A Descriptive and Analytic Look at Marx's Own Explanations for the Falling Rate of Profit (Long Version)

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Abstract: Marx's conclusions about the falling rate of profit have been analysed exhaustively. Usually this has been done by building models which broadly conform to Marx's views and then showing that his conclusions are either correct or, more frequently, that they can not be sustained. By contrast, this paper examines, both descriptively and analytically, Marx's arguments from the Hodgskin section of Theories of Surplus Value, the General Law section of the recently published Volume 33 of the Collected Works and Chapter 3 of Volume III of Capital. It also gives a new interpretation of Part III of this last work. The main conclusions are first, that Marx had an intrinsic explanation of the falling rate of profit but was unable to give it a satisfactory demonstration and second, that he had a number of subsidiary explanations of which the most important was resource scarcity. The paper closes with an assessment of the pedigree of various currents of Marxian thought on this issue.

I. Introduction.

Marx's conclusions about the falling rate of profit have been analysed exhaustively. Usually this has been done by building models which broadly conform to Marx's views and then showing that his conclusions are either correct or, more frequently, that they can not be sustained. By contrast, the detail of Marx's own arguments has not been looked at closely. This is partly because they are often obscure and difficult and partly because an important segment of them has only recently become available. This paper examines, both descriptively and analytically, Marx's arguments from the Hodgskin section of Theories of Surplus Value, the General Law section of the recently published Volume 33 of the Collected Works and Chapter 3 of Volume III of Capital. It also gives a new interpretation of Part III of this last work. The main conclusions are first, that

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Marx had an intrinsic explanation of the falling rate of profit but was unable to give it a satisfactory demonstration and second, that he had a number of subsidiary explanations of which the most important was resource scarcity. The paper closes with an assessment of the pedigree with respect to this issue of various currents of Marxian thought.

This project seems important for two reasons, one prosaic and the other controversial. First Marx is a major thinker and the falling rate of profit an important aspect of his thought. Thus mere completeness of intellectual history would seem to require a detailed description. Second Marx still has a large following so that a model that is called Marxian immediately acquires an enhanced interest. Of course the model must be authentically Marxian and of reasonable quality for this interest to be sustained. The detailed description of Marx's arguments is clearly an aid to authentication and is, as well, potentially useful in discovering approaches that are likely to lead to valid analytic support of Marx's general position.

The paper is organised as follows: Section II covers the writing of the 1862-3 period; section II provides formalisations; section IV gives a new interpretation of Part III of Volume III; section V deals with Chapter 3 of Volume III which was written in the 1870s; section VI contains the assessment and section VII concludes.

II.   The Arguments of 1863-3.

The problem Marx faced was the following: In a one sector model the rate of profit is

$$= \frac{S/V}{C/V + 1}$$

where S, C and V are surplus value, constant capital and variable capital.[1] Marx thought that the composition of capital C/V would increase over time, but he had it very clear that this would raise the rate of surplus value S/V. He had to explain why this would not be sufficient to cause the rate of profit to increase.

In October and November of 1862 Marx was writing out his comments on Thomas Hodgskin for the manuscript that would become Theories of Surplus Value (TSV). Hodgskin believed that the rate of profit would be forced to fall if capital grew faster than labour. Marx tried to set out this conclusion in a logical form and appeared to be

[1] The notation throughout is that of Roemer (1981).

surprised to find he could not. He broke off work on the manuscript and filled two hundred notebook pages mainly attempting to deal with this. In January of 1863 he returned to the Hodgskin section and then finished the rest of the manuscript. In English, TSV became available in 1971 but the two hundred notebook pages only appeared in 1991 with the publication of Volume 33 of the Collected Works (CW). This section covers the Hodgeskin section of TSV III pp.263-319 and the central section of the notebook pages, called the General Law for short, from the CW 33, pp.104-53.

1. Hodgskin.

Starting in section e p.128, Marx quotes Hodgskin as arguing that the rate of profit must fall as follows: If the rate of profit remained constant then capital and profits would grow faster than labour. The productivity of labour must grow so that output is sufficient to cover both the sustenance of the workers and the rapidly growing profits of the capitalists. Hodgskin simply does not think that it is possible for labour productivity to grow this rapidly. Rather "a balance must be struck and the rate of profit must fall."

Marx is broadly in agreement with this and after a few preliminary remarks he suddenly sets out his own theory of the causes of the fall in the rate of profit. He starts with a paragraph for the case in which the rate of surplus value is constant and another for the case in which it increases because the working day increases. Then, in what are arguably the three most important three paragraphs in all of Marx's writing on the falling rate of profit, he sets out his argument for the case in which the rate of surplus value rises because of the rise in the composition of capital has increased labour productivity, that is, the case of relative surplus value:

"3) If the normal working-day remains the same, surplus labour can be increased relatively by reducing the necessary labour time by[2] reducing the prices of the necessaries which the worker consumes, in comparison with the development of the productive power of labour. But this development of productive power reduces variable capital relative to constant. It is physically impossible that surplus labour-time of, say, two men, who displace twenty, can, by any conceivable increase of absolute of relative [surplus] labour-time, equal that of twenty. If each of the twenty men only work 2 hours of surplus labour a day, the total will be 40 hours of surplus labour, whereas the total life-span of two men amounts only to 48 hours in one day.

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[2] The actual text has "and" rather than "by".

The value of labour-power does not fall in the same degree as the productivity of labour or of capital increases. This increase in the productive power likewise increases the ratio between constant and variable capital in all branches of industry which do not produce necessaries (either directly or indirectly) without giving rise to any kind of alteration in the value of labour. The development of productive power is not even. It is the nature of capitalist production that it develops industry more rapidly than agriculture. This is not due to the nature of the land, but to the fact that land requires different social relations. Capitalist production turns towards the land only after its influence has exhausted it and after it has devastated its natural qualities. An additional factor is that, as a consequence of land ownership, agricultural products are expensive compared with other commodities, because they are sold at their value and not reduced to their cost-price. They form, however, the principal constituent of the necessaries. Furthermore, if one-tenth of the land is dearer to exploit than the other nine-tenths, these later are likewise hit "artificially" by this relative barrenness, as a result of the law of competition.

The rate of profit would have to grow if the productivity of capital[3] is to remain constant while accumulation of capital is taking place. The same worker, as long as capital yields 10 of surplus labour must, as soon as interest accumulates on interest and thus increases the capital employed, produce three fold, fourfold, five fold in progression of compound interest, which is a nonsense."(pp.300-1).

These three paragraphs are obscure in the extreme. The reason, I think, is that Marx is trying to give form to his intuition rather than setting out a formal analysis. However it is possible, it seems to me, to detect the argument that Marx was groping for. The general structure is as follows. In the first paragraph Marx sets out an argument for the falling rate of profit based on the reduction of labour causing a fall in surplus value. However he senses that the argument is defective. In the second he announces a condition for the rate of profit to fall and gives four separate arguments that the condition will be satisfied in the real world. Finally, in the short third paragraph he affirms that the rate of profit will, indeed, fall.

A formal model is needed to follow the detail. It is a one sector circulating capital model with fixed coefficients and one basic factor, labour. *A, L, b* and   are the capital

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[3] The actual text has "it" in place of "the productivity of capital".

and labour coefficients, the real wage, and the value of the good. In addition $\tilde{L}$ and K are the number of workers and the constant capital in physical units.

$$= L/(1-A), \ S = \tilde{L} - \ b L \tilde{L}, \ C = \ K, \ V = \ b \tilde{L}.$$

First one can see the argument that Marx was trying for in the first paragraph and its weakness.

$$= \frac{1 - \dfrac{L}{1-A} b \ \tilde{L}}{\dfrac{L}{1-A} K + \dfrac{L}{1-A} b\tilde{L}} \tag{1}$$

It is clear that a reduction of $\tilde{L}$ will lower surplus value and, if nothing else changes, (that is the denominator remains unchanged) the rate of profit will fall. This was the line Marx was following. But Marx emphasised that the reduction of $\tilde{L}$ would cause an increase in labour productivity and a fall in capital productivity, that is a fall in L and an increase in A, total capital may be lowered proportionally more than surplus value and the rate of profit may rise. Thus Marx needs both a condition that gives a limit to the permissible fall in L relative to the rise in A and an argument that the condition will be satisfied.

The first sentence of the second paragraph gives this condition although it requires a bit of an effort to see it. $P \quad 1/L$ is the productivity of labour and $\tilde{V} \quad b$ is the value of labour power. Using $K = (A/L)\tilde{L}$, (1) reduces to

$$= \frac{1}{A + bL} - 1. \tag{2}$$

Solving for A in terms of P and $\tilde{V}$ gives $A = 1 - b/\tilde{V}P$, substituting this in (2) gives

$$(P,\tilde{V}) = \frac{1}{1 - \dfrac{b}{P} \dfrac{1}{\tilde{V}} - 1} - 1$$

so that $\dfrac{}{P} < 0$ and $\dfrac{}{\tilde{V}} < 0$ [4]. Thus if $\tilde{V}$ does not fall "in the same degree" as P rises the rate of profit will fall. [5]

The sense is the following. A rise in productivity of labour with the value of labour constant implies a fall in the productivity of capital which is strong enough to lower the rate of profit. If the rise in the productivity of capital caused by the fall in the value of labour power is not sufficiently strong, the net effect will be a fall in the rate of profit.

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[4] $\tilde{V} < 1$ from (1) if > 0 is assumed.

Thus the first sentence is a complicated way of expressing a limit to the fall in L relative to the rise in A. Marx doing partial differentiation in his head strains ones credulity but I see no other interpretation.[6]

The rest of the paragraph sketches four distinct reasons why the condition may be satisfied. They concern, briefly, luxury goods, exhaustion of natural resources, monopoly in agriculture, and the development of less fertile land. All of these require a model of at least two sectors for their formal presentation. They are examined in section III below.

The third paragraph just expresses the conclusion, albeit in an indirect way: " the rate of profit would have to grow… which is a nonsense."

After setting out his theory thus, Marx writes a paragraph about the consequences of this for accumulation and then sets about to compare his ideas with those of Hodgskin. Hodgskin has capital growing faster than labour while Marx has labour decreasing and capital remaining constant. Marx says these are basically the same, trys to recast his argument in this form and disturbingly finds he can not do it. The example on p.304 is not carefully set out but can be reconstructed. There are initially two cases, I and II.

	C	V	S
I	£25	£25	£25
II	£175	£25	£25

The working day is 12 hours so that the worker produces £25 for his subsistence in 6 hours and £25 of surplus value in the other 6 hours. In each case the constant capital reproduces itself. Now Marx asks how many hours would have to be worked to maintain the 50% case I rate of profit in case II? The worker would have to produce £25 of subsistence plus £100 of surplus value. Since this would take 30 hours it is impossible and so the rate of profit must fall. Then Marx notes that this is assuming constant productivity, but that if the productivity rose so that the worker could produce

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£125 in 12 hours then the rate of profit would not fall.[7] Marx reacts to this by supposing productivity only rises so that the worker can produce £50 in 12 hours. Then, even if he worked 24 hours he could not produce the £125 so the rate of profit must fall.[8]

Marx clearly realises this is unsatisfactory because he immediately starts going over the example again but can find no way to make the argument stronger. After this he metaphorically backs up and makes another run at the problem but can make no headway. He then refers back to the three key paragraphs (p.312), makes three and half pages of random comments and, at this point, the break occurs.[9]

My interpretation of what happened is the following. When he started writing the section on Hodgskin, Marx thought that he could easily make the argument that the rise in the composition of capital would cause the rate of profit to fall. When he set out his position he sensed that there might be a weakness and hastily sketched the reinforcing arguments, but he was still confident that he could make his argument without excess effort. Then as he struggled to set out his argument in Hodgskin's form it slowly dawned on him that the problem was much deeper than he had suspected and he decided to break off the writing of TSV and sort out the problem of the falling rate of profit once and for all,

2. The General Law.

Marx has two types of explanations for the falling rate of profit: an intrinsic one which is based on the rising composition of capital and a number of subsidiary ones which involve phenomena like the worsening quality of resources. This section contains what I think is Marx's most sustained attempt to provide a justification for the intrinsic explanation. He first argues that the problem can be addressed in terms of a one sector model. After this he formulates the model first in value terms and then in terms of labour units. At last, following his deepening intuition, he tries to construct a model that combines both of these but fails because he can not handle the technical difficulties. The section ends with Marx, as it were, taking refuge in the subsidiary explanations.

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[7] In this case the value of labour-power falls from 5/10 of a day to 2/10 of a day, a fall of 60% rather than the 75% that Marx claims.

[8] These figures ate different from Marx's. He seems to think the rate of profit was 100% so that £200 of surplus value is needed. He allows the worker to produce £75 in 12 hours so that it is impossible to produce the required £225 in 24 hours.

[9] The method here and in some of the following sections is to give my interpretation of the structure of Marx's argument rather than lots of quotes. This seems to be only way to deal with all the material in a limited space. However it requires an act of faith on the part of the reader who, if he finds himself in disagreement, will have to turn to the original text.

a) The Intrinsic Explanation.

Marx starts by claiming that it is not necessary to enter into the competition of capitals. In a long preceding section Marx had talked about costs of production. In the opening paragraph of the General Law p.104 he summarises this as follows. Since the rate of profit can be got by taking the sum of surplus value over all industries and dividing this by the sum of total capital over all industries, the movements of the rate of profit can be understood without dealing with sectorial interactions. That is, the aim of the section is to explain the falling rate of profit in terms of a one sector model.

After this there follows what might be considered a warm up section with the following high points. Marx compares England and India. England has a larger composition of capital and, because of this, has a rate of surplus value which is double that of India but a rate of profit which is lower (p.107). He then emphasises that the development of the productive powers has two manifestations which work in opposite directions, the rise in C/V lowers the rate of profit while the rise in S/V raises it (p.109). But it is "strikingly apparent" that the first will predominate as is confirmed by experience, because the reduction of labour lowers surplus value (p.110). One might suppose from the style that Marx has convinced himself, but at the bottom of p.114 he starts the argument a fresh by saying "to bring this important question to a decisive conclusion the following must first be investigated:"

He then constructs an example in value terms in which the rate of profit rises. At first he does not acknowledge this, he produces a number of formulas, then switches back to another numerical example and finally acknowledges that the rate of profit has risen. He finishes by calculating what the rise in the rate of surplus value would have to be for the rate of profit to remain unchanged. In general he gives the impression that he is surprised that he can not demonstrate the necessity of a fall in the rate of profit in this framework. This impression is heightened because he then proceeds to glue in four large pages which start with the statement "let us first assemble the facts"(p.117).[10]

These four pages contain three separate attacks on the problem. The first is merely a continuation of the previous argument in terms of value categories. He sets out a number of examples in which he increases constant capital and discovers that the condition for constancy of the rate of profit is that surplus value and total capital grow

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[10] The transcription of these four pages is difficult to read since it has frequent gaps because the pages themselves have been damaged. Appendix 1 is a version of these pages with the gaps filled in. It is clear that this may distort what Marx wrote but there seems no other way to make the pages intelligible.

at the same rate and that this implies that the rate of surplus value grow at this rate as well (pp.117-9).

Finally convinced that this attack will not work, he shifts to the second. He lowers variable capital, either absolutely or relative to constant capital, and claims that surplus value will fall faster than total capital. He gives two reasons: first he equates a fall in variable capital to a fall in labour which will eventually reduce surplus value and second the development of the productive power is not uniform across sectors (p.120). The first reason is incomplete since he doesn't even mention the effect on total capital, while the second is only mentioned. Once again the style of the writing gives no indication that Marx is aware of these problems but it seems likely that he is.

He now draws a horizontal line, writes   and attempts to develop the second attack with more precision. He wants to construct an example in which workers are reduced and machinery increased in a way that keeps the value of total capital constant. First he implies, wrongly, that surplus value is the rate multiplied by the quantity of labour and notes that these two forces will act in opposite directions on surplus value. But then he reverts to surplus value itself rather than the product. His example shows that the reduction in the number of workers has definitely caused a fall in surplus value. The problem is that, since he has no value terms in his example, he can not calculate what has happened to the value of capital. He is forced to end with a limp statement about total capital remaining constant (pp.123-5).

Marx now appears to realise that he will have to have both workers and values in the same example if he wants to make his point. He writes   and begins to construct the model. Since this is Marx's major attempt to show that the rate of profit must fall, I will recount what happened in detail. He starts with an example which is set out in the first four columns of the following table. In case I C is 150 of raw material and 50 of

	V	C	$\tilde{L}$	b	S	
I	400	200	10	40	400	4/6
II	80	520	2	40	80	8/60

machinery while in case II C is 150 of raw material and 370 of machinery. The output in physical terms is the same in both cases. He notes that total capital is the same in both cases but that case II has a higher composition of capital. One can see where he is trying

to go. One could finish the example as follows: Suppose that the figures are hours and that each worker works 80 hours a week. Then one could fill in the S and in the table and have the rate of profit fall. By specifying b and especially $\tilde{L}$ he can force the surplus value to fall by making $\tilde{L}$ fall. This is the argument that Marx wanted to make.

But there is a problem. With physical output fixed, the value of a unit has fallen. Suppose output is 1000 units, then in case I the value of a unit is 1, and in case II 680/1000=0.68. Thus the case II wage in value terms should be reduced to keep the real wage constant. The simple argument that Marx hoped to make is not viable. But at least, in terms of this framework, the basic problem can be confronted (to last paragraph p.125).

Marx is aware of exactly this problem and reacts by trying to find the values or prices. He makes the natural assumption that machines are fixed capital, but this complicates his task in a way that he never manages to resolve. Specifically he assumes that the machine of case I lasts one year and that of case II ten years. He then calculates the price in case II as

$$326=(37+150+80+0.05\text{x}333)(1.05)$$

where 37 is the straight line depreciation of the machine, 0.05x333 is the interest payment on the un-depreciated part of the machine and (1.05) is the profit margin added by the capitalist.[11] This shows that Marx does not know how to calculate costs for the case of fixed capital[12] and he himself is aware that this method is problematic since he seems to change the method in the next case and is clearly worried, p.128 second full paragraph, by the fact that the interest payment enters into cost but not value. He ends the paragraph by noting that the price is much lower than in case I which should, by his method, be 600x1.05=630. It can not be emphasised too strongly that in this, Marx's main attempt to explain the falling rate of profit, he went to great lengths to keep the real wage constant (to the bottom of p.126).

He now sets out case III where C is composed of 150 of raw materials and a

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[11] Marx's example, as the editors note, is riddled with errors. The text here is consistent so that the numbers differ from those of the original text.

[12] This is not that straightforward. One way to do this would be to suppose that the capitalist can borrow and lend at the rate of interest, commits the present value of total costs at time 0, reinvests all revenue, and finally sets the price so that the ratio of the accumulated revenue at time 10 to the committed funds is $(1+r)^{10}$ where r is the profit margin.

	V	C	$\tilde{L}$	b
III	80	2150	2	40

machine which lasts either 10 or 100 years depending on the way the manuscript is interpreted. Using the same calculation as in case II, the price is either 546 or 366 depending on the interpretation.[13] There is no clear reason for the introduction of case III. If one takes the first interpretation, one possibility is that Marx was surprised by how cheap the good had become and was worried that the rate of profit would not fall so he wanted an example with a smaller fall. If one takes the second interpretation, then Marx wanted to see if lengthening the turnover time could raise the price, once again to get the rate of profit to fall. In any case, with case III, he has lost the line of his argument since total capital is no longer constant (to end of first paragraph on p.127).

Marx seems to realise this since he next gives a verbal discussion of the effect on price of lengthening turnover times when total capital is fixed. He says that there are two types of effects: The first merely lowers wear and tear and thus the price. The second combines this with an increase in machinery and a fall in workers in a way that causes output to fall and the price to rise. This can, perhaps, be interpreted as an attempt to redo the movement from case I to case II and then from case II to case III (second interpretation) but with total capital held constant. In any case, it is unconvincing and so far from the initial example that Marx admits that this discussion actually belongs elsewhere (to end of first full paragraph p.128).

Starting with the third paragraph of p.128, Marx makes a last attempt to revive his argument. But he can make no headway and ends, in the last full paragraph of p.130, talking about an American economist called Wayland. This is the end of what is the high water mark of all of Marx's attempts to provide a demonstration of the intrinsic explanation.

One can trace the stages of Marx's attack and see why it wouldn't have worked even if he had not been detained by technical difficulties. He started only with value terms writing

$$= \frac{S}{C+V} = \frac{S/V}{C/V+1}.$$

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[13] As noted, Marx calculates this case differently from the preceding one and does it in a way that is impossible to fathom. Since the method of case II is set out clearly, I have used it for this case as well.

He supposed that *C/V* would rise but could not show that this would imply a fall in
since he saw with extreme clarity that the rise in *C/V* would cause a rise in *S/V* . Next he tried to use the fall in labour with fixed capital to force a fall in the rate of profit. That is, given

$$= \frac{(1 - b)\tilde{L}}{K + b\tilde{L}}, \qquad = \frac{L}{1 - A} \; ,$$

Marx wanted to claim that the fall in $\tilde{L}$ with $K + b\tilde{L}$ fixed would cause to fall. However since he did not specify , he could not make the constant capital condition convincing. Finally he set up an example with *C, V,* $\tilde{L}$ and *b* specified. If he had proceeded in this way and had not tried to deal with fixed capital, he could have, in principle, completed his argument. But if he had done so he would have discovered that

$$= \frac{1}{A + bL} - 1$$

and that the rise in *A* and the fall in *L* can not guarantee the fall in *v* without further restrictions. In fact he might have convinced himself that this part of his intuition about the falling rate of profit was wrong.

b) The Subsidiary Explanations.

The rest of the section contains numerous random comments plus three will worked out, essentially two sector explanations of the fall of the rate of profit. The first is contained in the paragraph which starts at the bottom of p.130. It says that the rate of profit falls because low investment in agriculture means that productivity in this sector grows more slowly than the average. The second, which follows a similar line, appears between the horizontal lines on pp.133 and 136 and is much more detailed. The third which harks back to a point made in the discussion of Hodgskin is contained in the section of two paragraphs which starts at the bottom of p.148.

Marx starts this second explanation with a careful description of how an increase in productivity in a particular sector increases surplus value. The increase in productivity cheapens wage goods and this in turn lowers the value of labour power and thus raises surplus value. But he stresses the fact that a large increase in productivity in a particular sector, since its product is only a small part of worker consumption, will only have a small effect on the value of labour power and thus surplus value.

Then in the key paragraph he notes that if productive power grew evenly in all sectors, then surplus value would also grow at the same rate. But this doesn't happen for

two reasons: " The anarchy of competition" and natural conditions such as " the influence of the seasons,…, (the) exhaustion of forests, coal seams, mines and the like." He furthermore notes that productivity in agriculture, which is the main component of workers' consumption, grows more slowly than that of industry. He concludes that the growth of surplus value is always smaller than the growth of productive power of capital in all branches of industry.

There are three problems with this as an explanation of the fall in the rate of profit. First, Marx says productive power of capital when he ought to say labour. This, I think, is just a slip; probably Marx was thinking of the growth in the productivity of labour caused by the relative increase in capital. Second, Marx does not say that this is an explanation of the fall in the rate of profit. However it can be taken to be one since, in the summary on p.148 Marx clearly refers back to this section and states that if the rate of surplus value does not rise in proportion to the growth of productive power the rate of profit will decline. Finally there is the question of whether this condition is valid. The following calculations shows that it is. Let $e=S/V,$

$$e = \frac{(1 - bL - A)}{bL} = \frac{P}{b} - 1 - \frac{P}{b}A$$

$$A = 1 - \frac{b}{P}(1 + e)$$

$$= \frac{1}{1 - \frac{e}{P}b} - 1$$

where the last equation follows from (2). Thus if e/P falls so does the rate of profit. But its correctness (i.e. that the two reasons imply a fall in e/P) can only be accessed in the context of a formal two sector model.

The third explanation (pp.148-9) is that the increase in the composition of capital will also occur in sectors that ate unrelated to worker consumption. This will not increase the rate of surplus value and thus cause the rate of profit to fall. Marx had previously made this point in the section on Hodgskin. Here he adds that the growing cheapness may expand the range of worker consumption and thus weaken the effect.

To summarise, the section on the General Law contains Marx's failed major attempt to demonstrate the intrinsic explanation of the falling rate of profit plus a number of special two sector explanations whose validity remains to be accessed.

3. Conclusion.

One can speculate about how Marx felt about resource scarcity explanations and the intrinsic explanation of the falling rate of profit after he had finished the work on TSV.

With respect to the first, reference must be made to the section on Cherbulitz which Marx wrote as one of the final sections of TSV III. In this, Marx's explanation of the rise in the composition of capital partly consisted of a page long detailed description of growing resource scarcity (p.368). I think it is incontestable that at this point Marx thought that resource scarcity would cause the rate of profit to fall. Its mention in the comments on Hodgskin, and in the sections in the General Law, and finally its emphasis in the Cherbulitz piece allow no other conclusion.

With respect to the intrinsic explanation, on the one hand this argument fitted better with Marx's general view since it was a "barrier" that arose from the nature of capitalism itself. On the other hand Marx had just failed in his protracted attempt to demonstrate it. At least two positions are possible: first that he had changed his mind and now thought the proposition was incorrect and second, that he still thought the proposition was correct and hoped to demonstrate it in the future. I think the second is more likely for a number of reasons: first his problems were technical so that he could hope to resolve them, second when he came to write Part III of Volume III of Capital he gave the impression that he thought the intrinsic explanation was correct, and finally in the 1870s he had another go at demonstrating it.

III.   The Two Sector Explanations.

In the sections on Hodgskin and the General Law, apart from the intrinsic explanation, Marx gives four subsidiary explanations of the falling rate of profit. The formalisation of any of these requires at least a two sector model. This section provides this minimal formalisation of each of these explanations.

Before this is done, there are three methodological issues that must be resolved. First, the exercise will be performed for both the value and the price rates of profit. These are different in a two sector model but Marx thought they were the same. Thus, since it is not clear which case Marx was considering, both are set out.[14] Second, the value rate of profit depends on the composition of output. This will be taken as fixed since it does not seem to figure in Marx's arguments. Finally Marx's arguments will be judged as to whether they are correct for all, some, or no set of parameter values.

1) Luxury Goods.

In both the Hodgskin and the General Law sections, Marx argued that the existence of luxury goods would weaken the tendency of the rate of profit to rise since the cheapening of these goods would not lower the value of labour power. The simplest model for checking this is a two sector model where good 1 is a capital-consumption good and good 2 is a luxury good. Let $x_1 / x_2 = x$ be the output ratio.

The value rate of profit is

$$= \frac{{}_1(1 - A_{11} - bL_1)x + {}_2 - {}_1(A_{12} + bL_2)}{{}_1[(A_{11} + bL_1)x + A_{12} + bL_2]} \tag{3}$$

where $_1 = L_1/(1 - A_{11})$, $_2 = A_{12}\,_1 + L_2$ and $_i$ is the value of good i etc. Capitalist development is interpreted as $dA_{12} > 0$ and $dL_2 < 0$. Clearly $/ A_{12} < 0$. Letting D be the denominator of (3), an easy calculation shows

$$\overline{}_{L_2} = (1/ D^2)[\,_1/(1 - A_{11})][(A_{11}x + A_{12})(1 - A_{11} - bL_1)] > 0.$$

It follows that $d = \overline{}_{A_{12}}\, dA_{12} + \overline{}_{L_2}\, dL_2 < 0$. The intuition is that the rise in the capital input coefficient causes a fall in the rate of profit and surprisingly the fall in the labour coefficient does as well since the fall in the value of luxury goods dominates all other effects.

However with respect to the price rate of profit, one immediately sees that Marx's argument is wrong. The equations are:

$$p_1 = (1 + \ )(p_1 A_{11} + p_1 bL_1)$$
$$p_2 = (1 + \ )(p_1 A_{12} + p_1 bL_2)$$

where p_i is the price of the ith good and is the price rate of profit. The first equation determines , which does not depend on either A_{12} or L_2, while the second determines relative prices. Changes in the composition of capital of luxury goods do not effect the price rate of profit. It is well known that this result is generally valid.

These results are summarised as

Result 1. For the luxury good model just described, an increase in the composition of capital in the luxury good sector lowers the value rate of profit but leaves the price rate of profit unaffected.

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[14] It would be interesting to set out the single system version initiated by Foley (1986 chap.6) but a line must be drawn somewhere.

2. The Degradation of Natural Resources.

Marx merely mentions it in the General Law section but in the Hodgskin section he relates it to capitalism in a way that is not clear. In any case the simplest model in which this can be considered is one with two goods where good 1 is machinery and good 2 is food, and where the degradation is interpreted as $dA_{12} > 0$ and $dL_2 > 0$.

In this case the value rate of profit is

$$= \frac{\left(_1 - {}_1A_{11} - {}_2bL_1\right)x + {}_2 - {}_1A_{12} - {}_2bL_2}{\left(_1A_{11} + {}_2bL_1\right)x + \left(_1A_{12} + {}_2bL_2\right)}$$

where $_1 = L_1/(1 - A_{11})$ and $_2 = L_1A_{12}/(1 - A_{11}) + L_2$. When x is very large

$$\sim = \frac{_1 - {}_1A_{11} - {}_2bL_1}{_1A_{11} + {}_2bL_1}$$

with $d\tilde{}/d_2 < 0$. In this case, since $_2/\ A_{12} > 0$ and $_2/\ L_2 > 0$,

$$d \quad \frac{d\tilde{}}{d_2} \quad \frac{_2}{A_{12}} dA_{12} + \frac{_2}{L_2} dL_2 \quad < 0 .$$

On the other hand, when x and b are very small $\quad \tilde{} = \frac{_2 - {}_1A_{12}}{_1A_{12}} = \frac{L_2}{L_1A_{12}/(1 - A_{11})}$

so that $d \quad \frac{d\tilde{}}{\tilde{}} = \frac{dL_2}{L_2} - \frac{dA_{12}}{A_{12}} > 0 \quad$ if $\quad \frac{dL_2}{L_2} > \frac{dA_{12}}{A_{12}} \quad .$

Next consider the price rate of profit. The equations are

$$p_1 = (1 + \ )(p_1A_{11} + p_2bL_1)$$
$$p_2 = (1 + \ )(p_1A_{12} + p_2bL_2)^{\cdot}$$

Let the matrix be non-negative and indecomposable. Thus by the Frobenious-Perron theorem (See Roemer (1981) p.110.)   is decreasing in A_{12} and L_2 so that a fall in the quality of land causes the price rate of profit to fall.

All of this is summarised as

Result 2. In the fall in the quality of natural resources model just described, an increase in the input coefficients for the food industry has the following effects: First, it lowers the value rate of profit if production is concentrated on machines while it raises it in the opposite case if the product wage is low and the labour coefficient rises by proportionally more. Second, it always lowers the price rate of profit.

3. Slower Development of Agriculture.

Marx's third argument is difficult to interpret without using other references. In the comments of Hodgskin he merely notes that agricultural products are sold above their

cost price at their values while in the General Law section he notes that agriculture develops more slowly than industry. However in the section on Robertus in Part II of TSV pp.93-4, Marx provides a detailed description of the process. Initially value is equal to cost price in both agriculture and industry. Then the productive forces develop more rapidly in industry. This causes the cost price to rise above value in industry and fall below it in agriculture. However absolute rent appears in agriculture, because of the monopoly power of the landlords, so that the price of agricultural goods doesn't fall as the cost price does but stays equal to the value. Since workers' subsistence does not cheapen, the rate of profit falls.

There is a small surmountable problem in modeling this argument. Suppose a two sector model where sector 1 is machinery, sector 2 is food and both sectors use machinery and labour. The values are

$$_1 = L_1 / \left(1 - A_{11}\right) \tag{4}$$

$$_2 = L_1 A_{12} / \left(1 - A_{11}\right) + L_2 \tag{5}$$

The actual cost equals price equations contain both prices and values. To make these compatible let, m with dimension £/hour, be the monetary value of time. The equations of the system are

$$\tilde{p}_1 = \left(1 + \tilde{r}\right)\left(A_{11}\tilde{p}_1 + m \,_2 b L_1\right) \tag{6}$$

$$m \,_2 = \left(1 + \tilde{r}\right)\left(A_{12}\tilde{p}_1 + m \,_2 b L_2 + m \,_2 R\right) \tag{7}$$

$$\tilde{p}_1 + m \,_2 = \left(_1 + \,_2\right)u \tag{8}$$

where $\tilde{p}_1$ and $\tilde{r}$ are the cost price of machines in money and the rate of profit in the mixed system, R is the absolute rent in terms of food per unit output of food, $u =$£/hour to correct units and (8) is a convenient normalisation for prices. The system consists of equations (6)-(8) with unknowns $\tilde{p}_1$, $m \,_2$, $\tilde{r}$ and R with $_1$ and $_2$ given by (4) and (5). The problem is that the system is under-determined, (6)-(8) are not sufficient to determine the prices, the rate of profit and the level of absolute rent. The trouble with Marx's argument is that he defined absolute rent as the difference between cost price and value but didn't take into account that the absolute rent partially determines cost price.

This problem can be got around in the following way that is slightly fanciful but faithful to Marx's argument. Suppose that the landlords calculate the cost prices p_i, that is those that would arise if there was no absolute rent. That is they calculate p_2 from

$$p_1 = (1 + r)(p_1 A_{11} + p_2 b L_1) \qquad (9)$$

$$p_2 = (1 + r)(p_1 A_{11} + p_2 b L_2) \qquad (10)$$

$$p_1 + p_2 = (\nu_1 + \nu_2)u. \qquad (11)$$

Then they set the absolute rent equal to the difference between the value and the cost price of food, that is they set $R = \dfrac{u\nu_2 - p_2}{p_2}$. Once the absolute rent is determined in this manner, it can then be inserted in the equation (7) and $\tilde{p}_1$, m_2 and $\tilde{r}$ can be determined from equations (6)-(8).

One can then check Marx's statement as follows: Start with the initial situation of equal development in both sectors, that is $A_{11} = A_{12} \equiv A$ and $L_1 = L_2 \equiv L$. Faster development of industry is interpreted as $dA_{11} > 0$ and $dL_1 < 0$. This will effect $\tilde{r}$ directly through equations (4)-(8) and indirectly though its effect on R. The outcome is given by result 3[15].

Result 3. For the model of equations (4)-(11) , starting from the initial situation, $dA_{11} > 0$ and $dL_1 < 0$ imply:

i. $dR>0.$

ii. $d\tilde{r} < 0$ if $1 > 3(A + bL)$.

iii. $d\tilde{r} > 0$ is possible if $1 < 3(A + bL)$.

That is, the rapid development of industry always raises absolute rent; also it will cause the rate of profit to fall if the input of machines and food to feed workers is low relative to output but may not otherwise.

4. The Movement to Less Fertile Land.

Marx's forth argument seems to be taken directly from the notion that, as less fertile land is used, food becomes more expensive because of the increase in rent. The price equations for the second model can be used to calculate the effect on the price rate of profit. Let A_{12} and L_2 represent the capital and labour inputs on marginal land. Since this land pays no rent, the price of food is determined by this equation. Thus the rise in A_{12} and L_2 as more barren land comes into production, as argued before, causes the rate of profit to fall.

Now consider what happens to the value rate of profit when production expands so that worse land comes into use. The value role of profit is the total of surplus value divided by total capital. Marx emphasised that rent was also a part of surplus value so

this should be included in the total of surplus value. On the other hand, when one calculates variable capital, it is true that some of workers consumption is of food grown on good land whose individual value is lower than that grown on marginal land so that it is not clear what value should be assigned to this. Somewhat arbitrarily I will assign to all food the value of food produced on marginal land, that is the socially necessary labour.

Now suppose that there is good land and bad land and that initially only the good land is used. Then production expands and moves on to the bad land. There are two distinct phases. In the first one a very small percentage of the land used is bad land. In this movement the only change that occurs is that the social value of food increases. This has two effects: First the value rate of profit falls in both sectors which causes the value rate to fall. But second, the difference in the increase in variable capital between the two sectors changes their weights so that, if the weight of the sector with the higher sectorial profit rate rises, the aggregate rate may rise. If the second effect is strong the value rate of profit may rise in the first phase. In the second phase the percentage of all land that is bad grows. It is possible that the value rate of profit is higher on this than on the good land in which case there is a slow rise in the value rate of profit. This is summarised as result 4.[16]

Result 4. For the model just described with good and bad land, the movement on to bad land will have the following effects: First it will cause the price rate of profit to fall. Second, the value rate of profit may fall, but it is possible that it rises both when the bad land is initially brought into use and during the period when it becomes progressively more important.

5. Conclusion.

A summary of these results is provided in tabular form. In some cases the movement of the rate of profit depended on the values of the parameters and the size of the movements of the input coefficients. This is indicated by the expression "In some cases".

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[15] The proof is straightforward and is given in appendix 2 of Petith (2001).
[16] The proof is again straightforward and is given in appendix 3 of Petith (2001).

	Correct	In some cases	Incorrect
Luxury goods: value	x		
price			x
Degradation : value		x	
price	x		
Slower development		x	
Movement: value		x	
price	x		

In assessing this one should note that Marx's own aim was to show that the rate of profit must eventually fall. Thus he was successful only when correct appears. In this sense Marx's arguments are valid[17] in three out of four cases for at least one of the rates of profit.

IV.   Part III of Volume III of Capital.

Part III, for short, is generally taken to be the main source for Marx's explanation for the falling rate of profit. This section of the present paper gives a somewhat different interpretation. I will argue 1) that Part III does not contain an explanation of why the rate of profit falls, 2) that it contains clear references to previous work in which resource scarcity figured as a cause of the falling rate of profit and 3) that its purpose is to describe the consequences of the fall. The section finishes with a hypothesis about Marx's beliefs that makes these three points part of a consistent whole.

1.   The Absence of Explanations.

At first glance, the point that the section contains no explanations of the falling rate of profit is hard to sustain since there are three clear candidates. First, the rise in the rate of profit of the innovator and the eventual fall of the general rate that is mentioned in the last paragraph of p.233 and set out clearly starting on p.264. second, the constant rate of surplus value, rising composition of capital argument that opens the section on

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[17] Valid here means in terms of the framework that Marx was using. The question of whether capitalists would introduce a technique which lowered the profit rate does not seem to have occurred to him at this point. In any case this is more relevant for the intrinsic explanation rather than for the subsidiary ones. For example, if resources degrade, capitalists cannot return to the more profitable technique.

pp.211-6. And, finally, the falling maximum surplus value argument[18] which is mentioned many places like lines 7-13 p.235 and stated forcefully on p.247.

The first two are easily disposed of: the innovation argument is merely stated as a claim and no justification what so ever is given. The constant rate of surplus value rising composition of capital argument first appears as a formal demonstration. But immediately afterward it is admitted that the rate of surplus value may rise and it is merely claimed that this will be insufficient to overturn the result. Thus the first arguments reduce to unjustified claims.

The third, falling maximum surplus value argument, is more difficult to discard since the reasoning that supports it is apparently presented. However we know from the General Law section that Marx was aware that the reasoning he presented here was faulty. Thus this properly should be taken as an indication of the way Marx thought the eventual demonstration would go rather than a correct demonstration itself.

2. References to Resource Scarcity.

There are a number of sections of Part III that are taken virtually verbatim from longer sections in either TSV or the General Law section where the longer sections are arguments for the falling rate of profit based, usually, on resource scarcity.

First, in Part III Marx writes "Outside a few cases (for instance, if the productiveness of labour uniformly cheapens all elements of constant and variable capital) the rate of profit will fall, in spite of the higher rate of surplus value" p.226. But in the General Law he wrote "Only if productive power were to increase evenly in all branches of industry which directly of indirectly provide products for workers consumption could a proportional growth in surplus value correspond to a proportional increase in productive power" p.135. Marx later states correctly, as noted above[19], that this is a condition for the rate of profit not to fall.) He then gives a number of reasons that this doesn't happen, one of which is the deterioration of the resource base.

Next, in Part III on p.236 Marx wants to argue that the composition of capital will increase. Certainly the mass of capital increases relative to the number of workers but this may be outweighed by the fall in value of the constant capital. Specifically he takes the case of the modern spinner who works up more cotton with more machinery than his predecessor with a spinning wheel. However both the cotton and the machinery now

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[18] By this I mean the argument that, if the labour supply is reduced, surplus value must eventually fall in spite of an increase in productivity.
[19] See II.2.b) first paragraph.

have lower value. Marx states without argument that this will only slow the rise in the composition of capital. But on pp.365-9 of TSV III Marx deals with exactly the same spinners in much greater detail. He gives a complicated explanation about why the mass of machinery grows faster than the value falls. And then goes on for a full page about how growing resource scarcity means that the value of the raw materials can not fall as fast as their mass rises.

Finally there is a long section in Part III p.260, lines 7-19, that has been copied virtually verbatim from the General Law section, p.135, lines 10-22.

Volume III p.260	General Law p.135.
The fact that the development of productivity in different lines of industry proceeds at substantially different rates and frequently even in opposite directions is not due merely to the anarchy of competition and the peculiarity of the bourgeois mode of production.	Productive power increases in very different proportions in these different branches. Contrary movements often take place in these different spheres (This is due partly to the anarchy of competition and the specific nature of bourgeois production,
Productivity of labour is also bound up with natural conditions	partly to the fact that the productive power of labour is also tied to natural conditions
which frequently become less productive as productivity grows-inasmuch as the later depends on social conditions.	which often become less productive in the same proportion as productivity rises in so far as it depends on social conditions.)
Hence the opposite movements in these different spheres-progress here and retrogression there.	so that the productivity of labour rises in one sphere while it falls in another.
Consider the mere influence of the seasons, for instance, on which the bulk of raw materials depends for it mass, the exhaustion of forest lands, coal and iron mines etc,.	Think, for example of the simple influence of the seasons, on which the greater part of all the raw products of industry depends, the exhaustion of forests, coal seams , mines and the like.

The quote from Part III appears as the first paragraph of section IV, supplementary remarks. It is followed by another unconnected paragraph and then a long interposition

by Engels. On the other hand, as I argued above[20], the section in the General Law is part of a coherent argument that resource scarcity is one of the causes of the fall in the rate of profit.

In general it is difficult to escape the feeling that Marx make a conscious effort to avoid emphasising the role of resource scarcity.

3. The Consequences of the Falling Rate of Profit.

I support the claim that Part III is about the consequences of, rather than the reasons for, the fall in the rate of profit with a brief sketch of the three chapters. Chapter XIII, The Law a Such, opens with a five page claim that the rate of profit will fall. Then for the next fifteen pages, with the exception of one paragraph, it busies itself with a description of how the various value quantities change as the rate of profit falls with capitalist development. Chapter XIV, Counteracting Influences, then spends eight pages explaining why the rate of profit falls only slowly. Finally Chapter XV, Exposition of the Internal Contradictions of the Law, with the exception of two paragraphs, uses 19 pages to describe crises and the role played by the falling rate of profit. It finishes, in a somewhat raised tone, by saying that the falling rate of profit is the barrier that shows that capitalism is only a transitory mode of production.

4. Conclusion.

If one supposes that Marx still hoped to be able to demonstrate the intrinsic explanation satisfactorily then these three aspects coalesce into a consistent view of Part III. First the absence of a demonstration and the frequent reassurances follow immediately from the supposition. Second, that the topic was sufficiently important for Marx to set out the consequences before he had a firm demonstration is shown by his final conclusion. Finally, he occasionally drifted toward explanations which involved resource scarcity but pulled himself back since this explanation would have considerably weakened the main point he wanted to make. Moreover this was not dishonest since Marx believed that he would be able to give a proper demonstration of the intrinsic explanation. All this, it must be stressed, is based on supposition; but it does afford a coherent explanation of the main characteristics of Part III.

V. The Argument of the 1870s.

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[20] See paragraphs 2 and 3 of section II.2.b).

Marx made another attack on the falling rate of profit in his Mathematical Investigations of the 1870s. These were left in a disordered state and contained a large quantity of mathematical calculation.[21] Engels got a Cambridge mathematician friend to put the parts that were relevant in order and inserted them as Chapter 3 of Volume III of Capital. In it Marx takes the formula $=S/C=S/(C+V)$ and for twenty pages systematically varies individual variables or combinations of them and, in each case, observes the effect on the rates of profit and surplus value. The tediousness of the exercise makes it understandable that the chapter has lain un-discussed for over a hundred years, but there is gold buried beneath the detail.

The key example is on pp.56-8.

"Now, the variable capital may either rise of fall. Let us first take an example in which it rises. Let a certain capital be originally constituted and employed as follows:

$$I. \qquad 100_c + 20_v + 10_s; \ C = 120, \ s = 50\%, \ p = 8\tfrac{1}{3}\%.$$

Now let the variable capital rise to 30. In that case, according to our assumption, the constant capital must fall from 100 to 90 so that total capital remains unchanged at 120. The rate of surplus-value remaining constant at 50%, the surplus-value produced will then rise from 10 to 15. We shall then have:

$$II. \qquad 90_c + 30_v + 15_s; \ C = 120, \ s = 50\%, \ p = 12\tfrac{1}{2}\%.$$

Let us first proceed from the assumption that wages remain unchanged. Then the other factors of the rate of surplus-value, i.e., the working-day and the intensity of labour, must also remain unchanged. In that event the rise of v (from 20 to 30) can signify only that another half as many labourers are employed. Then the total value produced also rises one-half, from 30 to 45, and is distributed, just as before, 2/3 for wages and 1/3 for surplus-value. But at the same time, with the increase in the number of labourers, the constant capital, the value of the means of production, has fallen from 100 to 90. We have, then, a case of shrinkage of constant capital. Is such a case economically possible?

In agriculture and the extractive industries, which a decrease in labour productivity and, therefore, an increase in the number of employed labourers is quite comprehensible, this process is-on the basis and within the scope of capitalist production-attended by an increase, instead of a decrease, of constant capital. Even if the above fall of c were due merely to a fall in prices, an individual capital would be

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[21] The original work is available in German and French but not English. See Marx (1974) and Alcouffe

able to accomplish the transition from I to II only under very exceptional circumstances. But in the case of two independent capitals invested in different countries, or in different branches of agriculture of extractive industry, it would be nothing out of the ordinary if in one of the cases more labourers (and therefore more variable capital) were employed and worked with less valuable or scantier means of production than in the other case."

Marx now supposes the rise in v is due to a rise in wages and at the same time the working day increases proportionally. Then he returns to the original situation. " Now let us assume that the variable capital falls, instead of rising. Then we have but to reverse our example, taking II as the original capital, and passing from II to I.

II. $90_c + 30_v + 15_s$ then changes into

I. $100_c + 20_v + 10_s$, and it is evident that this transposition does not in the least alter any of the condition regulating the respective rates of profit and their mutual relation.

If v falls from 30 to 20 because 1/3 fewer labours are employed with the growing constant capital, then we have before us the normal case of modern industry, namely, an increasing productivity of labour, and the operation of a larger quantity of means of production but fewer labourers. That this movement is necessarily connected with a simultaneous drop in the rate of profit will be developed in the third part of this book."

I will make all my comments in terms of the second part of the example in which the composition of capital grows and the number of workers falls. This means that Marx's comments on the first part must be transposed. At first glance Marx's worries at the end of the third paragraph are difficult to understand. The constellation of a fall in labour, a rise in capital and a rise in labour productivity seems entirely natural. But if one supposes that Marx had the assumed constancy of the rate of surplus value in his mind, the source of his disquiet is clear. The rise in capital per man which raises productivity ought to increase the rate of surplus value.

One can also see the answer he is groping for. In the next paragraph he immediately mentions agriculture and the extractive industries. He then notes that it is unlikely that the change could occur in a given sector but that, for example, one might observe it between the same sector in two countries. It would seem that he is thinking of the following. If we compare agriculture in a country with fertile land and labour intensive cultivation with one with less fertile land and mechanised production, the rate of surplus

(1985).

value has not risen in spite of the mechanisation because of the lower fertility. The following formal analysis confirms the precision of Marx's intuition.

The structure of the argument is as follows. The value magnitudes are known, from these the physical magnitudes can be calculated, then a relation between output per man and capital per man is supposed and this shows that the rate of surplus value determines the composition of capital. The only way in which the composition can increase with the rate of surplus value constant is if a resource parameter is introduced into the production relation which degrades as the composition increases.

The details are as follows: the basic model consists of the four equations:

$$C = Ax$$
$$V = bLx$$
$$S = (1 - bL - A)$$
$$= L/(1 - A)$$

where C, V and S are known, b is given, x, the activity level, can be set to 1, and the unknowns are L, A and . The first three equations reduce to

$$k = \frac{A}{bL}$$

$$e = \frac{1 - bL - A}{bL}$$

where $k = C/V$. These give

$$L = \frac{1}{b(e + k + 1)}$$

$$A = \frac{k}{e + k + 1}$$

$$= \frac{1}{b(e + 1)}.$$

Write $A = K/Y$ and $L = \tilde{L}/Y$ where K, $\tilde{L}$, and Y are machines, labour and output in physical units. Set $b=1$, $K/\tilde{L} = A/L = k$. $Y/\tilde{L} = e + k + 1$. Suppose there is a technical relation between $Y/\tilde{L}$ and $K/\tilde{L}$, i.e. $Y/\tilde{L} = f(K/\tilde{L})$, or

$$e = f(k) - k - 1.$$

Note that $df/dk > 1$ because the model is one of circulating capital. Thus the composition of capital can not vary if the rate of surplus value is fixed. The way to make the model conform to Marx's description is to redefine the technical relation to include a fertility parameter r, $\tilde{f}(k,r)$ with $\partial\tilde{f}/\partial r > 0$. Then the condition that e is constant gives

$$\frac{dr}{dk} = -\frac{\tilde{f}/k - 1}{\tilde{f}/r} < 0,$$

that is, fertility falls as the composition of capital increases to allow the rate of surplus value to remain constant. The constancy of e ($=S/V$) and the rise of k ($=C/V$) then imply that the rate of profit falls.

The point that gives this section its importance is that Marx associates the example with the fall in the rate of profit described in Part III. If one accepts the interpretation given here, it means that by the 1870s Marx interpreted, for example, the fall in the rate of profit portrayed in the opening five pages of Part III as being caused by growing resource scarcity. And by implication, that he accepted that there was no intrinsic reason for the rate of profit to fall. I think this is overstated, but it hard to avoid the impression that Marx was drifting in this direction.

VI. The Pedigree of Various Treatments of the Falling Rate of Profit.

One can divide the various treatments of the falling rate of profit into four: those associated with the Okishio Theorem, the falling maximum surplus value argument, the rising wage mechanism and, finally, increasing resource scarcity.

1. The Okishio Theorem.

There is a long sequence of demonstrations, including that of   Okishio (1961), that show that Marx's rising innovational rate and eventually falling general rate of profit argument is wrong in terms of a linear, one basic factor model. But various authors append to their demonstrations the statement that Marx did not believe that resource scarcity would be responsible for the fall in the rate of profit. Roemer (1981 pp.87-8) contrasts Marx's view with that of Ricardo, Steedman (1977 pp.129) qualifies his statement with a reference to TSV and most strangely Samuelson (1957 p.894) says that Marx could not introduce land because it would conflict with his belief in the labour theory of value. The earlier part of this paper has shown that these claims are wrong in a straight forward sense, but they do contain an element of truth.

I have argued that Marx hoped that he would be able to provide a demonstration of his intrinsic explanation. What the Okishio authors have actually shown is that it is very unlikely that a demonstration can be provided and that virtually the only one of Marx's explanations of the falling rate of profit that remains viable is that which involves growing resource scarcity.

2. The Falling Maximum Surplus Value Argument.

Many authors mention this argument and justify it by citing Marx as an authority. Examples are Rosdolsky (1977 pp.408-10), Shaikh (1978 pp.240) and Okishio (1977). It is true that one can find this argument referred to many times in Marx's work. But, as has been detailed above, the one time that Marx attempted a rigorous demonstration of the falling rate of profit by this route, he failed. I have interpreted Marx's numerous references to the argument that appear after this failure as indications that he still thought the explanation could be demonstrated. If one accepts this interpretation, then it is a misrepresentation of Marx's position to cite him as an authority for this result.

3. The Rising Wage Argument.

There are a number of authors who have constructed Marxian models in which the mechanism that produces a long run falling rate of profit involves a rising wage. Examples are Laibman (1977), Foley (1986), Skillman (1997) and Dumènil and Lèvy (2001). Foley (pp.138-9) and Dumènil and Lèvy have actually argued that this reflects Marx's thought.

The principle source for this is the opening five pages of Part III where the rate of surplus value is held constant while the composition of capital rises. It has been know since the writings of Natalie Moszkowsha in the late 1920s that this implies a rise in the real wage if the rise in the composition of capital increases labour productivity. However the pedigree of this mechanism is undercut by two arguments. First Marx says in the next chapter of Part III "Nothing is more absurd… than to explain the falling the rate of profit by a rise in the wage" (p.240). Of course this does not carry much weight because he apparently has done exactly that in the preceding chapter. Second, and of much more importance, is that the rising real wage undermines one of Marx's basic building blocks, the value of labour power. Thus the argument in favour of the rising wage is problematic.

The contribution of this paper is to add two more negative points. The first is that in the General Law section, when Marx was trying to rigorously demonstrate the falling rate of profit, he was at great pains to keep the real wage constant. This is strong evidence that he did not associate the falling rate of profit with a rising wage. The second is that later, when Marx was reworking the arguments of Part III, he gave an explanation of the constant rate of surplus value in terms of increased resource scarcity

rather than rising wages. I think that these four reasons make it very unlikely that Marx had in mind a rising wage as a cause of the long run fall in the rate of profit.

4. The Resource Arguments.

The arguments of those writers who hold that resource scarcity was an important aspect of Marx's position on the rate of profit are not well known. For example, of the nine economists who responded to Brewer's (1995) purposely polemical attack, Hollander (1995) alone cited this as a factor. For this reason I will set out the views of this group in slightly greater detail.[22] It is convenient to relate these to the present paper via three issues. First, how is the argument substantiated? Second, is it claimed that this was Marx's main idea? And third, why is there only a slight mention of resources in Part III? At least six authors have written on this: Rosdolsky (1977) (He first mentioned the idea in (1956)), Meek (1967) (First appeared in (1960)), Lebowitz (1982), Perelman (1985), Moseley (1991) and Clarke (1994). Of these, Meek refers to Rosdolsky and Moseley to Lebowitz so only four have to be considered.

The first of these is Lebowitz (1982). He argues that there are barriers that are specific to capitalism and general barriers, and further that the rate of profit falls when a barrier is encountered. He gives a failure of aggregate demand as an example of a specific barrier. The main example of a general barrier is resource scarcity. He argues in a loose fashion that the rate of profit $S/(C+V)$ will fall if the ratio of dead to living labour $C/(V+S)$ rises and that this will rise if productivity rises more slowly in the sectors that produce the means of production. He then states that Marx believed that productivity would grow more slowly in these sectors because of resource problems and supports this claim with quotes from a large selection of Marx's writings. Finally he makes no comment on the absence of resources from Part III.[23]

The second author is Perelman (1985). His principle theme is not the fall in the rate of profit, but rather the development of Marx's view of resource scarcity as an economic problem. Marx thought that the role of capitalism was to develop the economy sufficiently so that socialism would become feasible and feared that resource problems might make this impossible. Until the cotton famine of 1862 he was optimistic

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[22] Although Hollander's main paper on the theme (1991) includes many quotes concerning resources, he does not appear here. The reason is that his conclusion, if I understand it correctly, is that Marx does not convincingly link resource scarcity to the falling rate of profit.

[23] Three additional points. First there are long quotes from the Cherbulitz section but nothing from that on Hodgskin. Second in Lebowitz (1976) he states that chapter 3 of volume III of Capital contains no argument for the falling rate of profit. Third, this is a good example of a rational reconstruction.

about the possibility that investment and scientific progress would resolve the problem. After this he changed his mind. (All Marx's writings referred to above date from after the change.) Perelman thinks that Marx thought that the falling rate of profit was connected with resource scarcity but doesn't try to locate specific arguments because he thinks that Marx had reasons not to state them explicitly.

"Scarcity may, in fact, play a much more significant role in Marx's theory than has hitherto been suspected. The frequency with which Marx associated difficulties in producing raw materials, a rising organic composition of capital, and a decline in the rate of profit provides a further clue to what Marx's intentions may have been. To have noted the importance of scarcity, even in a rich dialectical fashion, apparently meant less to Marx than the discovery of an important law of the falling rate of profit. Moreover, the issue of scarcity opened up serious political risks, not to mention that it could lend support to the enemies of socialism. Consequently, scarcity was downplayed, while the supposed law of the falling rate of profit was emphasised." (p.485)

The third author, Roman Rosdolsky (1977) is the first author to try to locate Marx's specific arguments. He reacts to the criticisms of Joan Robinson and Paul Sweezy that Marx supplied no argument that the composition of capital would rise and that he failed to take into account the rise in surplus value by citing first the section on Cherbulitz and then the one on Hodgskin (p.407-9). His interpretation of Hodgskin is non-analytic and slightly different from the one given above. He regards the section quoted above as two separate arguments: he presents the second paragraph as an argument based on resource scarcity and the first as a separate argument based on a falling maximal surplus value. (It must be admitted that Marx's statements are sketchy enough to bear either interpretation.) Finally Rosdolsky, probably because of his encyclopaedic knowledge of Marx's writing, sees no reason to explain the absence of resources in Part III.

The last author, Simon Clarke (1994), is the first of the six authors to have had access to the section of the General Law and the only writer, to my knowledge, to have subjected it to detailed analysis. He starts (p.213) by citing the criticisms that have been aimed at Marx, that the composition of capital may not rise, that the rate of surplus value will rise and that capitalists will not introduce a technique that lowers the profit rate. Clarke seems to accept that these criticisms undermine Marx's discussion but says that "Marx was not trying to develop a mechanical law, but locate the tendency for the rate of profit to fall within a wider context of the tendencies of capitalist development."

Clarke then turns to the first two criticisms. With regard to composition of capital he cites principally the section on Cherbulitz. With regard to the second, to defend Marx against the criticism that the rise in the rate of surplus value was not taken into account in the Part III argument, Clarke notes that Marx does mention the rise and claims that this is a reference back to earlier work that Marx would have assumed that the reader would be familiar with (p.217). Clarke then gives a series of quotes from the Hodgskin section and the resource part of the General Law section to show that Marx thought that resource problems would be responsible for the slower rise in the rate of surplus value.

Each of these works can be criticised in terms of the three issues from the standpoint of the present paper. That standpoint is that, first Marx made a specific argument that resources would be a cause of the falling rate of profit, that this cause was not the only one or even the most important and finally that Marx did not emphasise resource scarcity because the intrinsic explanation, which he still hoped to demonstrate, fitted better with his general philosophy.

Consider the first issue. Both Lebowitz and Perelman are weak here since neither cites a specific argument of Marx to support their claim. Although Perelman gives a reason that one should not be able to find one. Rosdolsky, just as the present paper, cites the Hodgskin section. The present paper goes further in providing an analytic presentation of Hodgskin and descriptions of the General Law and chapter 3 sections. Finally Clarke cites both the Hodgskin and the General Law sections and mentions chapter 3. However first he claims that Marx was not trying to find a mechanical law which belies, in my opinion, the huge effort Marx made in the General Law section to find what seems best described as a mechanical law.[24] And second he portrays the Hodgskin and General Law sections as if they were a single argument while it seems to me that they are distinct. However these are quibbles, the advance of the present paper is the analytic presentation and the discovery of chapter 3.

Now consider the second issue. Lebowitz, Perelman and Clarke all seem to rest their entire justification of Marx's theory on resource scarcity. It is not clear why since they were all undoubtedly aware of Marx's falling maximum surplus value argument. It is possible that it was left out because of its evident falsity in the post-Okishio theorem era. Furthermore, with regard to Perelman, Marx's main argument was far from the central theme of his paper. But Clarke's failure to mention Marx's titanic twenty page

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[24] Perelman's comments (1985 pp.484-5) and the references cited there are interesting in this context.

struggle in the General Law section must be considered a gap in a work that aimed at a general description of Marx's ideas. Finally Rosdolsky, alone, mentions the falling maximum surplus argument and in fact gives it great prominence. If anything he can be faulted for his unquestioning acceptance of its correctness. It is doubtful that he would have revised this attitude even if he had had access to the General Law section since his attitude towards Marx was generally uncritical.

Finally consider the third issue. Lebowitz and Rosdolsky do not attempt to explain the absence of emphasis of resources in Part III although Rosdolsky does imply that any reader worthy of the name should have been aware of the TSV material. Clarke argues that Marx would have assumed that the reader would refer back to his earlier work. In direct contrast, both Perelman and I argue for different reasons that Marx consciously suppressed references to resource scarcity in Part III. Stated boldly thus, it sounds like an accusation of intellectual dishonesty.[25] But what Perelman really is saying is that Marx chose an intellectual structure that would get him where he wanted to go and, as anyone who has done research knows, any other approach leads to chaos. My claim is merely that Marx suppressed the less important to heighten the impact of the more important, surely legitimate dramatic licence. In general the absence of emphasis of resources in Part III can not be taken to imply that Marx did not think that resource scarcity would be a cause of the fall of the rate of profit.

To summarise: the contribution of the present paper to this branch of the literature are, analytic presentations of Marx's arguments, a new reason for the absence of emphasis in Part III and the discovery of the argument of chapter 3.

VII. Conclusion.

The principle conclusions, which have been summarised in the introduction, indicate a direction for further research. The ideal Marxian model would be one in which the rate of profit fell, the composition of capital increased, the rate of surplus value rose and the wage was equal to the value of labour power; all this generated by some mechanism related to what I have called the intrinsic explanation. However this may not be possible. The attempts involving the temporal single system are in their infancy and look like facing extreme technical difficulties. The recent models involving rising real

wage are further advanced and are interesting descriptions of the actual economy. But viewed as Marxian, the rising real wage makes their pedigree doubtful. In this context there is a justification for developing models with Marxian characteristics that involve resource scarcity. Even though they would not be based on the intrinsic explanation that Marx had hoped for and thus are, one might say, second best, their key element would be one which figured strongly in Marx's accounts of the falling rate of profit.

Appendix 1. The completion of pages 995-998 of notebook XVI.[26]

Let us first assemble the facts.

$C=v+c$. $s=$ surplus value. $s'=$ rate of surplus value. $p'=$rate of profit. $s'=s/v$, $p'=s/C$ or $s/(v+c)$.

[Let us start with the following example.]

$C=800$. $c=200$. $v=600$. $s=120$. In this case, $c=(1/4)C$ and $v=(3/4)C$; $s'=120/600=20\%$. If c increases from 200 to 600, by a factor of three, C will rise from 800 to $1,200$, i.e. *by 50%.*

Since $c=1/4C$, its threefold increase causes it to grow from *1/4* to *3/4 (* by *2/4).* The total is now *(3/4)C+(3/4)C=(6/4)C.* It has

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therefore risen by [50%. Now consider variable capital]. It was originally $=(3/4)C$ *(=600),* so if it is tripled this brings it from *3/4* to *9/4*, from *600* to *1,800*, and it brings total capital to *2000([200+3x600=2000, (3/4)Cx3=(9/4)C)* which increases it] over and above the original capital *(6/4)C=1,000, (1,200+800=2000).* How far therefore the total capital becomes [larger with the] growth in c depends on the original proportion between c and v [as parts] of C. So the greater the proportion of $c:v$, or of $c:C$ $(c+v)$, the more does the total amount C grow though [a given proportional increase in c and] the more does the rate of profit fall and the greater is the growth in the rate of surplus value required for the rate of profit to remain the same. [Or the greater is the fall in the rate of profit caused by] the growth of total capital if the rate of surplus value is given.

[a)] In the case of an increase of C from *800* to *2,100*, of c from *200* to *600*, the constant capital is tripled and the total capital grows by [*400*, that is] by 50%. In this

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[25] A referee in a much earlier version of this paper thought that I had misrepresented Perelman. But careful rereading has convinced me that I have not. This is the reason for including the quote in the Perelman sketch.

[26] The square brackets indicate mainly my additions but occasionally those of the editors as well.

[27] Of Volume 33 of the CW.

case the rate of surplus value or *s'* continues to be 20% and *s=120*. But *p'=120/1,200=10%*. Surplus value and the rate of surplus value [have remained constant, but profits and the rate of profit] have fallen from *15* to *10,* i.e. by *1/3* or *33⅓%*. Why is there this difference, the rate of profit falls by *33⅓* [while total capital] grows by *50%* ? Because the relation of the rate of profit expresses it self as the inverse of the relation of the two capitals we have compared. [Total capital is either *800*] or *1200*. This growth is from *800:1,200=2:3* hence from *2:(2+1)* or by *50%*. The fall in the rate of profit expresses itself inversely, as a fall of [a fraction] from *120/800:120/1200=3:2;* hence as a fall of *1/3* of *33⅓%.*

The fall in the rate of profit therefore depends directly on the growth in the total capital, if the variable capital remains the same; its fall expresses itself in inverse proportion to the growth of the capital. If this grows from *2:3,* the rate of profit falls from *3:2*. Furthermore, if the variable capital remains the same, the growth of the total capital can only derive from the growth of the constant capital. However, the proportion in which a particular increase in constant capital causes the total capital to increase depends on the original ratio between *c* and *C* . This inverse relation explains in part why the rate of profit does not fall in the same proportion as the capital increases, even if the rate of surplus value remains the same. If *2* increases to *4* , that is a growth of *100%*. If *4* falls to *2,* that is a fall of *50%.*

b) If in the second case indicated above the rate of profit is to remain to same, the profit, hence the surplus value, will have to rise from *120* to *180* i.e. by *60* or *1/2* of *120,* rise by half its original magnitude. The surplus value would therefore have

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directly to grow in the same proportion as the total capital, by *50%,* therefore rising in a greater proportion than the fall in the rate of profit, surplus value remaining the same.

If *c* had risen to *1,200* instead of *600*, the total of capital would have risen to *1,800* for *C* would have risen by *1000,* hence by *125%*. [For the rate of profit to] remain the same, the total amount of surplus value = the total profit, would have had to rise to *270.* But *270:120* must [imply] a growth of *150* [in surplus value] or *125%* on top of *120. 120* on *120* is *100%,* and *30* on *120* is *1/4* or *25% (4x30=120),* [thus the total percentage change in surplus value is the same as the percentage change in total capital, *125%.*]

c) How in this case (b) would *s'* or [the rate of] surplus value have risen?

It was originally *120/600=20%* or *1/5* of variable capital. If capital grows to *1200* or *c* tripled, *180/600* or *30%*   or [in the proportion *30:20* which is the same as the proportional growth of total capital.] In the third case, if capital grows to *1800*, [the rate of surplus value is] *270/600=9/20* of the variable capital=*45%*. In [the preceding case the rate of] surplus value has risen from *20* to *30%* i.e. by *50%,* to the same degree as total capital has grown in this second case and the absolute surplus value [as well. Now consider this third case, the rate of surplus value has risen, in this] case, from *20* to *45* i.e. by *25*; but *25:20=11/4*   *(20+ 1/420* or *5)* hence *125%.* (This [calculation is based] on the growth of the increment not the relation of the numbers to each other as such.) The rate of surplus value would therefore have to [grow] directly [as the] total capital grew or in the same proportion as the absolute surplus value would have to grow for the rate of profit to remain unaltered with a growing [total capital stock. Finally consider a forth case in which constant capital grew to *3000* and total capital to *3600.*]

	Variable cap.			Constant cap	
Case I	600	Out of total	800=3/4C	`"`	200=1/4C
Case II	600	`"`	1200=2/4C	`"`	600=2/4C
Case III	600	`"`	1800=1/3[C]	`"`	1200=2/3C
Case IV	600	`"`	3600=1/6[C]	`"`	3000=5/6C

Surplus value or profit had to increase to *540*; the rate of surplus *=540/600, 9/10* or *90%. 90%* against *20* [*%* originally, an increase] of *70*. But *70* to *20* would be *350%.* The increase of capital would be *3600-800=2800,* similarly [*350%.*] In this case the rate of surplus labour *=9/10* of the total working day, hence given *10* hours of labour, *9* hours [are worked for the capitalist. The rise in the sum of constant and variable capital], although entirely corresponding to the growth of total capital, now express the rate of rise and fall inversely in the same value expression as the capital [increases]. If the capital rises from *2* to *4*, the rate of profit falls from *4* to *2*. The other rises by *100%,* that is the rate of surplus value must rise from *8* to *16* to keep the rate of profit constant.

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[Surplus value] and the rate of surplus value, which is an identical relation if variable capital remains the same, does not grow as capital [falls in relation to] total capital. There is absolutely no reason why the rise of productive power should observe exactly the same numerical ratio. It [insures that the size] of relative surplus value grows and

its growth is expressed in the ratio of the reduction in variable [to total capital], but not in the same ratio as this proportion declines. Productive power grows, hence surplus labour. Firstly there lies here [the heart of] the matter. One man may produce as much use value as 90. Never more than an average of 12 hours a day in value is [produced by a single worker], as this [yields] surplus value (of) never more than 12 hours -*x* , where *x* expressed the labour time necessary for his own production. The surplus value, [which is part of the total value the single worker produces, is limited by] the labour time which he himself works, not by the working days he replaces. If 90 men only worked only 1/2 an hour of surplus time a day, this would be [45] hours. If one man needed only one hour of necessary labour time he would never [produce] more than 11 hours of surplus value. [The proc]ess is double. It increases surplus labour of the working day but it also reduces the numerical coefficients of those working days, [all this with the fall in variable] capital. Secondly: The development of productive power is not uniform; certain branches of industry may themselves be more unproductive [this might seem to lower their individual profitability] but this is determined by the general productivity of capital.

[Generally absolute accumulation will increase as total capital increases even if the rate of profit falls. If we start] firstly at a stage of production which remains the same, without great revolutions in productive power, in proportion to its existing [variable capital, the quantity of constant capital is low. This case, with initial capital equal to *1* and a rate of profit equal to *100%*] only gives rise to a total capital of *2*, whereas *1000* at *10%* gives *1100. 1100* prod[uced is much larger than *2* although the rate of profit is lower. Now take a more detailed ex]ample of *800, v=600, c=200* and surplus value=*160* or a rate of profit equal to *20%*, a capital of *100,000* would give[*120,000* but, staying with a capital of *800,* this produces *160* . If as a proportion of total capital there was,] instead of *3/4*, only *1/6* variable (*3/4=18/24* and *1/6=4/24*) hence employs *14/24* or *7/12* less variable capital, relatively speaking [a fall of *18/24* to *4/24* is approximately a fall of *3/4* so that *1/4* of *20%* equals *5%*. Calculating the profit for the capital of *100,000* at] *5%* continues to be *5000*. His variable capital and the living labour employed by it would still be *16,666₁/₆* in total amount, hence [reverting to the first case with an initial capital of *800*, although a smaller proportion of the total capital,] it would still be nearly *28* times greater than the capital employed in the first case. But the rate of profit is determined, because the rate of surplus value is determined, by the ratio of the variable capital to the total capital. At simple interest

£100,000 would grow into *200,000* in *20* years, whereas *800* at *20%* would only produce an accumulation of *3,200* in *20* years (*160x20*.) In the second *20* years *200,000* at *5%* would grow to *400,000*. The other capital at *20%*, in contrast, would only grow to *12,800*.

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[   ] As a rule // see under surplus value for the exception: intensification of labour and therefore in fact increase of labour by machinery // machinery only creates relative surplus value through the curtailment of necessary labour time and therefore the prolongation of surplus labour time. This result is brought about by the cheapening of the commodities which enter directly of indirectly into the worker's consumption.

Surplus value is formed by two factors. Firstly the daily surplus labour of the individual worker. This determines the rate of surplus value, hence also the proportion in which variable capital is increased through the exchange with living labour. Secondly, the number of workers simultaneously exploited by capital or the number of simultaneous working days,

If the rate of surplus value is given, the magnitude of the surplus value-the surplus value itself as an independent magnitude-depends on the number of workers employed. If this [number and the number of simultaneous]working days is given, the magnitude of the surplus value depends on its rate.

[Machinery] now evidently has a tendency to affect the two factors of surplus value in opposite directions. It increases the rate [of surplus value and] reduces the number of workers// relatively anyway; with respect to a definite measure of capital, e.g. per cent //, whose labour [by this means] is exploited at an increased rate.

[Originally there were *12* workers and] each one provided *1* hour of surplus labour a day. By the employment of machinery *6* workers should each provide *2* hours of surplus labour a day [or a total of *12* hours a day.] In this case *6* workers provide *12* hours of surplus labour, just as previously *12* did. The time during which the *12* workers [work] every day, assuming [a norm]al working day of *12* hours, [can] be regarded as a total working day of *144* hours, of which [*132* hours are necessary labour] time, *12* surplus labour time. In the second case the total working day consists of *72* hours, of which *60* are necessary labour time, [*12* surplus labour time]. Since a total working day of *72* hours now contains as much surplus labour as the day of *144* hours, in the latter case [*6*

workers] appear [to be use]less, superfluous for the production of *12* hours of surplus value. They are therefore suppressed by the employment of machinery.

[We are considering a process]-which lies at the basis of all growth in relative surplus value-prolongation of surplus labour time through [curtailment of necessary] labour time; however, a process which was only employed previously in regard to the working day of the individual worker is now employed [on a quantity of hours] composed of the sum total of the working days of the workers simultaneously employed. The

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retranchement now takes [place as the process develops.] In the first case the sum total of hours of labour remains the same. It is merely their division between necessary and surplus labour, [between the time worked for themselves and for the capitalist,] which is altered. But now there is a change not only in the division of labour time but also in the sum total of labour time employed.

[*72* hours out of the] total working day of *144* hours e.g., which is no longer necessary, since the employment of machinery, to [produce] *12* hours of surplus labour. Superfluous, useless labour is removed. From the capitalist standpoint all labour is useless, i.e. unproductive, which is not necessary [for the production of surplus value,] which would therefore be required for the mere reproduction of the worker himself. In the above example *72* [hours were no longer necessary,] i.e. *6* days of labour. I.e. *6* of the *12* workers are dismissed. In the first case the magnitude remains [the same, that is the living labour remains (the actual] hours contained in it) the same. The division alone has changed. In the second case the magnitude changes-the total amount [of living labour changes and] the division of the same. In the first case the value remains the same while the surplus value increases. In the second case [there is a reduction] at the same time (of) the labour time objectified in the product, while the surplus value increases.

[In the first case a process] of simple co-operation and division of labour takes place. This is as with [an increase in production caused by having the workers use a better tool. The second case is different.] Relatively to the product, [that is to a fixed number of units], the number of workers is reduced[. Also there is a reduction of the number of] workers [relative to a fixed total] capital *C* [and also relative to] constant [capital *c* so that, regarding labour,] with machinery an absolute reduction (with regard to a particular capital) takes place. In certain branches of industry, agriculture [is an

exception, this] reduction is in fact always in advance, without being checked as in other branches of industry by the circumstance that, at the new rate [of introduction of machinery, the] old number of workers may be successively absorbed. But even an absolutely greater although relatively much smaller [number of workers would not change the general situation.]

The way in which the rate of profit is altered even in the case considered above, where the rate of surplus value grows in the same (or [a greater proportion]) than the fall in the number of workers, hence the fall in one factor finds compensation in the growth of the other through more [productivity]-hence the magnitude of the surplus value remains unchanged or even grows-depends on the proportion in which [the ratio between surplus value and total capital] is [affected by] a change in the components of total capital or on the proportion in which the change proceeds. [There is a tendency for the amount of machinery to increase relative to the number of workers employed. On the other hand,] the surplus value the capital makes can only derive from the number of workers it exploits, or from the number of workers who [remain employed after the new machinery has been installed. Modern] society-alias the class of capitalists as a whole-is affected by the setting free of the workers he has dismissed, [the ratio of machinery to employed workers rises for the economy as a whole.]

It is now an entirely self-evident general law that with the

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progressive employment of machinery, the magnitude [of surplus value can not] remain, but must fall; i.e. that the reduction of the number of the [workers] (in relation to a particular measure of capital) [must cause a fall in surplus value. The] reduction in number cannot be continuously counter balanced by a corresponding increase in the rate of surplus value. [There are limits to the part of] the working day of the individual worker (which) is exploited.

Assume that *50* workers provide only *2* hours of surplus [labour]; in that case the surplus value created by them $=100$. Assume further [that] if *10* men were replaced by *1*, *5* would replace the *50*. The total labour time $=5x12=72$ hours. The same for the total value of their product. The surplus [value] created by them [is]<than *72*, since only equal to *72* – the necessary labour time. Hence it is <than *100* by much more. There therefore takes place [a change], so large that the reduction in the absolute amount of labour which is employed, [cannot be compensated for] by an increase of equal size in the rate of surplus value-where surplus value therefore falls in spite of the growth in the

rate of surplus value. [This is the essential point.] A fall in the amount of surplus value-
or the total amount of surplus labour employed-must necessarily come about with the
development of machinery [and a reduction of the labour employed on it. By this] it is
[shown] here that capitalist production enters into contradiction with the development of
the productive forces and is by no means their absolute mode and final form.

//If the *50* workers could all be employed at the new rate, or even only   *25* perhaps,
surplus value would grow, and not only its rate, as compared with the earlier case.
Hence the importance of the scale on which machinery is employed, and its tendency to
employ as many workers as possible at the same time, combined with the tendency to
pay for as few necessary working days as possible.//(50) (150)

) Let us assume a capital of *600*. Let *400* of this be laid out in labour, *200* in
constant capital, instruments and raw material. Let the *400* represent *10* workers. If a
machine were to be employed, which together with the raw material=*520*, and if the
capital laid out in labour were only to be *80* now, *10* workers would be replaced by *2* or
5 by *1*. The total amount of capital laid out would remain the same, hence production
costs would remain the same. The *2* workers would not produce more surplus labour
time for each *12* hours that the *10* produced , for wages would have remained the same.
Nevertheless, the quantities of commodities produced under the changed conditions of
production might on

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certain presuppositions become cheaper, although it is presupposed that this quantity
has not increased, or that no more commodities are produced with the same capital
under the new process of production than were previously produced under the old one.
Since the same quantity of raw material has been worked on as before, *150*, the
machinery has now risen from *50* to *370*. // Namely *370* machinery, *150* raw material,
80 labour. *370+150+80=600.//*

   Assume now that the machinery employed has a turnover time //reproduction time//
of *10* years. Of the value employed, *37 (370/10)* would enter into the annual output of
commodities for the replacement, wear and tear, of the machinery. The sum total of the
production costs of the commodities //disregarding profit and surplus value here, as the
rate remains the same// would now be=*37+150+80=267*. The production cost of the
commodity under the old process=*600*, whereby we assume that the instruments which
enter into the process (estimated at *50*)must be renewed every year. The price of the

commodities would have been cheapened in the ratio *267:600*. To the extent that the commodity enters into the worker's consumption, its cheapening would bring about a reduction in the labour necessary for his reproduction and thereby an increase in the length of surplus labour time. //But initially, as in any employment of machines, capitalist II would admittedly sell cheaper than capitalist I, but not in the same proportion as his production costs had fallen. This is in fact an anticipation of the cheapening of the production costs of labour which occurs through machinery [. Thus , at first the reproduction costs of workers would not fall by the whole amount. If] his workers receive the same wages as previously, they can admittedly buy more commodities (more of the commodities they themselves have produced) but not in the proportion in which they have become more productive. It would be the same thing if the capitalist paid them in his own commodity, as if he were to give them a quantity which was admittedly larger, but smaller in the proportion to which this quantity expressed exchange value.// Even if we disregard the relation itself, and consider the empirical form, in which the capitalist calculates interest, say *5%*, on his total capital according to the part of it which has not been consumed. Then *5%* on *300* (the part of the capital not consumed in the first year)=*15*, or *5%* profit e.g., similarly *15*, therefore *30*. Thus the price of the commodities would come to *280+30=310,* still almost half as cheap as in the first case.

In fact only *370* thalers were laid out for fixed capital, *150* capital for raw material, and *80* for labour.

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However, if in order to replace *5* workers by one, the capital [had to increase by more, specifically] the machinery had to increase from *50* to perhaps *2000* instead of *370,* the total capital therefore rising to *2300,* the wear and tear contained in the commodity annually would=*2000/100=20*. Production costs would=*250*, with interest and profit of *150 . 250+150+80=480. 10%* on [*480* is *48*.] So in this case [the price is *480+48=528*], by the inequality [*528>310,* we see that raising the cost of the machine (to] *2000* again =[a part of the capital laid out) the price was increased. That is the price is increased by having] the machinery made dearer.

[A lengthening of the turn over time of capital can effect the price of a good] in two ways. [First there is an increase of] turnover time peculiar to fixed capital-mode of circulation-a much smaller aliquot part of it enters into the value [of the] product-than is really required for production. Only its wear and tear, the part of it that is worn out in

the course of a year, enters into the value of the product, because only this part really circulates. Hence the capital remains the same and there is only a change in the proportion of the capital [entering into the product relative to the] component of the capital laid out in labour, there is a cheapening of the product, the ultimate result of which is a cheap[ening in the pro]duction costs of labour and hence an increase in the rate of surplus value, i.e. surplus labour time.

[Second,] if the capital remains the same and there is also no increase in the surplus time (or no original reduction in wages), [then the price of the good may rise in equal] measure as the turnover time (reproduction time) of fixed capital declines in velocity.

[The fixed capital is increased by] the aliquot part of the old (variable) capital but (this) capital had rather to [be added to] so that the total capital might grow, the proportion of this growth, required for the number of workers [that is now smaller, will cause a situation to] occur, in which the commodity produced with the machine became dearer than that produced with hand labour [.]

[This conclusion is] posited on the assumption that the amount of commodities produced by the smaller number of workers is not larger, [but smaller] than the number produced without machinery or on the assumption that [the] capital with machinery does not produce more than previously (was produced) without it. […]

[Output could increase but then there would have to be a large increase in capital. With machinery it might be that the two] workers employed produced more than the *10* without it, they thus produce perhaps as much as *20*, [this is] always a definite number, but perhaps a greater number than they force out. In this case *1* replaced *10*, but one of them could perhaps only be employed if both were employed. In any case, the part of the capital laid out in [this operation] would have to be doubled, i.e. the magnitude of the capital could not remain unaltered.

[This happens when there is no cheapening of the product] but if the slow turnover time cheapens the product, even if the old capital increases again,

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and even still a smaller amount of commodities is produced. Then this is even more so in the other case.

This belongs to the section on production costs, just as the previous comments on surplus value must be treated under the heading "Surplus Value".

//The total amount of the capital advanced enters into the labour process, but only the part of the capital consumed during a particular period of the labour process enters into

the valorisation process or into the value of the product. (See Malthus.) Hence the smaller value or the greater cheapness of the commodities which are e.g. produced with the same capital of *500*, if *2/5* of this are fixed capital and *1/5* variable capital, than if the proportions are inverted. (Even if profit and interest are calculated on the whole of the capital, only an aliquot part of it enters into the value of the commodity, not the capital itself, as in the case in which the whole of the capital or the greatest part of it is laid out in living labour.) But the profit is calculated on the whole of the capital, including the unconsumed part of it. Although the unconsumed part of the capital does not enter into the value of the product of the individual capital considered for itself, it does enter into the average production costs of capitalist production, in the form of profit (interest), because it constitutes an element of the average profit, and an item in the calculation by means of which the capitalists divide among themselves the total surplus value of the capital.//

//The rate of profit depends upon, or is nothing other than, the ratio of the surplus value (considered as an absolute magnitude) to the magnitude of the capital advanced. But the surplus value itself-i.e. its absolute magnitude-may fall even though the rate of surplus value rises, and rises considerably. The amount of surplus value or its absolute magnitude must indeed fall, despite any rise whatever in the rate of surplus value, once the [loss] of surplus value of the labour which is displaced by machinery is greater than the total amount of value, or labour, which steps into its place. Or the surplus time of the displaced worker[s] is greater than the total labour time of the workers who replace them. Thus if *50* are replaced by *5*. And the surplus labour time of the *50* was *2* hours (with a normal working day of *12* hours).Their surplus labour time or the surplus value created by them=*100* hours. The total labour time of the value created [by the *5*] (hence the

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necessary labour time+surplus)=*60* hours. Assume that these *5* workers provide twice as much surplus time, or that surplus value=*4* hours every day for each of them. So that for *5* there are *20* hours. The rate of surplus value has grown by *100%*; the total amount of surplus value of the surplus value itself is only *4x5=20* hours. The surplus value is only *1/5* of the *100* created by the *50*, smaller by *80%*. If now *15* workers were employed at the new rate the amount of surplus value would rise to *60*, if *20* to *80,* if *25* to *100.* Half as many workers would have to be employed at the new rate in order to produce as much surplus value as at the old rate. But if *50* were employed, they would

produce twice as much, namely *200*. Not only the rate of surplus value, but also the surplus value itself would have doubled.// // Assume that the *5* only produced surplus value at the same rate as the *50*, hence only *10* hours. Then *50* workers would have to be employed just as before in order to produce the same surplus value, although they would produce *10* times as many commodities in the same time. This in the branches of industry where the product does not enter into the consumption of the workers themselves. Here the profit derives purely from the fact that the necessary labour time, over a certain average period, stands higher than the labour time needed by the capitalists who have introduced the new machinery; they therefore sell the commodity above its value. This is, however, different from sheer fraud. They sell it above the value it costs them, and below the value it costs society before the general introduction of the machinery. They sell the labour of their [workers for more hours than they work, their workers are productively] higher labour, but they buy it as yet at [the rate set by the output of the productively lower labour.]With the [general introduction of machinery the goods sell at their new value, that is] at the new rate. But there is also an increase in the quantity of goods a worker can buy which would be more significant [except that the value of the wage will also fall.]

Appendix 2. Proof of Result 3.

First consider the effect of dA_{11} and dL_1 on dR. From equations (9) and (10) with $p=p_1/p_2$

$$p = \frac{A_{11}p + bL_1}{A_{12}p + bL_2}.$$ (1A)

Thus in the initial situation $p=1$. From (4) and (5) $_1 = _2$ in the initial situation. From equation (11)

$$p_2 = \frac{1}{p+1}\left(_1 + _2\right)u$$

$$R = \frac{u_2}{p_2} - 1$$

so that in the initial situation

$$dR = 1/2\,dp + \frac{A-1}{}d_1$$ (2A)

because $d_2 = {}_{12}d_1$. From (1A)

$$p^2 A_{12} + p\left(bL_2 - A_{11}\right) - bL_1 = 0.$$

Thus, in the initial situation

$$dp = \frac{1}{A + bL}\left(dA_{11} + bdL_1\right).$$

(3A)

From (4)

$$d_{1} = \frac{L}{(1-A)^2}dA_{11} + \frac{1}{1-A}dL_1.$$

Substituting these expressions into (2A) and taking account of (4) or (5),

$$\frac{dR}{dA_{11}} = \frac{1}{2}\frac{1-A-bL}{A+bL} > 0$$

(4A)

$$\frac{dR}{dL_1} = \frac{1}{2}A\frac{A+bL-1}{(A+bL)L} < 0.$$

(5A)

This confirms i). This is crucial since $p_2 = _2$ so that $R=0$ in the initial situation (from $p=1$, $_1 = _2$, and (11)) so that if $dR<0$, one would have $R<0$ which can not happen according to Marx.

Now consider the effect of dA_{11} and dL_1 on $\tilde{r}$. Let $\tilde{p} = \tilde{p}_1 / m_2$. The same calculations that resulted in (3A) when applied to (6) and (7) give

$$d\tilde{p} = \frac{1}{A + bL}\left(dA_{11} + bdL_1 - dR\right).$$

(6A)

From (7)

$$d\tilde{r} = -\left(1 + \tilde{r}\right)^2\left(Ad\tilde{p} + dR\right)$$

so that

$$\frac{d\tilde{r}}{dA_{11}} < 0$$

from (4A) and (6A). Also

$$\frac{d\tilde{r}}{dL_1} = \left(1 + \tilde{r}\right)^2\frac{bA}{(A+bL)^2}\left[1 - 3(A+bL)\right]$$

This proves parts ii) and iii).

Appendix 3. Proof of result 4.

There are two sectors: 1 is machinery and 2 is food. There are two types of land, good and bad. The value of machines is $_1 = L_1/(1-A_{11})$, the value of food produced on land of type i is

$$_2^i = {}_1A_{12}^i + L_2^i \qquad i=g, b$$

where $i=g$ when only good land is used, etc., $A_{12}^g < A_{12}^b$, $L_2^g < L_2^b$ so that $_2^g < _2^b$.

$$K_m^i = {}_1A_{11} + {}_2^i bL_1 \qquad i = g,b$$
$$K_g^i = {}_1A_{12}^g + {}_2^i bL_2^g \qquad i = g,b$$
$$K_b^i = {}_1A_{12}^b + {}_2^b bL_2^b \qquad i = b$$

K_m^g is the total capital in machines per unit of output when only the good land is in use, K_m^b when both good and bad land are in use etc.. Note that $K_b^b > K_g^b$.

$$_m^i = \left({}_1 - K_m^i \right)/ K_m^i \qquad i = g, b$$
$$_g^i = \left({}_2^g - K_g^i \right)/ K_g^i \qquad i = g, b$$
$$_b^i = \left({}_2^b - K_b^i \right)/ K_b^i \qquad i = b$$

$_j^i$ is the rate of profit in sector j when, if $i=g$, only good land is in use etc.. Let x_i, $i=m$, g, b, be the output of machines and the outputs of food on good and bad lands. Let

$$_j^i = \frac{K_j^i x_j /\left(x_g + x_b\right)}{\underset{k=m,g,b}{K_k^i x_k /\left(x_g + x_b\right)}} \qquad i = g,b$$

$_j^i$ is the percentage of total capital in sector j when, if $i=b$, both good and bad land are in use etc.. Then

$$^i = \underset{j=m,g,b}{{}_j^i \, {}_j^i}$$

where ^{i} is the value rate of profit when, if $i=b$, both good and bad land is in use. It is supposed that x_g remains constant and that $dx_m >0$ and $dx_b >0$ are such that $x_m/(x_g + x_b)$ is constant, that is the production ratio is constant.

Now consider the situation in which production moves on to bad land but only an infinitesimal amount is used. Since dx_b and dx_m are infinitesimal, the $x_i /\left(x_g + x_b\right)$ are almost constant, whereas there are finite changes in the sectorial profit rates and the amounts of total capital. Let $K_j = K_j^b - K_j^g$, $\quad _j = {}_j^b - {}_j^g$, $\quad _j = {}_j^b - {}_j^g$ and $= {}^b - {}^g$. Clearly $K_j>0$, $_j<0$ while $_j$ may be positive or negative. We have

$$= {}_m \, {}_m^b + {}_g \, {}_g^g + {}_g \, {}_g^b + {}_m \, {}_m^g$$

because x_b is close to zero. There is a presumption that <0 since the first two terms are. But this result can be overturned for example if machinery has a higher rate of profit and the movement onto bad land raises the weight of machinery in total capital.

Now consider the effect of further growth of production of food onto bad land, $K_j= {}_j=0$ and only the changes in $x_i /(x_g+x_b)$ effect $_j^b$. Since $K_b^b > K_g^b$, the denominator of $_j^i$ rises which implies

$$d\,\pi^b_m < 0, \quad d\,\pi^b_g < 0.$$

To show that the rate of profit may rise, an example can be constructed in which $\pi^b_g < \pi^b_b$ and $\pi^b_m < \pi^b_b$.. Since $\pi^b_2 = {}_1A^b_{12} + L^b_2$,

$$\pi^b_g = \frac{\pi^g_2}{{}_1A^g_{12} + \pi^b_2 bL^g_2} - 1 = \frac{{}_1A^g_{12} + L^g_2}{{}_1A^g_{12} + \left({}_1A^b_{12} + L^b_2\right)bL^g_2} - 1.$$

Now to convert this into π^b_b we only have to raise L^g_2 to L^b_2 and similarly with A^g_{12}. Suppose that the movement of L_2 dominated. If b is close to zero it is clear that $\pi^b_g < \pi^b_b$. Next if $A_{12}=A_{11}$ and $L_1=L_2$ then $\pi_1 = \pi_2$ and $\pi^b_b = \pi^b_m$. If $b \to 0$, then

$$\pi^b_b \to \frac{\pi^b_2}{{}_1A^b_{12}}, \qquad \pi^b_m \to \frac{\pi_1}{{}_1A^b_{11}},$$

and since $\pi^b_2 = {}_1A^b_{12} + L^b_2$ an increase in L^b_2 produces the inequality $\pi^b_m < \pi^b_b$. Since the $d\,\pi^b_j$ sum to 0 it follows that

$$d\,\pi^b = \sum_{j=m,g,b} \pi^b_j d\,\pi^b_j = -\left(\pi^b_b - \pi^b_m\right)d\,\pi^b_m - \left(\pi^b_b - \pi^b_g\right)d\,\pi^b_g > 0,$$

that is the value rate of profit may rise as progressively more bad land is brought into use.

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