')$ where $(v + v')(S, P) \equiv v(S, P) + v'(S, P)$ for all $(S, P) \in ECL$. Similarly, given the game (N, v) and the scalar $\lambda \in \mathbb{R}$, the game $(N, \lambda v)$ is defined by $(\lambda v)(S, P) \equiv \lambda v(S, P)$ for all $(S, P) \in ECL$.

Let σ be a permutation of N. Then the σ *permutation* of the game (N, v), denoted by $(N, \sigma v)$ is defined by $(\sigma v)(S, P) \equiv v(\sigma S, \sigma P)$ for all $(S, P) \in ECL$.

Shapley [9] imposed three basic axioms a value φ should satisfy:

1. Linearity: A value φ satisfies the linearity axiom if:

   1.1. For any two games (N, v) and (N, v'), $\varphi(N, v + v') = \varphi(N, v) + \varphi(N, v')$.

   1.2. For any game (N, v) and any scalar $\lambda \in \mathbb{R}$, $\varphi(N, \lambda v) = \lambda \varphi(N, v)$.

2. Symmetry: A value φ satisfies the symmetry axiom if for any permutation σ of N, $\varphi(N, \sigma v) = \sigma \varphi(N, v)$.

3. Null player: A value φ satisfies the null player axiom if for any player i which is a null player in the game (N, v), $\varphi_i(N, v) = 0$.

Shapley [9] proved that these three basic axioms characterize a unique value in the class of games with no externalities where the worth of any coalition S does not depend on the organization of the other players. In games with no externalities, $v(S, P) = v(S, P')$ for every $S \subseteq N$ and $(S, P), (S, P') \in ECL$. Let us denote by $(N, \widehat{v})$ a game with no externalities, where $\widehat{v} : 2^N \to \mathbb{R}$ is a function that gives the worth of each coalition. The Shapley value ϕ can be written as:

$$\phi_i(N, \widehat{v}) = \sum_{S \subseteq N} \beta_i(S, n)\widehat{v}(S) \text{ for all } i \in N, \tag{1}$$

where we have denoted by $\beta_i(S, n)$ the following numbers:

$$\beta_i(S, n) = \begin{cases} \frac{(|S|-1)!(n-|S|)!}{n!} \text{ for all } S \subseteq N, \text{ if } i \in S \\ -\frac{|S|!(n-|S|-1)!}{n!} \text{ for all } S \subseteq N, \text{ if } i \in N \backslash S. \end{cases}$$

In games with externalities, the three proposed axioms do not characterize a unique sharing method. However, all the values satisfying the axioms must be linear combinations of the $v(S, P)$s. More precisely, any value $\varphi(N, v)$ can be written as:

$$\varphi_i(N, v) = \sum_{(S,P) \in ECL} \xi_i(S, P)v(S, P) = \text{ for all } i \in N.$$

We rewrite the previous equation as:

$$\varphi_i(N, v) = \sum_{(S,P) \in ECL} \alpha_i(S, P)\beta_i(S, n)v(S, P) \text{ for all } i \in N. \tag{2}$$

Symmetry is a property of anonymity: the payoff of a player is only derived from his influence on the worth of the coalitions, it does not depend on his “name”. The strong symmetry axiom strengthens the symmetry axiom by requiring that exchanging the names of the players inducing the same externality should not affect the payoff of any player. To formally state the axiom, we denote by $\sigma_{S,P}P$, with $P \ni S$, a new partition with $S \in \sigma_{S,P}P$, resulting from a permutation of the set $N \backslash S$. Given a coalition S and a partition P containing that coalition the $\sigma_{S,P}$ permutation of the game (N, v) denoted by $(N, \sigma_{S,P}v)$ is defined by $(\sigma_{S,P}v)(S, P) = v(S, \sigma_{S,P}P)$, $(\sigma_{S,P}v)(S, \sigma_{S,P}P) = v(S, P)$, and $(\sigma_{S,P}v)(R, Q) = v(R, Q)$ for all $(R, Q) \in ECL \backslash \{(S, P), (S, \sigma_{S,P}P)\}$.

2'. A value φ satisfies the strong symmetry axiom if:

(a) for any permutation σ of N, $\varphi(N, \sigma v) = \sigma\varphi(N, v)$,

(b) for any $(S, P) \in ECL$ and for any permutation $\sigma_{S,P}$, $\varphi(N, \sigma_{S,P} v) = \varphi(N, v)$.

Macho-Stadler *et al.* [3] have proven that any value φ satisfying linearity and null player axioms also satisfies the strong symmetry axiom if and only if it can be constructed through the average approach.

The "*average approach*" consists of, first constructing an average game $(N, \widetilde{v})$ associated with the Partition Function Game (N, v), by assigning to each coalition $S \subseteq N$ the average worth $\widetilde{v}(S) \equiv \sum_{P \ni S, P \in \mathcal{P}} \alpha(S, P) v(S, P)$, with

$$\sum_{P \ni S, P \in \mathcal{P}} \alpha(S, P) = 1. \tag{3}$$

We refer to $\alpha(S, P)$ as the "weight" of the partition P in the computation of the value of coalition $S \in P$. Second, the average approach constructs a value φ for the Partition Function Game (N, v) by taking the Shapley value of the game $(N, \widetilde{v})$. Therefore, if a value φ is obtained through the average approach then, for all $i \in N$,

$$\varphi_i(N, v) = \sum_{S \subseteq N} \beta_i(S, n) \widetilde{v}(S) = \sum_{S \subseteq N} \left[\beta_i(S, n) \sum_{P \ni S, P \in \mathcal{P}} \alpha(S, P) v(S, P) \right],$$

that is,

$$\varphi_i(N, v) = \sum_{(S,P) \in ECL} \alpha(S, P) \beta_i(S, n) v(S, P). \tag{4}$$

Note that, under strong symmetry, the weights $\alpha(S, P)$ do not depend on the player whose value we are computing. This is the difference between formulae (2) and (4). Moreover, due to the symmetry and null player axioms the weights must be symmetric (i.e., $\alpha(S, P)$ only depends on the sizes of the coalitions in P) and (Macho-Stadler *et al.* [3]) satisfy the following condition:

$$\alpha(S, P) = \sum_{R \in P \setminus S} \alpha(S \setminus \{i\}, (P \setminus (R, S)) \cup (R \cup \{i\}, S \setminus \{i\})), \tag{5}$$

for all $i \in S$ and for all $(S, P) \in ECL$ with $|S| > 1$. Macho-Stadler *et al.* [3] have also shown that any value constructed through the average approach with symmetric weights that satisfies condition (5) also satisfies the null player axiom.

The requirements of linearity, strong symmetry, and null player still do not yield a unique value for games with externalities. We denote by $\varphi(\alpha)$ *the value* satisfying the three previous axioms and constructed through the average approach with weights α, where α is a function from ECL to $\mathbb{R}$ that satisfies equations (3) and (5).

To illustrate the environment and the payoffs prescribed by the values obtained through the average approach, we now present a very stylized example with three players. Suppose that the characteristic function of the game $(\{A, B, C\}, v)$ is given by the following table where, for example, the second line indicates that the worth of the coalition $\{A, B\}$ in the partition $(\{A, B\}\{C\})$ is 18 and that of $\{C\}$ in the same partition is 6 :

P	worth(S)		
$\{A, B, C\}$	30		
$\{A, B\}\{C\}$	18	6	
$\{A, C\}\{B\}$	18	6	
$\{B, C\}\{A\}$	21	0	
$\{A\}\{B\}\{C\}$	6	9	9

Table 1: Characteristic function

This is a game with negative externalities, that is, a player's worth decreases when the other two players form a coalition. For example, player C's payoff decreases from 9 to 6 if the other two players join. This might reflect the worth of countries viewed as players in a tariff-setting game, when we interpret a coalition as a trade agreement.

The family of values proposed for this three player game is parametrized by the weights used to define the average game. The weights of all embedded coalitions in $\{A, B, C\}$ take the form,

$\alpha(\{A, B, C\}, \{A, B, C\}) = \alpha(\{i, j\}, (\{i, j\}, \{k\})) = 1$

$\alpha(\{k\}, (\{i, j\}, \{k\})) = a$

$\alpha(\{i\}, (\{i\}, \{j\}, \{k\})) = 1 - a,$

where $\{i, j, k\} = \{A, B, C\}$.

For any $a \in \mathbb{R}$, a different average game satisfying conditions (3) and (5) can be constructed and is given in the following table

S	worth(S)
$\{A,B,C\}$	30
$\{A,B\}$	18
$\{A,C\}$	18
$\{B,C\}$	21
$\{A\}$	$6(1-a)$
$\{B\}$	$6a+9(1-a)=9-3a$
$\{C\}$	$6a+9(1-a)=9-3a$

Table 2: Average game

Hence, for every $a \in \mathbb{R}$, a different value is obtained. However, we will consider only non-negative weights, which corresponds to $a \in [0,1]$. The use of weights larger than 1 often leads to objectionable results. In this example, the value assigned to the singleton coalition of player A, in the average game would be strictly negative (even though the value assigned to this coalition is non-negative for every possible partition). Furthermore, the value assigned to the singleton coalition of player B is strictly less than 6 (even though the value assigned to this coalition never falls below 6 for every possible partition).

Applying the Shapley value to the average games of our three-player example, gives the following family of values:

$\varphi_A = 8 - a,$

$\varphi_B = \varphi_C = 11 + \frac{a}{2}.$

In the following sections, we focus on values generated by a system of non-negative weights satisfying conditions (3) and (5) and propose two quite similar mechanisms that implement these values for general environments. The first mechanism is suitable for environments with negative externalities. The second mechanism is better suited to deal with situations where externalities are positive, that is, a coalition is better off when the rest of the players are grouped into large coalitions. The precise definitions of positive and negative externalities are given by:

Definition 1 *The game (N,v) has negative externalities if $v(S,P) \geq v(S,P')$ for every P' whose elements are given by a union of elements in P.*

Definition 2 *The game* (N, v) *has positive externalities if* $v(S, P) \leq v(S, P')$ *for every* P' *whose elements are given by a union of elements in* P*.*

Both types of environments, are also required to satisfy that the departure of a single player from a coalition results in efficiency losses. This is a mild requirement, commonly used and easily formulated in the literature on games with no externalities.[2] This property is usually referred to as zero-monotonicity. Formulating the property of zero-monotonicity for games with externalities is more delicate since the value of a player as well as the player's marginal contribution to a coalition depend on the partition specified for each situation. A natural extension of zero monotonicity to games with externalities is given in the next definition:

Definition 3 *The game* (N, v) *is* strictly zero-monotonic-A *if:*

$$v(S, P) > v(S \backslash \{i\}, (P \backslash S) \cup (S \backslash \{i\}, \{i\})) + v(\{i\}, (P \backslash S) \cup (S \backslash \{i\}, \{i\})),$$

for every $(S, P) \in ECL$ *and every* $i \in S$

Zero-monotonicity-A requires that the addition of a singleton player to a coalition is always beneficial, considering that the organization of the other players does not change.

3 The mechanism when externalities are negative

In this section, we analyze environments with negative externalities. Economic situations where there are negative externalities on the outsiders are, for example, research ventures that create efficiency gains, cost reducing alliances, or custom unions in international trade. For these environments, we introduce and analyze the mechanism, $M^{-}(\alpha)$. We show that it implements in Subgame Perfect Equilibrium (SPE) the value $\varphi(\alpha)$ in environments with negative externalities. In the next section we introduce the mechanism $M^{+}(\alpha)$, which addresses the problem in situations with positive externalities. Since the two mechanisms are very similar, we develop here their basic idea in a unified way. The informal description of the mechanisms is the following:

[2] See, for example, Winter [12], Pérez-Castrillo and Wettstein [7], Vidal-Puga and Bergantiños [10], and Vidal-Puga [11].

At each round, there is a set of "insiders" S and a set of "outsiders" $N\backslash S$. At the first round, the set of insiders is N. Each round is composed of two stages; the first stage is played among the insiders, the second one, if reached, is played among the outsiders. In the *insiders stage*, the players in S select a proposer among themselves through a multibidding procedure.[3] Once a proposer is chosen, he makes a proposal to the other members in S on the sharing of the (expected) benefits if they stay together (i.e., if they form the coalition S). If the proposal is rejected, the proposer joins the set of outsiders and the remaining insiders go to the next round. If the proposal is accepted, S forms and the organization of the outsiders is determined in the second stage, the outsiders stage. Note that in the case where $S = N$ the outsiders' stage is redundant.

At the *outsiders stage*, the set of players is $N\backslash S$, those agents whose proposals have been rejected at previous rounds of the mechanism. First, a "candidate partition" of N, including S, is randomly selected, where the probability of selecting a particular partition is the weight associated with this partition by the weight system α. Second, the members of each coalition in such a partition, other than S, play a game that determines whether the candidate partition (or some finer partition) is the final organization. Since the outsiders' game determines the outside option for the proposer, the main idea of this phase is that it should encourage the proposer in the insiders stage to make acceptable proposals. It is in this last phase that the mechanisms $M^{-}(\alpha)$ and $M^{+}(\alpha)$ differ since the nature of the externalities affects the payoffs (and then the incentives) of the outsiders. We denote this phase by $G^{-}(\alpha)$ and $G^{+}(\alpha)$. We now provide a formal description of the mechanism $M^{-}(\alpha)$.

The mechanism $M^{-}(\alpha)$

The mechanism $M^{-}(\alpha)$ proceeds in rounds. Each round is characterized by a coalition $S \subseteq N, S \neq \varnothing$. At the first round of the mechanism, $S = N$, if $s = |S| > 1$ the game goes to $I(S)$, the insiders' stage, otherwise it goes to $O(S)$, the outsiders' stage.

$I(S)$: *Insiders' stage*

$I(S)$.1: Each agent $i \in S$ makes bids $b_j^i \in \mathbb{R}$, for every $j \in S$, with $\sum_{j \in S} b_j^i = 0$. Agents' bids are simultaneous.

[3]The multibidding procedure follows the proposal analyzed in Pérez-Castrillo and Wettstein [7] and [8].

Define the aggregate bid to each player $i \in S$ by $B_i = \sum_{j \in S} b_i^j$. Let $\gamma_s = \text{argmax}_i(B_i)$ where an arbitrary tie-breaking rule is used in the case of a non-unique maximizer. Once the proposer γ_s has been chosen, every player $i \in S$ pays $b_{\gamma_s}^i$ and receives B_{γ_s}/s.

$I(S).2$: The proposer γ_s makes a proposal $x_i^{\gamma_s} \in \mathbb{R}$ to every $i \in S \backslash \{\gamma_s\}$.

$I(S).3$: The agents in $S \backslash \{\gamma_s\}$, sequentially, either accept or reject the offer. If an agent rejects it, then the offer is rejected and the game moves to the next round characterized by the coalition $S \backslash \{\gamma_s\}$. Otherwise, the offer is accepted, agent γ_s pays $x_i^{\gamma_s}$ to each agent $i \in S \backslash \{\gamma_s\}$, and then the final outcome is given after $O(S)$.

$O(S)$:*Outsiders stage*

A partition P, with $S \in P$, is chosen with probability $\alpha(S, P)$. Denote by T_{s+1} the coalition in P containing the last rejected proposer, γ_{s+1}.[4] A proposer $\beta(T)$ is randomly chosen for every $T \in P \backslash S$ with $|T| > 1$, the only restriction is that $\beta(T_{s+1}) \neq \gamma_{s+1}$, when $|T_{s+1}| > 1$. The agents in each such coalition T play the game $G(T)$, described below. The game $G(T_{s+1})$ is played first, the other $G(T)$ games are played sequentially following an arbitrary order.[5]

$G(T).1$: Player $\beta(T)$ makes a proposal $x_i^{\beta(T)} \in \mathbb{R}$ to every $i \in T \backslash \beta(T)$.

$G(T).2$: The agents in $T \backslash \beta(T)$, sequentially, either accept or reject the proposal. When an agent $\delta(T)$ rejects it, then the proposal is rejected. In this case, all the players in $T \backslash \delta(T)$ play the game $G(T \backslash \delta(T))$ with $\beta(T)$ as the proposer and player $\delta(T)$ stays as a singleton. Otherwise, the proposal is accepted, the coalition T is formed, and $\beta(T)$ pays $x_i^{\beta(T)}$ to every $i \in T \backslash \beta(T)$.

Following these games we obtain a partition $P(S)$ consisting of S, the coalitions resulting from the $G(T)$ games, and the singleton coalitions in P.

Outcome.

We denote by S^* the coalition of insiders which is formed and $P^* \equiv P(S^*)$ the final partition formed. Agent $i \in S^* \backslash \{\gamma_{s^*}\}$ obtains $x_i^{\gamma_{s^*}} + \sum_{k=s^*}^{n}(-b_{\gamma_k}^i + B_{\gamma_k}/k)$. Agent γ_{s^*} gets $v(S^*, P^*) - \sum_{i \in S \backslash \{\gamma_{s^*}\}} x_i^{\gamma_{s^*}} + \sum_{k=s^*}^{n}(-b_{\gamma_k}^{\gamma_{s^*}} + B_{\gamma_k}/k)$.[6]

[4] If $S = N$, then there is no γ_{s+1}, and the grand coalition N is chosen with probability 1.

[5] The fact that the sequence of games starts with $G(T_{s+1})$ is irrelevant for $M^-(\alpha)$, but plays an important role in the mechanism $M^+(\alpha)$. We introduce this requirement here to present both mechanisms in a more unified way.

[6] If $|S^*| = 1$, then there are no payments $x_i^{\gamma_{s^*}}$ since $S^* \backslash \{\gamma_{s^*}\} = \varnothing$.

The outsiders are $N \backslash S^* = \{\gamma_m\}_{m=s^*+1,...,n}$. The final outcome of player γ_m, for $m = s^*+1, ..., n$, is $v(\{\gamma_m\}, P^*)+\sum_{k=m}^{n}(-b_{\gamma_k}^{\gamma_m}+B_{\gamma_k}/k)$ if $\{\gamma_m\} \in P^*$. Otherwise, denote by T_m the coalition in P^* containing agent γ_m and by $\beta(T_m)$ the proposer in that coalition. The final payoff of player γ_m is $x_i^{\beta(T_m)} + \sum_{k=m}^{n}(-b_{\gamma_k}^{\gamma_m} + B_{\gamma_k}/k)$ if $\gamma_m \neq \beta(T_m)$ and $v(T_m, P^*) - \sum_{i \in T_m \backslash \gamma_m} x_i^{\gamma_m} + \sum_{k=m}^{n}(-b_{\gamma_k}^{\gamma_m} + B_{\gamma_k}/k)$ if $\gamma_m = \beta(T_m)$.

To characterize the equilibrium outcome of this mechanism, we study first the equilibrium outcome of the outsiders stage. The following Lemma outlines the main properties of the equilibrium outcome for this stage.

Lemma 1 *Consider a game (N, v) with negative externalities and which is strictly zero-monotonic-A. If the outsiders stage $O(S)$ is reached (hence $S \neq N$), then at the unique SPE of the game that starts at $O(S)$ the randomly chosen partition actually forms and the payoff of agent γ_{s+1} is $v(\{\gamma_{s+1}\}, (P \backslash T_{s+1}) \cup (\{\gamma_{s+1}\}, T_{s+1} \backslash \{\gamma_{s+1}\}))$, where we denote by T_{s+1} the coalition in P containing agent γ_{s+1}.*

Proof. We consider the subgame that starts at $O(S)$ with a partition P. At the stage $O(S)$, the games $G(T)$ are played sequentially. We first consider the equilibrium outcome in the case where T has to play $G(T)$,whereas the players in $N \backslash T$ have already formed their coalitions (T can be thought of as the "last coalition to play"). Denote by P' the partition consisting of T and the coalitions already formed by the players in $N \backslash T$. We prove, by induction on the size of T, the following property: at any SPE, the proposer $\beta(T)$ will propose $v(\{i\}, (P' \backslash T) \cup (T \backslash \{i\}, \{i\}))$ to each player i in $T \backslash \beta(T)$, and each player i will accept such an offer.

By strict zero-monotonicity-A the induction property must hold when $|T| = 2$. Proceeding by induction on the size of T we assume it holds when $|T| = m$ and show it holds when $|T| = m + 1$.

In the case where $|T| = m + 1$ any player j who rejects the offer knows he will receive $v(\{j\}, (P' \backslash T) \cup (T \backslash \{j\}, \{j\})$ since, by the induction argument, $T \backslash \{j\}$ will form. Hence, the equilibrium offer made by $\beta(T)$ (if he is interested in making an acceptable offer) to every player j will be $v(\{j\}, (P' \backslash T) \cup (T \backslash \{j\}, \{j\}))$. The additional payoff $\beta(T)$ gains in this case is $v(T, P') - \sum_{j \in T \backslash \beta(T)}(v(\{j\}, (P' \backslash T) \cup (T \backslash \{j\}, \{j\}))$.

If his offer is rejected by player i, the payoff of $\beta(T)$ is (by the induction property):

$$v(T\backslash\{i\},(P'\backslash T)\cup(T\backslash\{i\},\{i\}))-\sum_{j\in T\backslash\{i,\beta(T)\}}(v(\{j\},(P'\backslash T)\cup(T\backslash\{j,i\},\{j\},\{i\})).$$

To show that

$$v(T,P')-\sum_{j\in T\backslash\beta(T)}(v(\{j\},(P'\backslash T)\cup(T\backslash\{j\},\{j\}))>$$
$$v(T\backslash\{i\},(P'\backslash T)\cup(T\backslash\{i\},\{i\})-\sum_{j\in T\backslash\{i,\beta(T)\}}(v(\{j\},(P'\backslash T)\cup(T\backslash\{j,i\},\{j\},\{i\}))$$

note that

$$v(T,P')-v(T\backslash\{i\},(P'\backslash T)\cup(T\backslash\{i\},\{i\}))-v(\{i\},(P'\backslash T)\cup(T\backslash\{i\},\{i\}))>0$$

by the strict zero-monotonicity-A; whereas,

$$\sum_{j\in T\backslash\{i,\beta(T)\}}(v(\{j\},(P'\backslash T)\cup(T\backslash\{j\},\{j\}))-\sum_{j\in T\backslash\{i,\beta(T)\}}(v(\{j\},(P'\backslash T)\cup(T\backslash\{j,i\},\{j\},\{i\}))\leq 0$$

since the game is characterized by negative externalities.

Thus, the proposer would like his offer to be accepted and in equilibrium coalition T stays together. The proof of the first part of the lemma is completed by straightforward induction on the coalitions playing the $G(T)$ games.

We remark that the previous proof also supplies us with the unique SPE strategies in the $G(T)$ games. Moreover, suppose $S\neq N$ and consider the player γ_{s+1}. Either $\{\gamma_{s+1}\}\in P$ or $\gamma_{s+1}\neq\beta(T_{s+1})$. Given the strategies followed by all players in the $G(T)$ games, γ_{s+1} always obtains a payoff of $v(\{\gamma_{s+1}\},(P\backslash T_{s+1})\cup(\{\gamma_{s+1}\},T_{s+1}\backslash\{\gamma_{s+1}\}))$. ■

We can now show the following theorem:

Theorem 1 *If the game (N,v) has negative externalities and it is strictly zero-monotonic-A, then the mechanism $M^{-}(\alpha)$ implements in SPE the value $\varphi(\alpha)$.*

Proof. We first prove that every SPE of $M^{-}(\alpha)$ leads to a payoff vector coinciding with $\varphi(\alpha)$.

We define $\widehat{v}(R)=\sum_{P\ni R}\alpha(R,P)v(R,P)$ and denote by $\varphi_j(S,\widehat{v})$ the Shapley value of player $j\in S$ in the game with no externalities $(S,\widehat{v})$.

We fix the size n of the set of players N and proceed by induction over the number s of insiders, for $s = 1, ..., n$. The induction property is the following: for any set of insiders S of size s, if the game reaches the insiders stage $I(S)$, then at any SPE of $M^-(\alpha)$ the coalition S indeed forms and any player j in S receives from this stage onwards (i.e., without taking into account the payments made or received before the stage $I(S)$) the payoff $\varphi_j(S, \widehat{v})$.

($s = 1$) If there is one player in S, $S = \{j\}$, the rules of the mechanism $M^-(\alpha)$ imply that the game directly goes to the outsiders stage $O(\{j\})$, hence the coalition S indeed forms. Moreover, given Lemma 1, any chosen partition selected at stage $O(S)$ actually forms. Given that the probability that partition $P \ni \{j\}$ is chosen, is $\alpha(\{j\}, P)$, the expected payoff for j from this stage ($I(\{j\})$) on is given by:

$$\sum_{P \ni \{j\}} \alpha(\{j\}, P) v(\{j\}, P) = \widehat{v}(\{j\}) = \varphi_j(\{j\}, \widehat{v}).$$

We now assume the induction property holds for any set R with a number of players smaller than k and prove it also holds for any set S with k players.

($s = k$) We do the proof through a series of claims.

Claim 1. The following equation holds:

$$\widehat{v}(S) > \widehat{v}(S \backslash \{i\}) + \sum_{P' \ni S \backslash \{i\}} \alpha(S \backslash \{i\}, P') v(\{i\}, (P' \backslash R_{i,P'}) \cup (\{i\}, R_{i,P'} \backslash \{i\})), \qquad (6)$$

for any $i \in S$; where $R_{i,P'}$ denotes the coalition in P' containing i.

To prove Claim 1, we rewrite the right-hand side of equation (6), recalling that $\widehat{v}(S \backslash \{i\}) = \sum_{P' \ni S \backslash \{i\}} \alpha(S \backslash \{i\}, P') v(S \backslash \{i\}, P')$, as:

$$\sum_{P' \ni S \backslash \{i\}} \alpha(S \backslash \{i\}, P') \left[v(S \backslash \{i\}, P') + v(\{i\}, (P' \backslash R_{i,P'}) \cup (\{i\}, R_{i,P'} \backslash \{i\})) \right] =$$

$$\sum_{P \ni S} \sum_{\substack{P' = (P \backslash (R,S)) \cup (S \backslash \{i\}, R \cup \{i\}) \\ R \in P \backslash S}} \alpha(S \backslash \{i\}, P') \left[v(S \backslash \{i\}, P') + v(\{i\}, (P' \backslash (R \cup \{i\})) \cup (\{i\}, R)) \right].$$

(Note that the coalition $R_{i,P'}$ in equation (6) appears as coalition $R \cup \{i\}$ in the expression above.) Since $\widehat{v}(S) = \sum_{P \ni S} \alpha(S, P) v(S, P)$, a sufficient condition for equation (6) to hold is that, for all $i \in S$,

$$\alpha(S,P)v(S,P) > \sum_{\substack{P'=(P\setminus(R,S))\cup(S\setminus\{i\},R\cup\{i\})\\ R\in P\setminus S}} \alpha(S\setminus\{i\},P')\left[v(S\setminus\{i\},P') + v(\{i\},(P'\setminus(R\cup\{i\}))\cup(\{i\},R))\right],$$

for all $P \ni S$. Notice that

$$v(\{i\},(P'\setminus(R\cup\{i\}))\cup(\{i\},R)) = v(\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\})),$$

for all $R \in P\setminus S$. Moreover, since the game has negative externalities:

$$v(S\setminus\{i\},P') \leq v(S\setminus\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\}))$$

for all $P' = (P\setminus(R,S)) \cup (S\setminus\{i\}, R\cup\{i\})$. Hence,

$$\sum_{\substack{P'=(P\setminus(R,S))\cup(S\setminus\{i\},R\cup\{i\})\\ R\in P\setminus S}} \alpha(S\setminus\{i\},P')\left[v(S\setminus\{i\},P') + v(\{i\},(P'\setminus(R\cup\{i\}))\cup(\{i\},R))\right] \leq$$

$$\left[v(S\setminus\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\})) + v(\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\}))\right] \sum_{\substack{P'=(P\setminus(R,S))\cup(S\setminus\{i\},R\cup\{i\})\\ R\in P\setminus S}} \alpha(S\setminus\{i\},P').$$

Finally, we use equation (5) (which holds due to the fact we consider averaging methods that satisfy the null player axiom), to obtain that a sufficient condition for equation (6) to hold is:

$$v(S,P) > v(S\setminus\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\})) + v(\{i\},(P\setminus S)\cup(S\setminus\{i\},\{i\})),$$

for all $i \in S$; which holds because of strict zero-monotonicity-A.

Claim 2. In any SPE, a player $j \in S\setminus\{\gamma_s\}$ does not accept any offer below $\varphi_j(S\setminus\{\gamma_s\},\widehat{v})$, and the proposer γ_s does not make any offers exceeding these values.

The first part of the Claim is immediate by the induction hypothesis, since it states that $\varphi_j(S\setminus\{\gamma_s\},\widehat{v})$ is the payment that player $j \in S\setminus\{\gamma_s\}$ receives in case he rejects player γ_s's offer. Second, an offer strictly larger than $\varphi_j(S\setminus\{\gamma_s\},\widehat{v})$ cannot be part of an SPE strategy, since player γ_s can always undercut it, keeping it higher than $\varphi_j(S\setminus\{\gamma_s\},\widehat{v})$.

Claim 3. In any SPE, the proposer γ_s makes an offer $x_j^{\gamma_s} = \varphi_j(S\setminus\{\gamma_s\},\widehat{v})$ to every player $j \in S\setminus\{\gamma_s\}$ and these players accept the offer.

By Claim 2, the only possible SPE offer by the proposer γ_s that the others accept is $x_j^{\gamma_s} = \varphi_j(S\backslash\{\gamma_s\}, \widehat{v})$. We now prove that rejection of an offer cannot be part of SPE. By Lemma 1, total profits if the coalition S forms are $\widehat{v}(S)$. By the induction hypothesis, the profits that the coalition $S\backslash\{i\}$ obtains if player i rejects the offer are $\widehat{v}(S\backslash\{i\})$. Finally, Lemma 1 shows that the expected profit of the (supposedly) rejected proposer i is given by the second term on the RHS of equation (6). This implies, by Claim 1, that the total expected profit of the players in S in case of acceptance is larger than that in case of rejection. Hence, for any continuation payoff in the case of rejection, there exists an offer that is accepted by every player and gives a higher expected payoff to the proposer. Hence, in an SPE, the proposer in the insiders' stage must make an offer that is accepted.

Claim 4. In any SPE the aggregate bids are all zero, i.e., $B_i = 0$ for all $i \in S$. Moreover, any player $i \in S$ is indifferent with respect to the identity of the proposer.

The proof of this Claim is very similar to the proof of Claims (c) and (d) in the proof of Theorem 1 in Pérez-Castrillo and Wettstein [7]. The intuition underlying this result is easy to see in the case of a unique maximizer γ_s of the B_is where $B_{\gamma_s} > 0$. In such a case, any player i could slightly decrease his bid $b^i_{\gamma_s}$, γ_s would still be chosen as the proposer, and i would obtain a higher payoff.

Claim 5. In any SPE the actual payoff of player $i \in S$ from $I(S)$ on is:

$$\frac{1}{s}\sum_{j\in S\backslash\{i\}} \varphi_i(S\backslash\{j\}, \widehat{v}) + \frac{1}{s}\left[\widehat{v}(S) - \widehat{v}(S\backslash\{i\})\right].$$

By Claim 3, if player i is the proposer, his final payoff is $\widehat{v}(S) - \widehat{v}(S\backslash\{i\}) - b_i^i$; while the final payoff of player i is $\varphi_i(S\backslash\{j\}, \widehat{v}) - b_j^i$ if player $j \in S\backslash\{i\}$ is the proposer. Since $B^i = 0$, the sum of payoffs to player i over all possible choices of the proposer is given by:

$$\sum_{j\in S\backslash\{i\}} \varphi_i(S\backslash\{j\}, \widehat{v}) + \left[\widehat{v}(S) - \widehat{v}(S\backslash\{i\})\right].$$

By Claim 4 player i is indifferent between all s possible proposers and hence Claim 5 follows.

We note that

$$\varphi_i(S, \widehat{v}) = \frac{1}{s}\sum_{j\in S\backslash\{i\}} \varphi_i(S\backslash\{j\}, \widehat{v}) + \frac{1}{s}\left[\widehat{v}(S) - \widehat{v}(S\backslash\{i\})\right],$$

by a well-known property of the Shapley value.[7] Hence, the induction property holds for S.

We finally note that proving every SPE payoff of $M^{-}(\alpha)$ is $\varphi(\alpha)$ is nothing but the induction property applied to the set $S = N$. Indeed, the induction property implies that, at the SPE of the mechanism $M^{-}(\alpha)$, the agents accept the offer made by the proposer at the first round, and their final equilibrium payoff is given by $\varphi(N, \widehat{v})$, which corresponds to the value $\varphi(\alpha)$.

To prove that the value $\varphi(\alpha)$ is indeed an SPE outcome, we explicitly construct an SPE strategy profile that yields the value as an outcome. Consider the following strategies:

At $I(S).1$, each $i \in S$ announces $b^i_j = \varphi_j(S \backslash \{i\}, \widehat{v}) - \varphi_j(S, \widehat{v})$.

At $I(S).2$, player i, if he is the proposer, offers $\varphi_j(S \backslash \{i\}, \widehat{v})$ to every player $j \in S \backslash \{i\}$.

At $I(S).3$, player i, if player j is the proposer, accepts any offer greater than or equal to $\varphi_i(S \backslash \{j\}\}, \widehat{v})$ and rejects any offer strictly smaller than $\varphi_i(S \backslash \{j\}\}, \widehat{v})$.

At the $O(S)$, if reached, the players follow the strategies in the proof of Lemma 1.

These strategies clearly yield the value $\varphi_j(\alpha)$ to any player j who is not the proposer. Since these strategies also lead to the formation of the grand coalition, the proposer is left with the value prescribed for him, by $\varphi(\alpha)$. To show these are equilibrium strategies note that by Lemma 1 the $O(S)$ strategies are equilibrium strategies. Moreover, by the induction argument the $I(S).3$ strategies are optimal. At $I(S).2$ the offers made by the proposer are the best ones if the proposer indeed wants his offers to be accepted. The proposer's payoff when such offers are accepted is $\widehat{v}(S) - \widehat{v}(S \backslash \{i\})$, if the offer is rejected the payoff is given by: $\sum_{P' \ni S \backslash \{i\}} \alpha(S \backslash \{i\}, P') v(\{i\}, (P' \backslash R_{i,P'}) \cup (\{i\}, R_{i,P'} \backslash \{i\}))$.

By Claim 1 it is optimal for the proposer to make offers that are accepted. Hence the strategies specified for $I(S).2$ are optimal as well. The strategies for $I(S).1$ are optimal as well by arguments similar to those appearing in Theorem 1 in Pérez-Castrillo and Wettstein [7]. ■

It is worthwhile to emphasize that the proof of Theorem 1 also provides us with the explicit form of the unique SPE strategies of the mechanism $M^{-}(\alpha)$. The strategies are not complex, which adds further credibility to the implementation result and helps in the

[7] See, for instance, Myerson [6].

actual use of the mechanism.

4 The mechanism when externalities are positive

We consider now situations with positive externalities, such as collusive agreements or cartels that reduce market competition, R&D coalitions with spillovers, public goods' coalitions, or environmental agreements. For these environments, we design the mechanism $M^+(\alpha)$.

The mechanism $M^+(\alpha)$ is identical to $M^-(\alpha)$, except for a modification in the second stage of the $G(T)$ games. In $G^+(T_{s+1})$ the first player to answer the proposal by $\beta(T_{s+1})$ is player γ_{s+1} and if this player rejects the proposal, all the players in N remain as singletons. In $G^+(T)$ the players in $T\backslash\beta(T)$, sequentially, either accept or reject the proposal. When a player $\delta(T)$ rejects it, the game ends with the coalition T splitting into singletons. Otherwise, the games are the same as before.

The outcome function of the mechanism $M^+(\alpha)$ is thus the same as the outcome of $M^-(\alpha)$, except for the case in which an offer was rejected at one of the $G^+(T)$ games.

As it is the case with negative externalities, zero-monotonicity properties are essential for obtaining the implementation result. In addition to strict zero-monotonicity-A, we also need the following closely related property, where the values of the player and the player's marginal contribution are taken using a different reference partition. Namely, the additional worth for a coalition $S\backslash\{i\}$ when an agent i leaves another coalition R to join $S\backslash\{i\}$ is larger than the minimum worth of this player i (which, in games with positive externalities, corresponds to $v(\{i\},\{j\}_{j\in N})$).

Definition 4 *The game* (N,v) *is called* strictly zero-monotonic-B *if:*

$$v(S,P) > v(S\backslash\{i\},(P\backslash(S,R)\cup(S\backslash\{i\},R\cup\{i\})) + v(\{i\},\{j\}_{j\in N}),$$

for all $i\in S$, *for all* $R\in P\backslash S$, *for all* $(S,P)\in ECL$.

We note that zero-monotonicity-A and B are independent requirements. The zero-monotonicity-A considers the effect of adding a player i to a coalition, when that player otherwise is a singleton. Zero-monotonicity-B also considers the effect of adding player i to a coalition, but i could come from any other coalition. Hence B, it is more requiring in the sense it sets a larger set of conditions. On the other hand, the reference value to compare to the marginal effect of the player is different in both definitions. Zero-monotonicity-B requires the marginal effect to be greater than $v(\{i\}, \{j\}_{j \in N})$ which, when externalities are positive, is smaller than the worth of player i when the other coalitions remain intact, as required by zero-monotonicity-A. Clearly both requirements are equivalent for games with no externalities and coincide with zero monotonicity.

As we did in the previous section, we start by characterizing the unique SPE strategies at the outsiders stage.

Lemma 2 *Consider a game (N, v) with positive externalities which is strictly zero-monotonic A and B. If the outsiders stage $O(S)$ is reached (hence $S \neq N$), then at the unique SPE of the game that starts at $O(S)$ the randomly chosen partition actually forms. Moreover, the payoff of agent γ_{s+1} is $v(\{\gamma_{s+1}\}, \{j\}_{j \in N})$.*

Proof. The proof is similar to that of Lemma 1, and we indicate the adjustments needed in what follows. In this game, the proposer $\beta(T)$ of "the last coalition to play" T, if $\gamma_{s+1} \notin T$, proposes $v(\{i\}, (P' \backslash T) \cup \{k\}_{k \in T})$ to each player $i \in T \backslash \{\beta(T)\}$ and each player accepts this offer. If $\gamma_{s+1} \in T$, $\beta(T)$ proposes to each player i in $T \backslash \{\beta(T), \gamma_{s+1}\}$, $v(\{i\}, (P' \backslash T) \cup \{k\}_{k \in T})$, player γ_{s+1} is offered $v(\{\gamma_{s+1}\}, (k)_{k \in N})$, and each player accepts the offer made. To prove that proposing an acceptable proposal is the only SPE at this point, we show that the profit of the proposer is larger in case of acceptance than in the case of a rejection, that is (in the case where $\gamma_{s+1} \notin T$),

$$v(T, P') - \sum_{j \in T \backslash \beta(T)} (v(\{j\}, (P' \backslash T) \cup \{j\}_{j \in T}))) > v(\{\beta(T)\}, (P' \backslash T) \cup \{j\}_{j \in T})$$

or, in other words,

$$v(T, P') > \sum_{j \in T} (v(\{j\}, (P' \backslash T) \cup \{j\}_{j \in T}))).$$

To prove this inequality, we sequentially apply the strict zero-monotonicity-A and the property of positive externalities as follows:

$v(T, P') > v(T \backslash \{i\}, (P' \backslash T) \cup (T \backslash \{i\}, \{i\}) + v(\{i\}, (P' \backslash T) \cup (T \backslash \{i\}, \{i\}) >$
$v(T \backslash \{i, j\}, P' \backslash T \cup (T \backslash \{i, j\}, \{i\}, \{j\}) + v(\{j\}, P' \backslash T \cup (T \backslash \{i, j\}, \{i\}, \{j\})$

$+v(\{i\},(P'\backslash T)\cup(T\backslash\{i\},\{i\})>$

$v(T\backslash\{i,j\},P'\backslash T\cup(T\backslash\{i,j\},\{i\},\{j\})+v(\{j\},P'\backslash T\cup(T\backslash\{i,j\},\{i\},\{j\})$

$+v(\{i\},(P'\backslash T)\cup(T\backslash\{i,j\},\{i\},\{j\})$

The first two inequalities follow from the strict zero-monotonicity-A, whereas the last one follows having positive externalities. Proceeding in this manner we get the above inequality which implies that the proposer $\beta(T)$ in each coalition T, other then T_{s+1}, makes the offers $v(\{j\},(P'\backslash T)\cup\{j\}_{j\in T})$ to any $j\neq\beta(T)$.

It can be similarly shown that in the coalition T_{s+1}, offers to all players other than γ_{s+1} would be the same as in the other coalitions, whereas player γ_{s+1} is offered $v(\{\gamma_{s+1}\},(k)_{k\in N})$.

Thus, the proposer prefers his offer to be accepted, and in equilibrium coalition T stays together. The proof of the lemma is completed by straightforward induction on the coalitions playing the $G^{+}(T)$ games. As before this proof also supplies us with the unique SPE strategies. ■

The next theorem states the implementation result. It also provides us with the explicit form of the unique SPE strategies of the mechanism $M^{+}(\alpha)$.

Theorem 2 *If the game (N,v) has positive externalities and is strictly zero-monotonic-A and B, then the mechanism $M^{+}(\alpha)$ implements in SPE the value $\varphi(\alpha)$.*

Proof. The proof of this result is similar to the proof of Theorem 1. and we indicate the adjustments needed in what follows.

We substitute Claim 1 by Claim 1′:

Claim 1′. The following equation holds:

$$\widehat{v}(S)>\widehat{v}(S\backslash\{i\})+\sum_{\substack{P'\ni S\backslash\{i\}\\P'\ni\{i\}}}\alpha(S\backslash\{i\},P')v(\{i\},P')+\sum_{\substack{P'\ni S\backslash\{i\}\\\{i\}\notin P'}}\alpha(S\backslash\{i\},P')v(\{i\},\{j\}_{j\in N}),\tag{7}$$

for any $i\in S$.

We rewrite the right-hand side of equation (7) as:

$$\sum_{\substack{P'\ni S\backslash\{i\}\\P'\ni\{i\}}}\alpha(S\backslash\{i\},P')\left[v(S\backslash\{i\},P')+v(\{i\},P')\right]$$
$$+\sum_{\substack{P'\ni S\backslash\{i\}\\\{i\}\notin P'}}\alpha(S\backslash\{i\},P')\left[v(S\backslash\{i\},P')+v(\{i\},\{j\}_{j\in N})\right].$$

We write this as a sum over all $P \ni S$ noting that for every $P' \ni S\backslash\{i\}$, there exists a partition $P \ni S$ such that $P' = P\backslash(S,R) \cup (S\backslash\{i\}, R \cup \{i\})$ for some $R \in P$. When $P' \ni \{i\}$, $R = \varnothing$ and $v(S\backslash\{i\}, P') + v(\{i\}, P') < v(S,P)$ by strict zero-monotonicity-A. When $\{i\} \notin P'$,

$$v(S\backslash\{i\}, P') + v(\{i\}, \{j\}_{j \in N}) < v(S, (P'\backslash(R \cup \{i\}, S\backslash\{i\})) \cup (S,R)) = v(S,P)$$

by strict zero-monotonicity- B. Therefore, we can make a similar argument as the one we used in the proof of Claim 1 to complete the proof of Claim $1'$.

The rest of the claims and the reverse implication can be shown by the same arguments as in the proof of Theorem 1, we just need to take into account that in the proof of Claim 3 the expected profit of the (eventual) rejected proposer i is the sum of the last two terms on the RHS of equation (7). ■

5 Conclusion

In this paper we proposed mechanisms that implement a family of sharing methods for environments with externalities. The family of values implemented satisfies efficiency, anonymity, linearity and the null player property. It has the attractive feature of treating all externalities on the same footing by performing an averaging that is independent of players' names.

These mechanisms provide a non-cooperative foundation to the family of values suggested in Macho-Stadler *et al.* [3]. They may be used to apply any of these solutions in environments with externalities, thus overcoming the difficulties created by strategic behavior. Possible applications are the sharing of benefits achieved through environmental agreements, sharing the costs of constructing a communication or transportation network among several users and resolving wealth distribution disputes among family members or stock owners of a bankrupt firm.

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